Nanotechnology Perceptions
ISSN 1660-6795
www.nano-ntp.com

Stability Analysis of Composite Nanofluid in Porous Medium under Temperature Gradient

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Abstract: The stability analysis for an incompressible composite nanofluid in porous medium has been performed under temperature gradient. The fluid is supposed to be restrained within two infinite free boundaries. The normal mode analysis and the Galerkin method has been implicated for this study to obtain the Rayleigh number and to check the behavior of the fluid flow at stationary and oscillatory state. The presents results have been compared to the former results. It is perceived that the concerned parameters of the system Ln_1, Ln_2, N_{A1}, N_{A2} have stabilizing role whereas Rn_1, Rn_2 and $\boldsymbol{\varepsilon}$ plays destabilizing role in the system. Several comparisons have been carried out to check the nature of convection. A comparative study between composite and ordinary nanofluid in porous medium shows that the presence of composite particles in the fluid delays the convection. It is also noticed that the composite nanofluid in continuous medium sets in earlier rather than that in porous medium.

MSC 2020: 76A05, 76A10, 76D05, 76S05
Keywords: composite nanofluid, porous medium, temperature gradient.

1. Introduction

A composite nanofluid is a heat transfer medium (of high thermal conductivity) which is uniform and stable prepared by dispersing nano-sized metallic or non-metallic particles with fluid such as water, alcohol, and oil. Choi and Eastman [1] investigated the use of synthesized nanofluids, and observed a significant increase in the thermal conductivity of the fluid medium due to the suspension of nanoparticles. He then demonstrated the advantage of consuming nanoparticles in a fluid which makes the base for works given by Choi et al. [2], Liu et al. [3], Das et al. [4], Tyler et al. [5], Choi [6], Chandrasekhar [7]. Later following the work of Choi and Eastman [1], a lot of work has been done on thermal instability of nanofluids by Eastman et al. [8], Chand and Rana [9], Yadav et al. [10], Nield and Kuznetsov [11], Umavati and Beg [12] and Pundir et al. [13-14].

Convective heat transfer in trending fluids is a significant subdivision of fluid mechanics. It is widely used in many branches of engineering, medical, defense etc. Heat transfer improvement usually involves active and passive methods. Kasaeian et al. [15] have worked on a significant request of nanofluids in solar energy systems. Siddheshwar and Meenakshi [16] proposed a dimensional equation model to study Rayleigh–Bénard instability in nanofluids and discussed the improvement in heat transfer due to the nanoparticles present in base fluid. Jawdat et al. [17] deliberate Rayleigh–Bénard instability in nanofluids using a single-phase description of nanofluids and found that the presence of metal/metallic nanoparticles in water disrupts the chaotic motion. The Rayleigh-Bénard convection studied by Bhardwaj and Das [18] with copper oxide nanofluids under the influence of temperature and magnetic field concluded that turbulence can only be controlled by increasing the magnetic field by monitoring the Rayleigh number. Pundir et al. [19] worked on a double-diffusive instability in porous media for a rotatory nanofluid with couple-stress. Recently, Pundir et al. [20] performed a study on a couple stress nanofluid which is heated and soluted from below in a Darcy porous medium to examine the effect of magnetic field.

On the other hand, composite (hybrid) nanofluid is a newly type of nanofluid formed from the hybridization of more than one different type of nanofluids (NFs) with more amazing abilities promotion of freight transportation. Because of its amazing heat transport ability, a large number of researchers across the globe has conducted extensive research on composite (hybrid) nanofluid. In modern fluid technology, hybrid nanofluid (HNF) plays a very important role due to its best performance in thermal conductivity and heat transfer media as compared to single nano material particles with base fluid or nanofluid (NF). Few important studies on composite nanofluid were given by Rashad et al. [21], Kumar and Sarkar [22], Ghalambaz et al. [23], Kumar and Awasthi [24] and Pundir et al. [25]. Nazir et al. [26] proposed the Carreau Yasuda hybrid nanofluid using various nanoparticles mixed in base fluid to improve thermal efficiency momentum transport in a rotating cone by Ohm's law.

The present work deals with the stability analysis of composite nanofluid in porous medium which is subjected to temperature gradient. To the best of our knowledge, this problem has not been investigated yet. The study proposed by Kumar and Awasthi [24] considers a problem of thermal instability in a horizontal fluid layer with composite nano particles in continuous medium. For current study, we consider the horizontal composite nanofluid layer in a Darcy porous medium which is also one of the important aspects. In this work, we have investigated a composite nanofluid layer within porous medium to check the stability of the fluid flow under realistic conditions for which the normal mode method is applied. Here, it is assumed that the density of nanoparticles is less than that of the base fluid. The stability analysis is carried out for both the stationary and oscillatory state. The newly obtained results are compared with the former ones and have been depicted analytically and graphically.

2. Problem Formulation

A layer of composite nanofluid has been taken which is supposed to be restricted between two boundaries $z=0$ and $z=d$ in a porous medium. The value of temperature is $T=T_0$ at $z=0$ and $T=T_1$ at $z=d$. The layer is subjected to heating from below. At the boundary, nanoparticle flux is zero. The physical representation of the assumed situation is shown in

Figure 1.

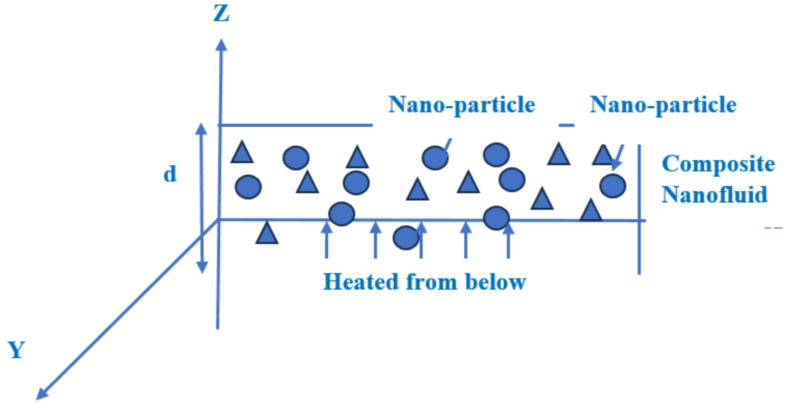

Figure 1: Geometrical Configuration

The equations governing the physical model are as follows:

$$\nabla \cdot q = 0 \tag{1}$$

$$\frac{\rho}{\varepsilon}\left(\frac{\partial q}{\partial t} + \frac{1}{\varepsilon}(q \cdot \nabla) q\right) = -\nabla p - \frac{\mu}{k_1} q + \left(\phi_1 \rho_{p_1} + \phi_2 \rho_{p_2} + (1 - \phi_1 - \phi_2)\rho\{1 - \beta_T (T - T_0)\}\right) g \tag{2}$$

$$(\rho c)_m \frac{\partial T}{\partial t} + (\rho c)_f (q \cdot \nabla) T = K \nabla^2 T + \varepsilon (\rho c)_{p_1}\left(D_{B_1} \nabla \phi_1 \cdot \nabla T + \frac{D_{T_1}}{T_0} \nabla T \cdot \nabla T\right) + \varepsilon (\rho c)_{p_2}\left(D_{B_2} \nabla \phi_2 \cdot \nabla T + \frac{D_{T_2}}{T_0} \nabla T \cdot \nabla T\right) \tag{3}$$

$$\frac{\partial \phi_1}{\partial t} + \frac{1}{\varepsilon}(q \cdot \nabla)\phi_1 = D_{B_1} \nabla^2 \phi_1 + \left(\frac{D_{T_1}}{T_0}\right)\nabla^2 T \tag{4}$$

$$\frac{\partial \phi_2}{\partial t} + \frac{1}{\varepsilon}(q \cdot \nabla)\phi_2 = D_{B_2} \nabla^2 \phi_2 + \left(\frac{D_{T_2}}{T_0}\right)\nabla^2 T \tag{5}$$

For this, the constant temperature is taken into consideration and the thermophoretic nanoparticle flux is taken as zero within the limits as given by [20].

The boundary conditions compatible to the problem [24] are

$$w = 0,\ \frac{\partial w}{\partial z} + \lambda_1 d \frac{\partial^2 w}{\partial z^2} = 0,\ \mathrm{T} = \mathrm{T}_0 + \Delta \mathrm{T},\ \phi_1 = \phi_{10}, \phi_2 = \phi_{20} \quad at\ z = 0 \tag{6}$$

$$w = 0,\ \frac{\partial w}{\partial z} - \lambda_2 d \frac{\partial^2 w}{\partial z^2} = 0,\ \mathrm{T} = \mathrm{T}_0,\ \phi_1 = \phi_{11}, \phi_2 = \phi_{21} \quad at\ z = d \tag{7}$$

where, d is the fluid layer's depth while λ_1 and λ_2 are parameters which attains zero value for rigid boundary and infinite for free boundary Also, k_1 is medium permeability of fluid.

Now let us non-dimensionalizing the variables as

$$(x_a, y_a, z_a) = \left(\frac{x, y, z}{d}\right), (u_a, v_a, w_a) = \left(\frac{u, v, w}{k_f}\right) d, t' = \frac{tk_f}{\sigma d^2},$$

$$p' = \frac{pk_1}{\mu k_f}, \phi_1' = \frac{(\phi_1 - \phi_{10})}{(\phi_{11} - \phi_{10})}, \phi_2' = \frac{(\phi_2 - \phi_{20})}{(\phi_{21} - \phi_{20})}, T' = \frac{(T - T_0)}{\Delta T}, \tag{8}$$

where $\alpha = \frac{k}{(\rho c)_f}$ is nanofluid thermal diffusivity and $\sigma = \frac{(\rho c)_m}{(\rho c)_f}$ where $(\rho c)_m$ and $(\rho c)_p$ are heat capacity of fluid and nano particle resp in porous medium, K is thermal conductivity.

On omitting a for convenience, we have the eq. (1)-(7) takes the form

$$\nabla' \cdot q' = 0 \tag{9}$$

$$\gamma_a \frac{\partial q'}{\partial t'} = -\nabla' p' - q' + R_a T' e_z - R_m e_z - R_{n_1} \phi_1' e_z - R_{n_2} \phi_2' e_z \tag{10}$$

$$\left(\frac{\partial T'}{\partial t'} + q' \cdot \nabla' T'\right) = \nabla'^2 T' + \frac{N_{B_1}}{L_{n_1}} \nabla' \phi_1' \cdot \nabla' T' + \frac{N_{A_1} N_{B_1}}{L_{n_1}} \nabla' T' \cdot \nabla' T' + \frac{N_{B_2}}{L_{n_2}} \nabla' \phi_2' \cdot \nabla' T' + \frac{N_{A_2} N_{B_2}}{L_{n_2}} \nabla' T' \cdot \nabla' T' \tag{11}$$

$$\frac{1}{\sigma} \frac{\partial \phi_1'}{\partial t'} + \frac{1}{\varepsilon} q' \cdot \nabla' \phi_1' = \frac{1}{L_{n_1}} \nabla'^2 \phi_1' + \frac{N_{A_1}}{L_{n_1}} \nabla'^2 T' \tag{12}$$

$$\frac{1}{\sigma} \frac{\partial \phi_1'}{\partial t'} + \frac{1}{\varepsilon} q' \cdot \nabla' \phi_2' = \frac{1}{L_{n_2}} \nabla'^2 \phi_2' + \frac{N_{A_2}}{L_{n_2}} \nabla'^2 T' \tag{13}$$

$$at\ z = 0, \quad \{w = 0, \phi_1' = 0, \phi_2' = 0, \frac{\partial w}{\partial z} + \lambda_1 d \frac{\partial^2 w}{\partial z^2} = 0, T' = 1\} \tag{14}$$

$$at\ z = 1, \quad \{w = 0, \phi_1' = 1, \phi_2' = 1, \frac{\partial w}{\partial z} - \lambda_2 d \frac{\partial^2 w}{\partial z^2} = 0, T' = 0\} \tag{15}$$

where,

$\gamma_a = \frac{k_1 \rho k_f}{\varepsilon \mu \sigma d^2}$; (Non-dimensional acceleration coefficient),

$R_m = \frac{\left(\rho_{p_1}\phi_{10} + \phi_{20}\rho_{p_2} + \rho\left(1-\phi_{10}-\phi_{20}\right)\right)k_1 g d}{\mu k_f}$;(Basic Density Rayleigh number) ,

$R_{n_1} = \frac{(\rho_{p_1} - \rho)(\phi_{11} - \phi_{10})\,\mathrm{d}\,g k_1}{\mu k_f}$;(for first nano-particle Concentration Rayleigh Number),

$R_{n_2} = \frac{(\rho_{p_2} - \rho)(\phi_{21} - \phi_{20})\,\mathrm{d}\,g k_1}{\mu k_f}$;(for second nano-particle Concentration Rayleigh Number),

$R_a = \frac{\left(1-\phi_{10}-\phi_{20}\right)\Delta T \rho \beta_T k_1 d}{\mu k_f}$; (the thermo-Rayleigh Number), $L_{n_1} = \frac{k_f}{D_{B_1}}$; (for first nano-particle thermo-nanofluid specify Lewis Number), $L_{n_2} = \frac{k_f}{D_{B_2}}$;(for second nano-particle thermo-nanofluid specify Lewis Number) $N_{A_1} = \frac{D_{T_1}\Delta T}{D_{B_1} T_0 (\phi_{11} - \phi_{10})}$; (for first nano-particle modify diffusivity ratio), $N_{A_2} = \frac{D_{T_2}\Delta T}{D_{B_2} T_0 (\phi_{21} - \phi_{20})}$; (for second nano-particle modify diffusivity ratio), $N_{B_1} = \frac{\varepsilon\left(\rho c\right)_{p_1}(\phi_{11} - \phi_{10})}{\left(\rho c\right)_f}$ and $N_{B_2} = \frac{\varepsilon\left(\rho c\right)_{p_2}(\phi_{21} - \phi_{20})}{\left(\rho c\right)_f}$ (the modify density increments for nano-particles).

3. Basic and Perturbation State

Succeeding Pundir et al. [20], Kumar and Awasthi [24], we assume that

$$u' = v' = w' = 0,\ p' = p_b\left(z\right),\ T' = T_b\left(z\right),\ \phi_1' = \phi_{1b}\left(z\right),\ \phi_2' = \phi_{2b}\left(z\right) \tag{16}$$

which reduces the basic solution as

$$T_b = 1 - z,\ \phi_{1b} = z \quad and \quad \phi_{2b} = z \tag{17}$$

We superimpose infinitesimal small perturbations into the basic state as

$$q'\left(u,v,w\right) = q''\left(u,v,w\right), T' = T_b + T'', \phi_1' = \phi_{1b} + \phi_1'', \phi_2' = \phi_{2b} + \phi_2'', p' = p_b + p'' \tag{18}$$

Here double dash quantities are quantities in perturbed state. Using eq. (18) in eq. (9) to (13) and linearizing the subsequent equations we obtain,

$$\nabla \cdot q'' = 0 \tag{19}$$

$$\gamma_a\left(\frac{\partial q''}{\partial t}\right)=-\nabla p''-q''+R_aT''\hat{e}_z-R_{n_1}\phi_1''\hat{e}_z-R_{n_2}\phi_2''\hat{e}_z \tag{20}$$

$$\frac{\partial T''}{\partial t}-w''=\nabla^2T''+\frac{N_{B_1}}{L_{n_1}}\left(\frac{\partial T''}{\partial z}-\frac{\partial\phi_1''}{\partial z}\right)-\frac{2N_{A_1}N_{B_1}}{L_{n_1}}\frac{\partial T''}{\partial z}+\frac{N_{B_2}}{L_{n_2}}\left(\frac{\partial T''}{\partial z}-\frac{\partial\phi_2''}{\partial z}\right)-\frac{2N_{A_2}N_{B_2}}{L_{n_2}}\frac{\partial T''}{\partial z} \tag{21}$$

$$\frac{1}{\sigma}\frac{\partial\phi_1''}{\partial t}+\frac{1}{\varepsilon}w''=\frac{1}{L_{n_1}}\nabla^2\phi_1''+\frac{N_{A_1}}{L_{n_1}}\nabla^2T'' \tag{22}$$

$$\frac{1}{\sigma}\frac{\partial\phi_2''}{\partial t}+\frac{1}{\varepsilon}w''=\frac{1}{L_{n_2}}\nabla^2\phi_2''+\frac{N_{A_2}}{L_{n_2}}\nabla^2T'' \tag{23}$$

with condition at the boundaries

$$at\ z=0\ \left\{w''=0,\frac{\partial w''}{\partial z}+\lambda_1\frac{\partial^2w''}{\partial z^2}=0,T''=0,\phi_1''=0\ \&\ \phi_2''=0\right\} \tag{24}$$

$$at\ z=1\ \left\{w''=0,\frac{\partial w''}{\partial z}-\lambda_2\frac{\partial^2w''}{\partial z^2}=0,T''=0,\phi_1''=0\ \&\ \phi_2''=0\right\} \tag{25}$$

It is noticed the factor Rm is absent in Eq. (19) - (23), since it is just an element of the necessary state pressure gradient.

Operating $\hat{e}_z\cdot$curl twice on (20) and by using curl twice = $\nabla\nabla-\nabla^2$ on eq. (18), we can eliminate p'' and get

$$\gamma_a\frac{\partial}{\partial t}\nabla^2w''=\nabla^2w''-R_a\nabla_H'^2T''-R_{n_1}\nabla_H'^2\phi_1''-R_{n_2}\nabla_H'^2\phi_2'' \tag{26}$$

where $\nabla_H'^2=\frac{\partial}{\partial x'^2}+\frac{\partial}{\partial y'^2}$.

4. Normal Mode Method

The perturbation into the normal modes takes the form

$$\left[w'',\phi_1'',T'',\phi_2''\right]=\left[W''(z),\varphi_1''(z),\Theta''(z),\varphi_2''(z)\right]\exp(nt).\exp\left(ik_xx+ik_yy\right) \tag{27}$$

where k_x,k_y are the wave numbers in x and y direction while $a^2=\left(k_x^2+k_y^2\right)$ respectively and n denote the frequency of the perturbation in nanofluid.

Using eq. (27) into eq. (26), (21)-(23), the subsequent eigenvalue problem takes place as follows

$$\left(\gamma_a n-1\right)\left(D^2-a^2\right)W\left(z\right)+a^2R_a\Theta\left(z\right)-a^2R_{n_1}\varphi_1\left(z\right)-a^2R_{n_2}\varphi_2\left(z\right)=0 \tag{28}$$

$$W\left(z\right)+\left(D^2-a^2-n+\frac{N_{B_1}}{L_{n_1}}D-\frac{2N_{A_1}N_{B_1}}{L_{n_1}}D+\frac{N_{B_2}}{L_{n_2}}D-\frac{2N_{A_2}N_{B_2}}{L_{n_2}}D\right)\Theta\left(z\right)$$

$$-\frac{N_{B_1}}{L_{n_1}}D\varphi_1\left(z\right)-\frac{N_{B_2}}{L_{n_2}}D\varphi_2\left(z\right)=0 \tag{29}$$

$$\frac{1}{\varepsilon}W\left(z\right)-\frac{N_{A_1}}{L_{n_1}}\left(D^2-a^2\right)\Theta\left(z\right)-\left(\frac{1}{L_{n_1}}\left(D^2-a^2\right)-\frac{n}{\sigma}\right)\varphi_1\left(z\right)=0 \tag{30}$$

$$\frac{1}{\varepsilon}W\left(z\right)-\frac{N_{A_2}}{L_{n_2}}\left(D^2-a^2\right)\Theta\left(z\right)-\left(\frac{1}{L_{n_2}}\left(D^2-a^2\right)-\frac{n}{\sigma}\right)\varphi_2\left(z\right)=0 \tag{31}$$

$$W=0, D^2W=0, \Theta=0, \varphi_1=0, \varphi_2=0 \ \ at\, z=0 \ and \ \ z=1 \tag{32}$$

where $D=\dfrac{d}{dz}$ and eq. (32) shows conditions for free boundary.

Method of Solution

Using the Galerkin method, we consider the solution of the form

$$\left[W,\Theta,\varphi_1,\varphi_2\right]=\left[W_0',\Theta_0',\varphi_{10}',\varphi_{20}'\right]\sin\left(\pi z\right) \tag{33}$$

Now using eq. (33) in eq's. (28)-(31) and integrating each equation, we get

$$\delta^2\left(\gamma_a n-1\right)W_0'-R_a a^2\Theta_0'+R_{n_1}a^2\varphi_{10}'+R_{n_2}a^2\varphi_{20}' \tag{34}$$

$$\mathrm{W}_0'-\left(\delta^2+n\right)\Theta_0'=0 \tag{35}$$

$$\frac{1}{\varepsilon}W_0'+\frac{N_{A_1}}{L_{n_1}}\delta^2\Theta_0'+\left(\frac{n}{\sigma}+\frac{\delta^2}{L_{n_1}}\right)\varphi_{10}'=0 \tag{36}$$

$$\frac{1}{\varepsilon}W_0'+\frac{N_{A_2}}{L_{n_2}}\delta^2\Theta_0'+\left(\frac{n}{\sigma}+\frac{\delta^2}{L_{n_2}}\right)\varphi_{20}'=0 \tag{37}$$

We obtain the matrix specified in eq. (38) and for a non-trivial solution of system of equations in $W_0', \Theta_0', \varphi_{10}'$ and φ_{20}', the det of matrix given in (38) should be zero.

$$\begin{pmatrix} \delta^2\left(\gamma_a n-1\right) & -R_a a^2 & R_{n_1} a^2 & R_{n_2} a^2 \\ 1 & -\left(\delta^2+n\right) & 0 & 0 \\ \frac{1}{\varepsilon} & \frac{N_{A_1}}{L_{n_1}}\delta^2 & \left(\frac{n}{\sigma}+\frac{\delta^2}{L_{n_1}}\right) & 0 \\ \frac{1}{\varepsilon} & \frac{N_{A_2}}{L_{n_2}}\delta^2 & 0 & \left(\frac{n}{\sigma}+\frac{\delta^2}{L_{n_2}}\right) \end{pmatrix}\begin{pmatrix} W_0{}' \\ \Theta_0{}' \\ \varphi_{10}{}' \\ \varphi_{20}{}' \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{38}$$

where total wave number $\delta^2=\pi^2+a^2$.

Stationary Convection

For the case when density of nanoparticles is less than that of base fluid $(\rho_{p_1},\rho_{p_2}<\rho_0)$.

$$R_a=\frac{n+\delta^2}{\sigma a^2}\left(\gamma_a n-1\right)\delta^2-\frac{\left\{L_{n_1}\left(n+\delta^2\right)+\varepsilon N_{A_1}\delta^2\right\}}{\varepsilon\left(nL_{n_1}+\sigma\delta^2\right)}R_{n_1}-\frac{\left\{L_{n_2}\left(n+\delta^2\right)+\varepsilon N_{A_2}\delta^2\right\}}{\varepsilon\left(nL_{n_2}+\sigma\delta^2\right)}R_{n_2} \tag{39}$$

For the case of non- oscillatory state, putting $n=0$ in equation (39) and solving, we get

$$R_a^{st}=-\frac{\delta^4}{a^2}-\left(\frac{L_{n_1}}{\varepsilon}+N_{A_1}\right)R_{n_1}-\left(\frac{L_{n_2}}{\varepsilon}+N_{A_2}\right)R_{n_2} \tag{40}$$

The size of critical cell for the onset of thermal instability is achieved from $\frac{\partial}{\partial a}R_a=0$. For stationary convection of composite nanofluid, critical Rayleigh no. given by

$$R_a^{st}=-4\pi^2-\left(\frac{L_{n_1}}{\varepsilon}+N_{A_1}\right)R_{n_1}-\left(\frac{L_{n_2}}{\varepsilon}+N_{A_2}\right)R_{n_2} \tag{41}$$

We noted that in Eq. (40) does not contain non-dimensional acceleration coefficient. This proves the acceleration in nanofluid is constrained by the regular fluid acceleration.

Oscillatory Convection

For oscillatory state we have $n=i\omega,\left(\omega>0\right)$, then from (38) Rayleigh number is given as

$$R_a=\frac{\left(SU+TV\right)+i\omega\left(TU-SV\right)}{U^2+V^2} \tag{42}$$

Where S, T, U and V are given by

$$S=\begin{Bmatrix}\varepsilon^2 L_{n_1}L_{n_2}\left(1+\gamma_a\delta^2\right)+\varepsilon^2\sigma\delta^4\left(L_{n_1}+L_{n_2}\right)-\gamma_a\varepsilon^2\sigma\delta^6\left(L_{n_1}+L_{n_2}+\sigma\right)\\ +a^2\sigma L_{n_1}L_{n_2}\left(R_{n_1}+R_{n_2}\right)\end{Bmatrix}\omega^2$$
$$-\left\{\varepsilon^2\sigma^2\delta^8+a^2\varepsilon\sigma^2\delta^4\left(R_{n_1}L_{n_1}+R_{n_2}L_{n_2}\right)+a^2\varepsilon^2\sigma^2\delta^4\left(R_{n_1}N_{A_1}+R_{n_2}N_{A_2}\right)\right\} \tag{43}$$

$$T=\left\{\varepsilon^2 L_{n_1}L_{n_2}\delta^2\left(1-\gamma_a\delta^2\right)-\gamma_a\varepsilon^2\sigma\delta^4\left(L_{n_1}+L_{n_2}\right)\right\}\omega^2$$
$$-\begin{Bmatrix}\varepsilon^2\sigma\delta^6\left(L_{n_1}+L_{n_2}\right)+\varepsilon^2\sigma^2\delta^6\left(1-\gamma_a\delta^2\right)+a^2\varepsilon\sigma\delta^2 L_{n_1}L_{n_2}\left(R_{n_1}+R_{n_2}\right)\\ +a^2\varepsilon^2\sigma\delta^2\left(R_{n_1}N_{A_1}L_{n_1}+R_{n_2}N_{A_2}L_{n_2}\right)+a^2\varepsilon\sigma^2\delta^2\left(R_{n_1}L_{n_1}+R_{n_2}L_{n_2}\right)\end{Bmatrix} \tag{44}$$

$$U=a^2\varepsilon^2\sigma^2\delta^4-\left(a^2\varepsilon^2 L_{n_1}L_{n_2}\right)\omega^2 \tag{45}$$

$$V=a^2\varepsilon^2\sigma\delta^2\left(L_{n_1}+L_{n_2}\right) \tag{46}$$

Rayleigh number R_a is always a real physical quantity, Eq. (42) says that we should necessarily have

$$TU-SV=0 \tag{47}$$

Or

$$E_1\lambda^2+E_2\lambda+E_3=0 \tag{48}$$

Where $\lambda=\omega^2$,

$$E_1=\frac{a^2\varepsilon^4\delta^2}{L_{n_1}L_{n_2}}\left\{\gamma_a\sigma\delta^2\left(L_{n_1}+L_{n_2}\right)-L_{n_1}L_{n_2}\left(1-\gamma_a\delta^2\right)\right\}$$

$$E_2=\frac{a^2\varepsilon^4\delta^2\sigma}{L_{n_1}L_{n_2}}\begin{Bmatrix}2\varepsilon\sigma\delta^4\left(1-\gamma_a\delta^2\right)+\varepsilon\delta^4\left(L_{n_1}+L_{n_2}\right)+a^2 L_{n_1}L_{n_2}\left(R_{n_1}+R_{n_2}\right)+\\ a^2\varepsilon\left(R_{n_1}N_{A_1}L_{n_1}+R_{n_2}N_{A_2}L_{n_2}\right)+a^2\sigma\left(R_{n_1}L_{n_1}+R_{n_2}L_{n_2}\right)-\varepsilon\left(1+\gamma_a\delta^2\right)\left(L_{n_1}+L_{n_2}\right)\end{Bmatrix}$$
$$-a^2\varepsilon^2\sigma\delta^2\left\{\varepsilon\sigma\delta^4\left(L_{n_1}+L_{n_2}\right)^2-\gamma_a\varepsilon\sigma\delta^6\left(L_{n_1}+L_{n_2}\right)^2\right\}$$

The lengthy expression of E_3 is not required in the analysis of over-stability, so we can leave E_3 here. Clearly, eq. (48) is a quadratic equation in $\lambda=\omega^2$, which must have two roots. The sum of the roots of eq. (48) is $-\frac{E_2}{E_1}$, therefore oscillatory convection will exist if either E_1 or E_2 is negative otherwise chances of oscillatory convection are debarred.

5. Results and Discussion

At stationary convection, for the stability analysis of the system, we calculate the derivatives $\frac{\partial R_a^{st}}{\partial Ln_1}, \frac{\partial R_a^{st}}{\partial Ln_2}, \frac{\partial R_a^{st}}{\partial N_{A_1}}, \frac{\partial R_a^{st}}{\partial N_{A_2}}, \frac{\partial R_a^{st}}{\partial Rn_1}, \frac{\partial R_a^{st}}{\partial Rn_2}, \frac{\partial R_a^{st}}{\partial \varepsilon}$ analytically.

From equation (40), for Ln_1, Ln_2, N_{A_1}, $N_{A_2} > 0$, Rn_1, $Rn_2 < 0$ and $0 < \varepsilon < 1$, we have $\frac{\partial R_a^{st}}{\partial Ln_1} > 0$, $\frac{\partial R_a^{st}}{\partial Ln_2} > 0$, $\frac{\partial R_a^{st}}{\partial N_{A_1}} > 0$, $\frac{\partial R_a^{st}}{\partial N_{A_2}} > 0$, $\frac{\partial R_a^{st}}{\partial Rn_1} < 0$, $\frac{\partial R_a^{st}}{\partial Rn_2} < 0$ and $\frac{\partial R_a^{st}}{\partial \varepsilon} < 0$. This clearly indicates that Ln_1, Ln_2, N_{A_1}, N_{A_2} stabilizes the system whereas Rn_1, Rn_2 and ε destabilizes the system.

For numerical computation of the results, we have plotted the graphs between stationary Rayleigh number R_a^{st} and wavenumber a for fixed values of parameters $Ln_1 = 100$, $Ln_2 = 100$, $N_{A_1} = 5$, $N_{A_2} = 5$, $Rn_1 = -4$, $Rn_2 = -4$ and $\varepsilon = 0.7$ by varying one of them with different values and keeping the remaining parameters fixed. The negative values of Rn_1 and Rn_2 denotes bottom-heavy distribution. Figure 2 and 3 represents that on increasing the first and second nano-particle thermo-nanofluid Lewis Number Ln_1 and Ln_2, the stationary Rayleigh number R_a^{st} increases which implies that Ln_1 and Ln_2 stabilizes the system. Figure 4 and 5 represents that on increasing the first and second nano-particle modify diffusivity ratio N_{A_1} and N_{A_2}, the stationary Rayleigh number R_a^{st} increases which implies that N_{A_1} and N_{A_2} stabilizes the system. Figure 6 and 7 represents that on increasing the first and second nano-particle concentration Rayleigh number Rn_1 and Rn_2, the stationary Rayleigh number R_a^{st} decreases which implies that Rn_1 and Rn_2 destabilizes the system. Figure 8 represents that on increasing the value of porosity parameter ε, the stationary Rayleigh number R_a^{st} decreases which implies that ε destabilizes the system.

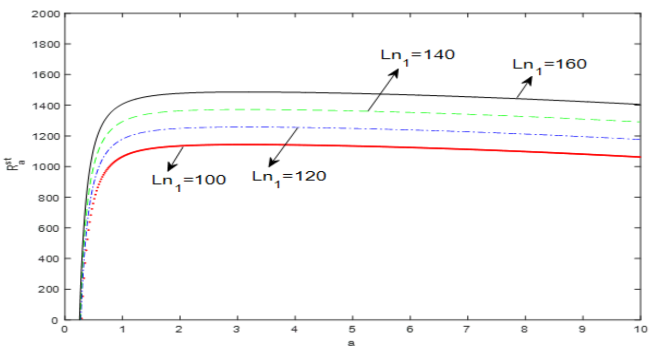

Figure 2: Variation of stationary Rayleigh number R_a^{st} with wavenumber a for different

values of Ln_1

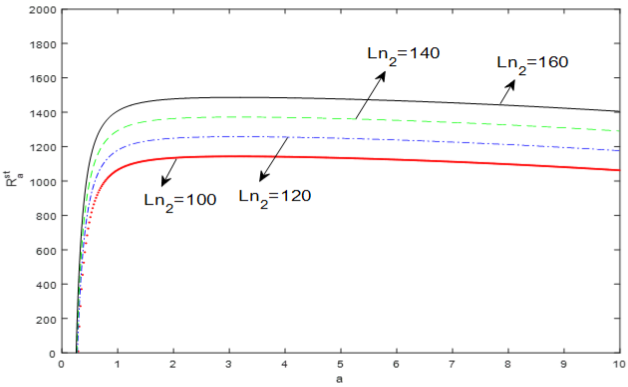

Figure 3: Variation of stationary Rayleigh number R_a^{st} with wavenumber a for different values of Ln_2

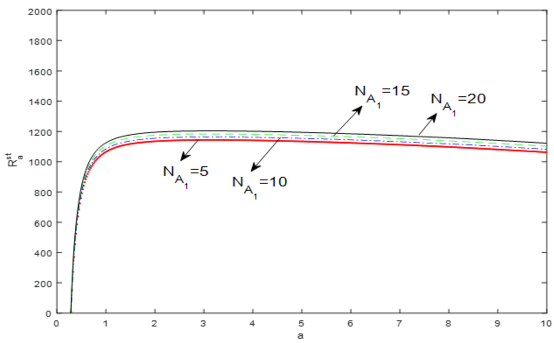

Figure 4: Variation of stationary Rayleigh number R_a^{st} with wavenumber a for different values of N_{A_1}

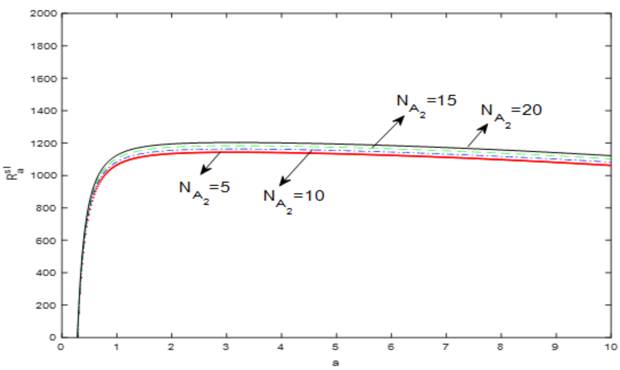

Figure 5: Variation of stationary Rayleigh number R_a^{st} with wavenumber a for different

values of N_{A_2}

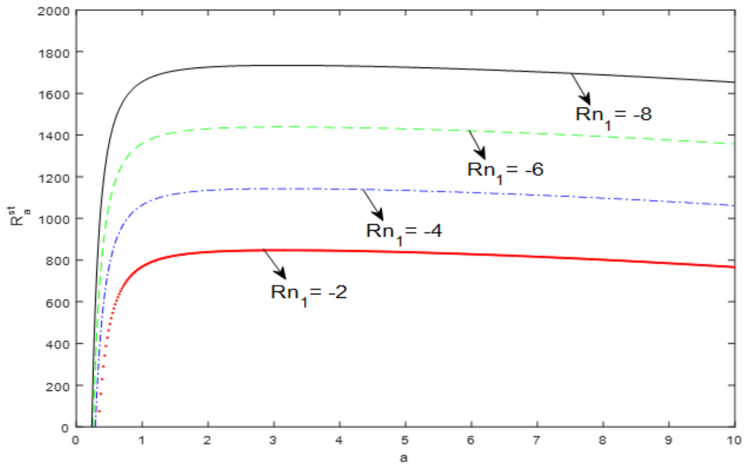

Figure 6: Variation of stationary Rayleigh number R_a^{st} with wavenumber a for different values of Rn_1

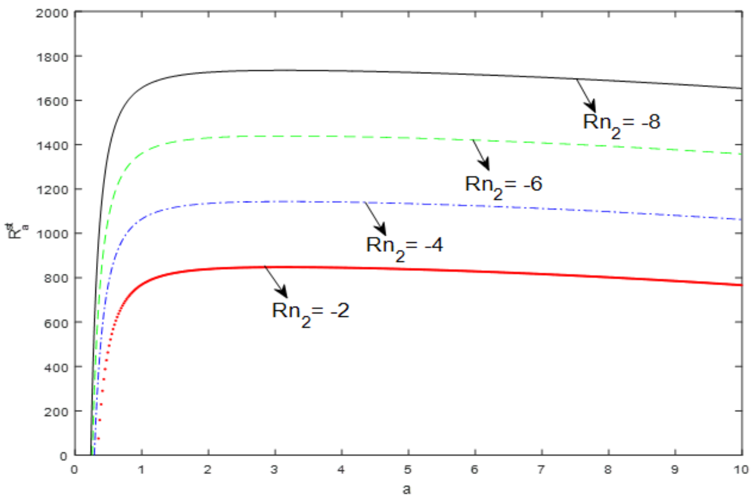

Figure 7: Variation of stationary Rayleigh number R_a^{st} with wavenumber a for different values of Rn_2

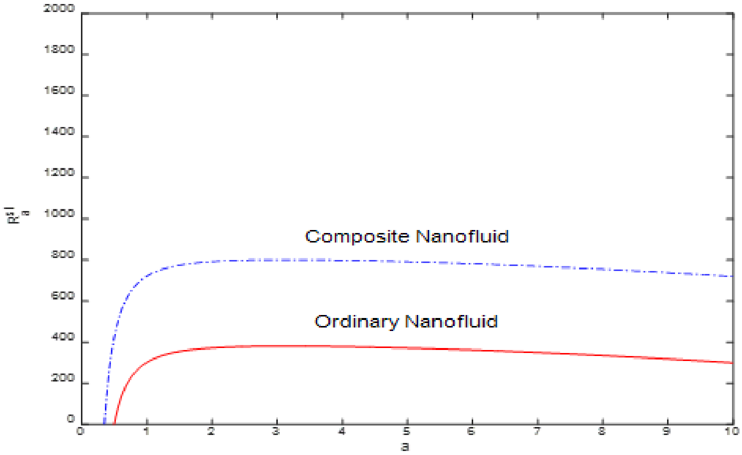

Figure 8: Variation of stationary Rayleigh number R_a^{st} with wavenumber a for different values of ε

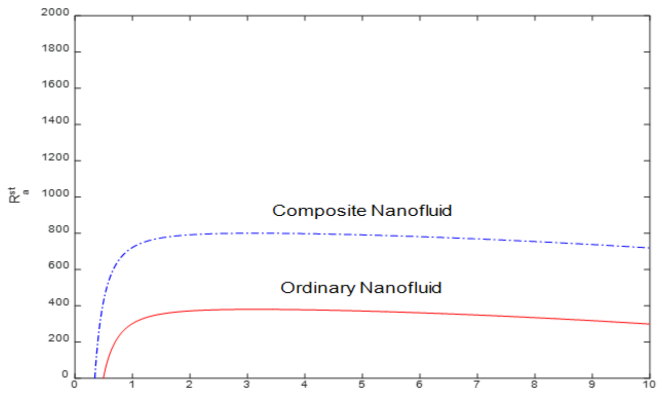

Figure 9: A Comparative study of ordinary and composite nanofluid in continuous medium

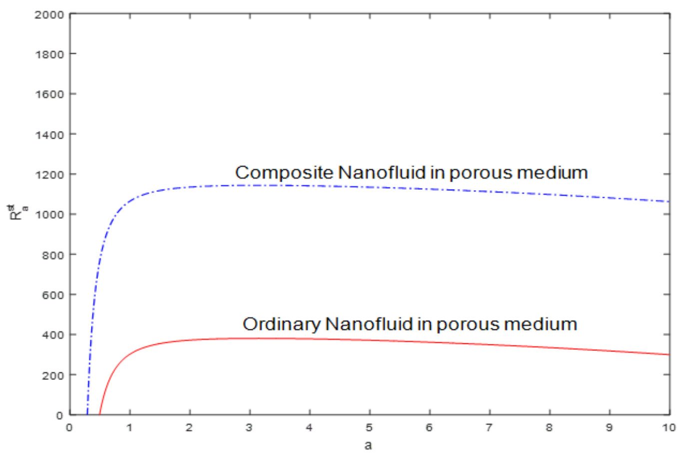

Figure 10: A Comparative study of ordinary and composite nanofluid in porous medium

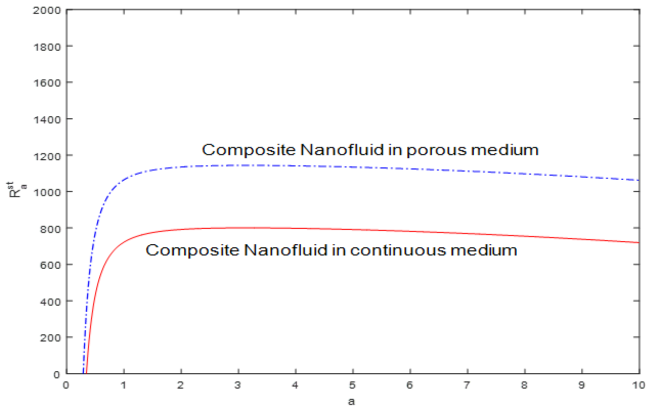

Figure 11: A Comparative study of composite nanofluid in continuous and porous medium

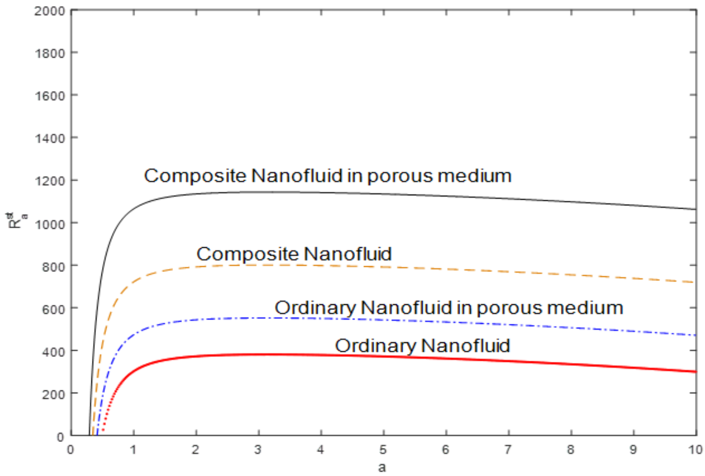

Figure 12: A Comparative study of ordinary and composite nanofluid in continuous and porous medium

Comparative studies of ordinary and composite nanofluid in continuous and porous medium have been shown through Figures 9-12. Figure 9 represents that for continuous medium, the stationary Rayleigh number R_a^{st} falls largely for ordinary nanofluid than that of composite indicating early onset of convection for ordinary nanofluid. Figure 10 represents that for porous medium as well, the stationary Rayleigh number R_a^{st} falls largely for ordinary nanofluid indicating early onset of convection in this case. Figure 11 represents the comparison of composite nanofluid in continuous and porous medium which shows that the stationary Rayleigh number R_a^{st} falls largely for composite nanofluid in continuous medium indicating early convection. Figure 12 represents the comparison of ordinay and composite nanofluid in both continuous and porous medium which shows that the stationary Rayleigh number R_a^{st} for ordinary nanofluid in continuous medium < R_a^{st} for ordinary nanofluid in porous medium <

R_a^{st} for composite nanofluid in continuous medium < R_a^{st} for composite nanofluid in porous medium.

6. Conclusion

The stability analysis of composite nanofluid in porous medium is performed which is subjected to temperature gradient. This study is performed to investigate the stability of the fluid flow under realistic conditions for which the normal mode method is applied. It is assumed that the density of nanoparticles is less than that of the base fluid i.e., bottom-heavy distribution is considered $(\rho_{p_1}, \rho_{p_2} < \rho_0)$. The value of Rayleigh number is obtained for stationary and oscillatory state to make the following observations:

(i) At stationary state, the acceleration in nanofluid is constrained by the regular fluid acceleration.

(ii) The condition for oscillatory convection is achieved.

(iii) It is found that Ln_1, Ln_2, N_{A_1} and N_{A_2} plays stabilizing role in the system.

(iv) Rn_1, Rn_2 and ε plays destabilizing role in the system.

(v) A comparative study between composite and ordinary nanofluid in porous medium indicates that presence of composite nanoparticles in fluid leads to delay in convection.

Acknowledgements

The corresponding author is thankful to UGC, New Delhi for providing financial assistance vide letter no. 44-1/2018(SA-III).

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