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Connected 2-Equitable Domination in Fuzzy Graphs

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A sub set $D \subseteq V(G)$ of a fuzzy graph $G(\sigma,\mu)$ is said to be equitable dominating set if each $v \in V - D$, there exists a vertex $u \in D$ such that $uv \in E(G)$ and $|\deg_s(u) - \deg_s(v)| \leq 1$. An equitable dominating set $D$of $V(G)$ is said to be 2-equitable dominating set in a fuzzy graphs $G(\sigma,\mu)$, if every vertex $v \in V - D$ there exists a vertex $u \in D$ or v is equitable dominated by at least two vertices in D. A 2-equitable dominating set $D \subseteq V(G)$ of a fuzzy graph $G(\sigma,\mu)$ is said to be connected 2-equitable dominating set if the induced subgraph $\langle D \rangle$ is connected. In this study, connected 2-equitable equitable dominating set, its number in fuzzy graphs are introduced. Bounds and some theorems related to connected 2-equitable equitable domination numbers are stated and proved.

Keywords: Fuzzy graph, equitable dominating set, equitable domination number, 2 - equitable dominating set, connected equitable dominating set and its number, connected 2 - equitable dominating set, connected 2 - equitable dominating number.

1. Introduction

Applications of fuzzy graph are include data mining, clustering, image capturing, networking, communications, planning, etc., L.A Zadeh [1] introduced fuzzy sets in 1965. Fuzzy graph theory was initiated by A. Rosenfeld [2] in 1975. Gurubaran et.,all[4] initiated the concept 2-equitable domination in fuzzy graphs in 2018. Complementary nil g-eccentric domination fuzzy graphs concepts introduced by Mohamed Ismayil and Muthupandiyan[5] in 2020.

S. Muthupandiyan and A. Mohamed Ismayil [7] introduced the concept isolate g-eccentric domination in fuzzy graph in 2023. John JC, Xavier P, Priyanka GB.[8] Divisor 2-equitable domination in fuzzy graphs in 2023. Muthupandiyan S, Ismayil AM[9] introduced the concept of connected g-eccentric domination in fuzzy graphs in 2022. Rabeeh Ahamed et.al.,[10] stated the concept complementary nil equitable domination in fuzzy graphs in 2024.

2. Basic Definitions

Definition 2.1[4]: A fuzzy graph $G = (\sigma, \mu)$is characterized with two functions ρ on V and μ on $E \subseteq V \times V$, where $\sigma: V \to [0,1]$ and $\mu: E \to [0,1]$ such that $\mu(x, y) \leq \rho(x) \wedge \rho(y)\ \forall x, y \in V$. We expect that V is finite and non-empty, μ is reflexive and symmetric. We indicate the crisp graph $G^* = (\sigma^*, \mu^*)$ of the fuzzy graph $G(\sigma, \mu)$ where $\sigma^* = \{x \in V: \rho(x) > 0\}$ and $\mu^* = \{(x, y) \in E: \mu(x, y) > 0\}$. The fuzzy graph $G = (\sigma, \mu)$ is called trivial in this case $|\rho *| = 1$.

Definition 2.2[4]: A path P of length n is a sequence of distinct nodes $u_0, u_1, \dots, u_n$ such that $\mu(u_{i-1}, u_i) > 0, i = 1,2, \dots, n$ and the degree of membership of a weakest arc is defined as its strength.

Definition 2.3[4]: An edge is said to be strong if its weight is equal to the strength of connectedness of its end nodes. Symbolically, $\mu(u, v) \geq CONN_{G-(u,v)}(u, v)$.

Definition 2.4[4]: The order and size of a fuzzy graph $G(\sigma, \mu)$ are defined by $p = \sum_{u \in V} \sigma(u)$ and $q = \sum_{uv \in E} \mu(u, v)$ respectively.

Definition 2.5[6]: Let $G(\sigma, \mu)$ be a fuzzy graph. The strong degree of a vertex $v \in \sigma^*$ is defined as the sum of membership values of all strong arcs incident at v and it is denoted by $d_s(v)$. Also, it is defined by $d_s(v) = \sum_{u \in N_s(v)} \mu(u, v)$ where $N_s(v)$ denotes the set of all strong neighbors of v.

Definition 2.6[6]: A fuzzy graph $G(\sigma, \mu)$ is connected if $CONNG(u, v) > 0$ where $CoNNG(u, v)$ is strength of connectedness between two vertices u, v in $G(\sigma, \mu)$.

Definition 2.7[6]: In a fuzzy graph $G(\sigma, \mu)$, strength of connectedness between two vertices $u, v \in V(G)$ is maximum strength of all paths between u, v in $V(G)$.

Definition 2.8[4]: A subset D of V is called a dominating set (DS) in G if for every $v \notin D$ there exist $u \in D$ such that u dominates v. The minimum scalar cardinality taken over all dominating set is called domination number and is denoted by the symbol γ. The maximum scalar cardinality of a minimal dominating set is called upper domination number and is denoted by the symbol Γ.

Definition 2.9[3]: A sub set $D \subseteq V(G)$ of a fuzzy graphs $G(\sigma, \mu)$ is said to be equitable dominating set (EDS) if each $v \in V - D$, there exists a vertex $u \in D$ such that $uv \in E(G)$ and $|\deg_s(u) - \deg_s(v)| \leq 1$.

Definition 2.10[3]: An equitable dominating set $D \subseteq V$ of a fuzzy graph $G = (\sigma, \mu)$ is called 2 – equitable dominating set if for every vertex $v \in V - D$ there exist $v \in D$ or v is equitable dominated by at least two vertices in D.

Definition 2.11[3]: Let $D \subseteq V(G)$ of a fuzzy graph G is an equitable dominating set. A set $D \subseteq V(G)$ of a fuzzy graph $G(\sigma, \mu)$ is said to be connected equitable dominating set, if the induced subgraph $\langle D \rangle$ is connected.

3. Main Results

Connected 2- Equitable Domination in Fuzzy Graphs

In this section discuss about connected 2- equitable dominating set and its number in fuzzy graphs. Bound and theorem related to connected 2- equitable domination number in fuzzy graphs are stated and proved.

Definition 3.1:

A 2-equitable dominating set $D \subseteq V(G)$ of a fuzzy graph $G(\sigma, \mu)$ is said to be connected 2-equitable dominating set if the induced subgraph $\langle D \rangle$ is connected. A connected 2 – equitable dominating set D is said to be minimal if no proper subset of D' is a connected 2 – equitable dominating set. The minimum scalar cardinality of a minimal connected 2 – equitable dominating set of G is called the connected 2 – equitable dominating number of G and is denoted by $\gamma_{c2eqd}(G)$. The maximum scalar cardinality of a minimal connected 2 – equitable dominating set of G is called an upper connected 2 – equitable dominating number of G and is denoted by $\Gamma_{c2eqd}(G)$.

Note 3.1: The minimum connected 2 - equitable dominating set is denoted by γ_{c2eqd}-set.

Example 3.1: Consider the fuzzy graph $G(\sigma, \mu)$.

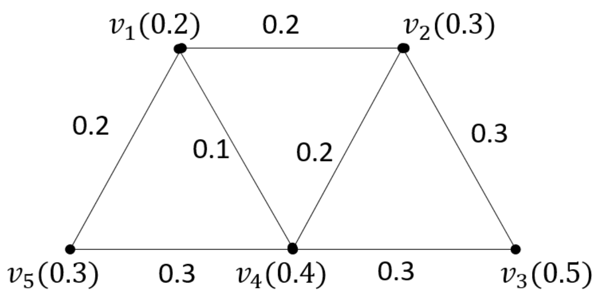

Figure: Connected 2-Equitable Dominating Set in a Fuzzy Graph

From the fuzzy graph given in example 3.1, the followings are observed.

1. The minimum connected 2- equitable dominating set is $D_1 = \{ v_1, v_2, v_4, \}$, then $\gamma_{c2eqd}(G) = 0.9$.

2. The upper connected 2- equitable dominating set is, $D_2 = \{ v_2, v_4, v_5 \}$, then $\Gamma_{c2eqd}(G) = 1.0$

Observation 3.1: For any connected fuzzy graphs $G(\sigma, \mu)$

1. $\gamma(G) \leq \gamma_{eqd}(G) \leq \gamma_{2eqd}(G) \leq \gamma_{c2eqd}(G)$

2. $\gamma_{c2eqd}(G) \leq \Gamma_{c2eqd}(G)$.

3. Obviously any minimum connected 2-equitable dominating set is also minimal but the converse is not true.

4. The complement of a connected 2-equitable dominating set is need not be connected.

5. Supper set of a connected 2-equitable dominating set is also a connected 2-equitable dominating set.

Proposition 3.1: For any fuzzy graph G with order p, then $\sum_{\substack{v_i, v_j \in G \\ v_i \neq v_j}} \min\left(\sigma(v_i), \sigma(v_j)\right) \leq \gamma_{c2eqd}(G) \leq p$.

Proof:
Let D be a connected dominating set of a fuzzy graph G having atleast two vertices has minimum of V which is a sum of minimum value of vertices $v_i, v_j \in D', \gamma_{c2eqd}(G) \leq p$ it is obviously true.

Theorem 3.1: Let G be a fuzzy graph$\gamma_{c2eqd}(G) = p$ iff the fuzzy graph G has adjacent to less than two vertices.

Proof:
Let G be a fuzzy graph, then $\gamma_{c2ed}(G) = p$ then definition of fuzzy graph has all vertices in dominating set D. which shows that every vertex in G has adjacent to less than two vertices. Conversely, G be a fuzzy graph has adjacent to less than two vertices then every vertex is in regular dominating set. Which is$\gamma_{c2eqd}(G) = p$.

Theorem 3.2: Let D is a minimal connected 2 - equitable dominating set, then $V - S$ contains minimal connected 2 - equitable dominating set if every vertex of V in a fuzzy graph G adjacent to more than two vertices in V.

Proof:
Let D be a minimal connected 2 - equitable set of G suppose that $V - D$ is not a connected 2 - equitable dominating set, then there exists at least one vertex $v \in D$ which is not an 2 equitable adjacent to any vertex in $V - D$. Therefore $V - D$ is connected 2 - equitable adjacent to at least two vertices in D then $D - \{v\}$ is a connected 2 - equitable dominating set which is a contradiction. Hence every vertex in D must be equitable adjacent to at least two vertices in $V - D$. Hence $V - D$ is an connected 2 - equitable dominating set which contains minimal connected 2 - equitable dominating set.

Theorem 3.3: Let G be a connected fuzzy graph has no non - equitable edge and H is spanning subgraph of G then $\gamma_{c2eqd}(G) \leq \gamma_{c2eqd}(H)$.

Proof:
Let G be a connected fuzzy graph and H is the spanning subgraph of H. consider D is minimum connected 2 - equitable dominating set of G, D also a connected 2 - equitable dominate all the vertices in $V(H) - D$ that is D is an connected 2 - equitable dominating set in H. Hence

$\gamma_{c2eqd}(G) \leq \gamma_{c2eqd}(H)$.

Theorem 3.4: For any fuzzy graph $G, \gamma_{2eqd} + \min\sigma(v_i) \leq \gamma_{c2eqd}(G)$, for $v_i \notin D'$.

Proof:

Let D be connected 2 - equitable dominating set with minimum cardinality γ_{c2ed} for any vertex $v_i \in D, D - \{v_i\}$ is 2 - equitable dominating set. Hence $\gamma_{2eqd} + \min\sigma(v_i) \leq \gamma_{c2eqd}(G)$.

Theorem 3.5: Let G be a fuzzy graph without isolated vertices. Then $\gamma(G) \leqslant \gamma_{c2eqd}(G)$

Proof:

Every connected 2- equitable dominating set is a connected dominating set. Thus $\gamma(G) \leqslant \gamma_{c2eqd}(G)$

Theorem 3.6 A connected 2 - equitable dominating set exists for any strong fuzzy graph G .

Proof.

Let $G = (\sigma, \mu)$ be a fuzzy graph. Suppose a strong fuzzy graph G has a connected 2- equitable dominating set, obviously it contains a 2 equitable dominating set D. Therefore every strong fuzzy graph has an connected 2 - equitable dominating set and it exists for strong fuzzy graph.

Theorem 3.7 For any fuzzy graph G $\gamma_{2eqd}(G) \leq \gamma(G) \leqslant \gamma_{c2eqd}(G)$ (G).

Proof.

It is clear that every inverse 2- equitable dominating set is a 2-equitable dominating set. we get $\gamma_{2eqd}(G) \leq \gamma_{c2eqd}(G)$.

Theorem 3.8 For a fuzzy graph $G = (\sigma, \mu)$ if γ_{c2eqd} is a minimum connected 2-equitable dominating set then $V - D$ is a dominating set of a fuzzy graph G.

Proof

Let v be any vertex in D, D is a connected 2-equitable set in G . Since G has no isolated vertex $v \in N_s(u)$.It is clearly every connected 2-equitable dominating set is a equitable dominating set such that $v \in V - S$. Hence every vertex of D dominates some of the vertices in $V - S$. Therefore, $V - D$ is a dominating set of fuzzy graph G.

Theorem 3.9

A connected2-equitable dominating set D of a fuzzy graph G is minimal if and only if for every vertex $u \in D$ one of the following conditions holds

(i) there exists vertex $v \in V - D$ such that $N_s(v) \cap D = \{u\}$

(ii) $N_s(u) \cap D = \emptyset$

Proof

Suppose that D_1 is a connected 2-equitable dominating set of a fuzzy graph G and (i) & (ii) not hold. Then for some vertex $u \in D$ there exists $v \in N_s(u) \cap D$. Therefore $D - \{u\}$ is an equitable dominating set of G, a contradiction with the minimality of D. Conversely, let for every $u \in D$ one of the conditions (i) or (ii) holds. Suppose that D_1 is not minimal, there exists

$u \in D_1$ such that $D - \{u\}$ is an equitable dominating set of a fuzzy graph G. This means there exists $v \in D - \{u\}$ which is equitable adjacent to u. Hence (ii) does not satisfy.

Theorem 3.10

For any fuzzy graph G without equitable isolated nodes, $\gamma(G) \leq \min\{\gamma(G), \gamma_{2ed}(G), \gamma_{c2eqd}(G)\}$

Proof

Every connected 2-equitable dominating set is a connected dominating sets of G and every connected dominating set is dominating set, similarly every 2-equitable dominating set is dominating set. Hence $\gamma(G) \leq \min\{\gamma(G), \gamma_{2ed}(G), \gamma_{c2eqd}(G)\}$.

Definition 3.2 A subset D of V is a neighborhood connected 2- equitable dominating set (nc2eqd-set) of G if for every node $v \in V - D$ there exists atleast two strong neighbors in . The neighborhood connected 2- equitable domination number of a fuzzy graph G denoted by $\gamma_{nc2eqd}(G)$ is the minimum cardinality of a nc2-eq dominating set of G. A Set D is a minimal neighborhood connected 2- equitable dominating set of a fuzzy graph , if for any vertex $u \in D$, $D - \{u\}$ is not a neighborhood connected 2-equitable dominating set of the fuzzy graph G.

Observation 3.2 For an fuzzy graph $G(\sigma, \mu)$

1. $\gamma_{eqd}(G) \leq \gamma_{nceqd}(G) \leq \gamma_{nc2eqd}(G)$.

2. $\gamma_{2eqd}(G) \leq \gamma_{nc2eqd}(G)$.

3. A supper set of a nc2ed-set is minimal.

Theorem 3.11 Every neighborhood connected 2-equitable dominating set of a fuzzy graph G is a

neighborhood connected equitable dominating set of a fuzzy graph G.

Proof:

Let D be a neighborhood connected 2- equitable dominating set of the fuzzy graph G. Then every node in $V - D$ has atleast two strong neighbors in S. (i.e) for every node $v \in V - D$, there exist minimum two nodes in D and both dominate V.Every node in $V - D$ is dominated by at least two nodes in D. Thus Dis a neighbourhood connected equitable dominating set of G.

Theorem 3.12 If G is a fuzzy graph then $\gamma_{nc2eqd}(G) \geq \gamma_{nceqd}(G)$.

Proof

By the previous theorem 3.11, every neighborhood connected 2-equitable dominating set of a fuzzy graph G is a neighborhood connected equitable dominating set of G. Thus every minimum nc2eqd-set of G is also (nceqd)- set of G. Therefore $\gamma_{nc2eqd}(G) \geq \gamma_{nceqd}(G)$.

Theorem 3.12 Every connected fuzzy graph G has minimum neighborhood connected equitable

2-dominating set Dthen $V - D$ need not be a neighborhood connected 2-equitable dominating set of G.

Proof:

Let D be a neighborhood connected 2-equitable dominating set of G and let $u \in V$. Suppose $|N_S(u)| = 1$ then u belongs to every neighborhood connected 2-equitable dominating set of . Thus v belongs to every minimum neighborhood connected 2 equitable dominating set of . Then $D - \{u\}$ has either no strong neighbor of u or only one strong neighbor of u. Thus $D - \{u\}$ does not have two strong neighbor for v. This implies that $V - D$ is not a neighborhood connected 2-equitable dominating set of . Suppose every node in S has atleast two strong neighbors in $V - D$. Then every node in D has atleast two strong neighbors in $V - D$. Thus $V - D$ is a neighborhood connected 2- equitable dominating set of G. Therefore $V - D$ need not be a neighborhood connected 2-equitable dominating set of G.

4. Conclusion

In this article, an connected 2 - equitable dominating set, its number in fuzzy graphs are obtained. Theorems related to an connected 2 - equitable dominating set and number in a fuzzy graph are stated and proved. Bounds and some points related an connected 2-equitable domination number are observed and discussed.

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