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System Safety in Markovian Process with Alpha Series Process Repairs in Stochastic Process

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Whenever the system states fall into one of three categories—Operating States, Failed-Safe States, or Failed-Unsafe States—a 3-state Markov model is employed to represent some safety-critical systems. The safety and security of the system are estimated using these models. When the system fails, a new model is constructed based on these models, with varying metrics for various fault states. The repair procedure is used for Failed-Safe States but not for the other failed states. Operating times, repair times, and coverage to various failure states are all impacted by frequent repairs. This repair procedure is described using the α – Series Process, which also yields various system safety indices.
Keywords: system safety; Markov process; α – Series.

DEFINITIONS

Definition 1: It is stated that X is stochastically greater than Y (or Y is stochastically less than X) given two random variables, X and Y, if

$$P(X > \alpha) \geq P(Y > \alpha) \text{ for all real } \alpha, \text{ and written as}$$

$X \geq_{st} Y$ or $Y \leq_{st} X$.

Definition 2: The sequence $\{\xi_n, n = 1,2, \dots\}$ is assumed to be a collection of independent, nonnegative random variables. $\{\xi_n, n = 1,2, \dots\}$ is referred to as a α – Series process if the distribution function of ξ_n is $S_n(t) = S(n^a t)$ for some $a > 0$, and all $n = 1,2, \dots, \{\xi_n, n = 1,2, \dots\}$ is stochastically increasing, meaning that

$$\xi_n \leq_{st} \xi_{n+1}, \; n = 1,2,\dots$$

1. Introduction

Under actuality, the failures result under various circumstances from various perspectives. Generally speaking, these failure scenarios fall into one of two groups under the safe view: failed-safe states or failed-unsafe states. These safety-critical systems are modelled using a 3-state Markov model in order to analyse these issues. Numerous studies have been conducted in this area, including those by Bukowsiki & Goble (2001), who each used Markov Chain to contribute to this type of model. Additionally, Cowing et al. (2004), DeLong et al. (2005), Inagaki & Ikebe (1989), and Zhou (1987) discussed the models' indices or metrics. This three-state model was recently established by Cui et al. (2006).

However, the majority of these studies only addressed the scenario in which the system returns to a state "as good as new" following repairs, or they did not address the effects of the maintenance. In actuality, the system may not be as good as new following the repair due to wear and age. Park (1979), Kijima (1989), and Brown & Proschan (1983) all contributed to the unsatisfactory maintenance. It might be more realistic to anticipate that a decaying system will have a lower lifespan following repair, but that the intervals between repairs will lengthen. Lam (1988) originally proposed a geometric process repair model to simulate such a system. Additionally, Zhang (1994) and Finkelstein (1993) created this repair model.

In [], the authors analyse the safe-critical system problem using the geometric process repair model and adjust the 3-state Markov model to better fit the practice.

In this article, we employ the partial product process repair model to analyze the problem of the safe-critical system and make the 3-state Markov model more accord to the practice. For most systems, when they fail, two kind's failed-states occur.

Repair measures are implemented right away for the Failed-Safe State, bringing the system back to a less-than-ideal condition. Since it could result in certain mishaps, remedial procedures are not suggested for the Failed-Unsafe State. Following a repair, the system's lifespan will decrease, but the duration of subsequent repairs will increase. The coverage of the two failed states varies depending on whether it is caused by natural or man-made forces. Typically, following the repair procedure, the coverage to Failed-Safe State may decrease. After multiple repairs, we believe the system is in an astatic condition. The repair process is halted when the system fails once more, whether it goes into the Failed-Safe or Failed-Unsafe states.

In Section 1, we will present a model based on the aforementioned scenarios. A few indexes for the system safety aspects are developed in Section 2. Finally, the application of the solutions to a specific issue is demonstrated with a numerical example.

2. ASSUMPTIONS OF THE MODEL

1) A fresh system is installed initially.

2) There are two failure states when the system is unavailable: There are two types of states: Failed-Safe and Failed-Unsafe. The system has failed to function, but in a safe way, and this

is known as the Failed-Safe State. When a system fails to function as intended and does so in an unsafe way, it is said to be in the Failed-Unsafe State.

3) The system cannot be "as good as new" after the repair, but it will be accepted as soon as it reaches a Failed-Safe State. Specifically, $X_n \leq_{st} X_{n-1} \leq_{st} \dots \leq_{st} X_2 \leq_{st} X_1$, A combination of X_1 and $Y_1 \leq_{st} Y_2 \leq_{st} \dots \leq_{st} Y_{n-1} \leq_{st} Y_n$,

The distribution functions of X_n and Y_n:, are, respectively,

$$F_n(t) = F(n^a t), G_n(t) = G\left(n^b t\right), a \geq 1, b \geq 1;$$

$$n = 1,2,3, \dots.$$

X_i and Y_i have the exponential distributions with $F_1(t) = 1 - e^{-\lambda t}$, $G_1(t) = 1 - e^{-\mu t}$ and X_n, Y_n are independent.

4) One mitigation coverage is $1 - C$, while the other mitigation coverage is C, which leads to a failed-safe condition.

5) The system cannot be "as safe as new" with the repairs. Specifically, $d^{n-1}C$ is the mitigation coverage of the Failed-Safe state following the N^{th} repair.

6) After the N^{th} repair, we believe the system is in an astatic state since the repairs make it more unsafe. If the system fails again, it either goes into the Failed-Safe state or the Failed-Unsafe state, and the repair process is halted. The repair procedure is terminated once the system is in the Failed-Unsafe State.

The system may be modelled using a Markov model as the Failed-Unsafe state is an absorbing state and the distributions of the sojourn intervals in the Operating States and Failed-Safe States are all exponential distributions. This model contains $2N + 3$ states based on assumptions 5) and 6). To keep things simple, let the operational states be the first, third,…,$(2N + 1)^{th}$ states, the failed-safe states be the second, fourth,…, $(2N + 2)^{th}$ states, and the absorbing $(2N + 3)^{th}$ state be the failed-unsafe state.

Figure 1 illustrates the connections between the states and the transition rates. The Markov model transition rate matrix A is obtained from figure 1. The differential equations are then resolved,

$$\frac{d}{dt}P(t) = P(t)A \qquad \dots (2)$$

We obtain $P(t)$, the Markov model transition probability matrix, with the initial condition $P(0) = 1$ (the identity matrix). The solution to (2) is

$$P(t) = \exp(At) \qquad \dots (3)$$

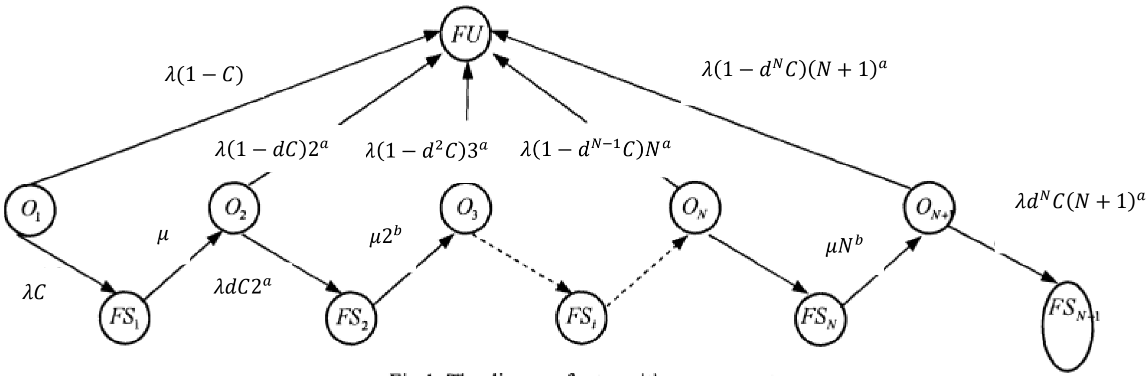


Fig 1. The diagram for transitions among states

A =

$$
\begin{bmatrix}
-\lambda & \lambda C & 0 & 0 & 0 & 0 & 0 & \cdots & \cdots & 0 & \lambda(1-C) \\
 & -\mu & \mu & 0 & 0 & 0 & 0 & \cdots & \cdots & 0 & 0 \\
 & & -\lambda 2^a & \lambda d C 2^a & 0 & 0 & 0 & \cdots & \cdots & 0 & \lambda(1-dC)2^a \\
 & & & -\mu 2^b & \mu 2^b & 0 & 0 & \cdots & \cdots & 0 & 0 \\
 & & & & -\lambda 3^a & \lambda d^2 C 3^a & 0 & \cdots & \cdots & 0 & \lambda(1-d^2C)3^a \\
 & & & & & -\mu 3^b & \mu 3^b & \cdots & \cdots & 0 & 0 \\
 & & & & & & \cdots & \cdots & \cdots & \cdots & \cdots \\
 & & & & & & -\lambda N^a & \lambda d^{N-1} C N^a & \cdots & 0 & \lambda\left(1-d^{N-1}C\right)N^a \\
 & & & & & & & -\mu N^b & \mu N^b & 0 & 0 \\
 & & & & & & & & & 0 & 0 \\
 & & & & & & & & & & 0
\end{bmatrix}
$$

So, from assumption 1), $p(0) = [1,0,\ldots,0]$, we get $p(t)$, probability vector for the Markov model at time t, from the following formula,

$$p(t) = p(0)P(t) \qquad \ldots (4)$$

But, it is difficult to get an analytic expression of $P(t)$ by using equation (3). Usually, for the numerical situation, we use the Laplace transform to get the following equation

$$P^*(s) = P(0)\,(sI - Q)^{-1} \qquad \ldots (5)$$

Thus the inverse Laplace transform of $P^*(s)$, we can get the formula of $P(t)$.

3. INDEXES FOR SYSTEM SAFETY

Once the $P(t)$ is fixed, we also know the expression of $p_i(t)$. Then we can find some important indexes for the system.

1. $S(t)$, Safe as a function of time

By assumption (7), we get

$$p_0(t) = \sum_{i=1}^{N+1} p_{2i-1}(t) \dots (6)$$

$$p_{rs}(t) = \sum_{i=1}^{N+1} p_{2i}(t) \dots (7)$$

and

$$p_{FU}(t) = p_{2N+3}(t) \dots (8)$$

$S(t)$ is the probability the system resides at the Operating States or Failed-Safe States at time. So,

$$S(t) = p_0(t) + p_{FS}(t) \qquad \dots (9)$$

2. C_{sys}, System coverage

System coverage C_{sys} is the probability that the transition will occur from the Operational State to either the Failed-Safe State or the Failed-Unsafe State, given

that the transition to a failure state does occur, when $t \to \infty$.

$$C_{sys} = \lim_{t\to\infty} \frac{p_{FS}(t)}{p_{FS}(t)+p_{FU}(t)} \qquad \dots (10)$$

3. MITUF, Mean TIme to Unsafe Failure

Let Z be the time to Unsafe Failure. Z can be expressed by X_n and Y_n. After i repairs, the probability that the system first enters the FU state is

$$C.\,dC \dots d^{i-1}C.\left(1 - d^iC\right) = d^{\frac{i(i-1)}{2}}C^i(1 - d^iC) \quad \dots (11)$$

and the time to Unsafe is

$$Z = \sum_{j=1}^{i}\left(X_j + Y_j\right) + X_{j+1},$$

For different situations, Z has different the expression. The details can be found in table 1.

Table.1 The expressions of Z for different situations.

Numbers of Repairs	0	1	…	N
Z	X_1	$X_1 + Y_1 + X_2$	…	$\sum_{i=1}^{N}(X_i + Y_i) + X_{n+1}$
probability	$1 - C$	$C(1 - dC)$		$d^{\frac{N(N-1)}{2}} C^N (1 - d^N C)$

Because X_n, Y_n are independent respectively, then we get

$$MTTUF = (I - C)E(X_i)$$

$$+\sum_{n=1}^{N}\left[d^{\frac{n(n-1)}{2}} C^n (1 - d^n C)\left(\sum_{i=1}^{n}(E(X_i) + E(Y_i)) + E(X_{n-1})\right)\right] \quad \ldots (12)$$

Details about these indexes are introduced by DeLong et a/(2005).

4. NUMERICAL EXAMPLE

We assume the new system's life X_1 and the first repair time all have the exponential distributions with parameters $\lambda = 3$ and $\mu = 2$ respectively. Let the ratio of the α – Series Process $a = 1.4, b = 0.4$ and the other parameters $C = 0.5, d = 0.8, N = 3$ respectively.

By equation (1), we have,

$$A = \begin{bmatrix} -3 & 1.5 & 0 & 0 & 0 & 0 & 0 & 0 & 1.5 \\ 0 & -2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -7.917 & 3.1668 & 0 & 0 & 0 & 0 & 4.7502 \\ 0 & 0 & 0 & -2.639 & 2.639 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -13.9666 & 4.4693 & 0 & 0 & 9.4973 \\ 0 & 0 & 0 & 0 & 0 & -3.1037 & 3.1037 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -13.9666 & 4.4693 & 9.4973 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$p_1(t) = e^{-3t},$$

$$p_2(t) = 1.5e^{-2t} - 1.5e^{-3t},$$

$$p_3(t) = 0.51e^{-2t} - 0.61e^{-3t} + 0.1e^{-7.92t}$$

$$p_4(t) = 2.51e^{-2t} - 7.8e^{-2.64t} + 5.35e^{-3t}$$

$$p_5(t) = 0.55e^{-2t} - 1.82e^{-2.64t} + 1.29e^{-3t} - 0.03e^{-7.92t}$$

$$p_6(t) = 2.24e^{-2t} - 17.48e^{-2.64t} + 55.51e^{-3t} - 40.29e^{-3.1t} + 0.03e^{-7.92t}$$

$$p_7(t) = 0.58e^{-2t} - 4.79e^{-2.64t} + 15.71e^{-3t} - 11.51e^{-3.1t} + 0.01e^{-7.92t}$$

$$p_8(t) = 0.02 - 1.3e^{-2t} + 8.11e^{-2.64t} - 23.4e^{-3t} + 16.58e^{-3.1t} - 0.01e^{-7.92t}$$

$$p_9(t) = 0.98 - 6.6e^{-2t} + 23.78e^{-2.64t} - 53.34e^{-3t} + 35.23e^{-3.1t} - 0.04e^{-7.92t}$$

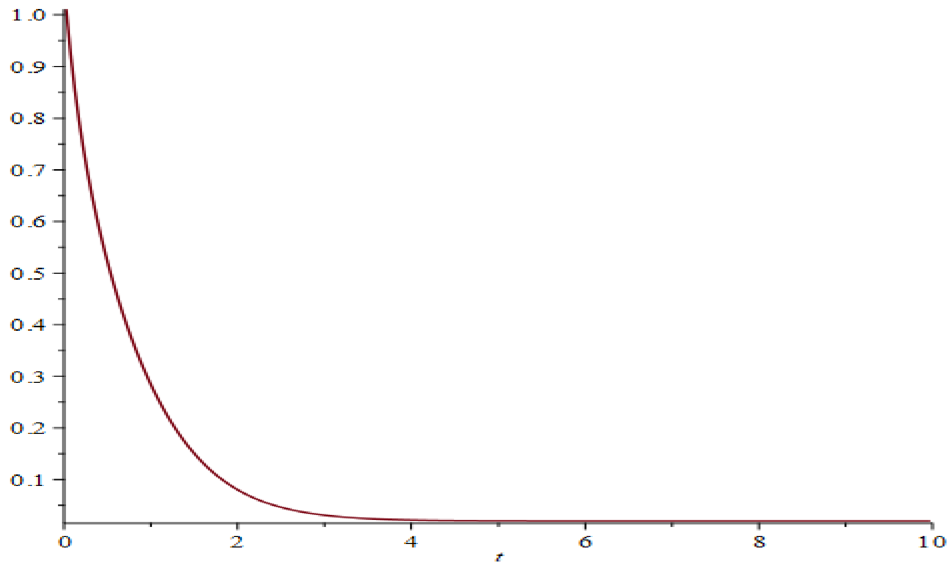
From equations (6), (7) and (8), we get the probabilities in the Operating States, the Failed-Safes State and the Failed-Unsafe States at time t, respectively,

$$p_0(t) = \sum_{i=1}^{3} p_{2i-1}(t) = p_1(t) + p_3(t) + p_5(t) + p_7(t)$$
$$= 1.64e^{-2t} - 6.61e^{-2.64t} + 17.39e^{-3t} - 11.51e^{-3.1t} + 0.08e^{-7.92t}$$

$$p_{FS}(t) = \sum_{i=1}^{3} p_2(t) = p_2(t) + p_4(t) + p_6(t) + p_8(t)$$
$$= 0.02 + 4.95e^{-2t} - 17.17e^{-2.64t} + 35.96e^{-3t} - 23.71e^{-3.1t} + 0.02e^{-7.92t}$$

$$p_{FU}(t) = p_9(t)$$
$$= 0.98 - 6.6e^{-2t} + 23.78e^{-2.64t} - 53.34e^{-3t} + 35.23e^{-3.1t} - 0.04e^{-7.92t}$$

$$S(t) = p_0(t) + p_{FS}(t)$$
$$= 0.02 + 6.59e^{-2t} - 23.78e^{-2.64t} + 53.35e^{-3t} - 35.22e^{-3.1t} + 0.1e^{-7.92t}$$



5. CONCLUSION

In practice, the repeatedly repairs may lead the system life becomes shorter while the consecutive repair times become longer and the coverage to the FS become smaller. In this paper, a new model is built to estimate the system's safety and security with the repeatedly imperfect repairs influence when the system in Failed-Safe States. This model has some significant meaning for evaluating the system safety performance under the imperfect repair condition.

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