

Seismic Design of Reinforced Concrete Structures Using Rigid-Plastic Method

Mahesh S. Salunkhe¹, S.N. Tande²

¹*Research Scholar, Department of Technology, Shivaji University, India,
Mss_tech@unishivaji.ac.in*

²*Professor, Walchand College of Engineering, Sangali, India*

The main goal of this project is to verify the applicability of the Rigid-Plastic Seismic Design (RPSD) approach for multi-story building frames. In areas where significant ground vibrations occur, this streamlined design process can be implemented. The approach is based on the premise that during strong ground motion, plastic deformations dominate, and the elastic component of deformations can be neglected. This study presents an algorithm for creating Rigid-Plastic Spectra (RPS) with various time histories. Comparisons between the Rigid-Plastic Seismic Design (RPSD) method and Non-Linear Time History Analysis (NLTHA) show a high degree of agreement when applied to reinforced concrete frames.

Keywords: inelastic seismic response; rigid-plastic spectrum; rigid-plastic oscillator; plastic response; elastic-plastic oscillator.

1. Introduction

Earthquake-resistant constructions are designed to yield plastically at critical points during intense seismic activity. In such circumstances, a structure's safety depends on its ability to withstand significant plastic deformations. The calculations required to determine the structure's plastic response are complex. Once yielding occurs, the structure's behavior is typically assumed to be fully plastic. During intense ground motion, the elastic component of displacement becomes negligible. For earthquakes of high magnitude, the elastic-plastic model may not accurately represent the behavior as compared to the rigid-plastic model. For oscillators with relatively low elastic limits and sufficient rigidity, the rigid-plastic model serves as a suitable alternative to the elastic-plastic oscillator. As the oscillator's natural period (T) decreases below 0.3 seconds and the earthquake magnitude increases, the maximum plastic displacement predicted by the elastic-plastic model converges with that of the rigid-plastic model.

The NLTHA-based Rigid Plastic Analysis (RPA) method is specifically applied to structures

expected to enter the inelastic range during severe seismic motion. In the RPA approach, the collapse behavior of the structure is first identified, while the remainder of the structure is treated as rigid. The hinges are assumed to exhibit rigid-plastic behavior. Rigid Plastic Spectra (RPS) are then used to determine the structural response, treating the structure as a generalized Single Degree of Freedom (SDOF) system that depends solely on its lateral strength. The rigid-plastic model assumes that non-hysteretic damping is unknown, and that energy dissipation occurs through plastic hinges.

Rigid-Plastic seismic Design Method

Step by step procedure of RPSD method is shown in Fig.1. The Rigid-Plastic Seismic Design (RPSD) method begins with the selection of a suitable collapse mechanism. During significant ground motion, the designer identifies the most effective way for the building to release energy, determining values for ϕ_i , m^* , and k . A suitable collapse mechanism must ensure widespread plastic dissipation throughout the structure while minimizing the non-linear deformation demand at plastic hinges to achieve the specified displacement. Additionally, it is essential to prevent local brittle failure to ensure structural safety. Next, the choice of a dynamic performance criterion, d_{max} , is made, which correlates the degree of plastic deformation at the yield zones with the extent of relative displacements in the structure, reflecting the overall damage.

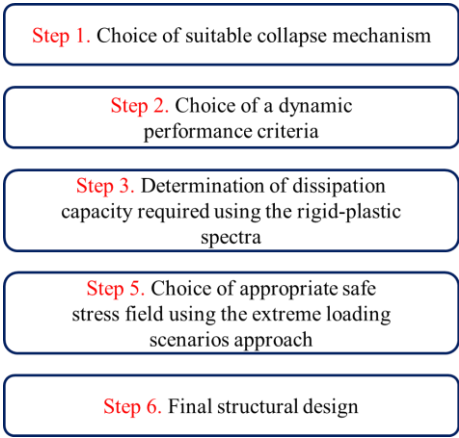


Fig.1 Step by step procedure of RPSD method

The required dissipation capacity is then defined using the rigid-plastic spectra. Given the known values of k and d_{max} , the process determines the dissipation capacity associated with F_y . This involves scaling the Generalized Rigid-Plastic Spectrum (GRPS) by a factor k in the d_{max} direction to produce the Spectral Rigid-Plastic Spectrum (SRPS). The performance criteria are applied to determine a_y , and subsequently, F_y is calculated. At this point, the parameters m^* , k , and F_y are fully defined, enabling the designer to formulate the structure's equations of motion and evaluate its dynamic response to ground motion. The next stage involves choosing an appropriate safe stress field using the extreme loading scenarios approach. This step consists of defining the external force fields using extreme loading scenarios, ensuring that the system remains statically determinate by satisfying specific requirements for the internal stress field, and achieving equilibrium between lateral force fields

and other loads acting during the earthquake. Additionally, the maximum strength demand at any critical point in the structure is determined.

Finally, the structural design is completed by calculating the seismic demand, both in terms of displacement and strength, at each point in the structure. Cross-sectional detailing is performed based on the anticipated ductility at the yield zones, ensuring that adequate stiffness is provided in the remaining portions of the structure. This guarantees that the structure meets performance requirements under seismic loading conditions while maintaining its overall safety and efficiency.

Example- Design of a 5-storey plane frame using RPSD

As an example, consider the design of a five-story planar frame using the Rigid-Plastic Seismic Design (RPSD) method. The frame geometry and mass distribution parameters are shown in Figure 2, while Table 1 provides the specific frame dimensions. For ground motion up to 1g, the design is based on the performance constraint that the maximum inter-story drift does not exceed 2.5%.

Building Data

- ALL concrete MIX – M25
- Steel- Fe415, Beam and columns size 230 x 380 mm
- Foundation on Medium soil
- NLTH analysis in X direction for 7 scaled Time Histories.
- Bay span is 4m
- $DL+LL=11\text{ kN/m}^2$

Table1:Building frame dimensions

Story no	B (mm) (in Y)	D (mm) (in X)	H _i (mm)	% steel
1	230	450	4000	1.3
2	230	450	3200	1.3
3	230	425	3200	1.1
4	230	425	3200	1.1
5	230	375	3200	0.8

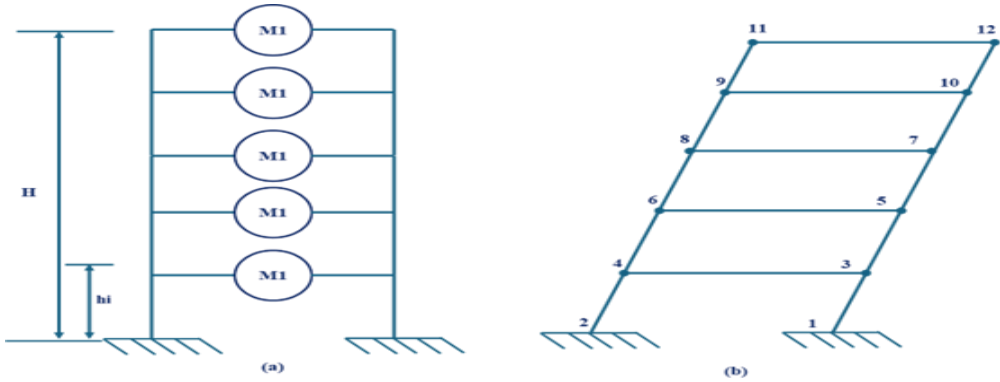


Fig.2 a)5-storey reinforced concrete plane frame b)suitable collapse mechanism

Table 2: Building Floor wise mass

Floor (i)	Mass, (mi)(ton)	Height, hi (m)
1	28.80	4
2	28.80	7.2
3	28.0	10.4
4	28.80	13.6
5	26.21	16.8

Seven-time histories are believed to represent the ground seismicity at the structure's implementation site.

Step 1 - Choice of a suitable collapse mechanism

Fig.2b illustrates the most appropriate collapse method for this straightforward arrangement. By making this decision, a sizable portion of the dissipation capacity is distributed among beam elements with a restricted axial force, and the plastic hinges experience the least amount of deformation while meeting the global displacement need. To prevent catastrophic failure—a sudden and complete collapse from which recovery is impossible—the weak beam-strong column paradigm is employed. As a result, we have:

$$\begin{aligned} \varphi_i &= h_i / H = \{0.238, 0.428, 0.619, 0.809, 1.00\} \\ m^* &= \sum_{i=1}^4 m_i \cdot \varphi_i^2 = 28.80 \times (0.238 + 0.428 + 0.619 + 0.809 + 1.00) = 118.82 \text{ ton} \\ k &= \frac{\sum_{i=1}^4 m_i \cdot \varphi_i}{\sum_{i=1}^4 m_i \cdot \varphi_i^2} = 28.80 / 118.82 \times (0.238 + 0.428 + 0.619 + 0.809 + 1.00) = 1.373 \end{aligned}$$

Step 2 --Choice of a dynamic performance criterion, d_{\max}

One way to express the dynamic performance requirement is in terms of d_{\max} , or the maximum displacement at the top.

$$d_{\max}=0.42 \text{ m} \Rightarrow \text{maximum drift} = 2.5\%$$

Step 3 -Definition of the required dissipation capacity using the rigid- plastic spectra

The GRPS curve of the El-Centro record is magnified by $k=1.373$ in the d_{\max} – direction to find the highest dynamic response in the X-direction of against the El-Centro record. Fig.3 (b) shows the scaled ground response spectra (SRPS) and Fig.3 (a) shows the ground response spectra (GRPS) of an El-Centro ground motion record.

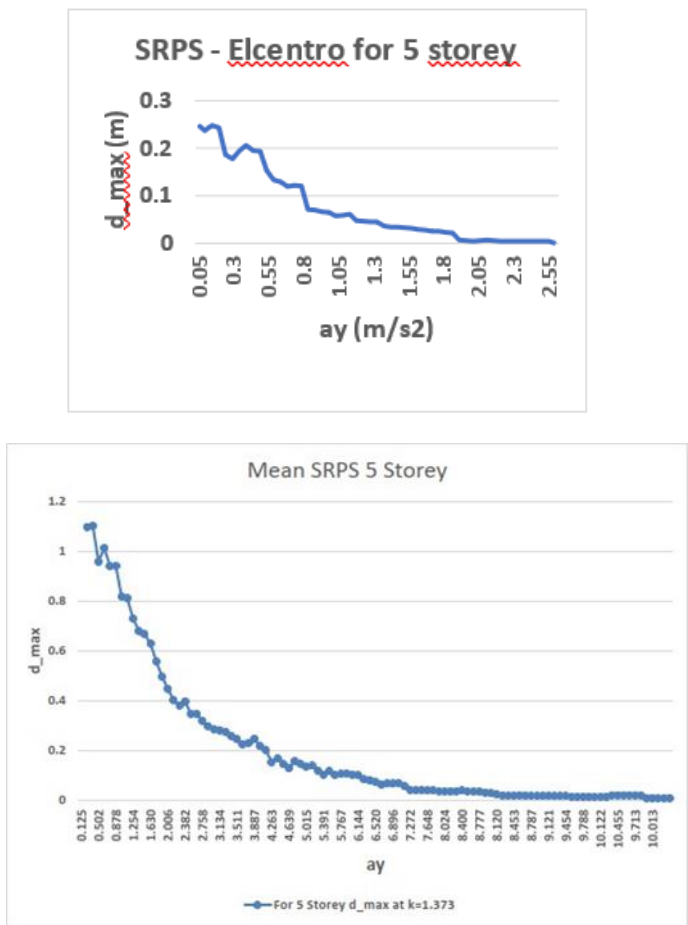


Fig.3 Transition from GRPS to SRPS

Table 3 -5 Story Plane Frame

Store y no	d_max %	h	hi	mi	gi	mi X gi	X gi^2	k	d_max	a_y	F_y*	PG A
	2.5	16. 8				86.55 1	63.04 1	1.373	0.42	2.81	243. 2	9.81
1	1059.08 3	4	4	28.8 0	0.23 8	6.857	1.633	27.429	d*_ma x	0.30591 3		
2	M*	3.2	7.2	28.8 0	0.42 8	12.34 3	5.290	88.869				
3	118.828 9	3.2	10. 4	28.8 0	0.61 9	17.82 9	11.03 7	185.41 7				
4	h*	3.2	13. 6	28.8 0	0.80 9	23.31 4	18.87 3	317.07 4				
5	12.2365 4	3.2	16. 8	26.2 1	1.00 0	26.20 8	26.20 8	440.29 4				

From the SRPS, we have

$$A_y=2.81m/s^2$$

$$F_y=kxm*ay=1.373 \times 118.82 \times 2.81=243.20$$

Step 4 - Choice of an appropriate safe stress field using the extreme loading scenarios approach

Table 4(a) -Lateral force fields for design purposes (KN)

Floor	Plastic Behavior				Slip Behavior	
	Mechanism in the positive direction		Mechanism in the negative direction		Mechanism in the positive direction	
	+PGA	-PGA	+PGA	-PGA	+PGA	-PGA
5	-238	261	-261	238	-249	249
4	-132	173	-173	132	-152	152
2	-26	85	-85	26	-56	56
3	80	-3	3	-80	41	-41
1	169	-82	82	-169	126	-126

Table 4(b) -Lateral moment fields for design purposes (KN-m)

Beams	Mp (kN.m)	Columns	Mp_centre (kN.m)	Mp_middle (kN.m)	Mp_corner (kN.m)
Floors 1-4	79.6305	GF	371.6092	297.2874	260.1265
Floors 5	59.7229	5F Top	185.8046	148.6437	130.0632

Table 5- Lateral force fields for plastic and slip behavior (kN)

Floor	Plastic Behavior				Slip Behavior	
	Mechanism in the positive direction		Mechanism in the negative direction			
	+PGA	-PGA	+PGA	-PGA	+PGA	-PGA
5	-164	217	-217	-164	-190	190

4	-69	164	-164	-69	-116	116
2	26	111	-111	-26	-42	42
3	121	58	-58	-121	-31	-31
1	197	5	-5	-197	-96	-96

The lateral external force and moment fields to be used for design purposes are calculated by table 3 and given in table 4 (a and b). In the present case, we shall make the following considerations regarding the safe stress field:

All yield moments in the plastic hinges at the ends of the beams M_p^B have the same magnitude M i.e. $M_p^B = M$

The yield moment at the base of the columns M_p^C is related to M_p^B by a factor X i.e. $M_p^C = X \cdot M_p^B$

Base shear force at each floor is equally distributed by both columns.

Here, lateral force demand on the structure is already determined. A particular choice of X is sufficient to determine the stress field in the structure at collapse.

$$M_p^* = F_y^* \cdot H = 243.20 \times 16.8 = 4085.76 \text{ kN.m} \quad (\text{Eq. 6})$$

$$M = \frac{M_p^*}{10 + 2X} \quad (\text{Eq. 7})$$

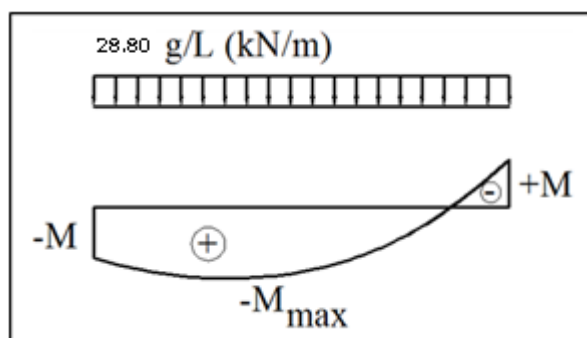


Fig. 4 Bending moment field in the beams when the collapse mechanism in the positive direction is activated

The greatest positive bending moment in these beams, M_{max} , is only 9% greater than M , according to static analysis, if $X=5$. Because there is no requirement to employ substantially differing longitudinal reinforcement along the whole length of the beam, this enables an economical design.

Thus, $M_p^B = 478 \text{ kN-m}$

$$X=4$$



$$M_p^C = 1912 \text{ kN-m}$$

To enforce the weak beam– strong column concept, we can increase the value of $X=6$ which yields:

Thus, $M_p^B = 382 \text{ kN-m}$

$$X=6$$



$M_p^C = 2294 \text{ kN-m}$

Even though it is still regarded as an economical design, the maximum positive bending moment in the beams is now 22% greater than the yield moment at the plastic hinges. The compressive axial force, shear force, and bending moment are denoted by M , V , and N , respectively. The first-floor columns have a significantly higher shear strength requirement, which should also be mentioned. Many building structural failures during strong earth motion can be attributed to this, which results in "soft-story mechanisms."

Table 6 Required strength in the columns for two values of X (kN-m)

Column	Cross- section	X=4				X=6			
		M N-M	V N	P_{\max} N	P_{\min} N	M N-M	V N	N_{\max}	N_{\min}
5 th Floor	Top	427	228	405	86	382	228	373	118
	Bottom	212				300			
4 th Floor	Top	478	228	405	86	382	228	373	118
	Bottom	267				300			
3 rd Floor	Top	745	346	809	172	554	346	745	236
	Bottom	763				954			
2 nd Floor	Top	680	355	1214	258	571	355	1118	354
	Bottom	1349				1636			
1 st Floor	Top	871	524	1618	344	1253	524	1490	472
	Bottom	1912				2294			

Step 5 – Final structural design

The design of the structure is based on the assumption that the yielding of the reinforcement is what causes the hinges to start acting plastically. In this case, 1.4 is the over strength factor. Assume that the width/height ratio β is 1 for columns and 0.6 for beams. The concrete's compressive cylinder strength is 25 MPa, and Grade Fe415 reinforcement steel is utilized. According to the two values of X and the anticipated ductility need, Table 6 details the various cross-sections of the structure.

All of the plastic hinges have sufficient rotational capacity to handle the maximum displacement demand, which equates to 2.5% drift as shown in table 7. Consequently, those elements' predicted rotation ductility demand is at least 7, meaning that plastic behavior will govern the dynamic response. As a result, it is anticipated that the RPSD method will yield precise estimates for this building's seismic reaction to the ground motion scenario.

Table 7 Cross-section detailing

Design	X=4				X=6			
	PHC	PHB	C	B	PHC	PHB	C	B
Flexural demand (kN-m)	1912	478	2677	669	2294	382	3212	535
Height (m)	0.8	0.6	0.8	0.6	0.85	0.55	0.85	0.55
Width (m)	0.8	0.35	0.8	0.35	0.85	0.35	0.85	0.35
Tensile Reinforcement steel area (mm ²)	5311	1770	7436	2478	5997	1543	8397	2162
Tensile reinforcement ration (%)	0.83	0.82	1.16	1.15	0.83	0.85	1.16	1.19

Shear force demand (kN)	524	405	734	567	524	373	734	522
Plastic hinge length (m)	0.93	0.39	-	-	1.09	0.34	-	-
Rotation capacity ($\times 10^{-2}$ rad)	6.46	3.57	-	-	7.11	3.46	-	-
Yield Rotation ($\times 10^{-2}$ rad)	0.32	0.17	-	-	0.35	0.16	-	-
Expected rotation ductility demand	8	15	-	-	7	15	-	-

PHC – Plastic hinges in columns; PHB – Plastic hinges in beams; C – Rest of the columns; B – Rest of the beams

2. Conclusion:

1. During strong ground motions, since plastic component of deformation is quite large as compared to elastic component, elastic deformations can be neglected. Hence rigid plastic relaxation can be assumed at the plastic hinge locations.
2. When pinching effect is considered for modified rigid- plastic relationship, Analyses are seen improved as compared to classic rigid-plastic relationship.
3. Rigid-Plastic Seismic Design (RPSD) can be analysed as simplified design procedure simple because very less computational effort is required. It is straight forward since there is an effective approximation between the dynamic response of the structure and strength distribution.
4. As compared Non-Linear Time History Analysis (NELTA), Rigid-Plastic Seismic Design (RPSD) is less computationally intensive and it takes very less computational time.
5. After choosing a suitable collapse mechanism, the frames are analyzed as Rigid-Plastic oscillators and dynamics response is found as soon as dissipation capacity at yield zone is known.
6. The required dissipation capacity of the structure associated with the generalized yield strength is estimated considering a predefined performance parameter by means of the rigid-plastic spectra and. The combination of both rigid-plastic spectra and the extreme loading scenarios yields a straight forward procedure towards final design at the structure.
7. Rigid-Plastic Seismic Design (RPSD) is predicting relatively less inter storey drift as compared to Non-Linear Time History Analysis (NLTHA). However, the design shear forces and bending moments are predicted on conservative side.
8. The top displacements predicted are quite close to Non-Linear Time History Analysis (NLTHA), however the base shear values predicted shows deviation due to sudden translations from rigid to plastic behaviour as opposed smooth transition in the case of elastoplastic models.

The maximum plastic displacement of an elastic-plastic oscillator in seismic ground motion can be predicted using a rigid-plastic oscillator. This conclusion is supported by the fact that the elastic-plastic oscillator is rigid and that a powerful earthquake causes the oscillator to

undergo significant plastic motion. A single parameter, the oscillator's yield strength, can make it easier to generate the rigid-plastic response spectrum. This spectrum can be used to calculate the maximum plastic displacement of any given earthquake in history

References

- [1] A. Paglietti and M. C. Porcu, "Rigid-plastic approximation to predict plastic motion under strong earthquakes," *Earthquake Engng Struct. Dyn.*, vol. 30, pp. 115–126, 2001.
- [2] J. L. Domingues Costa, R. Bento, V. Levchitch, and M. P. Nielsen, "Rigid-plastic seismic design of reinforced concrete structures," *Earthquake Engng Struct. Dyn.*, vol. 36, pp. 55–76, 2006.
- [3] C. Malaga-Chuquitaype, A. Y. Elghazouli, and R. Bento, "Seismic design and assessment of steel structures based on rigid-plastic response history analysis," *Earthquake Engng Struct. Dyn.*, 2009, doi: 10.1002/eqe.920.
- [4] J. L. Domingues Costa, M. Aono, T. Tchinose, R. Bento, and M. P. Nielsen, "The RPSD method and its application to the design of 12-story RC frame," in *Proc. First European Conf. Earthquake Engineering and Seismology*, Geneva, Switzerland, Sep. 2006, Paper No. 803.
- [5] M. C. Porcu and G. Carta, "Evaluating a rigid-plastic method to estimate the earthquake ductility demand on structures," in *Earthquake Resistant Engineering Structures VIII*, WIT Transactions on The Built Environment, vol. 120, 2011, doi: 10.2495/ERES110221.
- [6] J. L. Domingues Costa, *Rigid-Plastic Seismic Design of Reinforced Concrete Structures*, Ph.D. dissertation, Dept. Civil Eng., Technical Univ. Denmark, 2006.
- [7] M. Paz and W. Leigh, *Structural Dynamics: Theory and Computation*, 5th ed. New Delhi, India: CBS Publishers, 2004.
- [8] R. Bento, M. P. Nielsen, and A. Y. Elghazouli, "Rigid-plastic methods in seismic assessment: Current developments and challenges," *International Journal of Structural Engineering*, vol. 5, no. 4, pp. 312–325, 2014, doi: 10.1504/IJSTRUCTE.2014.064112.
- [9] G. Carta, M. C. Porcu, and A. Paglietti, "Plastic response mechanisms in multi-story structures under seismic loading," *Journal of Earthquake Engineering*, vol. 15, no. 2, pp. 245–265, 2013, doi: 10.1080/13632469.2012.708205.
- [10] A. Krawinkler and E. Miranda, "Performance-based earthquake engineering," in *Proc. 12th World Conference on Earthquake Engineering*, Auckland, New Zealand, 2000, Paper No. 2831.
- [11] H. Tagawa, "Development of rigid-plastic analysis for steel frame structures subjected to seismic loading," *Structural Engineering and Mechanics*, vol. 45, no. 2, pp. 233–250, 2013, doi: 10.12989/sem.2013.45.2.233.
- [12] F. G. Molina and L. M. F. Ribeiro, "Incremental dynamic analysis for rigid-plastic systems," *Earthquake Engineering and Structural Dynamics*, vol. 40, no. 6, pp. 751–769, 2011, doi: 10.1002/eqe.1065.
- [13] N. Makris and S. G. Black, "Dimensional analysis of rigid-plastic oscillators under pulse-like ground motions," *Journal of Engineering Mechanics*, vol. 130, no. 9, pp. 1006–1018, 2004, doi: 10.1061/(ASCE)0733-9399(2004)130:9(1006).
- [14] T. Paulay and M. J. N. Priestley, *Seismic Design of Reinforced Concrete and Masonry Buildings*. New York, NY: Wiley, 1992.
- [15] J. F. Hajjar, "Composite steel and concrete structural systems for seismic design," *Progress in Structural Engineering and Materials*, vol. 4, no. 3, pp. 259–274, 2002, doi: 10.1002/pse.121.
- [16] C. Amadio, M. Fragiaco, and F. Macorini, "Seismic response of composite steel-concrete structures under rigid-plastic assumptions," *Engineering Structures*, vol. 30, no. 3, pp. 632–642, 2008, doi: 10.1016/j.engstruct.2007.05.010.
- [17] S. Mazzoni, F. McKenna, M. H. Scott, and G. L. Fenves, "OpenSees command language manual," *Pacific Earthquake Engineering Research Center*, Univ. California, Berkeley, CA, 2006.
- [18] J. Bozorgnia and V. V. Bertero, *Earthquake Engineering: From Engineering Seismology to Performance-Based Engineering*. Boca Raton, FL: CRC Press, 2004.
- [19] K. J. Elwood and J. O. J. Moehle, "Dynamic collapse of reinforced concrete frames during earthquakes," *Earthquake Engineering and Structural Dynamics*, vol. 37, no. 5, pp. 733–752, 2008, doi: 10.1002/eqe.782.
- [20] H. Krawinkler, "Seismic design and performance: From theory to practice," *Structures Congress 2006: Structural Engineering and Public Safety*, St. Louis, MO, USA, 2006, pp. 1–12.