# Heat Transfer Problem Solving Techniques in Materials Engineering: A Numerical Approach and Practical Applications

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This paper addresses the application of a heat conduction equation in a one-dimensional bar in the context of materials engineering. The equation considers the temperature distribution along the bar as a function of various parameters, including material density, specific heat, diffusivity and thermal conductivity, which can be constant or dependent on temperature. In addition, internal and external heat sources are taken into account, as well as heat transfer on the surface of the bar. The problem of solving this equation is closely related to understanding how heat is transferred through different materials and structures, which is fundamental in materials engineering. Knowledge of the temperature distribution in a given material allows us to optimize its design and performance, as well as predict its thermal behavior in various applications, from heating and cooling systems to manufacturing processes and materials analysis. This study contributes to the field of materials engineering by providing a deeper understanding of heat transfer processes in specific materials, which can lead to improvements in the design and efficiency of thermal systems, as well as the development of new materials. with optimized thermal properties.

**Keywords:** Conductivity, Heat Transfer, Materials Engineering, Temperature Distribution, Firing Method.

## 1. Introduction

Heat transfer plays a fundamental role in many aspects of materials engineering, from the design of heating and cooling systems, to the manufacturing of electronic devices and the optimization of industrial processes. Understanding how heat is transferred through different materials and structures is crucial for developing more efficient technologies and improving the performance of thermal systems.

In this context, this work focuses on the study of the temperature distribution in solid materials, specifically in a one-dimensional bar, by solving a heat conduction equation. This equation describes how the temperature varies depending on the position along the bar, considering various parameters such as the density of the material, specific heat, conductivity and thermal diffusivity.

The problem is formulated as a non-homogeneous Sturm-Liouville problem with Robin-type boundary conditions, which allows modeling both internal and external heat sources, as well as the heat transfer on the surface of the bar. Solving this equation provides valuable information about the temperature distribution in the material, which in turn can be used to optimize the design of thermal systems, predict the thermal behavior of materials under different conditions, and improve energy efficiency in a variety of applications. industrial applications.

This study contributes to the field of materials engineering by providing a deeper understanding of heat transfer processes in solid materials by offering tools for the design and analysis of more efficient and reliable thermal systems. Throughout this work, practical applications of the heat conduction equation in materials engineering will be explored, as well as possible areas of future research to continue advancing in this field.

Heat transfer is a ubiquitous phenomenon in materials engineering, with applications ranging from the design of cooling systems to the manufacturing of electronic devices. At the heart of this discipline is the study of how temperature propagates through different media, being essential to understanding and optimizing the performance of a wide range of technologies.

Heat transfer is a crucial phenomenon in a wide range of real-life applications, from the manufacturing of electronic devices to the optimization of industrial processes and the design of heating and cooling systems. In materials engineering, understanding how heat spreads through different media is essential to improving the performance, efficiency and durability of various products and processes.

Imagine, for example, the importance of temperature control in the automotive industry. Vehicle engines generate a large amount of heat during operation and it is crucial to ensure that this heat is dissipated effectively to prevent overheating and failure of critical components. Additionally, in automobile manufacturing, a variety of materials are used that have different thermal properties, such as metals in the engine, plastics in the interior, and glass in windows. Understanding how heat behaves in these materials is essential to designing efficient cooling systems and preventing temperature problems that can affect vehicle performance and safety.

Another example is found in the aerospace industry, where the materials used in the construction of aircraft must withstand extreme temperature conditions during flight. From the fuselage to the engines and propulsion systems, each component must be carefully designed to withstand the heat generated by friction and aerodynamic drag. Heat transfer plays a crucial role in determining the thermal resistance of these materials and preventing structural deterioration due to thermal expansion and fatigue..

Furthermore, in the electronics industry, thermal management is essential to ensure the performance and reliability of devices such as computers, mobile phones and communication equipment. Electronic circuits generate heat during operation, and excess heat can cause component failure and reduce the life of the device. Therefore, engineers must design efficient cooling systems, such as heat sinks and fans, to control temperature and prevent heat damage.

In summary, heat transfer is a fundamental aspect of materials engineering with vital applications in numerous real-life fields. Understanding how heat behaves in different materials and structures is essential for the development of more efficient, safe and reliable technologies that improve our quality of life and drive progress in modern society.

In this context, this work focuses on the analysis of the temperature distribution in solid materials, particularly in a one-dimensional bar. Modeling this process is achieved by solving a heat conduction equation, which describes how temperature varies as a function of position along the bar. This equation, influenced by parameters such as material density, specific heat, thermal conductivity and diffusivity, provides a detailed view of how heat behaves in the material.

The problem is formulated as a non-homogeneous Sturm-Liouville problem with Robin-type boundary conditions, which allows modeling various internal and external heat sources, as well as the heat transfer on the surface of the bar. The solution of this equation provides crucial information about the temperature distribution in the material, which can be used to optimize the design of thermal systems, predict the thermal behavior of materials under different conditions and improve energy efficiency in a variety of applications. industrial.

This study aims to contribute to the field of materials engineering by providing a deeper understanding of heat transfer processes in solid materials by offering practical tools for the design and analysis of thermal systems. Throughout this work, concrete applications of the heat conduction equation in materials engineering will be explored, as well as possible directions for future research in this exciting interdisciplinary field.

## 2. Mathematical model.

Solving the problem using the shot method:

$Dadas\ f, w\ en\ (\theta, L), encontrar\ \ u: (0, L) \to \mathbb{R}^{+} tal\ que:$

$$\begin{cases} \rho . C . \alpha . u - \frac{d}{dx}\left(K(u)\frac{du}{dx}\right) - C_{sur} . \left(\frac{2}{r}\right)(u_{sur} - u) = f + w\ ; \\ u(x_0) = 0 \\ u(x_L) = 0 \end{cases}$$

$$en\ \ x_0 \leq x \leq x_L \tag{1}$$

The PVF (1) is a non-homogeneous Sturm-Liouville problem with Robin-type boundary conditions.

The K(u) function is the thermal conductivity function, which can be considered constant or linear with respect to u (temperature). Both cases will be treated:

*Caso a)* K(u)constant: $K(u) = a + b\,;\quad con\ a,b\ \in\ \mathbb{R}_+^*$

In this case, the term$\frac{d}{dx}\left(K(u)\frac{du}{dx}\right)$can be replaced by$\frac{d}{dx}\left((a+b)\frac{du}{dx}\right)$

$$\frac{d}{dx}\left((a+b)\frac{du}{dx}\right) = (a+b)\frac{d^2u}{dx^2}$$

Then, substituting

$$\rho.C.\alpha.u - (a+b)\frac{d^2u}{dx^2} - b\frac{du}{dx} - C_{sur}.\left(\frac{2}{r}\right)(u_{sur} - u) = f + w\,;$$

$$\frac{d^2u}{dx^2} = \frac{1}{a+b}\left(\left(\rho.C.\alpha. + C_{sur}.\left(\frac{2}{r}\right)\right)u - \left(C_{sur}.\left(\frac{2}{r}\right)u_{sur} + f + w\right)\right);$$

Then the PVF(1) can be rewritten as:

$$\begin{cases} \frac{d^2u}{dx^2} = \frac{1}{a+b}\left(\left(\rho.C.\alpha. + C_{sur}.\left(\frac{2}{r}\right)\right)u - \left(C_{sur}.\left(\frac{2}{r}\right)u_{sur} + f + w\right)\right); \\ u(x_0) = 0 \\ u(x_L) = 0 \end{cases}$$

$$en\ x_0 \le x \le x_L$$

Using the notation$u'' = \frac{d^2u}{dx^2}\ ;\quad u' = \frac{du}{dx}$you have:

$$\begin{cases} u'' = \frac{1}{a+b}\left(\left(\rho.C.\alpha. + C_{sur}.\left(\frac{2}{r}\right)\right)u - (C_{sur}.\left(\frac{2}{r}\right)u_{sur} + f + w)\right)\,; \\ u(x_0) = 0 \\ u(x_L) = 0 \end{cases} \qquad (2)$$

Let the linear function be:

$$f(x,u,u') = \left(\left(\rho.C.\alpha. + C_{sur}.\left(\frac{2}{r}\right)\right)u - \left(C_{sur}.\left(\frac{2}{r}\right)u_{sur} + f + w\right)\right)\frac{1}{a+b}$$

Then, the linear PVF that we want to solve is:

$Dadas\ f,w\ \ encontrar\ \ u{:}\,(0,L) \to \mathbb{R}^+ tal\ que{:}$

$$\begin{cases} u'' = f(x,u,u')\,;\ en\ x_0 \le x \le x_L \\ u(x_0) = 0 \\ u(x_L) = 0 \end{cases} \qquad (3)$$

To solve the linear PVF (3) by the linear shot method, the following PVI must be solved

$$\begin{cases} u'' = \left(\left(\rho.C.\alpha. + C_{sur}.\left(\frac{2}{r}\right)\right)u - \left(C_{sur}.\left(\frac{2}{r}\right)u_{sur} + f + w\right)\right)\frac{1}{a+b}\ ; \\ u(x_0) = 0 \\ u'(x_0) = 0 \end{cases}$$

$$en\ \ x_0 \leq x \leq x_L$$

(4)

and

$$\begin{cases} u'' = \frac{1}{a+b}\left(\rho.C.\alpha. + C_{sur}.\left(\frac{2}{r}\right)\right)u\ ;\ \ en\ \ x_0 \leq x \leq x_L \\ u(x_0) = 0 \\ u'(x_0) = 1 \end{cases} \quad (5)$$

To know the term f+w that appears in the PVI (4), the analytical solution of the problem is used, which is:

$$u(x) = \frac{ae^{-\Delta t}}{\rho Cb} C_{sur}\left(\frac{2}{r}\right) sen\left(\frac{\pi}{L}x\right) senh\left(\frac{\pi}{L}x\right)$$

u'(x) and u"(x) were determined

$$u'(x) = \frac{ae^{-\Delta t}}{\rho Cb} C_{sur}\left(\frac{2}{r}\right)\frac{\pi}{L}\left(sen\left(\frac{\pi}{L}x\right)cosh\left(\frac{\pi}{L}x\right) + cos\left(\frac{\pi}{L}x\right)senh\left(\frac{\pi}{L}x\right)\right)$$

$$u''(x) = \frac{ae^{-\Delta t}}{\rho Cb} C_{sur}\left(\frac{2}{r}\right)\left(\frac{\pi}{L}\right)^2 2cos\left(\frac{\pi}{L}x\right)cosh\left(\frac{\pi}{L}x\right)$$

Then you have

$$f + w = \rho.C.\alpha.u - (a+b)u'' - C_{sur}.\left(\frac{2}{r}\right)(u_{sur} - u)$$

### 3. Experimental Results

Solving the problem using the shot method:

*Dadas* $f, w$ *en* $(\theta, L)$*, encontrar* $u: (0, L) \to \mathbb{R}^+$ *tal que*:

$$\begin{cases} \rho.C.\alpha.u - \frac{d}{dx}\left(K(u)\frac{du}{dx}\right) - C_{sur}.\left(\frac{2}{r}\right)(u_{sur} - u) = f + w\ ; \quad en\ \ x_0 \leq x \leq x_L \\ u(x_0) = 0 \\ u(x_L) = 0 \end{cases}$$

$$\begin{cases} \rho.C.\alpha.u - \frac{d}{dx}\left(K(u)\frac{du}{dx}\right) - C_{sur}.\left(\frac{2}{r}\right)(u_{sur} - u) = f + w\ ; \quad en\ \ x_0 \leq x \leq x_L \\ u(x_0) = 0 \\ u(x_L) = 0 \end{cases} \quad (1)$$

The PVF (1) is a non-homogeneous Sturm-Liouville problem with Robin-type boundary conditions.

The K(u) function is the thermal conductivity function, which can be considered constant or linear with respect to u (temperature). Both cases will be treated:

*Caso a)* K(u)constant: $K(u) = a + b\,;\quad con\ a,b\ \in\ \mathbb{R}_{+}^{*}$

In this case, the term $\frac{d}{dx}\left(K(u)\frac{du}{dx}\right)$ can be replaced by $\frac{d}{dx}\left((a+b)\frac{du}{dx}\right)$

$$\frac{d}{dx}\left((a+b)\frac{du}{dx}\right) = (a+b)\frac{d^2u}{dx^2}$$

Then, substituting

$$\rho.C.\alpha.u - (a+b)\frac{d^2u}{dx^2} - b\frac{du}{dx} - C_{sur}.\left(\frac{2}{r}\right)(u_{sur} - u) = f + w\,;$$

$$\frac{d^2u}{dx^2} = \frac{1}{a+b}\left(\left(\rho.C.\alpha. + C_{sur}.\left(\frac{2}{r}\right)\right)u - \left(C_{sur}.\left(\frac{2}{r}\right)u_{sur} + f + w\right)\right);$$

Then the PVF(1) can be rewritten as:

$$\begin{cases} \dfrac{d^2u}{dx^2} = \dfrac{1}{a+b}\left(\left(\rho.C.\alpha. + C_{sur}.\left(\frac{2}{r}\right)\right)u - \left(C_{sur}.\left(\frac{2}{r}\right)u_{sur} + f + w\right)\right); \quad en\ x_0 \le x \le x_L \\ u(x_0) = 0 \\ u(x_L) = 0 \end{cases}$$

Using the notation $u'' = \frac{d^2u}{dx^2}\ ;\quad u' = \frac{du}{dx}$ you have:

$$\begin{cases} u'' = \frac{1}{a+b}\left(\left(\rho.C.\alpha. + C_{sur}.\left(\frac{2}{r}\right)\right)u - (C_{sur}.\left(\frac{2}{r}\right)u_{sur} + f + w)\right)\ ;\ en\ x_0 \le x \le x_L \\ u(x_0) = 0 \\ u(x_L) = 0 \end{cases} \qquad (2)$$

Let the linear function be:

$$f(x,u,u') = \left(\left(\rho.C.\alpha. + C_{sur}.\left(\frac{2}{r}\right)\right)u - \left(C_{sur}.\left(\frac{2}{r}\right)u_{sur} + f + w\right)\right)\frac{1}{a+b}$$

Then, the linear PVF that we want to solve is:

$Dadas\ f,w\ encontrar\ u: (0,L) \to \mathbb{R}^{+} tal\ que:$

$$\begin{cases} u'' = f(x,u,u')\ ;\ en\ x_0 \le x \le x_L \\ u(x_0) = 0 \\ u(x_L) = 0 \end{cases} \qquad (3)$$

To solve the linear PVF (3) by the linear shot method, the following PVI must be solved

$$\begin{cases} u'' = \left(\left(\rho.C.\alpha. + C_{sur}.\left(\frac{2}{r}\right)\right)u - \left(C_{sur}.\left(\frac{2}{r}\right)u_{sur} + f + w\right)\right)\frac{1}{a+b}\ ; \\ u(x_0) = 0 \\ u'(x_0) = 0 \end{cases}$$

(4) $\qquad en\ x_0 \le x \le x_L$

AND

$$\begin{cases} u'' = \frac{1}{a+b}\left(\rho.C.\alpha.+C_{sur}.\left(\frac{2}{r}\right)\right)u\,; \;\; en \;\; x_0 \leq x \leq x_L \\ u(x_0) = 0 \\ u'(x_0) = 1 \end{cases} \quad (5)$$

To know the term f+w that appears in the PVI (4), the analytical solution of the problem is used, which is:

$$u(x) = \frac{ae^{-\Delta t}}{\rho Cb} C_{sur}\left(\frac{2}{r}\right) sen\left(\frac{\pi}{L}x\right) senh\left(\frac{\pi}{L}x\right)$$

They were determinedu'(x) and u"(x)

$$u'(x) = \frac{ae^{-\Delta t}}{\rho Cb} C_{sur}\left(\frac{2}{r}\right)\frac{\pi}{L}\left(sen\left(\frac{\pi}{L}x\right)cosh\left(\frac{\pi}{L}x\right)\right.$$

$$\left. + cos\left(\frac{\pi}{L}x\right)senh\left(\frac{\pi}{L}x\right)\right)$$

$$u''(x) = \frac{ae^{-\Delta t}}{\rho Cb} C_{sur}\left(\frac{2}{r}\right)\left(\frac{\pi}{L}\right)^2 2cos\left(\frac{\pi}{L}x\right)cosh\left(\frac{\pi}{L}x\right)$$

Then you have

$$f + w = \rho.C.\alpha.u - (a+b)u'' - C_{sur}.\left(\frac{2}{r}\right)(u_{sur} - u)$$

In this second experiment it is observed that the method is sensitive to the value of the ODE parameters, by varying the value of the parameter$\Delta t$errors are generated in the approximate solution. This sensitivity comes from the Runge Kutta method, since it is sensitive to the parameters, especially if they are very low values, due to rounding errors.

For the nonlinear case we can observe a similar behavior.

*Caso b)* K(u)linear with respect to u:

$$K(u) = a + b.u\,; \;\; con \;\; a, b \in \mathbb{R}_+^*$$

In this case, the term$\frac{d}{dx}\left(K(u)\frac{du}{dx}\right)$can be replaced by$\frac{d}{dx}\left((a+bu)\frac{du}{dx}\right)$

$$\frac{d}{dx}\left((a+bu)\frac{du}{dx}\right) = (a+bu)\frac{d^2u}{dx^2} + b\frac{du}{dx}$$

Then, substituting

$$\rho.C.\alpha.u - (a+bu)\frac{d^2u}{dx^2} - b\frac{du}{dx} - C_{sur}.\left(\frac{2}{r}\right)(u_{sur} - u) = f + w\,;$$

$$\frac{d^2u}{dx^2} = \left(\left(\rho.C.\alpha.+C_{sur}.\left(\frac{2}{r}\right)\right)u - b\frac{du}{dx}\right.$$

$$\left. - \left(C_{sur}.\left(\frac{2}{r}\right).u_{sur} + f + w\right)\right)\frac{1}{a+bu}\,;$$

Then the PVF(1) can be rewritten as:

$$\begin{cases} \frac{d^2u}{dx^2} = \left(\left(\rho.C.\alpha. + C_{sur}.\left(\frac{2}{r}\right)\right)u - b\frac{du}{dx} - (u_{sur} + f + w)\right)\frac{1}{a+bu}; \quad en\ x_0 \leq x \leq x_L \\ u(x_0) = 0 \\ u(x_L) = 0 \end{cases}$$

Using the notation$u'' = \frac{d^2u}{dx^2}$ ; $u' = \frac{du}{dx}$you have:

$$\begin{cases} u'' = \left(\left(\rho.C.\alpha. + C_{sur}.\left(\frac{2}{r}\right)\right)u - bu' - (u_{sur} + f + w)\right)\frac{1}{a+bu}\ ; \quad en\ x_0 \leq x \leq x_L \\ u(x_0) = 0 \\ u(x_L) = 0 \end{cases} \quad (6)$$

Let the nonlinear function be:

$$f(x,u,u') = \left(\left(\rho.C.\alpha. + C_{sur}.\left(\frac{2}{r}\right)\right)u - bu' - (u_{sur} + f + w)\right)\frac{1}{a+bu}$$

Then, the nonlinear PVF that we want to solve is:

$Dadas\ f, w\ en\ (\theta, L), encontrar\ \ u: (0, L) \to \mathbb{R}^+ tal\ que:$

$$\begin{cases} u'' = f(x,u,u')\ ;\ en\ x_0 \leq x \leq x_L \\ u(x_0) = 0 \\ u(x_L) = 0 \end{cases} \quad (7)$$

To solve the PVF (3) by the nonlinear firing method, the PVF must be rewritten as a sequence of PVI of the form:

$$\begin{cases} u'' = f(x, u(x,t_k), u'(x,t_k))\ ;\ en\ x_0 \leq x \leq x_L \\ u(x_0) = 0 \\ u'(x_0) = t_k \end{cases} \quad (8)$$

In order to use the Newton method, within the shot method, and obtain new values of tk, it is necessary to determine$\frac{du(L,t_k)}{dt_k}$, for this we take the partial derivative of the ODE in (6) with respect to tk

$$\frac{\partial u''(x,t_k)}{\partial t_k} = \frac{\partial f(x,u(x,t_k),u'(x,t_k))}{\partial x}\frac{\partial x}{\partial t_k}$$

$$+ \frac{\partial f(x,u(x,t_k),u'(x,t_k))}{\partial u}\frac{\partial u(x,t_k)}{\partial t_k}$$

$$+ \frac{\partial f(x,u(x,t_k),u'(x,t_k))}{\partial u'}\frac{\partial u'(x,t_k)}{\partial t_k}$$

Since x is independent of tk, we have:$\frac{\partial x}{\partial t_k} = 0$,

Therefore, we have the following

$$\frac{\partial u''(x,t_k)}{\partial t_k}=\frac{\partial f\left(x,u(x,t_k),u'(x,t_k)\right)}{\partial u}\frac{\partial u(x,t_k)}{\partial t_k}\cdot$$
$$+\frac{\partial f\left(x,u(x,t_k),u'(x,t_k)\right)}{\partial u'}\frac{\partial u'(x,t_k)}{\partial t_k}$$

with initial conditions: $\frac{\partial u(x,t_k)}{\partial t_k}=0;\frac{\partial u'(x,t_k)}{\partial t_k}=1$

Making a change of variable in the previous differential equation, making

$z(x,t_k)=\frac{\partial u(x,t_k)}{\partial t_k}$ and $z'(x,t_k)=\frac{\partial u\prime(x,t_k)}{\partial t_k}$

then, we can rewrite it as the following PVI $\begin{cases} z''=\frac{\partial f(x,u(x,t_k),u'(x,t_k))}{\partial u}z+\frac{\partial f(x,u(x,t_k),u'(x,t_k))}{\partial u'}z' \\ z(x_0,t_k)=0 \\ z'(x_0,t_k)=1 \end{cases}$

(9)

Determining the terms $\frac{\partial f(x,u(x,t_k),u'(x,t_k))}{\partial u}$ and $\frac{\partial f(x,u(x,t_k),u'(x,t_k))}{\partial u'}$ one has

$$\frac{\partial f(x,u(x,t_k),u'(x,t_k))}{\partial u}=\left(a\left(\rho.C.\alpha+C_{sur}.\left(\frac{2}{r}\right)\right)\right.$$
$$\left.+b\left(f+w+C_{sur}.\left(\frac{2}{r}\right)U_{sur}\right)+b^2u'\right)\frac{1}{(a+b.u)^2}$$

$$\frac{\partial f(x,u(x,t_k),u'(x,t_k))}{\partial u'}=\frac{-b}{a+b.u}$$

To know the term f+w that appears in the PVI, the analytical solution of the problem is used, which is:

$$u(x)=\frac{ae^{-\Delta t}}{\rho Cb}C_{sur}\left(\frac{2}{r}\right)sen\left(\frac{\pi}{L}x\right)senh\left(\frac{\pi}{L}x\right)$$

u'(x) and u"(x) were determined

$$u'(x)=\frac{ae^{-\Delta t}}{\rho Cb}C_{sur}\left(\frac{2}{r}\right)\frac{\pi}{L}\left(sen\left(\frac{\pi}{L}x\right)cosh\left(\frac{\pi}{L}x\right)+cos\left(\frac{\pi}{L}x\right)senh\left(\frac{\pi}{L}x\right)\right)$$

$$u''(x)=\frac{ae^{-\Delta t}}{\rho Cb}C_{sur}\left(\frac{2}{r}\right)\left(\frac{\pi}{L}\right)^2 2cos\left(\frac{\pi}{L}x\right)cosh\left(\frac{\pi}{L}x\right)$$

Then you have

$$f+w=\rho.C.\alpha.u-(a+bu)u''-bu'-C_{sur}.\left(\frac{2}{r}\right)(u_{sur}-u)$$

Experimental Results

In the trigger method algorithm, the PVIs (8) and (9) were programmed.

The EDO parameter values appear in the code.

Experiment 3

The values are taken$\Delta t = 1;\ t_0 = \frac{u(x_L)-u(x_0)}{x_L-x_0} = 0$slope of the line that passes through$(x_0, u_0)$and $(x_L, u_L)$,

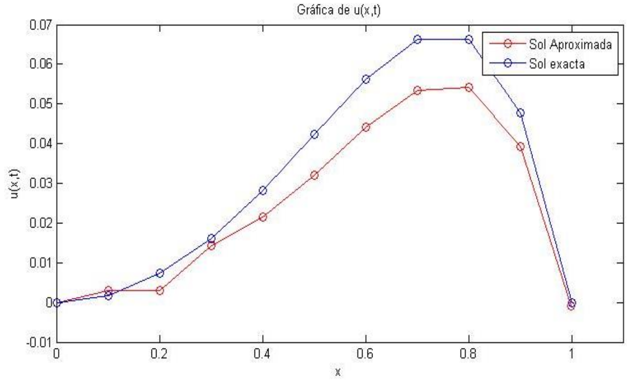
Tolerance in Newton's method 10-3

| Approximate Solution | Exact Solution | Absolut mistake |
|---|---|---|
| 0 | 0 | 0 |
| 0.003082560752516 | 0.001815215791059 | 0.001267344961457 |
| 0.003069552155903 | 0.007249075536276 | 0.004179523380373 |
| 0.014310852459054 | 0.016195561662498 | 0.001884709203443 |
| 0.021391557007402 | 0.028243380713850 | 0.006851823706449 |
| 0.032149977523862 | 0.042330027707463 | 0.010180050183601 |
| 0.044191509754773 | 0.056279233571348 | 0.012087723816575 |
| 0.053319348643586 | 0.066266200681326 | 0.012946852037740 |
| 0.054138119461820 | 0.066298899090203 | 0.012160779628383 |
| 0.039324114561730 | 0.047867827890861 | 0.008543713329130 |
| -0.000862353836318 | 0.000000000000000 | 0.000862353836318 |

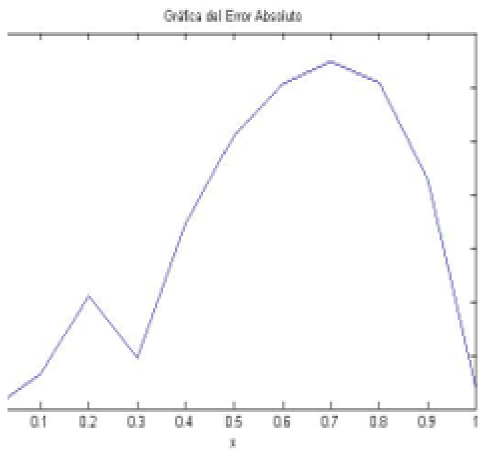
The number of iterations used was k=27

Relative error= 0.195280045602644

Graph 7. Comparison between approximate solution and exact solution in experiment 3




Graph 8. Absolute Error in experiment 3




Experiment 4

The values are taken$\Delta t = 0.1;\ t_0 = 0$

Tolerance in Newton's method 10-3

The number of iterations exceeds 30,000 iterations.

Experiment 5

The values are taken$\Delta t = 1;\ t_0 = 0.01$

Tolerance in Newton's method 10-3

| Approximate Solution | Exact Solution | Absolut mistake |
|---|---|---|
| 0 | 0 | 0 |
| 0.003082560752516 | 0.001815215791059 | 0.001267344961457 |
| 0.003069552155903 | 0.007249075536276 | 0.004179523380373 |
| 0.014310852459063 | 0.016195561662498 | 0.001884709203435 |
| 0.021391557007589 | 0.028243380713850 | 0.006851823706261 |
| 0.032149977528832 | 0.042330027707463 | 0.010180050178631 |
| 0.044191509866012 | 0.056279233571348 | 0.012087723705335 |
| 0.053319350632846 | 0.066266200681326 | 0.0129468500484 80 |
| 0.054138151051494 | 0.066298899090203 | 0.012160748038710 |
| 0.039324645215393 | 0.047867827890861 | 0.008543182675468 |
| -0.000848466146607 | 0.000000000000000 | 0.000848466146607 |

The number of iterations used was k=36

Relative error= 0.195280015598220

Experiment 6

The values are taken$\Delta t = 1;\ t_0 = 0.05$.

Tolerance in Newton's method 10-3

The number of iterations exceeds 30,000 iterations

Note: Using a tolerance in the Newton method <10-3 in this case study causes the shot method to require more than 30,000 iterations.

When comparing the experiments, it is observed that the nonlinear trigger method is sensitive to the value of the parameters presented by the ODE. Between Experiment 3 and Experiment 4 it can be seen that when changing the value of the parameter□t the method requires too many iterations to solve the problem. This sensitivity to the value of the parameters comes from the Runge-Kutta (RK) method that is used to solve the PVI associated with the original problem. Runge-Kutta methods are sensitive to parameter values, especially when the ODEs come from real problems and are highly nonlinear; this is due to rounding errors that affect the method. In real problems the ODE parameters must be well bounded, since there are ranges for said parameters in which the RK methods fail; while for other ranges of the same parameters the RK methods work well.

The influence of the choice of the first value for tk is also observed; Between Experiment 3 and Experiment 5, it can be seen that the number of iterations necessary to solve the problem increases from 27 iterations to 36 iterations when changing the value of tk= t0=0 to t0=0.01, while for Experiment 6 more are required. of 30000 iterated to solve the problem by taking t0=0.05. This indicates that the nonlinear trigger method requires a good initial approximation for the value of the first derivative in the original PVF.

## 4. CONCLUSIONS AND RECOMMENDATIONS

The present study focused on the resolution of a heat transfer problem in a one-dimensional bar using the shot method, addressing two specific cases: one with constant thermal conductivity and another with linear thermal conductivity with respect to temperature.

Numerical experiments were carried out to analyze the performance of the method in solving these problems.

For the case of constant thermal conductivity, the experimental results showed a good agreement between the approximate solution and the exact solution of the problem, with relatively low errors. It was observed that the linear trigger method was effective in reproducing the solution of the problem satisfactorily, although sensitivity to the choice of parameters was evident, especially in the size of the integration step $\Delta t$.

Regarding the case of linear thermal conductivity with respect to temperature, the results showed that the nonlinear trigger method was also capable of reproducing the solution of the problem, but a greater sensitivity to the parameters and a greater dependence on the initial approximation was observed. for the first derivative in the original problem. It was evident that small changes in the parameters could significantly affect the performance of the method, suggesting the need for careful selection of the parameters and initial conditions to guarantee the convergence of the method.

In general, it is concluded that both the linear and nonlinear firing methods are useful tools to solve heat transfer problems in materials engineering. However, it is important to take into account the sensitivity of the method to the initial parameters and conditions, as well as the need to appropriately adjust these values to ensure convergence and accuracy of the solution. It is recommended to perform a detailed analysis of the problem parameters and perform sensitive tests to ensure reliable and accurate results.

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