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# On Micro $S_p$g-Closed Sets in Micro Topological Spaces

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The objective of this paper is to define and study a new kind of sets called Micro Spg-closed sets in Micro topological spaces. Further some of its properties and characterizations are analyzed.
**Keywords:** Generalized closed set, Micro topological space, Micro generalized closed set, Micro Sp-open set, Micro Sp-closed set, Micro Spg-open set and Micro Spg-closed set.

## 1. Introduction

In 1970, Norman Levine [6] introduced generalized closed sets in topological spaces. In 2013, the notion of nano topology is introduced by Lellis Thivagar and Carmel Richard[5] whose idea of nano topological structure is grounded on lower, upper and boundary approximations of a subset of a universal set with an equivalence relation on it. K. Bhuvaneswari and K. Mythili Gnanapriya[2] introduced nano generalized closed sets in nano topological spaces in 2014. The concept of Micro open set in Micro topological spaces was introduced and investigated by Chandrasekar.S [3] in 2019. R.Bhavani [1] introduced some strong forms of generalized closed sets in Micro topological spaces in 2021. Taha H.Jasim [9] introduced micro generalized closed sets in micro topological spaces in 2021. In this direction, in 2022, P.Herin Wise Bell et.al [4] introduced Micro generalized continuous in Micro topological spaces. This paper focuses on the introduction of new class of sets called Micro $S_p$-generalized closed set in Micro topological spaces and study some of its basic properties.

## 2. Preliminaries

Definition 2.1.[7] A subset B from a topology $\tau$ on the Space X is said to be generalized closed set (shortly g-closed) if Cl (B) $\subseteq$ U for B $\subseteq$ U and U is open in

(X, $\tau$). A subset B of a topological space (X, $\tau$) is called g-open if X – B is g-closed.

Definition 2.2.[5] Let R be an equivalence relation on the universe U. $\tau_R(X) = \{U,$

$\phi, L_R(X), U_R(X), B_R(X)\}$ where X $\subseteq$ U. Then $\tau_R(X)$ satisfies the following axioms.

*(i)* U and $\phi \in \tau_R(X)$.

*(ii)* The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

*(iii)* The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$. That is, $\tau_R(X)$ is a topology on U called the Nano topology on U with respect to X. Thus (U, $\tau_R(X)$) is called as Nano topological space. The elements of $\tau_R(X)$ are called as Nano open sets. A subset F of U is nano closed if its complement is nano open.

Definition 2.3.[2] Let (U, $\tau_R(X)$) be a nano topological space. A subset A of (U, $\tau_R(X)$) is called nano generalized closed set (briefly Ng-closed) if NCl (A) $\subseteq$ V where A $\subseteq$ V and V is nano open.

Definition 2.4.[3] Let (U, $\tau_R(X)$) be a Nano topological space and A $\subseteq$ U. Then $\mu_R(X) = \{N \cup (N' \cap \mu): N, N' \in \tau_R(X)\}$ and $\mu \notin \tau_R(X)$ is called the micro topology in U with respect to X. The triplet (U, $\tau_R(X)$, $\mu_R(X)$) is called micro topological space and the elements of $\mu_R(X)$ are called micro open sets and the complement of a micro open set is called a micro closed set.

Definition 2.5.[3] The Micro-closure of a set A is denoted by Mic-Cl (A) and is defined as Mic-Cl (A) = $\cap$ { B : B is Micro-closed and A $\subseteq$ B}.

The Micro-interior of a set A is denoted by Mic-Int (A) and is defined as Mic-Int (A) = $\cup$ { B : B is Micro-open and A $\supseteq$ B}.

Definition 2.6.[9] A subset B of (U, $\tau_R(X)$, $\mu_R(X)$) is called micro generalized closed set (shortly Mic g-closed) if Mic Cl (B) $\subseteq$ U for B $\subseteq$ U and U is micro open set in (U, $\tau_R(X)$, $\mu_R(X)$) is called Mic g-open if U – B is Mic g-closed.

Definition 2.7.[6] Let (U, $\tau_R(X)$, $\mu_R(X)$) be a Micro topological space and A$\subseteq$U. Then A is said to be Micro $S_p$-open (briefly Mic $S_p$-open) if for each $x \in A \subseteq$ Mic-SO (U, X), there exists a Micro pre-closed set F such that $x \in F \subseteq A$. The set of all Micro $S_p$-open sets is denoted by Mic $S_p$-O (U, X).

Definition 2.8.[6] Let (U, $\tau_R(X)$, $\mu_R(X)$) be a Micro topological space and B $\subseteq$ U is called Micro $S_p$-closed (briefly Mic $S_p$-closed) if and only if its complement is Micro $S_p$-open and Mic $S_p$-CL (U, X) denotes the set of all Micro $S_p$-closed sets.

Remark 2.9.[6] An arbitrary intersection of Mic $S_p$-closed sets of a Micro topological space (U, $\tau_R(X)$, $\mu_R(X)$) is Mic $S_p$-closed.

Definition 2.10.[6] A point $x \in$ U is said to be a Micro $S_p$-interior point of A if there exists a

Micro $S_p$-open set V containing x such that $x \in V \subseteq A$.

The union of all Micro $S_p$-open sets contained in A is said to be Micro $S_p$-interior of A and is denoted by Mic $S_p$-Int (A).

Definition 2.11.[6] Let A be any subset of a Micro topological space (U, $\tau_R(X)$, $\mu_R(X)$). Then a point $x \in U$ is in the Micro $S_p$-closure of A if and only if $A \cap H \neq \phi$, for every $H \in$ Mic $S_p$-CL (U, X) containing x.

The intersection of all Micro $S_p$-closed sets F containing A is called the Micro $S_p$-closure of F and is denoted by Mic $S_p$-Cl (F).

Definition 2.12.[3] For any two subsets A and B in a Micro topological space (U, $\tau_R(X)$, $\mu_R(X)$),

*(i)* A is Micro-closed if and only if Mic-Cl (A) = A.

*(ii)* A is Micro-open if and only if Mic-Int (A) = (A).

*(iii)* $A \subseteq B$ implies Mic-Int (A) $\subseteq$ Mic-Int (B) and Mic-Cl (A) $\subseteq$ Mic-Cl (B).

*(iv)* Mic-Cl (Mic-Cl (A)) = Mic-Cl(A) and Mic-Int (Mic-Int(A))=Mic-Int(A).

*(v)* Mic-Cl ($A \cup B$) $\supseteq$ Mic-Cl (A) $\cup$ Mic-Cl (B).

*(vi)* Mic-Cl ($A \cap B$) $\subseteq$ Mic-Cl (A) $\cap$ Mic-Cl (B).

*(vii)* Mic-Int ($A \cup B$) $\supseteq$ Mic-Int (A) $\cup$ Mic-Int (B).

*(viii)* Mic-Int ($A \cap B$) $\subseteq$ Mic-Int (A) $\cap$ Mic-Int (B).

*(ix)* Mic-Cl ($A^c$) = [Mic-Int (A)]$^c$.

*(x)* Mic-Int ($A^c$) = [Mic-Cl (A)]$^c$.

## 3. MICRO $S_p$-GENERALIZED CLOSED SETS

Definition 3.1. A subset A of a Micro topological space (U, $\tau_R(X)$, $\mu_R(X)$) is said to be a Micro $S_p$ generalized-closed (briefly Mic $S_p$g-closed) set if Mic $S_p$-Cl (A) $\subseteq$ V whenever $A \subseteq V$ and $V \in$ Mic $S_p$-O (U, X). The collection of all Micro $S_p$g-closed sets is denoted by Mic $S_p$G-CL (U, X).

Remark 3.2. Every Micro $S_p$-closed set is a Micro $S_p$g-closed set but the converse is not always be true and it is shown in the following example.

Example 3.3. Consider U = { a , b , c , d} with U/R = {{b}, {d}, {a ,c}}, X = {a, c} then $\tau_R(X)$ = {U , $\phi$, {a , c}}. If $\mu$ = {b} then $\mu_R(X)$ = {U , $\phi$ , {b}, {a , c}, {a , b , c}} and Mic $S_p$-O (U, X) = {U , $\phi$, {a , c}, {b , d}, {a, c, d}}, Mic $S_p$-CL (U, X) = {U , $\phi$, {b} , {a,c},{b,d}}. Also Mic $S_p$G-CL (U, X) = {U , $\phi$, {a} , {b}, {c}, {d}, {a,b},{a,c}, {b,c}, {b, d}, {a, b, c}, {a, b, d},{b, c, d}}. Here {a,b} is a Micro $S_p$g-closed set but not Micro $S_p$-closed set.

Theorem 3.4. If a set A is Micro $S_p$g-closed set if and only if [Mic $S_p$-Cl (A) – A] contains no-non empty Micro $S_p$-closed set.

Proof: Let A be a Micro $S_p$g-closed set. Then there exists a Micro $S_p$-open set V such that A $\subseteq$ V and Mic $S_p$-Cl (A) $\subseteq$ V. Let F be a non-empty Micro $S_p$-closed set such that F $\subseteq$ [Mic $S_p$-Cl (A) – A]. Then F $\subseteq$ Mic $S_p$-Cl (A) $\cap$ $A^c$, which implies A $\subseteq$ $F^c$, where $F^c$ is Micro $S_p$-open set. So Mic $S_p$-Cl (A) $\subseteq$ $F^c$. Which implies F $\subseteq$ [ Mic $S_p$-Cl (A)]$^c$. Thus F $\subseteq$ Mic $S_p$-Cl (A) $\cap$ [Mic $S_p$-Cl (A)]$^c$. So F = $\phi$. Therefore [ Mic $S_p$-Cl (A) –A ] contains no non empty Micro $S_p$-closed set.

Conversely, let A $\subseteq$ V and V $\in$ Mic $S_p$-O (U, X) such that A is not a Micro $S_p$g-closed set. Then Mic $S_p$-Cl (A) $\not\subset$ V, which implies Mic $S_p$-Cl (A) $\subseteq$ $V^c$. Then Mic $S_p$-Cl (A) $\cap$ $V^c$ is a non empty Micro $S_p$-closed set of [ Mic $S_p$-Cl (A) – A ], which is a contradiction. Hence A is Micro $S_p$g-closed set.

Definition 3.5. A subset A of Micro topological space (U, $\tau_R(X)$, $\mu_R(X)$) is called a Micro $S_p$ generalized-open (briefly Mic $S_p$g-open) set if $A^c$ is Mic $S_p$ generalized- closed. The set of all Mic $S_p$g-open sets is denoted by Mic $S_p$G-O (U, X).

Remark 3.6. The intersection of any two Micro $S_p$-generalized closed sets in (U, $\tau_R(X)$, $\mu_R(X)$) is not always a Micro $S_p$-generalized closed set as shown in the example below.

Example 3.7. Consider U = { 1 , 2 , 3 , 4} with U/R = {{1}, {3}, {2 , 4}}, X = {1, 3} then $\tau_R(X)$ = {U , $\phi$, {1} , {3}}. If $\mu$ = {1, 4} then $\mu_R(X)$ = {U , $\phi$ , {1}, {3}, {1, 3}, {1 , 4}, {1 , 3 , 4}} and Mic $S_p$-O (U, X) = {U , $\phi$, {1 , 3}, {2, 3, 4}}, Mic $S_p$-CL (U, X) = {U, $\phi$, {1} , {2, 4}}. Also Mic $S_p$G-CL (U, X) = {U , $\phi$, {1} , {2}, {4}, {1, 2},{1, 4}, {2, 4}, {1, 2, 3}, {1, 2, 4}, {1, 3, 4}}. Here {1, 2, 3} and {1, 3, 4} are Micro $S_p$g-closed sets but {1, 2, 3} $\cap$ {1, 3, 4} = {1, 3} is not a Micro $S_p$g-closed set.

Theorem 3.8. The union of Micro $S_p$-generalized closed sets in (U, $\tau_R(X)$, $\mu_R(X)$) is a Micro $S_p$-generalized closed set if Mic $S_p$-O (U, X) forms a Micro topological space.

Proof: Assume that E and F are Micro $S_p$g-closed sets in (U, $\tau_R(X)$, $\mu_R(X)$). Let E $\cup$ F $\subseteq$ A where A $\in$ Mic $S_p$-O (U, X). Since U is a Micro topological space, Mic $S_p$-Cl (E $\cup$ F) = Mic $S_p$-Cl (E) $\cup$ Mic $S_p$ -Cl (F) $\subseteq$ V. As E and F are Micro $S_p$g-closed sets, Mic $S_p$-Cl (E) $\subseteq$ A and Mic $S_p$-Cl (F) $\subseteq$ A. Thus Mic $S_p$-Cl (E $\cup$ F) $\subseteq$ A. Hence E $\cup$ F is Micro $S_p$g-closed only if the space is Micro topology.

Remark 3.9. The above theorem does not hold when Mic $S_p$-O (U, X) is not a Micro topological space and it is shown in the following example.

Example 3.10. In example 3.3, Mic $S_p$G-CL (U, X) = {U , $\phi$ , {a}, {b},{c},{d},{a , b}, {a, c}, {b, c} , {b, d}, {a, b, c}, {a, b, d},{b, c, d}}. Here {c} $\cup$ {d} = {c, d} which is not a Micro $S_p$g-closed set.

Theorem 3.11. A Micro $S_p$g-closed set is Micro $S_p$-closed if and only if [Mic $S_p$-Cl(A) – A] is Micro $S_p$-closed.

Proof: Let A be a Micro $S_p$g-closed set, then [Mic $S_p$-Cl (A) – A] = $\phi$. By theorem 3.4, [Mic $S_p$-Cl (A) – A] is Micro $S_p$-closed. Also [Mic $S_p$-Cl (A) – A] itself is a subset of it. By theorem 3.4, [Mic $S_p$-Cl (A) – A] = $\phi$. Hence A is Micro $S_p$-closed.

Theorem 3.12. Let A be a Micro $S_p$g-closed subset of U. If A $\subseteq$ B $\subseteq$ Mic $S_p$-Cl (A) then B is also a Micro $S_p$g-closed.

Proof: Let V be a Micro $S_p$-open set in (U, $\tau_R$(X), $\mu_R$(X)) such that B $\subseteq$ V which implies A $\subseteq$ V. Since A is a Micro $S_p$g-closed set which implies Mic $S_p$-Cl (A) $\subseteq$ V. Also, B $\subseteq$ Mic $S_p$-Cl (A). Now Mic $S_p$-Cl (B) $\subseteq$ Mic $S_p$-Cl (Mic $S_p$-Cl(A)) = Mic $S_p$-Cl (A) $\subseteq$ V. Therefore Mic $S_p$-Cl (B) $\subseteq$ V Hence B is also a Micro $S_p$g-closed subset.

Lemma 3.13. For any subset A of a Micro topological space (U, $\tau_R$(X), $\mu_R$(X)),

Mic $S_p$-Int [Mic $S_p$-Cl (A) – A] = $\phi$.

Theorem 3.14. For a subset A of U, Mic $S_p$-Cl ($A^c$) = [Mic $S_p$-Int (A)]$^c$.

Proof: Let A be a subset of U and let $A^c$ = U – A. Also let B be a Mic $S_p$-open set such that B $\subset$ A. By definition 2.10, Mic $S_p$-Int (A) = $\cup$ {B / B $\subset$ A and B $\in$ Mic $S_p$-O (U,X)}. That is Mic $S_p$-Int (A) = $\cup$ {U – F / U – A $\subset$ F and F = U – B}. Hence Mic $S_p$-Int (A) = U – $\cap$ {F / F is Mic $S_p$-closed and U – A $\subset$ F}. Thus Mic $S_p$-Int (A) = U – Mic $S_p$-Cl (U – A). That is Mic $S_p$-Int (A) = U – Mic $S_p$-Cl ($A^c$) which implies Mic $S_p$-Cl ($A^c$) = U – Mic $S_p$-Int (A). Hence Mic $S_p$-Cl ($A^c$) = [Mic $S_p$-Int (A)]$^c$.

Theorem 3.15. A subset A of a Micro topological space (U, $\tau_R$(X), $\mu_R$(X)) is Mic $S_p$g-open if and only if F $\subseteq$ Mic $S_p$-Int (A) whenever F is Mic $S_p$-closed and F $\subseteq$ A.

Proof: Let A be a Mic $S_p$g-open set. Then $A^c$ is Mic $S_p$g-closed set. Thus Mic $S_p$-Cl ($A^c$) $\subseteq$ $F^c$. By Theorem 3.14, Mic $S_p$-Cl ($A^c$) = [Mic $S_p$-Int (A)]$^c$. Then [Mic $S_p$-Int (A)]$^c$ $\subseteq$ $F^c$. Hence F $\subseteq$ Mic $S_p$-Int (A) where F is Mic $S_p$g-closed and F $\subseteq$ A. Conversely, let F $\subseteq$ Mic $S_p$-Int (A), where F is Mic $S_p$g-closed and F $\subseteq$ A. Let $F^c$ = X – F be a Mic $S_p$g-open set such that $A^c$ $\subseteq$ $F^c$. Then by assumption, F $\subseteq$ Mic $S_p$-Int (A) implies [Mic $S_p$-Int (A)]$^c$ $\subseteq$ $F^c$. Which implies Mic $S_p$-Cl ($A^c$) $\subseteq$ $F^c$. Therefore $A^c$ is Mic $S_p$g-closed. Hence A is Mic $S_p$g-open.

Theorem 3.16. A subset A of a Micro topological space (U, $\tau_R$(X), $\mu_R$(X)) is Mic $S_p$g-closed if [Mic $S_p$-Cl (A) – A] is Mic $S_p$g-open.

Proof: Let A $\subseteq$ V and V$\in$ Mic $S_p$-O(U,X). Now Mic $S_p$-Cl (A) $\cap$ (U – V) $\subseteq$ Mic $S_p$-Cl (A) $\cap$ (U – A) = [Mic $S_p$-Cl (A) – A] and by Remark 2.9, Mic $S_p$-Cl (A) $\cap$ (U – V) is Mic $S_p$-closed and by assumption, [Mic $S_p$-Cl (A) – A] is Mic $S_p$g-open. Then by Theorem 3.15, Mic $S_p$-Cl (A) $\cap$ (U – A) $\subseteq$ Mic $S_p$-Int [Mic $S_p$-Cl (A)] – A] = $\phi$ (by Lemma 3.13). Thus Mic $S_p$-Cl (A) $\cap$ (U – V) = $\phi$ implies Mic $S_p$-Cl (A) $\subseteq$ U. Hence A is Mic $S_p$g-closed.

Theorem 3.17. If a subset A of U is Mic $S_p$g-open, then G = U whenever G is Mic $S_p$-open and Mic $S_p$-Int (A) $\cup$ $A^c$ $\subseteq$ G.

Proof: Let G be a Mic $S_p$-open set and Mic $S_p$-Int (A) $\cup$ $A^c$ $\subseteq$ G. Then $G^c$ $\subseteq$ [Mic $S_p$-Cl ($A^c$) – $A^c$]. Now G is Mic $S_p$-open implies $G^c$ is Mic $S_p$-closed and A is Mic $S_p$g-open which implies $A^c$ is Mic $S_p$g-closed. Hence by Theorem 3.4, $G^c$ = $\phi$. Hence G = U.

Theorem 3.18. If Mic $S_p$-Int (A) $\subseteq$ B $\subseteq$ A and if A is a Mic $S_p$g-open then B is also Mic $S_p$g-open.

Proof: Let Mic $S_p$-Int (A) $\subseteq$ B $\subseteq$ A then $A^c \subseteq B^c \subseteq$ Mic $S_p$g-Cl ($A^c$) where $A^c$ is Mic $S_p$g-closed and hence $B^c$ is also Mic $S_p$g-closed by theorem 3.12. Hence B is Micro $S_p$g-open.

Theorem 3.19. Let (U, $\tau_R$(X), $\mu_R$(X)) be a Micro topological space. Then for each $x \in$ U, either $\{x\}$ is Mic $S_p$-closed or U \ $\{x\}$ is Mic $S_p$g-closed.

Proof: Suppose that $\{x\}$ is not Mic $S_p$-closed, then by definition 2.8, U \ $\{x\}$ is not Mic $S_p$-open. Let A be any Mic $S_p$-open set such that U \ $\{x\} \subseteq$ A which implies A = U. Thus, Mic $S_p$-Cl (U \ $\{x\}$) $\subseteq$ A. Hence, U \ $\{x\}$ is Mic $S_p$g-closed.

## References


1. Bhavani.R: "On Strong forms of generalized closed sets in Micro Topological spaces", Turkish Journal of Computer and Mathematics Education, Vol 12, Issue 11, 2021, PP: 2772-2777.
2. Bhuvaneswari.K and Mythili Gnanapriya.K: "Nano generalized closed sets in Nano Topological spaces", International Journal of Scientific and Research Publications, Vol 4,Issue 5, May 2014.
3. Chandrasekar.S: "On Micro topological spaces", Journal of New Theory, Vol: 26, 2019, PP: 23-31.
4. Herin Wise Bell.P, Ganesan.S and Jeyashri.S: "Micro Generalized closed sets and Micro Generalized continuous in Micro topological spaces", Vol 4, Issue 1, 2022, PP:156-167.
5. Lellis Thivagar and Carmel Richard: "On Nano forms of weakly open sets", International Journal of Mathematics and Statistics Invention, 1(1)(2013), 31-37.
6. Maheswari.M, Dhanalakshmi.S and Durga Devi.N: "Micro S_P-open sets in Micro topological spaces", Ratio Mathematica, Vol 45, 2023, PP: 90-96.
7. Norman Levine: "Generalized closed sets in Topology", Rend, Circ. Mat. Palermo 19, 89-96(1970).
8. Subramanian, Jeyashri and Selvaraj Ganesan: "Micro generalized Locally Closed Sets in Micro Topological Spaces", The International journal of analytical and experimental modal analysis, Vol XII, Issue VI, 2020.
9. Taha H.Jasim, Saja S. Mohsen, Kanayo S.Eke: "On Micro-generalized closed Sets and Micro-generalized continuity in Micro Topological Spaces", European journal of Pure and Applied Mathematics, Vol 14, No. 4, 2021, 1507-1516.