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# Ultrasoft Longitudinal photons Energy at Next to-leading order in Hot Scalar QED

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We present the calculation of the compact analytic expression for the next-to-leading order to the retarded self-energy for slow-moving longitudinal photons with soft momentum. We use the real-time formalism (RTF) in the context of hard-thermal-loop summed perturbation of massless scalar quantum electrodynamics (SQED) at high temperature. We derive the expressions of the effective propagators in RTF that contribute to the complete next-to-leading order contribution of retarded longitudinal-photon self-energy. The next-to-leading order of the energy is related to the real part of the retarded longitudinal photons self-energy.
**Keywords:** Soft Photons Energy, Resummation, Hard Thermal Loop NLO, Effective expansion, Real time formalism, SQED.

## 1. Introduction

One runs into difficulties when one applies standard perturbation theory to gauge theories at high temperature: physical quantities become gauge dependent [1-4]. The problem was resolved in [5-12], which showed that in order to calculate consistently at high temperature, we have to use an effective perturbation that sums the so-called hard thermal loops (HTL) into dressed propagators and vertices [5-12], it is additionally required in abelian theory as electrodynamics (QED) [12], scalar electrodynamics (SQED) [13] and in $\phi^4$-theory [14]. In this framework, the zero-momentum transverse gluon damping rate was shown to be finite and positive [15].

However, subsequent studies of the behavior of the gluon and quark damping rates in the imaginary-time formalism have indicated that there are difficulties in the infrared sector [16-

24]. A similar observation has been done in the context of scalar electrodynamics [25]. To avoid the analytic continuation from Matsubara frequencies to real energies and all that comes with it, which, in a sophisticated calculation like the determination of the dispersion relations, can blur the analytic behavior of the physical quantities, we use the real-time formalism to calculate the next-to-leading order longitudinal photon energies. To look further into the infrared behavior, we have determined the next-to-leading contributions to the longitudinal gluon and the retarded quark self-energy in the context of HTL summed perturbation of massless QCD at high temperature using real time formalism [26-28], also the retarded fermion self-energy in the context of HTL summed perturbation of massless QED has been calculated [29].

To tackle this specific issue in more depth, we embark in this work on the investigation of the next-to-leading order dispersion relation for longitudinal photon with ultrasoft momentum p in the context of next-to-leading order HTL perturbation of scalar QED. We will examine the next-to-leading order energy.

In the present work, we show the main steps leading to the determination of an analytic expression for the next-to-leading contribution to the retarded longitudinal photon self-energy in massless Scalar QED at high temperature using the real-time formalism of finite-temperature quantum field theory [30-33]. We derive the expressions of the effective propagators in RTF that contribute to the complete next-to-leading order contribution of retarded longitudinal-photon self-energy which their real part permit to determine the NLO energy for slow-moving longitudinal-photon at high temperature.

## 2. EFFECTIVE PROPAGATORS AND VERTICES

The transverse and longitudinal Hard Thermal Loop in photon self energy are obtained by summing the contribution of diagrams shown in Figure 1 where $P \sim eT, K \sim T$

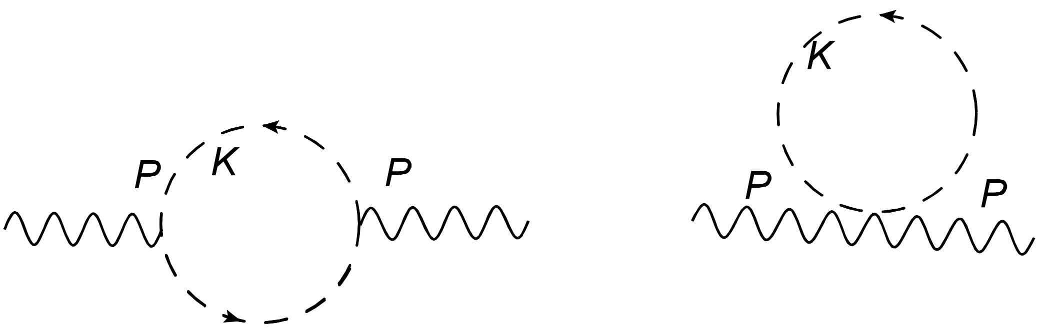


Figure1: Hard thermal loop contribution to photon self energy

These are expressed by:

$$\delta\Pi_T^{r/a}(K) = \frac{3}{2}m^2\frac{k_0^2}{k^2}\left[1-\left(1-\frac{k^2}{k_0^2}\right)\frac{k_0}{k}ln\frac{k_0+k\pm i\varepsilon}{k_0-k\pm i\varepsilon}\right];$$

$$\delta\Pi_L^{r/a}(K) = -3m^2\left[1-\frac{k_0}{2k}ln\frac{k_0+k\pm i\varepsilon}{k_0-k\pm i\varepsilon}\right],$$

with $m = \frac{1}{9}e^2T^2$ the photon thermal mass to lowest order.

The HTL resummed photon propagator followed from the resummation of the HTL photon self energy, it is given by:

$$\Delta^{\mu\nu}_{ra/ar}(K) = P^{\mu\nu}_{T}\frac{1}{\delta\Pi^{t}_{ra/ar} - K^2 \mp i\,sgn(k_0)\varepsilon} + P^{\mu\nu}_{L}\frac{1}{\delta\Pi^{l}_{ra/ar} - K^2 \mp i\,sgn(k_0)\varepsilon}, \quad (2)$$

where $P^{\mu\nu}_{T,L}$ are the usual transverse and longitudinal projectors respectively.

The HTL contribution to the scalar self-energy is obtained by summing the one loop diagrams Figure 2 and it is expressed by:

$$\delta\Sigma_{htl}(K) = m_s^2 \ ,$$

(3) $m_s = \ eT/2$

with the scalar thermal mass.

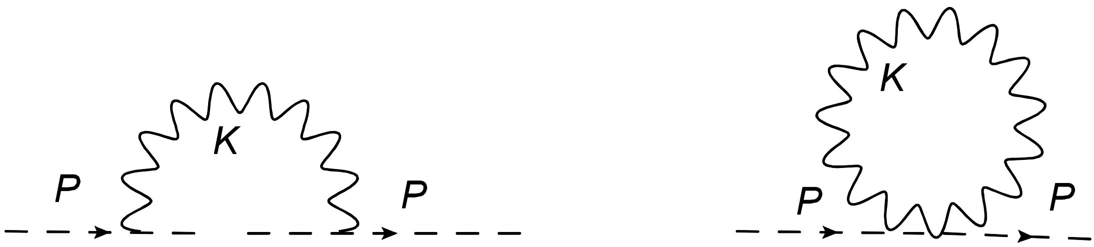


Figure2: Hard thermal loop contribution to scalar self energy

Hence, the dressed scalar propagator is given by:

$$\Delta_{ar,ra}^{-1}(K) = K^2 - m_s^2 \pm i\,sgn(k_0)\varepsilon$$

$$\Delta_{F}^{-1}(K) = -2\pi i\left[1 + 2n_B(|k_0|)\right]\delta\left(\left(K^2 - m_s^2\right)\right), \quad (4)$$

where $n_B(k_0) = 1/\left(\exp(k_0/T) - 1\right)$ the Bose distribution function.

The effectif vertex with one photon and two external scalar lines is:

$$\Gamma^{\mu}(P,Q) = -e(P+Q)^{\mu}, \quad (5)$$

with Q incoming, P outgoing; and the effectif vertex between two photons and two scalars

$$\Gamma^{\mu\nu}(P,Q) = -ie^2 g^{\mu\nu}, \quad (6)$$

i.e the vertices are unaffected by HTL.

## 3. LONGITUDINAL PHOTONS DAMPING AND ENERGY AT NEXT-TO-LEADING ORDER IN HOT SQED

The longitudinal photons dispersion relation reads:

$$p^2 - \delta\Pi^l_{ra/ar}(\Omega_l, p) - {}^*\Pi^l_{ra/ar}(\Omega_l, p) = 0, \tag{7}$$

where ${}^*\Pi^l$ is the next-to-leading order longitudinal photon self-energy.

The longitudinal photons energy at next-to-leading-order is expressed with:

$$\mathrm{Re}\,\Omega_l(p) = \omega_l(p) - \frac{\mathrm{Re}\,{}^*\Pi^l_{ra/ra}(P)}{\frac{\partial}{\partial\omega}\delta\Pi^l_{ra/ar}(P)\Big|_{\omega=\omega_l}} \tag{8}$$

So, to determine the longitudinal photons damping rate to lowest order and their energy at next-to-leading-order, we have to calculate the effective longitudinal photons self-energy. The diagrams that contribute to next-to-leading-order longitudinal photons self energy are the following two diagrams Figure 3, in which the internal momenta are soft.

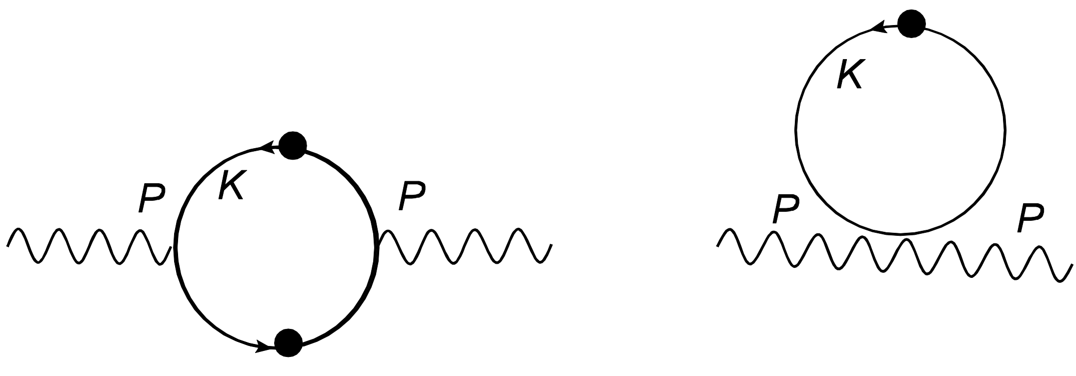


Figure3: NLO HTL-summed photon self-energy

The next-to-leading-order effective longitudinal photons self-energy in the Keldysh basis is given by:

$$\begin{aligned}{}^*\Pi^L_{ab}(P) = \int \frac{d^4K}{(2\pi)^4}\Big[ & {}^*\Gamma^{\mu\nu}_{ab\alpha\beta}(Q,-Q,K,-K)\ {}^*\Delta_{\alpha\beta}(K) \\ & + {}^*\Gamma^{\mu}_{a\alpha\beta}(K,K-Q)\ {}^*\Delta_{\alpha\alpha'}(K)\,{}^*\Gamma^{\nu}_{\alpha' b\beta'}(K-Q,K)\ {}^*\Delta_{\beta\beta'}(K-Q)\Big]\end{aligned} \tag{9}$$

where K is the soft internal momentum and Q=P-K.

The contribution of the first term to the retarded self energy using the real time formalism can be written as:

$$^{tad*}\Pi_R^L(P) = -ie^2 \int \frac{d^4K}{(2\pi)^4} \left( {}^*\Delta_F(K) + {}^*\Delta_R(K) + {}^*\Delta_A(K) \right) \tag{10}$$

After integrating over $k_0$ by means of $\delta$ function and trivially over the angle reduces to:

$$^{tad*}\Pi_R^L(P) = -\frac{e^2}{\pi^2} \int_0^\infty dk \frac{k^2}{\omega_k} n_B(\omega_k) \tag{11}$$

where. $\omega_k^2 = k^2 + m_s^2$

The contribution of the second term is given by:

$$^{2*}\Pi_R^L(P) = \frac{e^2}{2} \int \frac{d^4K}{(2\pi)^4} (2k_0 + p_0)^2 \left[ {}^*\Delta_F(Q)\, {}^*\Delta_R(K) + {}^*\Delta_A(Q)\, {}^*\Delta_F(K) \right] \tag{12}$$

The integration over $k_0$ is done by means of $\delta$ function and we get the following result for ${}^*\Delta_F(Q)\, {}^*\Delta_R(K)$ contribution:

$$I_{FR} = \frac{-ie^2}{4\pi^2} \int_0^\infty \frac{k^2}{\omega_q} \int \frac{d\Omega}{4\pi} \left\{ \frac{(p_0 + 2\omega_q)^2 (1 + 2n_B(\omega_q))}{p_0^2 + 2p_0\left((\vec{k} - \vec{p})^2 + m^2\right)^{1/2} - 2\vec{k}.\vec{p} + p^2 + i\varepsilon} + \frac{(p_0 - 2\omega_q)^2 (1 + 2n_B(\omega_q))}{p_0^2 - 2p_0\left((\vec{k} - \vec{p})^2 + m^2\right)^{1/2} - 2\vec{k}.\vec{p} + p^2 + i\, sgn(p_0 - \omega_q)\varepsilon} \right\} \tag{13}$$

and the following result for ${}^*\Delta_A(Q)\, {}^*\Delta_F(K)$ contribution:

$$I_{AF} = \frac{-ie^2}{4\pi^2} \int_0^\infty \frac{k^2}{\omega_k} \int \frac{d\Omega}{4\pi} \left\{ \frac{(2\omega_k - p_0)^2 (1 + 2n_B(\omega_k))}{p_0^2 - 2p_0 k + 2\vec{k}.\vec{p} - p^2 - i\, sgn(\omega_k - p_0)\varepsilon} + \frac{(2\omega_k + p_0)^2 (1 + 2n_B(\omega_k))}{p_0^2 + 2p_0\omega_k + 2\vec{k}.\vec{p} - p^2 + i\varepsilon} \right\} \tag{14}$$

The sum of the three terms given by (11), (13) and (14) gives the NLO longitudinal photon self-energy which determines the NLO longitudinal dispersion relation. In the lowest order, the solution of the longitudinal photons dispersion relation $\Omega_l(p)$ are real. In Figure 4, we have shown the variation of NLO longitudinal photon energy with p by using expression (8). We remark that in the limit of vanishing p, we found the same results existing in Kraemmer et al [13]

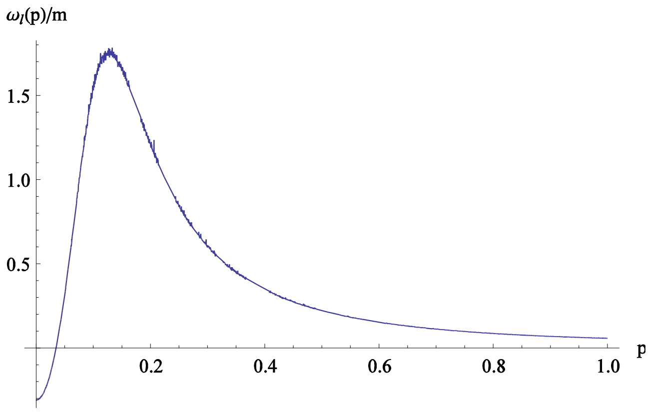


Figure4: The longitudinal photon energy variation with p

## 4. CONCLUSION

In the present work, we have studied the NLO longitudinal photon self-energy and their corresponding dispersion relations using the HTL resummation. This has been done using the real time formalism of Keldysh indices and the considered longitudinal photons are the slow moving ones. As discussed in the text, NLO dispersion laws gives us the physical quantities like NLO energy and this observable comes from the zeros of the HTL dressed longitudinal photons propagators. We have derived a compact analytic expression for the complete next-to-leading contribution to the retarded longitudinal photons self-energy in the context of hard-thermal-loop summed perturbation of Scalar QED at high temperature. These expressions have been manipulated, mainly numerically, to determine the next-to- leading order energy for slow-moving longitudinal photons at high temperature.

Declarations

Conflict of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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