

Mathematical Model to the Optimal Setting of Solar Panels Achieve the Maximum Solar Energy

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Solar panels are one of the most promising renewable energy technologies in building energy supply. In installing solar panels on the roof of buildings, it is important to know the optimal slope angle in order to have the most annual, seasonal, monthly or daily energy. In this research, using mathematical and isotropic models, the optimal tilt angle of solar panels to receive the maximum amount of average monthly solar radiation in the Sohar, Sultanate Oman and Hillah, Iraq have been obtained. In all the used models, the effects of reflected radiation of the earth are also taken into account. In this research, mathematical model is used to obtain the optimal slope angle and the optimal crest side angle in Sohar, Sultanate Oman and Hillah, Iraq. Using the isotropic model, optimal angles for different months of the year, seasons and the whole year have been introduced. To harvest the most annual radiant energy, the panel should be tilted close to the location's latitude. It has increased by 22%, 8%, and 4% in the Sohar by setting panels to the best monthly, seasonal, and yearly angle. In the Hillah, the variation increases by 23%, 9%, and 4%, respectively. It may be deduced that the maximum energy received is related to the zero azimuth angle, and that as the azimuth angle (both positive and negative) increases, so does the quantity of received energy.

Keywords: Solar energy; mathematical model; optimal angle; solar panel.

1. Introduction

With the gradual depletion of non-renewable energy sources and the rising concerns over global warming and climate change, the demand for renewable energy sources like solar energy has surged tremendously in recent years [1]–[4]. Solar panels are one of the most popular and effective means of harvesting solar energy [5]–[7]. However, to make the most of solar energy, it is crucial to optimize the settings of solar panels according to the environmental and operational factors [8]–[10]. Human has been using solar energy for a very long time as a source of renewable, cost-free power, despite the fact that this practice is harmful to the environment [11], [12]. As a result of the development of solar collectors, this limitless, unaltered, and cost-free source of energy can now be employed to a greater level, which in turn lessens the demand for fossil fuels. Iraq and Oman are countries that is situated in the solar belt and is recognized as one of the countries that receives a considerable amount of solar energy on a yearly basis [13]. It is vital to utilize a range of energy sources in the event that issues arise as a consequence of an unforeseen incident; as a result, embracing solar energy is one more motivation to do so [14]. A further reason is the relatively high cost of utilizing fossil fuels, which is increasing, as well as the high cost of equipment, such as turbines, and their high depreciation due to the humid weather in the region. Another reason is that there is a shortage of alternative energy sources in the region. Information on the amount of solar radiation at various positions can be obtained by measuring the total radiation on a surface that is horizontal. This information is typically reported in the form of total daily radiation accumulated over the course of the year. Because the greatest amount of direct solar radiation is received by a surface when it is perpendicular to the direction in which the sun's radiation is traveling, the flat surface must be oriented so that it is almost perpendicular to the direction in which the radiation is traveling in order to absorb the greatest amount of energy [15]–[18]. This can be accomplished by the utilization of sun trackers, which are devices that track the sun for a limited amount of time; however, the primary challenge lies in the high cost of producing such devices. Adjusting the angle of inclination of the surface on a daily basis is a useful technique that can be applied to flat solar collectors and photovoltaic cells. Given that the information about solar radiation that is currently available is for a horizontal surface, the quantity of energy that should be used to calculate the amount of energy that should be fed to the surface should be based on the radiation information that is for a horizontal surface [19].

Numerous research studies have been conducted on the optimal settings of solar panels to achieve maximum solar energy [20]–[23]. These studies have revealed that various environmental and operational factors significantly impact the performance of solar panels efficiency. Some of the critical factors include the orientation and tilt angle of solar panels, the angle of incidence, and the shading effect of surrounding objects. Research has shown that adjusting the orientation and tilt angle of solar panels based on the latitude and longitude of their installation site can enhance their energy output. Moreover, the angle of incidence, which is the angle of the incident sunlight with respect to the surface of the solar panel, also affects the efficiency of solar panels. Thus, optimizing the angle of incidence by adjusting the orientation and tilt angle of solar panels based on the position of the sun in the sky can improve the solar energy output. The shading effect of surrounding objects like trees,

buildings, and other structures can also significantly impact the solar energy output of solar panels. Therefore, it is crucial to consider the placement of solar panels from shaded areas and design anti-shading systems to reduce the shading effect.

Most of the studies conducted in this field indicate that the optimal tilt angle and tilt angle in the Northern Hemisphere for south-facing solar panels are dependent on latitude [24], [25].

As a novel strategy, this study includes a mathematical solution to the problem Grasshopper plugin for the optimal deflection angle of solar panels in vertical mode on all slopes. The value and requirement of employing the panels in various modes in this approach were determined by programming and setting the numerical index. The calculations for this solution will be validated. As a result, the best deviation of the panels for specific arrays and perpendicular to the facade in each of its various orientations have been discovered. In addition, the angle's applicability and value in the orientation were determined. This article emphasizes the significance of choosing the deviation angle that is most suited for east and west orientations.

2. Method

The optimal angle for placement of solar panels is subject to variation due to the discrepancy in the sun's radiant energy across different geographical locations, and is dependent on several variables. Therefore, it is not feasible to provide a universal recommendation for the angle of placement that would be optimal for all panels. Consequently, a parametric solution was devised and implemented to compute the optimal deviation angle and the relevant metrics outlined in "Grasshopper". This approach enables the assessment of the impacts of variable modifications in conjunction with the computation. This section will introduce the position of the sun in the sky, Calculation of solar radiation outside the Earth's atmosphere, Solar radiation on the surface of the earth.

2.1. Sun Position

It is required to evaluate the position of the sun at different times of the day in order to establish the best angle at which solar panels should be positioned in order to obtain the most amount of energy. This can be done by observing the sun's path across the sky. Equations are provided specifically for this application. The position of the sun is investigated with the help of spherical coordinates in Figure 1. In order to determine the angles and position of the sun, the first and second equations are employed in the calculation.

$$\sin \alpha = \cos \varphi \cos \delta \cos \omega + \sin \varphi \sin \delta \quad (1)$$

$$\sin \theta = \frac{\cos \delta \cos \omega}{\cos \alpha} \quad (2)$$

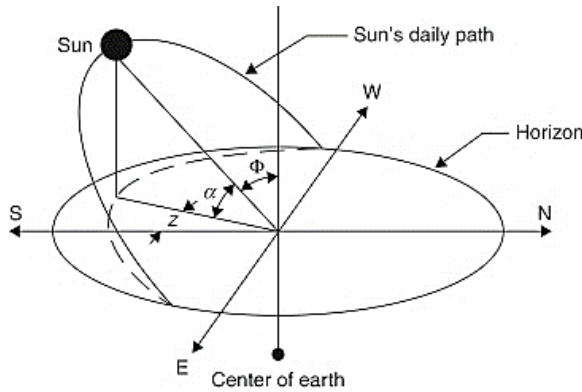


Figure 1. The position of the sun in the atmosphere

where, ϕ is the latitude of the place and δ is the sun's deviation angle, which is the position of the sun at solar noon relative to the equatorial plane, and ω is the solar hour angle, which are obtained by Equations 3 and 4, respectively.

$$\delta = 23.45 \sin \left(360 \frac{\pi}{365} \right) \quad (3)$$

$$\omega = 15(t - 12) \quad (4)$$

2.2. Solar radiation model

The amount of solar energy radiation that is received on a plane that is outside of the atmosphere at any given time can be computed with the help of Equation 5 [26].

$$G_{oh} = G_{sc} \left[1 + 0.033 \cos \left(\frac{360(n+81)}{365} \right) \right] \times (\sin \phi \sin \delta + \cos \phi \cos \delta \cos \omega). \quad (5)$$

In this regard, G_{oh} is the intensity of the solar radiation on a horizontal plane in m and G_{sc} is the solar radiation, which shows the average amount of solar radiation per unit of time per unit of surface, perpendicular to the direction of radiation on the outer surface of the earth's atmosphere [27].

From the time of sunrise to the time of sunset, as determined by Equation 6, the radiation is equal to the integral of Equation 5, which is equal to one day's worth of radiation.

$$H_{oh} = \frac{24 \times 3600}{\pi} G_{sc} \left[1 + 0.033 \cos \left(\frac{360(n + 81)}{365} \right) \right] \times \left(\cos \delta \cos \varphi \sin \omega_s + \frac{2\pi\omega_s}{360} \sin \varphi \sin \delta \right) \quad (6)$$

The unite of the H_{oh} is J/m^2 day.

ω_s is the hour angle of sunrise and sunset is calculated from Equation 7 and is negative for sunrise and positive for sunset.

$$\cos \omega_s = -\tan \delta \tan \varphi \quad (7)$$

Equation 8 calculates the solar radiation in the atmosphere over a period of time ($\omega_1 - \omega_2$).

$$I_{oh} = \frac{24 \times 3600}{2\pi} G_{sc} \left[1 + 0.033 \cos \left(\frac{360(n + 81)}{365} \right) \right] \times \left(\cos \delta \cos \varphi (\sin \omega_2 - \sin \omega_1) + \frac{2\pi(\omega_1 - \omega_2)}{360} \sin \varphi \sin \delta \right) \quad (9)$$

The solar radiation break up into their component parts as they move through the earth's atmosphere. Some of the light is absorbed and dispersed by ozone molecules, water particles, and other airborne particles, while the remaining light reaches the surface of the earth. As a direct consequence of this, the amount of solar energy that is absorbed by the surface of the earth is lower than the amount that is absorbed when the atmosphere is not present. According to Equation 9, the total radiation that reaches the surface of the earth is equal to the sum of the direct and scattered components of that radiation.

$$I_d + I_b = I_h \quad (9)$$

I_h total radiation, I_b direct radiation and I_d scattered solar radiation on the horizontal surface or the earth in terms of J/m^2h [29].

On the other hand, the total radiation can be calculated using the Equation 10, which takes into account both the radiation that is present outside of the atmosphere as well as the hourly coefficient of air purity, k_t .

$$k_t = I_h / I_{oh} \quad (10)$$

In addition, Equation 11 is provided for k_t .

$$\begin{aligned}
 k_t &= \left[a + b \cos \frac{2\pi}{24} (t - 12) \right] \bar{K}_T \\
 a &= 0.404 + 0.5016 \sin (\omega_s - 60) \\
 b &= 0.6606 - 0.4767 \sin (\omega_s - 60)
 \end{aligned}
 \tag{11}$$

On an inclined plane, direct radiation, scattered solar radiation, and reflected radiation from the ground are the three components that make up the incoming radiation. Equation 12 can be used to calculate the total solar radiation received by an inclined plane over the course of one hour.

$$I_T = I_b R_b + I_d \frac{1 + \cos \beta}{2} + S_g (I_b + I_d) \frac{1 - \cos \beta}{2}
 \tag{12}$$

which S_g is the perception that the receiver has of the reflection coefficient of the ground that is all around them. The S_g is typically set to a low value; ($S_g = 0.1$) is chosen for normal days, and ($S_g = 0.8$) is chosen for the situation in front of the steam or dust receiver.

3. RESULT AND DISCUSSION

To acquire the necessary parameters for the mathematical models, the data utilized in the preceding section was obtained from the Meteorological Organization located in Sohar and Hillah. Sohar, Oman is situated at a geographic coordinate of 56.69° longitude and 24.35° latitude. Hillah, Iraq is situated at a longitude of 44.43° and a latitude of 32.48° (Figure 2).

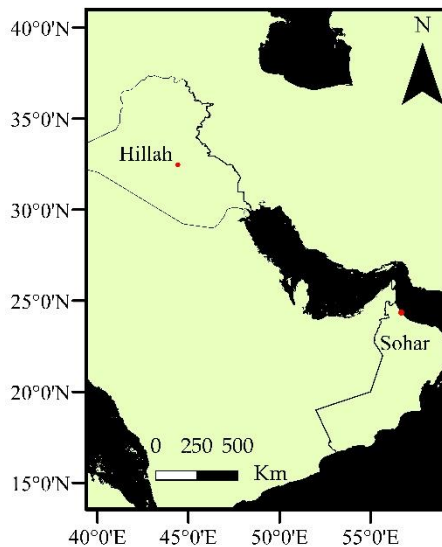


Figure 2. The case studies location

There is sufficient sunlight during year as a result of the region having a climate that is hot and humid in both areas. Because of this, it is possible to fulfill a sizeable portion of the cities energy requirements by developing an understanding of the optimal angles at which solar panels should be positioned on systems that draw power solar. Input for this work consists of monthly average data collected on a daily basis for the horizontal surface. It is important to keep in mind that the reflection coefficient of the earth is thought to be equal to $\rho_g = 0.3$. All of the radiation values that are measured on a horizontal surface are derived from radiometric data that was collected in 2017 and is available at the Sohar Meteorological Department and in 2019 for Hillah. The values for the amount of solar radiation energy received by a horizontal surface in Sohar and Hillah during each month of the year are displayed in Figure 3, respectively. The amount of maximum energy that is received from the sun at its peak is 1034 W/m^2 and 1050 W/m^2 for the Hillah and Sohar in the May, respectively.

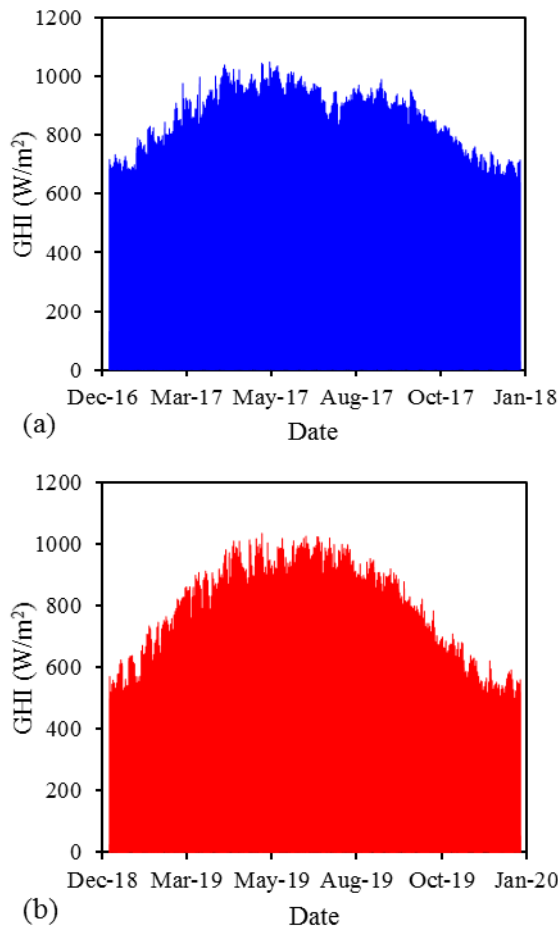


Figure 3. Daily total radiant energy reached the surface in a) Sohar in 2017, b) Hillah in 2019.

Table 1 breaks down the data by month, season, and year to show the optimal angle as well as the amount of radiation energy that reaches the inclined surface. The optimal angle values for the season are found by averaging the optimal angle values for each month that falls within the same season. The optimal angles for the months of May, June, and July are all negative, whereas the optimal angles for the remaining months of the year are all positive in the Sohar. However, in the Hillah the optimal angles for the months of June and July are negative and remain positive. The amount of energy that is received from the sun at its peak is 30.15 and 29.25 MJ/m² for in the June in the Sohar and Hillah, respectively. While the amount of energy that is received at its minimum is 20.91 MJ/m² in the month of January for Sohar and 19.04 MJ/m² in the month of January for Hillah.

Table 1. The values of the optimal tilt angle and the monthly average daily energy reached to the surface of the solar panel based on the optimal angle

Month	Sohar		Hillah	
	$\beta_{opt(m)}(^{\circ})$	$H_{opt(m)}$ (MJ/m ²)	$\beta_{opt(m)}(^{\circ})$	$H_{opt(m)}$ (MJ/m ²)
January	50.67	20.91	57.91	19.40
February	40.62	22.64	48.51	22.39
March	28.35	20.44	36.48	20.34
April	14.35	22.53	22.48	22.02
May	-1.35	27.48	6.21	27.08
June	-12.82	30.15	-5.74	29.25
July	-8.33	28.95	-1.09	27.56
August	6.62	27.74	14.51	27.91
September	22.35	27.97	30.48	27.22
October	36.35	24.65	44.48	24.39
November	47.65	24.71	55.21	24.88
December	55.18	26.85	62.26	25.29

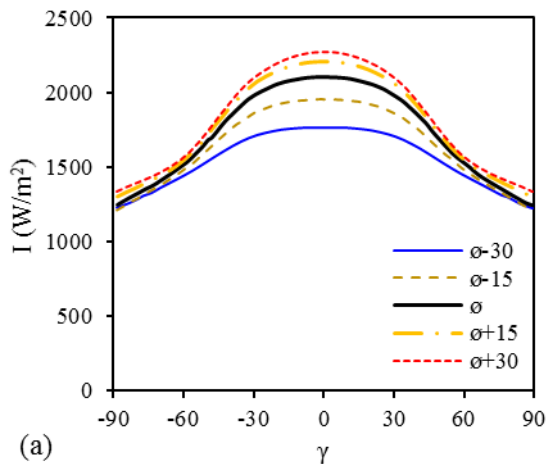
The system is to receive the maximum radiation energy annually, the months of maximum and minimum energy intake are September and January, with energy intakes of 21.27 MJ/m² and 20.15 MJ/m², respectively. It is possible to calculate the total energy received per year using various optimization methods and on the horizon surface, the corresponding results of which can be seen in Table 2.

Table 2. Annual amounts of energy received from the sun on a surface under the optimal angle in Sohar, Oman and Hillah, Iraq

Optimal angle		$\beta_{opt(m)}$	$\beta_{opt(s)}$	$\beta_{opt(y)}$	$\beta=0$
H_{total} (MJ/m ²)	Sohar	30.76	27.18	26.12	25.10
	Hillah	29.51	26.13	25.03	23.91

Examining the data in Table 2 enables one to compare the amount of energy received from the sun in one year on the surface of the horizon to the amount of energy received from the sun in one year using the optimal angles related to the month, season, and year. This allows one to determine the amount of energy received from the sun in one year using the optimal angles related to each month, season, and year. By setting panels to the optimal angle of monthly, seasonally, and yearly, it has increased by 22%, 8% and 4%, respectively in the Sohar. While in the Hillah this variation increase by 23%, 9%, 4% respectively. As was mentioned earlier, the isotropic model is only considered correct when γ equals 0.

The findings obtained from two non-isotropic models in order to investigate the effect of γ . The values of the average radiation energy that was reached by the inclined surface are specified in Figures 4 and 5 for the months of January and July. These values are given for five angles of the apex side, γ , different 10, 20, 30, 40, and 50, using the mathematical model for four slope angles (ϕ), ($\phi-10$), ($\phi+10$), ($\phi-30$) and ($\phi+30$).



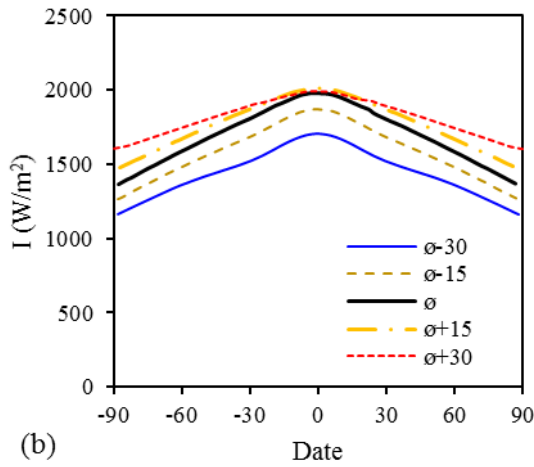
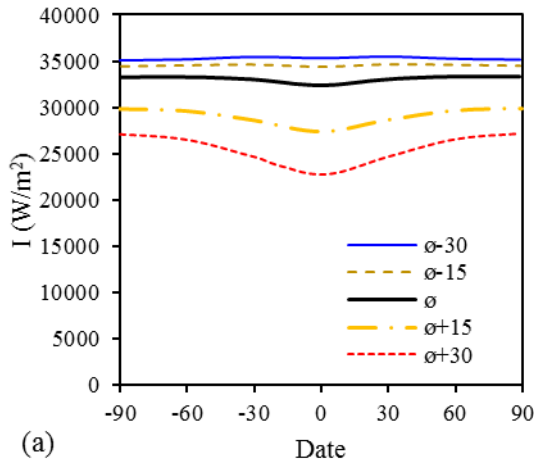


Figure 4. Amounts of total radiant energy in January Using the mathematical model a) Sohar, Oman and b) Hillah, Iraq.

According to Figures 4 and 5, it can be concluded that the maximum energy received is related to the zero azimuth angle, and with the increase of the azimuth angle (both positive and negative), the amount of received energy decreases.



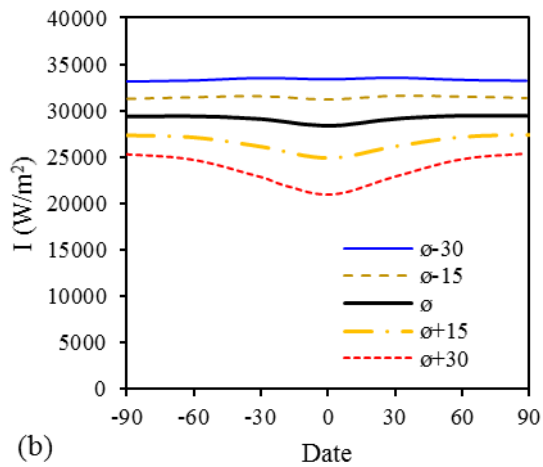


Figure 5. Amounts of total radiant energy in July Using the mathematical model a) Sohar, Oman and b) Hillah, Iraq.

4. Conclusion

The best deflection angle of solar panels in vertical mode on all slopes is a problem that has a mathematical solution, which is included in this study's Grasshopper plugin. By programming and establishing the numerical index, it was possible to assess the importance and need of using the panels in different ways in this manner. This solution's calculations will be verified. As a consequence, it has been determined which panel deviation is optimal for certain arrays and perpendicular to the facade in all of its possible orientations. The application of the angle and its importance in the orientation were also established. The importance of selecting the deviation angle best suited for east and west orientations is emphasized in this study.

The findings showed that the system is to receive the most radiation energy yearly, with the highest and lowest energy intakes occurring in the months of September and January, respectively, with intakes of 21.27 MJ/m² and 20.15 MJ/m². The total energy received each year may be calculated using a variety of optimization techniques and on the horizon surface. In the meanwhile, a single-axis system It has grown by 22%, 8%, and 4% in the Sohar by adjusting panels to the best angle for the month, the season, and the year. In contrast, this variance increased by 23%, 9%, and 4%, respectively, in the Hillah. While in double-axis, the greatest energy received is correlated with the azimuth angle of zero, and as azimuth angle increases (both positively and negatively), less energy is received.

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