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Application of Point Shelter Set in Optimization

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Point shelter sets play an important role in graph theory and areas like discrete optimization. It is widely studied and discussed due to its applications in real world problem and various engineering field such as fault diagnosis, resource allocation in networks, sensor placement, reliability and resilience analysis, network design, VLSI design, PMU placement for full observability in electric network, telecommunication network, social network graph, transportation graphs, work allocation and circuit design. Some of applications have been discussed here. Results regarding edge addition and substraction may help to programming these optimization problem for stake holders. Overall, Point shelter set provides a mathematical framework to solve optimization problems related to node coverage and connectivity in various electrical engineering applications, leading to more efficient and reliable systems.

Keywords: Point shelter set, Point shelter number, Plus number , Minus number, minimal and minimum Point shelter set.

1. Introduction

The concept of Point shelter set was introduced by R.C.Brigham, P.Z.Chinn and R.D.Dutton[1] and several authors including [2] have studied the topic and derived interesting results regarding critical graphs.D. D. Pandya and D. K. Thakkar have discussed the topic focusing on critical graphs with help of edge addition, edge removal and bondage number. In this paper, concept of point shelter set has been discussed for the criticality of the graph and considering the situation of optimization relating graphs with engineering field situation with mathematical modeling. Optimization to the problem will be quick solution with help of flow chart or programming. Results regarding edge addition target to decrease this number which is the backbone of optimization process. In such particular situation when one edge is not enough to be added then reinforcement number or minus number will be study of interest.

2. Definitions

Definition 2.1 : Point shelter set [1]

A set ψ of $\alpha(\Omega)$ is called Point shelter set of the graph Ω if every edge has at least one end point in ψ.

Definition 2.1.2.3 : Point shelter number [1]

The cardinality of the minimum Point shelter set of graph Ω is known as the point shelter number expressed by $\Delta^0(\Omega)$

Definition 2.4 : Plus Number

The Smallest number of edges whose removal increase Point shelter number in the resultant graph is called Plus number.

Definition 2.5 : Minus Number

The Smallest number of edges whose addition decrease Point shelter number in the resultant graph is called Minus number.

3. Edge Removal effect

Theorem 3.1 : $\Delta^0(\Omega-\phi)\le\Delta^0(\Omega)$ for the graph Ω and an edge ϕ of the graph Ω.

Proof: If we consider ψ to be Point shelter set of graph Ω then it is applicable to be point shelter set of the graph $\Omega-\phi$ also which proves $\Delta^0(\Omega-\phi)\le|\psi|=\Delta^0(\Omega)$

Theorem 3.2 : If there exist a minimum point shelter set ψ of the graph Ω such that for an edge $\phi=v^1v^2$, all adjacent vertices of v^1 except possibly v^2 belongs to ψ and all adjacent vertices of v^2 except possibly v^1 belongs to ψ iff $\phi=v^1v^2\in\beta^-_{ps}$ where β^-_{ps} is the set of edges whose removal decrease point shelter number.

Proof : $\Rightarrow$ Consider ψ as a point shelter set of graph Ω which involves a point v^1. Now if we remove v^1 from ψ then call it the new set ψ_1. Now every adjacent vertices of v^1 except possible v^2 belongs to ψ which indicates that every adjacent vertices of v^1 in $\Omega-v^1v^2$ belongs to ψ. Now if v^2 doesn't belong to ψ then every adjacent vertices of v^2 in $\Omega-v^1v^2$ belongs to ψ_1. So ψ_1 is point shelter set in new graph.

$\Leftarrow$ Assume that $\phi=v^1v^2\in\beta^-_{ps}$. Suppose ψ_1 to be minimum point shelter set in the graph $\Omega-v^1v^2$ means ψ_1 is not point shelter set in Ω which indicates both vertices do not belong to ψ_1. Now if we make a new set ψ by adding v^1 with ψ_1 then ψ is now minimum point shelter set in the graph Ω. Suppose v^3 is adjacent to v^1 in $\Omega-v^1v^2$. Now ψ_1 is point shelter set in graph $\Omega-v^1v^2$ and v^3 is in ψ_1 and hence v^3 is in ψ. Which proves the theorem.

Theorem 3.3 : If for every a minimum point shelter set ψ of the graph Ω such that for an edge $\phi = v^1 v^2$, there exist an adjacent vertex of v^1 except possibly v^2 does not belong to ψ and there exist an adjacent vertex of v^2 except possibly v^1 does not belong to ψ iff $\phi = v^1 v^2 \in \beta_{ps}^{0}$ where β_{ps}^{0} is the set of edges whose removal does not point shelter number.

Illustration 3.5 : Considering $\phi = 34 \in \beta_{ps}^{0}$ for the given graph which shows effect of edge removal on point shelter number

Figure 3.5.1

Illustration 3.6 : The example of the graph which shows removal of any edge from the graph does not change point shelter number.

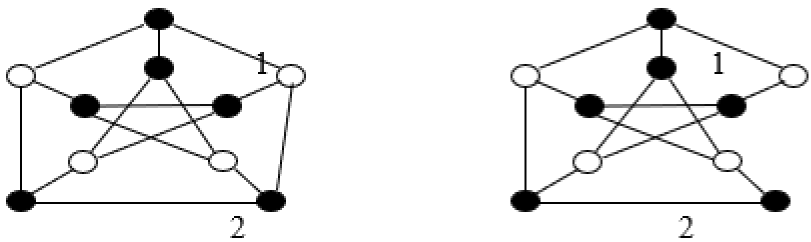


Figure 3.6.1

Illustration 3.7 : $\phi = 12 \in \beta_{ps}^{-}$ for the given graph which shows effect of edge removal on point shelter number

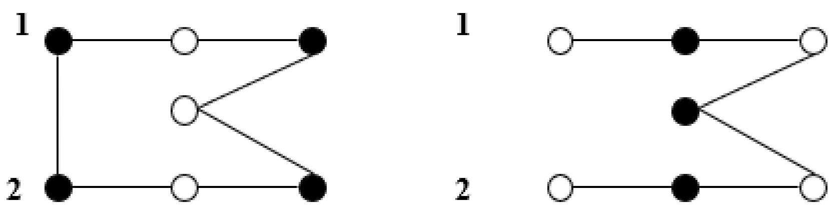


Figure 3.7.1

4. Relation Between Vertices and Edges

Lemma 4.1 : $\Delta^0(\Omega - v^2) \leq \Delta^0(\Omega - \phi) \leq \Delta^0(\Omega)$ for the graph Ω and an edge $\phi = v^1 v^2$ of the graph Ω.

Proof: Considering the theorem, we require to prove only $\Delta^0(\Omega - v^2) \le \Delta^0(\Omega - \phi)$

Suppose ψ be point shelter set in the graph $\Omega - v^1 v^2$.

Now for the cases v^2 doesn't belong to ψ though v^1 belongs to or doesn't belong to ψ, ψ will be point shelter set of the graph $\Omega - v^2$

But if v^2 belongs to ψ though v^1 doesn't belong to ψ, considering the edge pq of the graph $\Omega - v^2$ then $p \ne v^2 \,\&\, q \ne v^2$. Now if $p = v^1$ then $v^1 q$ edge of graph $\Omega - v^1 v^2$ has one end point in ψ. Similarly $q = v^1$ then $v^1 p$ edge of graph $\Omega - v^1 v^2$ has one end point in ψ.

Now for the case, both vertices v^1 & v^2 belong to ψ, defining new s et ψ_1 as $\Omega - v^2$. Now if pq is an edge of $\Omega - v^2$ then $p \ne v^1 \,\&\, q \ne v^2$. Then pq is an edge of the graph $\Omega - v^1 v^2$ not involving the vertex v^2 but ψ is the point shelter set of $\Omega - v^1 v^2$, $p \in \psi \; or \; q \in \psi$ and equivalently $p \in \psi_1 \; or \; q \in \psi_1$. Thus ψ_1 is point shelter set in $\Omega - v^2$ which proves $\Delta^0(\Omega - v^2) \le \Delta^0(\Omega - \phi)$

Theorem 4.2 : If $\phi = v^1 v^2 \in \beta^{-}_{ps}$ where β^{-}_{ps} is the set of edges whose removal decrease point shelter number then $v^1 \in \alpha^{-}_{ps}$ and $v^2 \in \alpha^{-}_{ps}$

Proof: Let $\phi = v^1 v^2 \in \beta^{-}_{ps}$ then by the theorem, $\Delta^0(\Omega - v^2) \le \Delta^0(\Omega - \phi) \le \Delta^0(\Omega)$ and $\Delta^0(\Omega - v^1) \le \Delta^0(\Omega - \phi) \le \Delta^0(\Omega)$ which proves $v^1 \in \alpha^{-}_{ps}$ and $v^2 \in \alpha^{-}_{ps}$

Theorem 4.3 : For the graph Ω, $v^2 \in \alpha(\Omega)$ then $v^2 \in \alpha^{-}_{ps}$ iff there exist a point shelter set ψ_1 such that $v^2 \in \psi_1$

Theorem 4.4 : For the graph Ω, $v^2 \in \alpha(\Omega)$ then $v^2 \in \alpha^{0}_{ps}$ iff for every point shelter set ψ_1 such that $v^2 \notin \psi_1$

Corollary 4.5 : If $v^2 \in \alpha^{0}_{ps}$ then for every edge $\varepsilon = v^1 v^2$, $\varepsilon \in \beta^{-}_{ps}$

Proof : Since $v^2 \in \alpha^{0}_{ps}$, $\Delta^0(\Omega - v^2) = \Delta^0(\Omega)$ and hence $\varepsilon \in \beta^{-}_{ps}$

Illustration 4.6 : Relating the vertices and edges with the fact, $1 \in \alpha^{-}_{ps}$ and $2 \in \alpha^{-}_{ps}$ but $e = 12 \notin \beta_{ps}^{-}$ Infact $e = 12 \in \beta_{ps}^{0}$.

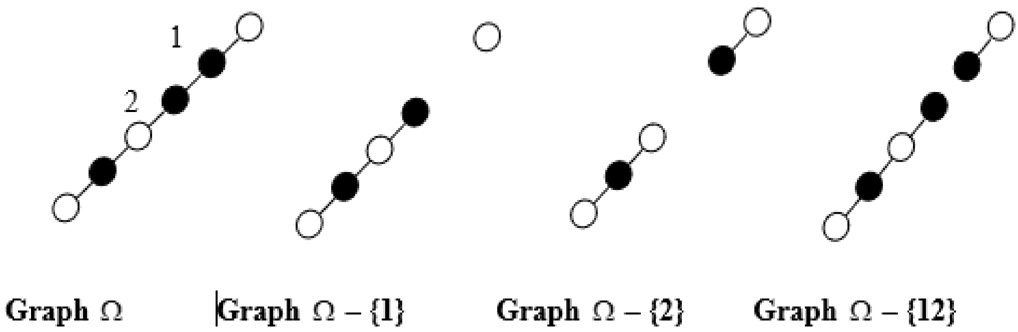


Figure 4.6.1

Illustration 4.7 : Here we can observe that $e = 12 \in \beta_{ps}^{0}$

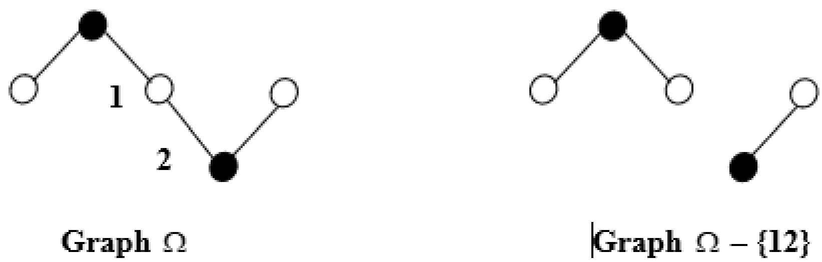


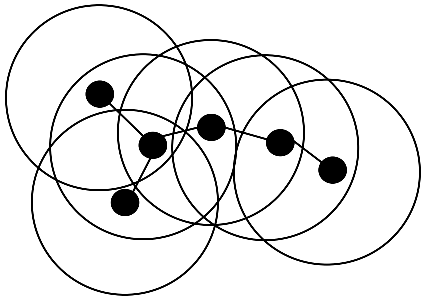
Figure 4.7.1

5. Application of Point Shelter Set

We are discussing some of the many applications in which applications of all above results of edge adding, edge removal , vertex removal, vertex addition and their relation formula results into optimization with greater efficientcy. Programming for the same will be quick solution for the stake holders.

Application 5.1: Telecommunications Networks

In designing telecommunications networks, Point shelter set can be used to ensure that every point (or vertex) in the network is covered by transmitters or receivers. This ensures reliable connectivity and efficient use of resources. Our results regarding edge addition, edge removal may helpful to optimize the problem with flowchart or programming. Sometimes if we want to optimize and removal of single edge is not sufficient than negative number or positive number will come into the picture.



Application 5.2: Sensor Networks

Tim Nieberg [5] submitted thesis on Independent and Dominating Sets in Wireless Communication Graphs .In the context of sensor networks, where sensors are deployed to monitor an area, vertex Point shelter set helps ensure that every part of the area is monitored by at least one sensor. This is crucial for applications like environmental monitoring, security systems, and industrial process monitoring.

Dominating sets of small cardinality are frequently used for backbone structures in communication networks, e.g. to obtain efficient multi-hop routing protocols. For the optimization problems of seeking independent sets of large cardinality (Maximum Independent Set problem), and seeking dominating sets of small cardinality (Minimum Dominating Set problem) on graphs. Application of Ad-hoc network required Maximum Independence Number and Independent Dominating set. In thesis Tim Nieberg has used algorithm to find this number. Our results regarding edge removal and edge addition can help to construct a network as per minimality or maximality. Programming for the same using the results will increase efficiency of work quickly.

Application 5.3 Social Network Theory

In her Ph. D dissertation, Kelleher presented research on dominating sets in social network graphs.[6].

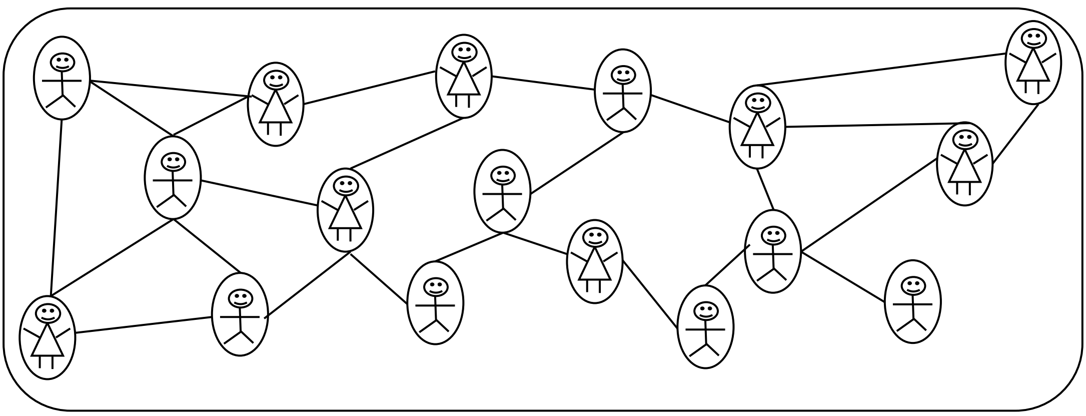


Figure 5.3.1

In social network theory one studies relationships that exist among members of a group. The people in such a group are called actors. These relationships are typically defined in terms of one or more dichotomous properties, that is, a property that for any two actors unambiguously either holds or does not hold. Given such a property, one can construct a social network graph, in which thee vertices represent the actors and an edge between two vertices indicates that the property in question holds between the corresponding actors. Bondage number and reinforcement number come into picture for the optimization.

Application 5.4 : Network Graphs

Railway network is the example of the largest physical graph in which stations are vertices and rail tracks are edges. Similarly bus routing in state , BRTS bus network in the city, virtual graph of social relation, work assignment problems, allocation of electric poles, water connection or broad casting tower require to minimize it for the cost cutting as well as ease of work. Programming for such network graphs by converting technical situation into mathematical modeling will give easiest solution.

Application 5.5 : VLSI Design

In designing integrated circuits, Point shelter set can be used to minimize the number of test points required to ensure that all components (vertices) are tested adequately. This helps in improving the reliability and manufacturability of the circuits.

Application 5.6 : Optimization of Network Design

When designing new electrical grids or communication networks, vertex covering can optimize the layout by identifying essential nodes (substations, nodes with high connectivity) that must be included to ensure efficient and reliable network operation.

Application 5.7 : Supply Chain Management

In logistics and supply chain management, vertex covering can help optimize the placement of distribution centers or warehouses to ensure that every region (vertex) is adequately serviced, minimizing transportation costs and delivery times.

Application 5.8: Circuit Design and Testing

In the design and testing of electronic circuits, identifying minimal sets of components (vertices) that need to be tested or replaced to ensure the circuit functions correctly is crucial. Vertex covering helps in determining the smallest number of test points or components that need to be verified or replaced to maintain circuit functionality.

Application 5.9 : Reliability and Resilience Analysis

Assessing the reliability and resilience of electrical grids or communication networks involves understanding how failures propagate through the system. Vertex covering can identify critical nodes whose failure could significantly impact network reliability, helping engineers design more robust systems.

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