SD-Shell Nuclei's Longitudinal Form Factors Calculated Using Core Polarization and MSDI for the Coulomb Electron Scattering

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By utilizing the nuclear potential, the nucleon-nucleon interaction (NN) can be determined through the examination of inelastic electron scattering and longitudinal form factors. The longitudinal form factors, which involve both momentum and angular momentum, were analyzed in the context of the longitudinal inelastic shell model. The effects of core polarization (CP) were also taken into account using NuShellX with a harmonic oscillator (HO) potential to investigate the nuclei of 26Mg, 22Ne, and 30Si. The calculation involved determining the single-particle matrix elements and the inelastic electron-scattering form factors, which were then compared to experimental data. The results indicate that the model space adequately explains the form factors in NushellX, while also demonstrating good agreement between the NuShellX core polarization results for longitudinal form factors and the available experimental data.

Keywords: Electron Scattering, MSDI, Form factor, P-shell, SD-shell, NushellX program.

1. Introduction

At the subatomic level, understanding nuclear structure is pivotal in understanding matter's fundamental properties. This paper details our use of M3Y core polarization method and MSDI for Coulomb electron scattering to calculate the longitudinal form factor of the SD shell core. Our focus is on selected states of p-shell and sd-shell cores such as 10B, ^{6}Li, and 29Si aiming at improving our knowledge on core structure. The estimation of the longitudinal shape factor holds information related to stability and behavior of nuclei, especially those within p-shells and sd-shells. We are introducing an extension of model space to all orbitals in 1p for an accurate calculation of the same factor. Coulomb electron scattering can be analyzed using

M3Y nuclear polarization method from different atomic nuclei which serve as hosters of p-shell electrons. The M3Y nuclear polarization technique has been used to study ground state nuclei like Li6, C12, O18, F19 and Ne20 in order to gain an understanding of nuclear structure [4]. The M3Y core method provides useful information on nuclear energy dynamics by looking at the shape factor of inelastic longitudinal electron scattering excited states such as 2+ and 4+ in the core of the shell model (2p3/2 1f 5/2 2p1/2) [5]. CP effects play a significant role in these calculations thus, using a modified interaction along with the efficient two-body Michigan three-block Yukawa (M3Y) nucleon interaction enhances the accuracy of results [6]. When conducting electron scattering experiments on SD shell cores, analysis of Coulomb scattering behavior via MSDI method combined with core polarization can only be realized effectively if proper attention is given to details because it forms an important part of such observations. Human researchers have learned much from looking at boring old ground state nuclei such as Li6, C12, O18, F19 and Ne20 using those fancy MSDI methods [4]. When you put the two together thoughMSDI and nuclear polarization it's like seeing all sides of an elephant. We get a complete understanding of electron scattering phenomena in SD shell cores (oh so many shells!) along with glimpses into those mysterious nuclear processes [4]. No need for tea leaves here! With careful calculations plus simulations galore, MSDI method stands strong as a lighthouse beaming light on dark stormy seas revealing hidden rocks or rather details about electron scattering dynamics in these nuclei (more data for our ever-growing library). Let's make a toast then: Coulomb electron scattering coming straight outta SD shell core because this is indeed one unique cocktail party invitation where only special guests can reveal complexity of core structure through experimental techniques while studying Coulomb shape factor of E4 junction in SD shell nuclei. Oh, but wait let's not forget those pesky nuclear polarization effects by hexapole giant resonance that might rain on our parade with confetti showing charge distribution and properties like color ribbons on these festive guests! Ah yes, insights into the charge distribution and properties. This sure sounds like a fun game: pinning tail on donkey; except here we're pinning properties onto nuclei while blindfolded scientists try to hit bulls-eye. The use of electron scattering as a research mechanism has made it possible to investigate the nuclear structure of odd-A nuclei with proton holes in the outermost orbitals an approach that brings us closer to understanding the details of nuclear properties and behavior [7]. Coulomb electron scattering phenomena analyzed in detail open up new horizons for data richness which can help us expand our knowledge on nuclear structure and dynamics [8]. Results obtained through calculations for the longitudinal shape factor of SD shell core turn out to be highly informative about basic properties of such atomic systems [4]. Similarly, other innovative methods proposed by researchers include aerodynamic effects using a catamaran with transverse steps under varying conditions (demonstrating diverse applications) leading researchers down uncharted paths towards different fields of study [9]. The primary objective was simple— developing an understanding of nuclear structure. Scientists use a comparison of calculated shape factors with experimental data to validate the reliability and accuracy of the theoretical model. It also helps in enhancing our knowledge on electron scattering phenomena in SD shell cores through Coulomb electron scattering MSDI (referenced as paper [10]). Determining the longitudinal shape factors is significant research work for nuclear physicists that belong to the p and sd shells aiming at defining scientific rationale plus objectives (referred to as paper [11]). Longitudinal cohort studies can be seen waving benefits towards understanding stability and behavioral aspects among nuclei within different shells (referred to as paper [12]). This paper discusses the study's potential significance, its likely impact on other areas of research, which would help promote nuclear physics research (referred to as paper [13]). In determining the shape factor longitudinally, this paper is able to develop a complete and detailed comprehension of electron scatterings phenomena in SD shell cores. As well as providing information on core structure and dynamics for the wider scientific community. The field of electron scattering research does face many challenges but at the same time it also offers a very broad perspective for its further development because recent experimental work has allowed us to compare our theoretical models with low energy angular differential. From such experimental observations we gather valuable insights on intricacy of nuclear dynamics which is not easily deducible through other means Discussions regarding future prospects of this field include new theoretical possibilities that need to be explored along with advanced experimental setups emphasizing therefore high precision calculations, which should always be aimed for in any electron scattering work as part of their contribution toward sustaining growth and progress within electron scattering research. A path rich with technical challenges— problems to be solved dealing with the energy and luminosity ranges, for electron scattering experiments [16]. However, overcoming these obstacles is where innovation takes place: looking at these challe

2. Theory of electron scattering

When momentum transfer and defined multilateralism are present, the electron scattering form factor can be understood as follows: [6],

$$\left|F_\lambda(q)\right|^2=\frac{1}{2j_i+1}\left(\frac{4\pi}{Z^2}\right)\left|\left\langle\Gamma_f\ \|\|\ \hat{T}_\lambda^\xi\ \|\|\ \Gamma_i\right\rangle\right|^2\left|\ F_{f.s}F_{\ c.m}\ \right|^2 \quad (1)$$

The revision of the center of mass is determined by the mass number, while the fixed modification of nucleon size and the scale factors for the harmonic oscillator are denoted as b. The theory of Microscopics incorporates shell-model functions of the wave and higher-energy configuration, extending the concept of hole-particle intrusion and accounting for the influence of core polarization (CP) in form factors. The electron's decreased scattering operator is represented by the matrix component. The input from the fp-model space (p) and the inclusion of core polarization (CP) are indicated in the following notation [6].

$$\left\langle\Gamma_f\ \|\|\ \hat{T}_\lambda^\xi\ \|\|\ \Gamma_i\right\rangle=\left\langle\Gamma_f\ \|\|\ \hat{T}_\lambda^\xi\ \|\|\ \Gamma_i\right\rangle_{ms}+\left\langle\Gamma_f\ \|\|\ \delta\hat{T}_\lambda^\xi\ \|\|\ \Gamma_i\right\rangle_{cp} \quad (2)$$

The transverse form factors for the longitudinal, electric, and magnetic fields (L, E, and M) should be chosen accordingly. It has been noted that quantum numbers in both coordinate space and isospace can be denoted by Greek symbols , and . The acceptance of the single-particle matrix element as the fp-shell model-space (MS) element, represented by a linear combination, is permitted [7]. .[

$$\left\langle\Gamma_f\ \|\|\ \hat{T}_\lambda^\xi\ \|\|\ \Gamma_i\right\rangle_{ms}=\sum_{\alpha_f\alpha_i}\chi^\lambda\Gamma_f\Gamma_i(\alpha_f,\alpha_i)\left\langle\ \alpha_f\ \|\|\ \hat{T}_\lambda^\xi\ \|\|\ \alpha_i\ \right\rangle \quad (3)$$

where $\chi^{\lambda}\Gamma_f\Gamma_i(\alpha_f,\alpha_i)$ one body density matrix element's structure factors are provided by:

$$\chi^{\lambda}\Gamma_f\Gamma_i(\alpha_f,\alpha_i)=\frac{\left\langle \Gamma_f \ \| \ \left[a^{+}(\alpha_f)\otimes\tilde{a}(\alpha_i)\right]^{\lambda} \ \| \ \Gamma_i \right\rangle}{\sqrt{2\lambda+1}} \tag{4}$$

Both the and the label are single particle states in the p-shell model space (MS). As a result, the matrix of core polarization (CP) for the element is as follows:

$$\left\langle \Gamma_f \ \| \ \delta\hat{T}_{\lambda}^{\xi} \ \| \ \Gamma_i \right\rangle_{cp}=\sum_{\alpha_f\alpha_i}\chi^{\lambda}\Gamma_f\Gamma_i(\alpha_f,\alpha_i)\left\langle \alpha_f \ \| \ \delta T_{\lambda}^{\xi} \ \| \ \alpha_i \right\rangle \tag{5}$$

The single-particle matrix aspect for higher-energy configuration is given by [8] up to the first level expansion method.

$$\left\langle \alpha_f \ \| \ \delta T_{\lambda}^{\xi} \ \| \ \alpha_i \right\rangle = \left\langle \alpha_f \ \| \ \hat{T}_{\lambda}^{\xi}\frac{Q}{E_i-H_o}V_{res} \ \| \ \alpha_i \right\rangle+\left\langle \alpha\Gamma_f \ \| \ V_{res}\frac{Q}{E_f-H_o}\hat{T}_{\lambda}^{\xi} \ \| \ \alpha\Gamma_i \right\rangle \tag{6}$$

Outside of the model's space, operator Q is the space projection operator. In both E_i and E_f . is the initial and final states, the energies are of and. Equation (6) can be used to write the two terms for the remaining relationship, MSDI and M3Y, as [9] on the side of the right wrist.

$$\sum_{\alpha_1\alpha_2\Gamma}\frac{(-1)^{\alpha_i+\alpha_2+\Gamma}}{e_{\alpha_i}-e_{\alpha_f}-e_{\alpha_1}+e_{\alpha_2}}(2\Gamma+1)\begin{Bmatrix}\alpha_f & \alpha_i & \lambda\\ \alpha_2 & \alpha_1 & \Gamma\end{Bmatrix}\sqrt{(1+\delta_{\alpha_1\alpha_f})(1+\delta_{\alpha_2\alpha_i})}\left\langle\alpha_2\| \ T_{\lambda} \ \|\alpha_1\right\rangle$$
$$\times\left\langle\alpha_f\alpha_1\left|V_{res}\right|\alpha_i\alpha_2\right\rangle_{\Gamma}$$
$$+ \text{ Terms with } \alpha_1 \text{ and } \alpha_2 \text{ exchanged with an over all minus sign,} \tag{7}$$

Where e particle states and hole states interact to manipulate the energy of a single particle. Every state α_1 and α_2 is protected by the conditions of the core orbits and hole-states.

$$e_{nlj}=(2n+l-\frac{1}{2})\eta\omega+\begin{cases}-\frac{1}{2}(l+1)\left\langle f(r)\right\rangle_{nl} & for \quad j=l-\frac{1}{2},\\ \frac{1}{2}l\left\langle f(r)\right\rangle_{nl} & for \quad j=l+\frac{1}{2},\end{cases} \tag{8}$$

$\left\langle f(r)\right\rangle_{nl}\approx-20A^{-2/3}$ and $h\omega=45A^{-1/3}-25A^{-2/3}$. The electric of transition's strength is calculated from:

$$B(C\lambda) = \frac{|(2\lambda+1)!!|^2}{4\pi} \frac{Z^2}{k^\lambda} |F_\lambda(k)|^2 \quad (9)$$

where wave number is expressed as k = Ex /ħc

3. Results and discussion

3.1 The nucleus ^{26}Mg

By utilizing the model-space (MS) and core-polarization (CP) techniques, we can analyze the isoscalar change in the ^{26}Mg C2 transition. Specifically, we focus on the excitation energy of 1.809 MeV in the ^{26}Mg nucleus, where the transition occurs from the ground state (Jπ = 0+, T = 1) to the excited state (Jπ = 2_1^+, T = 1). In order to understand the potential influence of harmonic oscillators, we incorporate the size-parameter b, which is set at 1.8 fm for the single-particle wave functions. These wave functions are then combined with the interaction matrix elements. Notably, As seen in Fig 1, the model space (MS) (without the cp effect) exhibits deviation from experimental data in every momentum transfer.

In all regions of the momentum transfer, the core polarization shows good agreement with the experimental data; the inclusion of cp effects improves the C2 form factor noticeably. As we can see from Fig. 1, the entire model space and core polarization of the experimental data are underestimated.Radhi, R.A. et al. [17

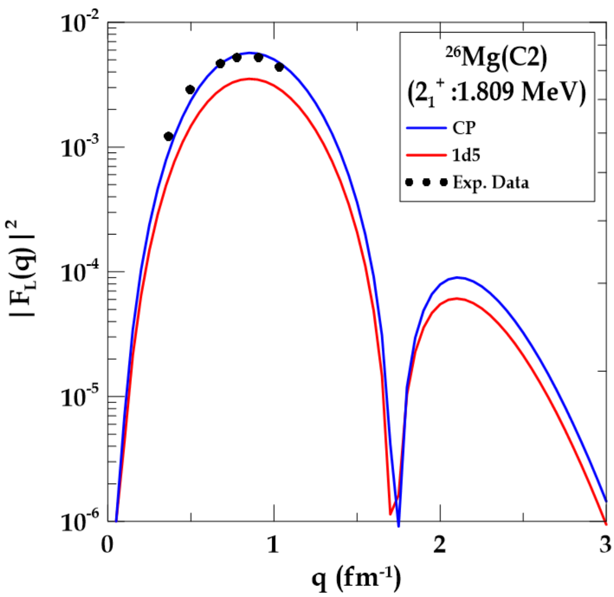

Figure 1: The longitudinal C2 graph of the ^{26}Mg isotope form factor. EX = (1.809) MeV represents the isoscalar transformation from the ground state (J^π = 0+ and T = 1) to (J^π = 2_1^+ and T = 1).

3.2 The nucleus ^{22}Ne

To measure the transformation of the ground state (Jπ = 0+, T = 1) to the (Jπ = 2_1^+, T = 1) state at EX = (1.275) MeV, the single-particle wave harmonic-oscillator (HO) equations are utilized with a size parameter (b) of 1.63 fm. The interaction matrix (OBDM) elements shown in Fig. 2 are employed to assess both the model-space (MS) and core-polarization (CP) results. Fig. 2, the model space (MS) (without the cp effect) exhibits deviation from experimental data in every momentum transfer.

In all regions of the momentum transfer, the core polarization shows good agreement with the experimental data; the inclusion of cp effects improves the C2 form factor noticeably. As we can see from Fig. 2, every region of momentum transfer and the model space and core polarization of the experimental data are underestimated. Radhi, R.A. et al. [17].

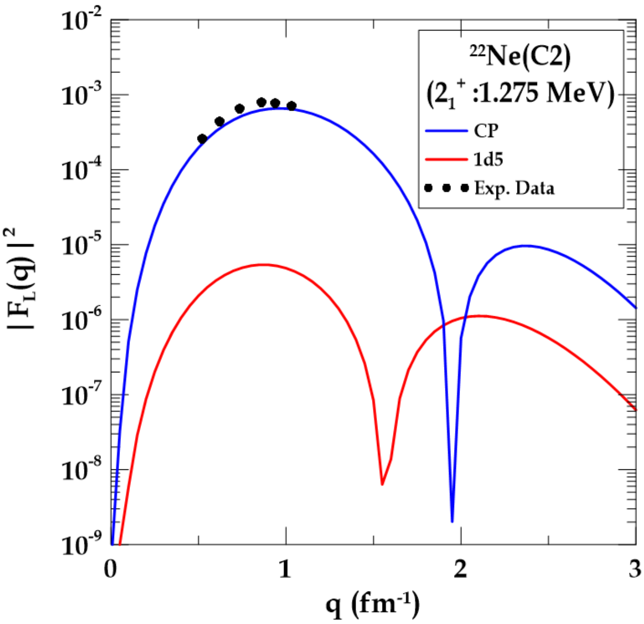

Figure 2: The ^{22}Ne isotope's longitudinal form factor during the C2 transition. The isoscalar transformation from the ground of state ($J^\pi = 0^+$, T = 1) to the state ($J^\pi = 2^+_1$, T = 1) takes place at EX = (1.275) MeV.

3.3 The nucleus ^{30}Si

The measurements of the isoscalar change for the nucleus 30Si C2 transition at 30Si nuclei with an excitation energy of 2.235 MeV are conducted using both model-space (MS) and core-polarization (CP) techniques. The transition occurs from the ground state (Jπ = 0+, T = 1) to the excited state (Jπ = 2^+_1, T = 1). To investigate the potential influence of harmonic oscillators, the size-parameter b is applied to single-particle wave functions, resulting in a value of 1.59 fm using interaction matrix elements. As seen in Fig.3. The model space (MS) (without the cp effect) exhibits deviation from experimental data and slight overexposure to experimental data in all momentum transfer.

As we can see from Fig. 3, the core polarization of the experimental data is underestimated throughout the model space, and the inclusion of cp effects significantly increases the C2 form factor. The core polarization is also in good agreement with the experiment data in all regions of momentum transfer. Radhi, R.A. et al. [17].

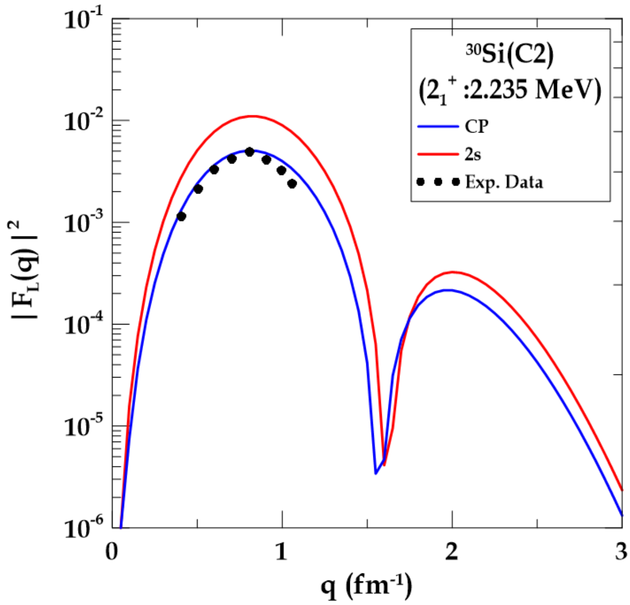

Figure 3: The ^{30}Si isotope's longitudinal form factor during the C2 transition. The isoscalar transformation from the ground of state ($J^{\pi} = 0^{+}$, T = 1) to the state ($J^{\pi} = 2_1^{+}$, T = 1) takes place at EX = (2.235) MeV.

4. Conclusions

The harmonic oscillator (HO) is a suitable choice as an effective residual interaction with some effective charges of protons and neutrons in the shape factor calculations. The electron scattering shape factor and C2 transform level are less effective for the SD shell model. Since experimental data in the high momentum transfer region cannot reproduce these two models, high configuration effects outside the SD shell model space are incorporated into a microscopic theory called core polarization (CP) effects.

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