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Effect of Fixed Bended Plate on Obliquely Incident Waves in Deep Water

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Present analysis is concerned with the problem of incident waves progressing obliquely towards a fixed bended plate, in deep water. Standard perturbation method along with the application of inverse Fourier sine transform technique is used to obtain the first order correction to the velocity potential that involves an unknown function. The correction to the velocity potential is also found by considering two special shapes of the bended plate.

Keywords: water waves, bended plate, velocity potential, perturbation theory, irrotational flow.

1. Introduction

The objective of the present investigation is to obtain an analytical solution for the water wave potential of obliquely incident water waves progressing towards a bended plate, in deep water. The problem involving a vertical wall or barrier in a single liquid or in a two layered liquid media is a subject of considerable research interest among scientists and engineers as the problem is a special case of the well-known sloping beach problem. However existing literature on problems involving a curved wall or barrier, even for a single liquid is rather limited. The problem in this area was first considered by Shaw (cf. [1]) where he used a technique based on perturbation theory that involves the solution of a singular integral equation to find the first order corrections to the reflection and transmission coefficients associated with a surface piercing nearly vertical barrier in deep water. Chakrabarty (cf. [2]) studied the problem of incoming surface water waves against a cliff which is periodically corrugated with a small amplitude including surface tension effect at the free surface. An analysis involving a Fourier sine transform of some special type was used to solve the problem approximately. Mandal and Kar (cf. [3]) considered the problem of reflection of water waves by a nearly vertical wall and they employed a technique based on a simplified perturbational analysis. Since then, few attempts have been made to study this class of water wave problems and few of its' generalizations by employing different mathematical methods (cf. [1]-[9]).

The study is related with the problem of obliquely incident water waves progressing towards a bended plate in deep water. In this topic, no reflection of waves by the plate is assumed. Generally, the plate bound wave conveys certain energy with it and is totally reflected back, if there is no mechanism to absorb (or dissipate) the incoming energy in an inviscid fluid system (cf. [10]) with a rigid plate. For the present study, the assumption of no reflection of waves by the plate can be justified by introducing a source/sink type behavior in the potential function at the origin, i.e., where the free surface of the liquid meets the plate, which requires logarithmic singularity in the potential function at the origin (cf. [11]). Assuming an appropriate expression for the first order correction to the velocity potential describing the motion of the liquid, which involves an unknown function, the problem is tackled for solution by using linear theory. The method of solution involves: (i) standard perturbation technique, giving a sequence of boundary value problems (BVPs), (ii) the known solution of the corresponding vertical plate problem (cf. [12])), and (iii) the inverse Fourier sine transform technique (cf. [13]). The unknown function is finally obtained in terms of integrals involving the shape of the bended plate. Considering two particular shapes of the bended plate, viz. $\delta(y) = y\exp(-\lambda y), \lambda > 0$ and $\delta(y) = a\sin\lambda y$, the first order correction to the velocity potential in each shape is also obtained.

2. Formulation of the problem:

We consider that a train of water waves progressing towards a homogeneous, inviscid, incompressible liquid of density ρ is incident, obliquely, on a bended plate. Cartesian co-ordinate system is chosen in which the y-axis is taken vertically downwards into the liquid medium, the undisturbed free surface of the liquid is given by $y = 0, x \geq 0$, the position of the curved wall is B: $x = \varepsilon\delta(y), 0 < y < \infty$, where ε is a small dimensionless quantity and $\delta(y)$ is bounded and continuous in $0 < y < \infty$ satisfying $c(0) = 0$. The origin is taken at a point on the line of intersection where the curved wall and the free surface meet. Assuming irrotational motion, there exists a velocity potential $\Phi(x, y, z, t)$, in the liquid region which represent progressive waves moving towards the shore line (i.e., the z-axis) such that the wave crests at large distance from the shore tend to straight line which make an arbitrary angle α with the z-axis. Thus, for periodic motion, we may write

$$\Phi(x, y, z, t) = \mathrm{Re}[\phi(x, y)\exp\{-i(\sigma t + \mu z)\}]$$

where $\mu = K\sin\alpha$, $K = \sigma^2/g$ is the wave number, g is the acceleration due to gravity and σ is the circular frequency.

Then the function $\phi(x, y)$ satisfies:

the two-dimensional modified Helmholtz's equation

$$(\nabla^2 - \mu^2)\,\phi = 0 \qquad \text{in the liquid region,} \tag{2.1}$$

where ∇^2 is the two-dimensional Laplacian,

the free surface conditions

$$K\phi + \frac{\partial\phi}{\partial y} = 0 \ \text{ on } \quad y = 0\,, \tag{2.2}$$

as the plate is rigid and fixed, the condition of vanishing of the normal component of velocity at the plate is

$$\frac{\partial \phi}{\partial n} = 0 \quad \text{on} \quad B: x = \varepsilon\delta(y), y > 0, \qquad (2.3)$$

where n denotes the outward drawn normal to the surface of the plate,

the condition of no motion at infinite depth gives

$$\nabla\phi \to 0 \quad \text{as} \quad y \to \infty, \qquad (2.4)$$

as no reflection of the incoming waves by the plate is assumed, a source/sink type behavior of the potential function at the shore-line is to be considered, so that

$$\phi \to \ln r \quad \text{as} \quad r = (x^2 + y^2)^{\frac{1}{2}} \to 0.$$

(2.5)

Finally, as $x \to \infty$, ϕ represents incoming waves progressing towards the plate, so that

$$\phi \to \exp(-Ky - i\nu x) \qquad \text{as} \qquad x \to \infty, \; y > 0,$$

(2.6)

where $\nu = K\cos\theta$.

Assuming that the parameter ε is very small, and neglecting $O(\varepsilon^2)$ terms, the boundary condition (2.3) on the bended plate

$x = \varepsilon\,\delta(y)$ can be expressed, in approximate form, on $x = 0$ as (cf. [4])

$$\frac{\partial \phi}{\partial x}(0, y) - \varepsilon \frac{d}{dy}\left\{\delta(y)\frac{\partial \phi}{\partial y}(0, y)\right\} = 0 \;\; \text{for} \;\; 0 < y < \infty. \qquad (2.7)$$

3. Solution of the problem:

The approximate boundary condition (2.7) indicates that we may assume the following perturbational expansion, in terms of the small parameter ε, for the unknown function $\phi(x, y)$:

$$\phi(x, y, \varepsilon) = \phi_0(x, y) + \varepsilon\phi_1(x, y) + O(\varepsilon^2).$$

(3.1)

Utilizing the expansion for $\phi(x, y)$ given by (3.1) into the original boundary value problem described by (2.1), (2.2), (2.4), (2.6) and (2.7), we obtain, after equating the coefficients of ε^0 and ε from both sides of the results derived thus, that the functions $\phi_0(x, y)$ and $\phi_1(x, y)$ must satisfy the following two independent boundary value problems (BVPs):

BVP-I: The problem is to find the function $\phi_0(x, y)$ satisfying:

$$(\nabla^2 - \mu^2)\phi_0 = 0 \quad \text{in the liquid region,}$$

$$K\phi_0 + \frac{\partial \phi_0}{\partial y} = 0 \text{ on } \; y = 0, x > 0,$$

$$\frac{\partial \phi_0}{\partial x} = 0 \;\text{ on } \; x = 0, \qquad 0 < y < \infty,$$

$$\nabla\phi_0 \to 0 \;\text{ as }\; y \to \infty,$$

$$\phi_0 \to \ln r \;\text{ as }\; r \to 0,$$

$$\phi_0 \to \exp(-Ky - i\nu x) \;\text{ as }\; x \to \infty.$$

BVP-II: The problem is to obtain $\phi_1(x, y)$ satisfying:

$$(\nabla^2 - \mu^2)\phi_1 = 0 \;\text{ in the region }\; x > 0,\; y > 0,$$

$$K\phi_1 + \frac{\partial \phi_1}{\partial y} = 0 \quad \text{ on } \; y = 0, x > 0,$$

$$\frac{\partial \phi_1}{\partial x}(0, y) = \frac{d}{dy}\left\{\delta(y)\frac{\partial \phi_0}{\partial y}(0, y)\right\} = u(y), \text{say}, \quad \text{on } \; x = 0, 0 < y < \infty, \tag{3.2}$$

$$\nabla\phi_1 \to 0 \;\text{ as }\; y \to \infty,$$

$$\phi_1 \;\text{ is bounded as }\; r \to 0,$$

$$\phi_1 \to 0 \;\text{ as }\; x \to \infty.$$

Of the two problems, BVP-I and BVP-II, for the functions $\phi_0(x, y)$ and $\phi_1(x, y)$, the solution of the problem BVP-I, which corresponds to the vertical cliff problem, is known (cf. [12), and is given by

$$\phi_0(x, y) = \exp(-Ky - i\nu x) + \frac{i\cos\theta}{\pi}\int_0^\infty \frac{k(k\cos ky - K\sin ky)}{k_1(k^2 + K^2)}\exp(-k_1 x)dk. \tag{3.3}$$

where $k_1 = (k^2 + \mu^2)^{\frac{1}{2}}$.

Though the complete solution of BVP-I for the function $\phi_0(x, y)$ is well known, however, it is not easy to find the function $\phi_1(x, y)$ which solves the BVP-II completely. To solve BVP-II, let us imagine

$$\phi_1(x, y) = \int_0^\infty f(k)(k\cos ky - K\sin ky)\exp(-k_1 x)dk, \tag{3.4}$$

where A(k) is unknown and is to be determined.

From (3.4) we obtain

$$\frac{\partial \phi_1}{\partial x}(0, y) = -\int_o^\infty k_1\, f(k)(k \cos ky - K \sin ky)\, dk.$$

Thus utilizing (3.2) we find

$$\int_o^\infty \boldsymbol{k_1\, f(k)(k \cos ky - K \sin ky)\, dk} = -\boldsymbol{u(y)},$$

which, after some elementary manipulation, reduces to

$$\int_o^\infty \boldsymbol{k_1\, f(k) \sin ky\, dk} = \boldsymbol{v(y)},$$

where

$$\boldsymbol{v(y)} = \boldsymbol{exp(Ky) \int_y^\infty u(p)\, exp(-Kp)\, dp}. \qquad \mathbf{(3.5)}$$

Therefore, by inverse Fourier sine transform, we get

$$\boldsymbol{k_1\, f(k)} = \boldsymbol{\frac{2}{\pi} \int_o^\infty v(y) \sin ky\, dy}.$$

Substituting for $\mathbf{g(y)}$ from (3.5), and changing the order of integration, we obtain

$$\frac{\boldsymbol{\pi}}{\mathbf{2}} \mathbf{k_1\, A(k)} = \int_{\mathbf{0}}^\infty \mathbf{u(p)\, exp(-Kp)} \left\{ \int_{\mathbf{0}}^{\mathbf{p}} \sin ky \exp(Ky) dy \right\} dp. \qquad (3.6)$$

It can be easily shown that

$$\int_0^p \sin ky \exp(Ky) dy = \frac{(K \sin kp - k \cos kp) \exp(Kp) + k}{k^2 + K^2}, \qquad (3.7)$$

so that using (3.7) in (3.6) we find

$$f(k) = \frac{2}{\pi k_1(k^2+K^2)}\left\{\int_0^\infty (K\sin kp - k\cos kp)\, u(p)\, dp + k\int_0^\infty u(p)\exp(-Kp)\, dp\right\}. \qquad (3.8)$$

Substituting for $u(y)$ in (3.8), we obtain after some manipulation (Appendix-I):

$$f(k) = \frac{2k}{\pi k_1(k^2+K^2)}\Bigg[K\int_0^\infty \eta(k,y)\,\delta(y)\exp(-Ky)dy + \frac{i\cos\alpha}{\pi}\int_0^\infty \eta(k,y)\,\delta(y)\left\{\int_0^\infty \frac{q^2\eta(q,y)}{q_1(q^2+K^2)}dq\right\}dy$$
$$-K^2\int_0^\infty \delta(y)\exp(-2Ky)dy - \frac{iK\cos\alpha}{\pi}\int_0^\infty \delta(y)\exp(-Ky)\left\{\int_0^\infty \frac{q^2\eta(q,y)}{q_1(q^2+K^2)}dq\right\}dy\Bigg] \qquad (3.9)$$

where $\eta(k,y) = k\sin ky + K\cos ky$ and $q_1 = (q^2+\mu^2)^{\frac{1}{2}}$.

For a particular shape of the bended plate, $f(k)$ can be obtained explicitly and hence utilizing (3.4), we find $\phi_1(x,y)$ i.e. the first order correction of the potential function $\phi(x,y)$.

4. Special shapes of the bended plate:

LET US CONSIDER TWO SPECIAL SHAPES FOR THE BENDED PLATE VIZ. (I) $\Delta(Y) = Y\exp(-\Lambda Y)$, FOR $\Lambda > 0$ AND (II) $\Delta(Y) = A\sin\Lambda Y$, (AS CONSIDERED BY CHAKRABARTI [2]).

CASE - I: $\delta(y) = y\exp(-\lambda y), \lambda > 0$.

In this case we obtain (see Appendix - II):

$$f(k) = \frac{k}{\pi k_1(k^2+K^2)}\Bigg\{\frac{2K\{(K+2\lambda)(k^2+K^2)+K\lambda^2\}}{\{(\lambda+K)^2+k^2\}^2} + \frac{i\cos\alpha}{\pi}\int_0^\infty \frac{q^2}{q_1(l^2+K^2)}\left[\frac{\lambda^2-(k-q)^2}{\{\lambda^2+(k-q)^2\}^2}(K^2+kq) + \frac{\lambda^2-(k+q)^2}{\{\lambda^2+(k+q)^2\}^2}(K^2-kq)\right]dq$$
$$-\frac{2K^2}{(\lambda+2K)^2} + \frac{2iK\lambda\cos\alpha}{\pi}\int_0^\infty \frac{q^2}{q_1(q^2+K^2)}\left[\frac{(k+q)^2}{\{\lambda^2+(k+q)^2\}^2} + \frac{(k-q)^2}{\{\lambda^2+(k-q)^2\}^2}\right]dq$$

$$-\frac{2iK\cos\alpha}{\pi}\int_0^\infty \frac{q^2}{q_1(q^2+K^2)}\left[\frac{2q^2(\lambda+k)}{\{(\lambda+K)^2+q^2\}^2} + K\frac{(\lambda+K)^2-q^2}{\{(\lambda+K)^2+q^2\}^2}\right]dq\Bigg\}. \quad (4.1)$$

CASE - II: $\delta(y) = a\sin\lambda y$.

In this case we find (see Appendix - III):

$$f(k) = \frac{k}{\pi q_1(k^2+K^2)}\left[\lambda a K^2\left\{\frac{1}{(\lambda+k)^2+K^2} + \frac{1}{(\lambda-k)^2+K^2} - \frac{2}{\lambda^2+4K^2}\right\}\right.$$
$$\left. + \frac{ia\cos\alpha}{\pi}\int_0^\infty \frac{q^2}{q_1(q^2+K^2)}\left\{\frac{K^2(\lambda+k)+kq^2}{(\lambda+k)^2-q^2} + \frac{K^2(\lambda-k)-kq^2}{(\lambda-k)^2-q^2} - \frac{\lambda K^2}{(\lambda+q)^2+K^2} - \frac{\lambda K^2}{(\lambda-q)^2+K^2}\right\}dq\right]. \quad (4.2)$$

5. Conclusions:

The problem of surface water waves incident obliquely towards a bended plate, in deep water, is discussed by an approximate procedure. In this technique, the boundary condition on the bended plate is first replaced by an approximate condition on the corresponding vertical plate. The first order correction to the water wave potential is determined by a perturbational analysis followed by the inverse Fourier sine transform technique. Analytical expression for this correction is also found by assuming two different shapes of the bended plate viz. i) $\delta(y) = y\exp(-\lambda y), \lambda > 0$ and ii) $\delta(y) = a\sin\lambda y$. The main advantage of the problem considered in this paper is that the approximate solution in connection with the corresponding two-dimensional problem can also be derived simply by the substitution of $\alpha = 0$. This problem is a simplified mathematical model of the well-known sloping beach problem arising in oceanography. The problem considered here may further be developed.

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APPENDIX-I:

The analytical expression for $f(k)$ can be found from (3.8). Substituting for $u(p)$ from (3.2) we find

$$\int_0^\infty (K\sin kp - k\cos kp)\, u(p)dp = -kI_1 + KI_2, \qquad (A1.1)$$

where

$$I_1 = \int_0^\infty \frac{d}{dp}\left\{\delta(p)\frac{\partial\phi_0(0,p)}{\partial p}\right\}\cos kp\, dp, \qquad (A1.2)$$

and

$$I_2 = \int_0^\infty \frac{d}{dp}\left\{\delta(p)\frac{\partial\phi_0(0,p)}{\partial p}\right\}\sin kp\, dp, \qquad (A1.3)$$

Utilizing the known expression for $\phi_0(0,p)$ obtained from (3.3) and exploiting the conditions $\delta(0) = 0$ and $\nabla\phi_0 \to 0$ as $y \to \infty$, it can be shown that

$$I_1 = I_3 + I_4 \qquad (A1.4)$$

and

$$I_2 = I_5 + I_6 \qquad (A1.5)$$

where

$$I_3 = -kK\int_0^\infty \delta(p)\exp(-Kp)\sin kp\, dp, \qquad (A1.6)$$

$$I_4 = -\frac{ik\cos\alpha}{\pi}\int_0^\infty \delta(p)\left\{\int_0^\infty \frac{q^2(q\sin pq + K\cos pq)}{q_1(q^2+K^2)}dp\right\}\sin pq\, dp, \qquad (A1.7)$$

$$I_5 = kK\int_0^\infty \delta(p)\exp(-Kp)\sin kp\, dp, \qquad (A1.8)$$

$$I_6 = \frac{ik\cos\alpha}{\pi}\int_0^\infty \delta(p)\left\{\int_0^\infty \frac{q^2(q\sin pq + K\cos pq)}{q_1(q^2+K^2)}dq\right\}\cos kp\, dp. \quad (A1.9)$$

Similarly, using the known expression for $f(s)$ we find

$$\int_0^\infty u(p)\exp(-Kp)dp = \int_0^\infty \frac{d}{dp}\left\{\delta(p)\frac{\partial\phi_0(0,p)}{\partial p}\right\}\exp(-Kp)dp.$$

Using the known expression for $\phi_0(0,p)$ obtained from (3.3) and exploiting the conditions $\delta(0) = 0$ and $\nabla\phi_0 \to 0$ as $y \to \infty$, we find the above integral reduces to:

$$I_7 + I_8$$

where

$$I_7 = -K^2\int_0^\infty \delta(p)\exp(-2Kp)\, dp, \quad (A1.10)$$

and

$$I_8 = -\frac{iK\cos\alpha}{\pi}\int_0^\infty \delta(p)\left\{\int_0^\infty \frac{q^2(q\sin pq + K\cos pq)}{q_1(q^2+K^2)}dq\right\}\exp(-Kp)\, dp. \quad (A1.11)$$

Using I_1, I_2, I_7, I_8 thus obtained, the analytical expression for $f(k)$ is found, which is given by (3.9).

APPENDIX-II:

Explicit calculation of various integrals for $\delta(y) = y\exp(-\lambda y), \lambda > 0$.

Assuming $\delta(y) = y\exp(-\lambda y)$, in the integrals represented by I_1 to I_8, defined in Appendix-I, we find

$$I_3 = -kK\int_0^\infty y\exp\{-(\lambda+K)y)\}\sin ky\, dy$$
$$= -\frac{2Kk^2(\lambda+K)}{\{(\lambda+K)^2+k^2\}^2}, \quad (A2.1)$$

$$I_4 = -\frac{ik\cos\alpha}{\pi}\int_0^\infty y\exp(-\lambda y)\left\{\int_0^\infty \frac{q^2(q\sin qy + K\cos qy)}{q_1(q^2+K^2)}dq\right\}\sin ky\, dy$$
$$= -\frac{ik\cos\alpha}{\pi}\int_0^\infty \frac{q^2}{q_1(q^2+K^2)}\left\{\int_0^\infty y(q\sin qy + K\cos qy)\sin ky\exp(-\lambda y)dy\right\}dq. \quad (A2.2)$$

The inner integral of I_4 is evaluated as

$$\int_0^\infty y(q\sin qy + K\cos qy)\sin ky\exp(-\lambda y)dy$$
$$= \frac{q}{2}\left[\frac{\lambda^2-(k-q)^2}{\{\lambda^2+(k-q)^2\}^2} - \frac{\lambda^2-(k+q)^2}{\{\lambda^2+(k+q)^2\}^2}\right] + \frac{K}{2}\left[\frac{2\lambda(k+q)}{\{\lambda^2+(k+q)^2\}^2} + \frac{2\lambda(k-q)}{\{\lambda^2+(k-q)^2\}^2}\right]. \quad (A2.3)$$

So that we get from (A2.2)

$$I_4 = -\frac{ik\cos\alpha}{2\pi}\int_0^\infty \frac{q^3}{q_1(q^2+K^2)}\left[\frac{\lambda^2-(k-q)^2}{\{\lambda^2+(k-q)^2\}^2} - \frac{\lambda^2-(k+q)^2}{\{\lambda^2+(k+q)^2\}^2}\right]dq$$

$$-\frac{ikK\lambda\cos\alpha}{\pi}\int_0^\infty \frac{q^2}{q_1(q^2+K^2)}\left[\frac{(k+q)}{\{\lambda^2+(k+q)^2\}^2}+\frac{(k-q)}{\{\lambda^2+(k-q)^2\}^2}\right]dq. \qquad (A2.4)$$

Therefore, using (A2.1) and (A2.4) in (A1.4) we find

$$I_1=-\frac{2Kk^2(\lambda+K)}{\{(\lambda+K)^2+k^2\}^2}-\frac{ik\cos\alpha}{2\pi}\int_0^\infty \frac{l^3}{q_1(q^2+K^2)}\left[\frac{\lambda^2-(k-q)^2}{\{\lambda^2+(k-q)^2\}^2}-\frac{\lambda^2-(k+q)^2}{\{\lambda^2+(k+q)^2\}^2}\right]dq$$
$$-\frac{ikK\lambda\cos\alpha}{\pi}\int_0^\infty \frac{q^2}{q_1(q^2+K^2)}\left[\frac{(k+q)}{\{\lambda^2+(k+q)^2\}^2}+\frac{(k-q)}{\{\lambda^2+(k-q)^2\}^2}\right]dq. \qquad (A2.5)$$

Also,

$$I_5=kK\int_0^\infty y\exp\{-(\lambda+K)y)\}\cos ky\,dy=\frac{kK\{(\lambda+K)^2-k^2\}}{\{(\lambda+K)^2+k^2\}^2}, \qquad (A2.6)$$

$$I_6=\frac{ik\cos\alpha}{\pi}\int_0^\infty y\exp(-\lambda y)\left\{\int_0^\infty \frac{q^2(q\ \sin qy+K\cos qy)}{q_1(q^2+K^2)}dq\right\}\cos ky\,dy$$
$$=\frac{ik\cos\alpha}{\pi}\int_0^\infty \frac{q^2}{q_1(q^2+K^2)}\left\{\int_0^\infty y(q\sin qy+K\cos qy)\cos ky\exp(-\lambda y)dy\right\}dq. \qquad (A2.7)$$

Inner integral of I_6 is

$$\int_0^\infty y(q\sin qy+K\cos qy)\cos ky\exp(-\lambda y)\,dy=\lambda q\left[\frac{(q+k)}{\{\lambda^2+(q+k)^2\}^2}+\frac{(q-k)}{\{\lambda^2+(q-k)^2\}^2}\right]+\frac{K}{2}\left[\frac{\lambda^2-(q+k)^2}{\{\lambda^2+(q+k)^2\}^2}+\frac{\lambda^2-(q-k)^2}{\{\lambda^2+(q-k)^2\}^2}\right], \qquad (A2.8)$$

so that using (A2.8) into (A2.7) we get

$$J_6=\frac{ik\lambda\cos\alpha}{\pi}\int_0^\infty \frac{q^3}{q_1(q^2+K^2)}\left[\frac{(q+k)}{\{\lambda^2+(q+k)^2\}^2}+\frac{(q-k)}{\{\lambda^2+(q-k)^2\}^2}\right]dq$$
$$+\frac{ikK\cos\alpha}{2\pi}\int_0^\infty \frac{q^2}{q_1(q^2+K^2)}\left[\frac{\lambda^2-(q+k)^2}{\{\lambda^2+(q+k)^2\}^2}+\frac{\lambda^2-(q-k)^2}{\{\lambda^2+(q-k)^2\}^2}\right]dq. \qquad (A2.9)$$

Therefore, from (A1.5) we find, after using (A2.6) and (A2.9)

$$J_2 = \frac{kK\{(\lambda+K)^2-k^2\}}{\{(\lambda+K)^2+k^2\}^2} + \frac{ik\lambda\cos\alpha}{\pi}\int_0^\infty \frac{l^2}{(l^2+K^2)(l^2+\nu^2)^{\frac{1}{2}}}\left[\frac{(l+k)}{\{\lambda^2+(l+k)^2\}^2}+\frac{(l-k)}{\{\lambda^2+(l-k)^2\}^2}\right]dl + \frac{ikK\cos\theta}{2\pi}\int_0^\infty \frac{l^2}{(l^2+K^2)(l^2+\nu^2)^{\frac{1}{2}}}\left[\frac{\lambda^2-(l+k)^2}{\{\lambda^2+(l+k)^2\}^2} + \frac{\lambda^2-(l-k)^2}{\{\lambda^2+(l-k)^2\}^2}\right]dl. \qquad (A2.10)$$

Using (A2.5) and (A2.10) into (A1.1), we obtain

$$\int_0^\infty f(y)(K\sin ky - k\cos ky)dy = \frac{kK\{(K+2\lambda)(K^2+k^2)+K\lambda^2\}}{\{(\lambda+K)^2+k^2\}^2} + \frac{ik\ \cos\theta\alpha}{2\pi}\int_0^\infty \frac{q^2}{q_1(q^2+K^2)}\left[\frac{\lambda^2-(\mathrm{k}-q)^2}{\{\lambda^2+(k-q)^2\}^2}(K^2+kq)+\frac{\lambda^2-(k+q)^2}{\{\lambda^2+(k+q)^2\}^2}(K^2-kq)\right]dq + \frac{ikK\lambda\cos\alpha}{\pi}\int_0^\infty \frac{q^2}{q_1(q^2+K^2)}\left[\frac{(k+q)^2}{\{\lambda^2+(k+q)^2\}^2} + \frac{(k-q)^2}{\{\lambda^2+(k-q)^2\}^2}\right]dq. \qquad (A2.11)$$

Also

$$I_7 = -K^2\int_0^\infty y\exp\{-(\lambda+2K)y)\}dy = -\frac{K^2}{(\lambda+2K)^2}, \qquad (A2.12)$$

$$I_8 = -\frac{iK\cos\alpha}{\pi}\int_0^\infty y\exp(-\lambda y)\left\{\int_0^\infty \frac{q^2(q\sin qy+K\cos qy)}{q_1(q^2+K^2)}dq\right\}\exp(-Ky)dy = -\frac{iK\cos\alpha}{\pi}\int_0^\infty \frac{q^2}{q_1(q^2+K^2)}\left\{\int_0^\infty y(q\sin qy+K\cos qy)\exp\{-(\lambda + K)y)\}dy\right\}dq. \qquad (A2.13)$$

Inner integral of I_8 reduces to

$$\frac{2q^2(\lambda+k)}{\{(\lambda+K)^2+q^2\}^2} + K\frac{(\lambda+K)^2-q^2}{\{(\lambda+K)^2+q^2\}^2} \qquad (A2.14)$$

Therefore

$$J_8 = -\frac{iK\cos\alpha}{\pi}\int_0^\infty \frac{q^2}{q_1(q^2+K^2)}\left[\frac{2q^2(\lambda+k)}{\{(\lambda+K)^2+q^2\}^2} + K\frac{(\lambda+K)^2-q^2}{\{(\lambda+K)^2+q^2\}^2}\right]dq. \qquad (A2.15)$$

Using (A2.12) and (A2.15), we find

$$I_7 + I_8 = -\frac{K^2}{(\lambda + 2K)^2} - \frac{iK\cos\alpha}{\pi}\int_0^\infty \frac{q^2}{q_1(q^2+K^2)}\left[\frac{2q^2(\lambda+k)}{\{(\lambda+K)^2+q^2\}^2} + K\frac{(\lambda+K)^2-q^2}{\{(\lambda+K)^2+q^2\}^2}\right]dq. \quad (A2.16)$$

Thus, exploiting (A2.11) and (A2.16) into (3.8) we find the analytical expression for $f(k)$ which is given by (4.1).

APPENDIX-III:

Explicit calculation of various integrals for $\delta(y) = a\sin\lambda y$.

When $\delta(y) = \mathrm{a}\sin\lambda y$, we find

$$I_3 = -kaK\int_0^\infty \sin\lambda y \sin ky \exp(-Ky)dy = -\frac{kaK^2}{2}\left[\frac{1}{(\lambda-k)^2+K^2} - \frac{1}{(\lambda+k)^2+K^2}\right], \quad (A3.1)$$

and

$$I_4 = -\frac{iak\cos\alpha}{\pi}\int_0^\infty \sin\lambda y \sin \mathrm{ky}\left\{\int_0^\infty \frac{q^2(q\sin qy + K\cos qy)}{q_1(q^2+K^2)}dq\right\}dy = -\frac{iak\cos\alpha}{\pi}\int_0^\infty \frac{q^2}{q_1(q^2+K^2)}\left\{\int_0^\infty \sin\lambda y\sin ky\,(q\sin qy + K\cos qy)dy\right\}dq. \quad (A3.2)$$

Using a convergence factor of the type used by Evans and Morris ([12]), the inner integral of I_4 is evaluated, and is given by

$$\frac{q^2}{2}\left[\frac{1}{q^2-(\lambda-k)^2} - \frac{1}{q^2-(\lambda+k)^2}\right]$$

so that from (A3.2) we get

$$I_4 = -\frac{iak\cos\alpha}{2\pi}\int_0^\infty \frac{q^4}{q_1(q^2+K^2)}\left[\frac{1}{q^2-(\lambda-k)^2} - \frac{1}{q^2-(\lambda+k)^2}\right]dq. \quad (A3.3)$$

Therefore, using (A3.1) and (A3.3) into (A1.4) we get

$$I_1 = -\frac{kaK^2}{2}\left[\frac{1}{(\lambda-k)^2+K^2} - \frac{1}{(\lambda+k)^2+K^2}\right] - \frac{iak\cos\alpha}{2\pi}\int_0^\infty \frac{q^4}{q_1(q^2+K^2)}\left[\frac{1}{q^2-(\lambda-k)^2} - \frac{1}{q^2-(\lambda+k)^2}\right]dq. \quad (A3.4)$$

Also,

$$I_5 = kaK\int_0^\infty \sin\lambda y \cos \mathrm{ky} \exp(-Ky)dy$$

$$= \frac{kaK}{2}\left[\frac{\lambda+k}{(\lambda+k)^2+K^2}+\frac{\lambda-k}{(\lambda-k)^2+K^2}\right] \qquad (A3.5)$$

and

$$I_6 = \frac{iak\cos\alpha}{\pi}\int_0^\infty \sin\lambda y\cos \mathrm{ky}\left\{\int_0^\infty \frac{q^2(q\sin qy+K\cos qy)}{q_1(\mathrm{q}+K^2)}dq\right\}dy$$

$$= \frac{iak\cos\alpha}{\pi}\int_0^\infty \frac{q^2}{q_1(q^2+K^2)}\left\{\int_0^\infty \sin\lambda y\cos ky\,(q\sin qy + K\cos qy)\,dy\right\}dq. \qquad (A3.6)$$

Using a convergence factor as used by Evans and Morris ([12]), the inner integral of (A3.6) is evaluated and is given by

$$\frac{K}{2}\left[\frac{\lambda+k}{(\lambda+k)^2-q^2}+\frac{\lambda-k}{(\lambda-k)^2-q^2}\right].$$

Using the above result into (A3.6) we obtain

$$I_6 = \frac{iakK\cos\alpha}{2\pi}\int_0^\infty \frac{q^2}{q_1(q^2+K^2)}\left[\frac{\lambda+k}{(\lambda+k)^2-q^2}+\frac{\lambda-k}{(\lambda-k)^2-q^2}\right]dq. \qquad (A3.7)$$

Therefore from (A1.5), after using (A3.5) and (A3.7), we find

$$I_2 = \frac{kaK}{2}\left[\frac{\lambda+k}{(\lambda+k)^2+K^2}+\frac{\lambda-k}{(\lambda-k)^2+K^2}\right] + \frac{iakK\cos\alpha}{2\pi}\int_0^\infty \frac{q^2}{q_1(q^2+K^2)}\left[\frac{\lambda+k}{(\lambda+k)^2-q^2}+\frac{\lambda-k}{(\lambda-k)^2-q^2}\right]dq. \qquad (A3.8)$$

Thus, using (A3.4) and (A3.8) into (A1.1) we obtain

$$\int_0^\infty u(y)(K\sin ky - k\cos ky)dy = \frac{\lambda akK^2}{2}\left[\frac{1}{(\lambda+k)^2+K^2}+\frac{1}{(\lambda-k)^2+K^2}\right] + \frac{iak\cos\alpha}{2\pi}\int_0^\infty \frac{q^2}{q_1(q^2+K^2)}\left[\frac{K^2(\lambda+k)+kq^2}{(\lambda+k)^2-q^2}+\frac{K^2(\lambda-k)-kq^2}{(\lambda-k)^2-q^2}\right]dq. \qquad (A3.9)$$

Also

$$I_7 = -aK^2 \int_0^\infty \sin\lambda y \exp(-2Ky)dy = -\frac{aK^2\lambda}{\lambda^2+4K^2} \quad \text{(A3.10)}$$

and

$$I_8 = -\frac{iaK\cos\alpha}{\pi}\int_0^\infty \sin\lambda y\left\{\int_0^\infty \frac{q^2(q\sin qy + K\cos qy)}{q_1(q^2+K^2)}dq\right\}\exp(-Ky)dy$$
$$= -\frac{iaK\cos\alpha}{\pi}\int_0^\infty \frac{q^2}{q_1(q^2+K^2)}\left\{\int_0^\infty \sin\lambda y\,(q\sin qy + K\cos qy)\exp(-Ky)dy\right\}dq. \quad (A3.11)$$

It can be easily shown that

$$\int_0^\infty \sin\lambda y\,(q\sin qy + K\cos qy)\exp(-Ky)dy = \frac{qK}{2}\left[\frac{1}{(\lambda-q)^2+K^2} - \frac{1}{(\lambda+q)^2+K^2}\right] + \frac{K}{2}\left[\frac{\lambda+q}{(\lambda+q)^2+K^2} + \frac{\lambda-q}{(\lambda-q)^2+K^2}\right].$$

So that from (A3.11) we get

$$I_8 = -\frac{iaK^2\cos\alpha}{2\pi}\int_0^\infty \frac{q^2}{q_1(q^2+K^2)}\left[q\left\{\frac{1}{(\lambda-q)^2+K^2} - \frac{1}{(\lambda+q)^2+K^2}\right\} + \left\{\frac{\lambda+q}{(\lambda+q)^2+K^2} + \frac{\lambda-q}{(\lambda-q)^2+K^2}\right\}\right]dq \quad (A3.12)$$

Therefore, using (A3.10) and (A3.12) we obtain

$$I_7 + I_8 = -\frac{aK^2\lambda}{\lambda^2+4K^2} - \frac{iaK^2\cos\alpha}{2\pi}\int_0^\infty \frac{q^2}{q_1(q^2+K^2)}\left[q\left\{\frac{1}{(\lambda-q)^2+K^2} - \frac{1}{(\lambda+q)^2+K^2}\right\} + \left\{\frac{\lambda+q}{(\lambda+q)^2+K^2} + \frac{\lambda-q}{(\lambda-q)^2+K^2}\right\}\right]$$
$$= -\frac{aK^2\lambda}{\lambda^2+4K^2} - \frac{iaK^2\lambda\cos\alpha}{2\pi}\int_0^\infty \frac{q^2}{q_1(q^2+K^2)}\left[\frac{1}{(\lambda-q)^2+K^2} + \frac{1}{(\lambda+q)^2+K^2}\right]dq. \quad (A3.13)$$

Finally, using (A3.9) and (A3.13) into (3.8) we obtain the analytical expression for $f(k)$, which is given by (4.2).