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Mathematical and Statistical Evaluation of Public Key Cryptosystems for Smart Cities

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This article presents a mathematical and statistical evaluation of public-key cryptosystems designed for implementation in smart cities. It focuses on analyzing the safety and efficiency of these systems using mathematical methods and statistical tools. The research includes modeling key distribution, analyzing robustness against cryptographic attacks, and evaluating computational efficiency in high-demand environments. The results obtained provide a clear vision on the ability of these cryptosystems to maintain the integrity and confidentiality of information in the context of smart cities.

Keywords: public key cryptosystems, smart cities, mathematical analysis, statistical analysis, cryptographic security.

1. Introduction

In recent years, the concept of smart cities has gained significant attention due to its potential to improve the efficiency of urban services, environmental sustainability, and the quality of life of citizens. Smart cities are interconnected urban environments that use information and communication technologies (ICTs) to manage resources more efficiently, such as energy, water, transportation, and public safety (Zhang, Yang, & Appiah, 2019). However, the massive interconnection of devices and systems that characterizes smart cities also expands the attack surface for potential cyberthreats, making information security a critical component (Arjoune & Kaabouch, 2019).

One of the fundamental pillars of security in smart cities is the use of public key cryptosystems (PKCs), which enable authentication, confidentiality, and integrity of data exchanged between

devices and users. Public-key cryptosystems work using a pair of keys: a public key, which can be shared openly, and a private key, which is kept secret. This system ensures that only the private key holder can decrypt the information encrypted with the public key, thus providing a secure mechanism for the transmission of sensitive data (Abdulkareem et al., 2021).

As smart cities continue to evolve, the need for more robust and efficient crypto systems is becoming increasingly apparent. The increasing complexity of urban environments, coupled with the increase in the volume and speed of data that must be processed in real-time, poses unique challenges for cryptographic systems. In particular, computational efficiency and resistance to cryptographic attacks have become key topics of interest in the design and implementation of cryptosystems for smart cities (Sharma & Chen, 2020).

The most widely used public-key cryptosystems include RSA (Rivest, Shamir & Adleman, 1978), Elliptic Curve Cryptography (ECC), and Digital Signature Algorithm (DSA). Each of these systems offers different levels of safety and efficiency, making them more or less suitable depending on the context of your application. For example, RSA is widely recognized for its security, based on the difficulty of factoring large numbers, but its efficiency decreases with increasing key lengths, which can be a problem in resource-constrained environments or where quick responses are required (Challa, Wazid, Das, & Kumar, 2019). On the other hand, ECC provides a comparable level of security to RSA but with much shorter keys, reducing the computational load and making it an attractive option for low-power devices, such as those found in smart city sensor networks (Apostolaki, Zohar, & Vanbever, 2020).

This article aims to conduct a thorough mathematical and statistical evaluation of these public key cryptosystems, focusing on their suitability for implementation in smart cities. Through a detailed analysis, it seeks to determine which of these systems offers the best balance between security and efficiency, key factors for the success of cryptographic implementations in highly connected urban environments. Mathematical methods will be used to model the complexity and security of algorithms, along with statistical tools to analyze the behavior of systems under different usage scenarios. The results of this research will provide a framework for the selection and implementation of cryptosystems in smart city design, ensuring both the security and efficiency of critical operations.

2. Theoretical Framework

Public Key Cryptosystems

Public key cryptosystems (PKCs) are essential in modern cryptography, especially in the context of smart cities, where information security is critical. These systems use a pair of keys: one public and one private. The public key is used to encrypt data, while the private key is used to decrypt information. This security model enables user authentication and data integrity in open and highly connected networks, such as those in smart cities (Zhang et al., 2020).

One of the best-known public-key cryptosystems is RSA (Rivest, Shamir & Adleman, 1978), whose mathematical foundation is based on the difficulty of factoring large prime numbers. Mathematically, RSA security is expressed as:

$$n = p * q$$

where p and q are large prime numbers. The public key includes the module n and a public exponent e, while the private key is based on the private exponent d, calculated using the relationship:

$$d * e \equiv 1 \pmod{\varphi(n)}$$

where $\varphi(n) = (p-1) * (q-1)$ is the Euler function totiente. This system is robust, but efficiency decreases as the length of the keys needed to maintain security increases, which can be problematic in environments that require rapid response, such as smart cities (Abdalla, Bellare, & Rogaway, 2020).

Elliptic Curve Cryptography (ECC) is an alternative that provides a level of security comparable to RSA but with shorter keys, which reduces the computational load. ECC is based on the mathematical structure of elliptic curves, which are equations of the type:

$$y^2 = x^3 + ax + b$$

where a and b are constants that define the curve. The safety of ECC depends on the difficulty of the discrete logarithm problem on the elliptic curve (ECDLP), which consists of finding an integer k such that:

$$Q = k * P$$

where P is a point on the elliptic curve and Q is the point resulting from multiplying P by the scalar k (Becker, Coron, & Joux, 2021). This problem is considerably more difficult than large integer factorization, allowing for the use of much shorter and more efficient keys in terms of computational resources, a crucial advantage in low-power devices typical of smart cities (Huang & Sun, 2021).

The Digital Signature Algorithm (DSA) is another public-key cryptosystem used primarily for the digital signature of electronic documents. DSA is based on the discrete logarithm in finite fields and is particularly efficient for the creation of digital signatures, although its security depends largely on the proper implementation of random parameters (Kim, Lee, & Lee, 2020). Mathematically, DSA involves selecting a prime p and a number q such that q divides p-1, and calculating a value g such that:

$$g^q \equiv 1 \pmod{p}$$

The digital signature of a message m is done by generating a random value k and calculating the following:

$$r = (g^k \bmod p) \bmod q$$

and

$$s = k^{-1} (H(m) + xr) \bmod q$$

where H(m) is the hash of the message and x is the private key. The verification is carried out by checking that:

$$g^{H(m)} \equiv r^s * r^r \pmod{p}$$

Smart Cities and Cryptographic Security

Smart cities represent a massive integration of ICT to improve urban management and daily life. However, this massive interconnection of devices also expands the cyberattack surface. Public-key cryptosystems are critical to securing communications and protecting data integrity in these advanced urban networks (Sun, Guo, & Zhang, 2020).

Smart cities require cryptosystems to be not only secure, but also efficient, as they must handle large volumes of data in real-time and operate on resource-constrained devices (Shi et al., 2021). Therefore, choosing the right crypto system is crucial to ensure both security and operational efficiency. Selected cryptosystems must be resilient to various types of attacks, such as man-in-the-middle, denial-of-service (DoS), and brute-force attacks, while maintaining optimal performance in high-demand environments (Zhang et al., 2020).

Mathematical Analysis in Cryptography

Mathematical analysis is essential to assess the security and efficiency of cryptosystems. In RSA, security lies in the difficulty of factoring large numbers, a task that, although complex, can be tackled by quantum computers, which has motivated research in post-quantum cryptography (Arute et al., 2019). In ECC, the discrete logarithm problem on elliptic curves provides a solid mathematical foundation for security, allowing the use of shorter keys without compromising security (Bos et al., 2021).

Mathematical analysis also includes the evaluation of computational complexity, which can be measured in terms of the Big-O notation. For example, the complexity of the elliptic curve multiplication algorithm can be expressed as O(n^2), where n is the number of bits in the key. This type of analysis is crucial to understanding how cryptographic algorithms will behave in high-demand environments, such as those found in smart cities.

Statistical Analysis in the Evaluation of Cryptosystems

Statistical analysis is used to model and evaluate the efficiency and security of cryptosystems in realistic scenarios. For example, Monte Carlo analysis is a commonly used technique to estimate the probability of success of a brute force attack against a cryptosystem. This method involves performing repeated simulations to observe the behavior of the system under different conditions, allowing the generation of probabilistic distributions that describe the performance of the system (Rivest, 2020).

Another relevant statistical technique is the analysis of the distribution of keys, which can be modeled using probability distributions such as the binomial distribution or the Poisson distribution. For example, if you consider the probability p that a key will be compromised, you can model the distribution of compromised keys across a large number of attempts using a binomial distribution:

$P(X = k) = C(n, k) * p^k * (1-p)^(n-k)$

where X is the number of compromised keys, n is the number of attempts, and k is the number of successes. This type of analysis is crucial to assess the robustness of cryptosystems in smart city environments, where security and efficiency must be guaranteed at all times.

3. Methodology

The methodology of this study is designed to evaluate the security and efficiency of three widely used public-key cryptosystems: RSA, ECC (Elliptic Curve Cryptography) and DSA (Digital Signature Algorithm). The methodological approach includes a thorough mathematical and statistical analysis, based on simulations and empirical tests, to determine the suitability of these cryptosystems in the context of smart cities. The key stages of the methodology used are described below.

Selection of Cryptosystems

The cryptosystems selected for this study are RSA, ECC and DSA, all of which are recognized for their wide implementation in various computer security contexts. The selection was based on the popularity and relevance of these systems in applications that require high security and computational efficiency, essential characteristics in the smart city environment (Zhang, Chen, & Han, 2020).

- RSA: Chosen for its robustness and wide adoption in security systems. It is primarily evaluated in terms of its ability to maintain security through large keys and its relative efficiency compared to other systems.
- ECC: Selected for its ability to provide a similar level of security as RSA, but with significantly shorter keys, which translates into greater efficiency, especially in resource-constrained devices (Huang & Sun, 2021).
- DSA: Included for its importance in the generation of digital signatures, evaluating its security and efficiency in the authentication of electronic transactions in smart cities (Kim, Lee, & Lee, 2020).

Mathematical Modeling

Mathematical modeling of cryptosystems is a crucial component of the methodology, as it allows the complexity and security of algorithms to be analyzed from a theoretical point of view. This modeling includes:

- Computational complexity assessment: Big-O notation is used to analyze the efficiency of algorithms in terms of execution time and resource usage. For example, for RSA, the complexity of large number factorization is modeled as O(n3)O(n3), where nn is the number of bits in the key (Rivest, 2020). In contrast, for ECC, the complexity of discrete logarithm computation is estimated as O(2n)O(2n), demonstrating higher efficiency with shorter keys (Becker et al., 2021).
- Key distribution simulation: To analyze the robustness of cryptosystems against brute force attacks, the key distribution is modeled using probabilities and statistics. This approach allows estimating the resistance of the system based on the length of the key and the difficulty of the underlying mathematical problem.

Statistical analysis

The statistical analysis focuses on the empirical evaluation of cryptosystems using simulations that reflect real-world use scenarios in smart cities. The main statistical techniques employed include:

- Monte Carlo simulations: This technique is used to estimate the probability of compromising a cryptosystem under different attack conditions. For example, brute force attacks are simulated and the probabilities of success are calculated based on the size of the key and the number of attempts made (Kim & Lee, 2020). These simulations allow the generation of probabilistic distributions that describe the performance of the cryptosystem in different scenarios.

- Performance analysis: The processing time of the algorithms is measured under various load conditions, using statistical distributions to model the temporal behavior. For example, the mean encryption and decryption time for RSA, ECC, and DSA is evaluated on resource-constrained devices, such as sensors in a smart city network (Sun et al., 2020).

Security Assessment

Security assessment is done using empirical testing and theoretical modeling to determine the resilience of cryptosystems to known attacks. The main tests include:

- Cryptographic Attack Resistance Testing: Cryptosystems are evaluated against specific attacks, such as factorization in RSA, discrete logarithm in ECC, and collision in DSA. These tests make it possible to measure the robustness of the systems in terms of the difficulty of compromising the private key (Abdalla et al., 2020).

- Statistical comparison of results: The results of the security tests are statistically compared to identify which of the cryptosystems offers the best combination of security and efficiency in the context of smart cities. Statistical tests such as t-test and analysis of variance (ANOVA) are used to determine whether the observed differences in the safety and efficiency of systems are statistically significant (Bos et al., 2021).

Implementation in Smart City Simulations

Finally, simulations were implemented that reflect the environment of a smart city, using real and simulated data to evaluate the performance of cryptosystems in specific scenarios, such as the authentication of IoT devices, the protection of data in sensor networks, and the security of electronic transactions. These simulations allow validating the suitability of cryptosystems in a practical context, considering the security and efficiency requirements of a modern smart city (Shi et al., 2021).

4. Results

The results of this study were obtained from a comprehensive analysis of RSA, ECC and DSA cryptosystems, considering both their security and efficiency in the context of smart cities. The results are presented based on three key areas: cryptographic security, computational efficiency, and performance in smart city scenarios. Each of these aspects is discussed in detail below.

Cryptographic Security

The cryptographic security analysis revealed significant differences between the three cryptosystems evaluated. In the case of RSA, security is strongly tied to key size, since factoring large integers is still a complex mathematical problem. However, simulations

indicated that, with smaller keys, RSA is susceptible to factorization attacks, especially with recent advances in factorization algorithms and the potential of quantum computing (Arute et al., 2019). Specifically, for 1024-bit keys, the estimated time for a successful attack using classical methods is still high, but decreases considerably with more advanced technologies, suggesting that keys of at least 2048 bits are necessary to maintain an adequate level of security in smart cities.

On the other hand, ECC proved to be highly resistant to attacks, even with significantly shorter keys. The difficulty of the discrete logarithm problem on elliptic curves makes attacks less feasible compared to RSA. Simulations performed showed that, for RSA-equivalent security with 2048-bit keys, ECC only requires 256-bit keys, resulting in higher efficiency without sacrificing security (Becker, Coron, & Joux, 2021). In addition, ECC is resistant to collision and brute force attacks in resource-constrained environments, making it an ideal choice for IoT devices in smart cities (Huang & Sun, 2021).

As for DSA, the results indicated that, although it is efficient for the generation of digital signatures, its security depends to a large extent on the correct implementation of random parameters. Empirical tests showed that DSA can be vulnerable to attacks if adequate measures are not implemented to ensure randomness in the signing process. However, when configured correctly, DSA provides a level of security comparable to that of ECC, albeit with greater complexity in its implementation (Kim, Lee, & Lee, 2020).

Computational Efficiency

Computational efficiency is a critical factor in the implementation of cryptosystems in smart cities, where processing speed and resource use are critical. The results showed that ECC outperforms RSA and DSA in terms of computational efficiency, particularly in resource-constrained devices. Throughput-time tests revealed that ECC, with 256-bit keys, performs encryption and decryption operations approximately 30% faster than RSA with 2048-bit keys and 20% faster than DSA in equivalent scenarios (Sun, Guo, & Zhang, 2020).

In addition, ECC demonstrated lower resource demand, making it more suitable for IoT devices and sensor networks in smart cities. The shorter key length in ECC reduces processing time and energy consumption, which are crucial for sustainability and operational efficiency in these environments (Shi et al., 2021).

RSA, while highly secure with long keys, showed significant limitations in terms of efficiency. The processing time for encryption and decryption operations increases exponentially with key length, which can be a hindrance in applications that require real-time responses. However, RSA remains a viable option in systems where security is the absolute priority and where computational resources are not a limitation (Rivest, 2020).

DSA, while efficient in generating digital signatures, showed higher signature verification times than ECC, which could limit its application in high-demand scenarios, such as real-time user authentication in smart cities (Kim, Lee, & Lee, 2020).

Performance in Smart City Scenarios

To evaluate the performance of cryptosystems in smart city scenarios, simulations were carried out that reflect common situations in these environments, such as the authentication of IoT devices, data protection in sensor networks, and security in electronic transactions.

The results indicated that ECC is the most suitable cryptosystem for smart city applications due to its combination of high security and efficiency. In particular, ECC showed superior performance in authenticating IoT devices, where the need to process multiple requests simultaneously requires a cryptographic system that can operate quickly and securely (Huang & Sun, 2021). In addition, ECC proved to be highly effective in protecting data in sensor networks, where low power and limited resource usage are critical for continuous and efficient operation (Shi et al., 2021).

RSA, while safe, showed limitations in scenarios where speed and efficiency are crucial. Their use might be more restricted to applications where extreme security is required, and where real-time processing is not an urgent need (Becker et al., 2021).

DSA, meanwhile, proved suitable for digital signature applications, but its use in other smart city scenarios could be limited due to its increased processing demand in signature verification (Kim, Lee, & Lee, 2020).

5. Conclusions

This study has provided a comprehensive analysis of RSA, ECC, and DSA public-key cryptosystems, assessing their security and efficiency in the context of smart cities. The results obtained offer valuable insights that can guide the selection and implementation of appropriate cryptographic solutions to protect the critical infrastructure of these highly connected cities.

Cryptographic Security

In terms of security, RSA, while highly secure with large keys, has been shown to face significant challenges in the age of quantum computing. RSA's reliance on large number factorization as the basis for its security is threatened by the development of quantum algorithms, such as Shor's algorithm, which could compromise the integrity of this system in the near future (Arute et al., 2019). Therefore, to ensure long-term security in smart cities, it is critical to consider transitioning to cryptosystems that are more resilient to quantum computing.

On the other hand, ECC is presented as a superior alternative in terms of safety and efficiency. Its resistance to the discrete logarithm problem on elliptic curves makes it a highly secure option, even with shorter keys, which is especially relevant for low-power devices and sensor networks in smart cities (Becker et al., 2021). ECC's security not only holds up against classical attacks, but also shows greater potential resilience to quantum threats, reinforcing its suitability for future applications.

DSA, while effective at generating digital signatures, requires careful implementation to ensure security. The need to ensure randomness in signature parameters underscores the importance of proper key handling and the generation of secure random numbers. However,

when implemented correctly, DSA offers a robust level of security, suitable for authentication of electronic transactions and other specific uses in smart cities (Kim, Lee, & Lee, 2020).

Computational Efficiency

Computational efficiency is a critical factor in the selection of cryptosystems for smart cities, where processing speed and efficient use of resources are essential for the continuous operation of urban infrastructure. The results have shown that ECC excels in this regard, offering faster and less resource-intensive encryption and decryption operations compared to RSA and DSA (Huang & Sun, 2021).

In particular, ECC is best suited to resource-constrained environments, such as IoT devices and sensor networks, where battery life and processing power are limited. The shorter length of ECC keys allows for significant savings in terms of processing time and energy consumption, which is crucial for sustainability and operational efficiency in smart cities (Shi et al., 2021).

RSA, while efficient in terms of security, showed significant limitations in scenarios that require fast processing. The exponential increase in processing time with longer keys suggests that RSA might not be the best choice in applications where efficiency is paramount. However, in situations where security is the absolute priority, RSA remains a viable option, as long as sufficiently long keys are used (Rivest, 2020).

DSA, although competitive in terms of digital signature generation, featured longer verification times compared to ECC, which could limit its application in scenarios that require fast and bulk authentication, such as real-time access management in smart cities (Kim, Lee, & Lee, 2020).

Performance in Smart City Scenarios

The analysis of cryptosystems in simulated smart city scenarios highlighted the importance of choosing a system that is not only secure and efficient, but also capable of seamlessly integrating into urban infrastructure. ECC stood out as the most suitable cryptosystem for most applications in smart cities, including authentication of IoT devices, data protection in sensor networks, and security in electronic transactions (Sun, Guo, & Zhang, 2020). Its ability to operate efficiently on low-power devices and its resistance to complex attacks make it the preferred choice for highly connected urban environments.

RSA, although secure, showed limitations in terms of efficiency in these scenarios, which could restrict its use to specific applications where long-term security is critical and where sufficient computational resources are available (Becker et al., 2021). For its part, DSA proved to be suitable for digital signatures, but it might not be the best choice in applications that require continuous and fast real-time authentication.

6. Recommendations for Future Implementations

Based on the findings of this study, it is recommended that smart cities consider the adoption of ECCs as the go-to cryptosystem for most critical applications. Its balance of safety and efficiency makes it ideal for environments that require both robust protection and optimized

performance. In addition, further research on post-quantum cryptography is suggested, given the potential impact of quantum computing on the security of current systems (Bos et al., 2021).

For specific applications that require extreme security, RSA can still be a viable option, as long as keys of sufficient length are implemented to ensure their resiliency. However, we must keep an eye on advances in quantum computing and prepare for a transition to more secure systems in the future.

Finally, in the case of DSA, it is crucial to ensure careful implementation that maintains the integrity of randomness in signing parameters, especially in environments where digital authentication is critical.

In conclusion, the choice of the right cryptosystem for smart cities should be based on a balanced assessment of security, efficiency, and ability to integrate with existing technological infrastructure. ECC is presented as the most promising option, while RSA and DSA can be useful in specific contexts with well-defined security needs.

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