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Path Ideas in Fuzzy Incidence Graphs with Multi-Bipolarity

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Incidence graphs can be used to simulate some non-deterministic intercommunication networks with extra node-edge interactions. The bipolar fuzzy graph is a tool for bipolarity or thinking from two opposing perspectives. It may be used to handle many real-world problems more successfully. The matrix structure and comprehension of the m-BPFIG are proposed in this paper. Numerous properties are included in the definition of the m-BPFISG. Incidence pairings, paths, and connectivities among pairs are displayed in m-BPFIGs. We study the properties of different kinds of cut and strong pairs in m-BPFIGs.

Keywords: m-BPFIG, networks.

1. Introduction

A brief discussion of the fundamental concepts of routes, circuits, complete and strong fuzzy graph, and their applications may be found in [3]. Generalized fuzzy graphs strong arcs and pathways, together with their practical uses, are provided in [4]. Fuzzy incidence graphs were initially presented by Dinesh [1]. Numerous node kinds and fuzzy incidence graph characteristics have been covered in [5, 6]. Poulik and Ghorai [2]studied connectivity concepts in bipolar graphs. We have only used recognized terms and concepts in this work. Readers are led to other symbols, language, and usage not included in the research [7-12].

Certain non-deterministic intercommunication networks with additional node-edge interactions can be simulated using incidence graphs. Numerous issues in the actual world may be handled more skillfully using it. This study proposes the matrix structure and understanding of the m-BPFIG. The m-BPFISG specification includes a large number of attributes. m-BPFIGs show incidence pairings, pathways, and connectivities between pairs. We examine the characteristics of various types of strong and cut pairings in m-BPFIGs.

2. Preliminaries

Here are a few fundamental definitions and characteristics.

Definition 2.1: An m-BPFG of a graph $\xi^* = (N, E)$ is a pair $\xi = (N, Q, R)$ where $Q = \langle [P_h o\Psi_Q^+, P_h o\Psi_Q^-]_{h=1}^m \rangle$, $P_h o\Psi_Q^+ : N \to [0,1]$ and $P_h o\Psi_Q^- : N \to [-1,0]$ an m-BPFS is an m-BPFS on N and $R = \langle [P_h o\Psi_R^+, P_h o\Psi_R^-]_{h=1}^m \rangle$, $P_h o\Psi_R^+ : \overleftrightarrow{N^2} \to [0,1]$ and $P_h o\Psi_Q^- : \overleftrightarrow{N^2} \to [-1,0]$ in an m-BPFS in $\overleftrightarrow{N^2}$ such that

$$P_h o\Psi_R^+(qr) \le \min\{P_h o\Psi_Q^+(q), P_h o\Psi_Q^+(r)\},$$

$$P_h o\Psi_R^-(qr) \ge \max\{P_h o\Psi_Q^-(q), P_h o\Psi_Q^-(r)\}$$

for all $qr \in \overleftrightarrow{N^2}, h = 1, 2, \cdots, m$ and $P_h o\Psi_R^+(qr) = P_h o\Psi_R^-(qr) = 0$ for all $qr \in \overleftrightarrow{N^2} - E$.

Definition 2.2: An m-BPFG $\xi^* = (N^*, Q^*, R^*)$ is called an m-BPFSG of an m-BPFG of $\xi = (N, Q, R)$ if $N^* \subseteq N$, $E^* \subseteq E$ such that $P_h o\Psi_Q^+(f) = P_h o\Psi_{Q^*}^+(f)$, $P_h o\Psi_Q^-(f) = P_h o\Psi_{Q^*}^-(f)$, $\forall f \in N^*$ and $P_h o\Psi_{R^*}^+(ef) = P_h o\Psi_R^+(ef), P_h o\Psi_{R^*}^-(ef) = P_h o\Psi_R^-(ef)$, $h = 1, 2, \cdots, m$, $\forall ef \in \xi^*$.

Definition 2.3: Let $\xi = (N, Q, R)$ be an m-BPFG and $e, f \in N$.

- A path $L: e = l_0, l_1, \cdots, l_{r-1}, l_r = f$ in ξ is a sequence of different nodes such that $[(P_h o\Psi_R^+(l_{i-1}l_i) > 0, P_h o\Psi_R^-(l_{i-1}l_i) < 0)]_{h=1}^m, i = 1, 2, \cdots, r$ and the length of the path is r.
- If $L: e = l_0, l_1, \cdots, l_{r-1}, l_r = f$ be a path of length r between e and f, then $([P_h o\Psi_R^+(ef)]_{h=1}^m)^r$ and $([P_h o\Psi_R^-(ef)]_{h=1}^m)^r$ are defined as $([P_h o\Psi_R^+(ef)]_{h=1}^m)^r = \sup\{[P_h o\Psi_R^+(el_1) \wedge P_h o\Psi_R^+(l_1 l_2) \wedge \cdots \wedge P_h o\Psi_R^+(l_{r-1}f)]_{h=1}^m\}$ and

$$([P_h o\Psi_R^-(ef)]_{h=1}^m)^r = \inf\{[P_h o\Psi_R^-(el_1) \vee P_h o\Psi_R^-(l_1 l_2) \vee \cdots \vee P_h o\Psi_R^-(l_{r-1}f)]_{h=1}^m\}.$$

$(([P_h o\Psi_R^+(ef)]_{h=1}^m)^\infty, ([P_h o\Psi_R^-(ef)]_{h=1}^m)^\infty)$ is called to be the energy of connectedness between two nodes e and f in ξ, where $([P_h o\Psi_R^+(ef)]_{h=1}^m)^\infty = \mathrm{Sup}_{r\in N}\{([P_h o\Psi_R^+(ef)]_{h=1}^m)^r\}$ and $([P_h o\Psi_R^-(ef)]_{h=1}^m)^\infty = \inf_{r\in N}\{([P_h o\Psi_R^-(ef)]_{h=1}^m)^r\}$.

- $[P_h o\Psi_R^+(ef)]_{h=1}^m \ge ([P_h o\Psi_R^+(ef)]_{h=1}^m)^\infty$ and $[P_h o\Psi_R^-(ef)]_{h=1}^m \le ([P_h o\Psi_R^-(ef)]_{h=1}^m)^\infty$ then the arc ef in ξ is called to be a strong arc. A path $e - f$ is strong path if all arcs on the path are strong.

Definition 2.4: For an m-BPFG ξ, if $[P_h o\Psi_R^+(ef)]_{h=1}^m \ge ([P_h o\Psi_R^+(ef)]_{h=1}^m)^\infty$ and $[P_h o\Psi_R^-(ef)]_{h=1}^m \le ([P_h o\Psi_R^-(ef)]_{h=1}^m)^\infty$, then the edge ef is called to be a strong edge of ξ.

3. m-Bipolar Fuzzy Incidence Graph

We defined m-BPFIG and its matrix form in this part. Examples are provided to illustrate the properties of m-BPFISGs.

Definition 3.1: Let $\xi = (N, Q, R)$ be an m-BPFG. Let $\gamma = \langle [P_h o\gamma^+, P_h o\gamma^-]_{h=1}^m \rangle$, $P_h o\gamma^+ : N \times E \to [0,1]$ and $P_h o\gamma^- : N \times E \to [-1,0]$ such that

$$P_h o\gamma^+(q, qr) \le \min\{P_h o\Psi_Q^+(q), P_h o\Psi_R^+(qr)\},$$

$$P_h o\gamma^-(q, qr) \ge \max\{P_h o\Psi_Q^-(q), P_h o\Psi_R^-(qr)\},$$

for all $q \in N$, $qr \in E, h = 1, 2, \cdots, m$. Then γ is said to be an m-BPFI of the m-BPFG of ξ and $\breve{\xi} = (N, Q, R, \gamma)$ is said to be an m-BPFIG.

Example 3.2: Consider an m-BPFG $\xi = (N, Q, R)$ is shown in Figure 1. Here $N = \{l_1, l_2, l_3, l_4\}$ and $\overleftrightarrow{N^2} = \{l_1 l_2, l_1 l_3, l_1 l_4, l_2 l_3, l_3 l_4\}$.

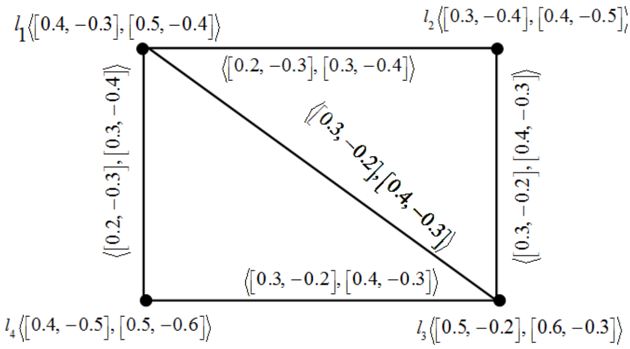

Figure 1. m-BPFG

Now, From Figure 2., we get $P_h o\gamma^+(l_1, l_1 l_2) \le \min\{P_h o\Psi_Q^+(l_1), P_h o\Psi_R^+(l_1 l_2)\}$,

$P_h o\gamma^-(l_1, l_1 l_2) \ge \max\{P_h o\Psi_Q^-(l_1), P_h o\Psi_R^-(l_1, l_1 l_2)\}$,

for all $l_1 \in N$, $l_1 l_2 \in E, h = 1, 2, \cdots, m$. Hence $\breve{\xi} = (N, Q, R, \gamma)$ is an m-BPFIG.

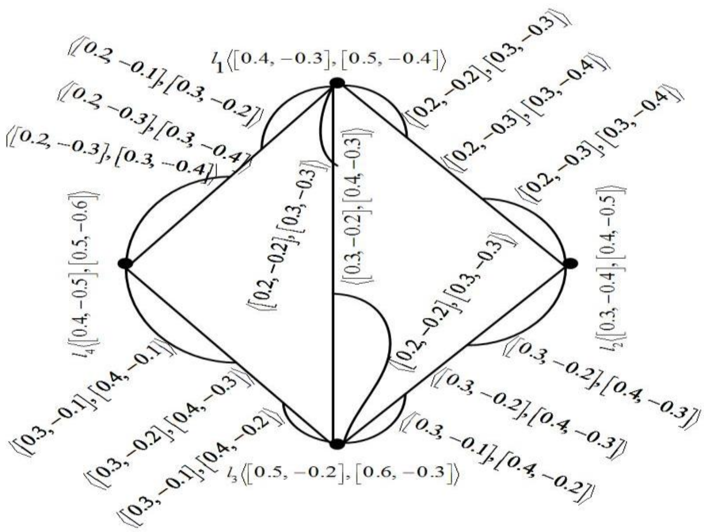

Figure 2: m-BPFIG $\breve{\xi}$ of the m-BPFG ξ

Definition 3.3: Let $\breve{\xi} = (N, Q, R, \gamma)$ be a m-BPFIG and γ be the m-BPFI $\breve{\xi}$. Consider $\breve{\xi}$ has n nodes $l_1, l_2, l_3, \cdots, l_n$ and m edges $k_1, k_2, k_3, \cdots, k_m$. The matrix form of the m-BPFI γ of $\breve{\xi}$ is represented as $[\gamma_{ij}]_{n\times m}$ and is defined as

$$[\gamma_{ij}]_{n\times m} = \begin{matrix} & k_1 & k_2 & k_3 & \cdots & k_m \\ l_1 & \gamma_{11} & \gamma_{12} & \gamma_{13} & \cdots & \gamma_{1m} \\ l_2 & \gamma_{21} & \gamma_{22} & \gamma_{23} & \cdots & \gamma_{2m} \\ . & . & . & . & \cdots & . \\ . & . & . & . & \cdots & . \\ l_n & \gamma_{n1} & \gamma_{n2} & \gamma_{n3} & \cdots & \gamma_{nm} \end{matrix}$$

where $\gamma_{ij} = \langle [P_h o\gamma^+(l_i, l_i l_j), P_h o\gamma^-(l_i, l_i l_j)]_{h=1}^{m} \rangle$.

Definition 3.4: An m-BPFIG $\breve{H} = (N^*, Q^*, R^*, \gamma^*)$ is called to be an m-BPFIPSG of an m-BPFIG $\breve{\xi} = (N, Q, R, \gamma)$ if $P_h o\Psi^+_{Q^*}(e) \leq P_h o\Psi^+_{Q}(e), P_h o\Psi^-_{Q^*}(e) \geq P_h o\Psi^-_{Q}(e), P_h o\Psi^+_{R^*}(ef) \leq P_h o\Psi^+_{R}(ef), P_h o\Psi^-_{R^*}(ef) \geq P_h o\Psi^-_{R}(ef), P_h o\gamma^{*+}(e, ef) \leq P_h o\gamma^+(e, ef)$ and $P_h o\gamma^{*-}(e, ef) \geq P_h o\gamma^-(e, ef), \forall e \in N, ef \in E, h = 1, 2, \cdots, m$.

Again, $\breve{H}$ is called to be a m-BPFISG of an m-BPFIG $\breve{\xi}$ if $N^* \subseteq N, E^* \subseteq E, \gamma^* \subseteq \gamma, P_h o\Psi^+_{Q^*}(e) = P_h o\Psi^+_{Q}(e)$, $P_h o\Psi^-_{Q^*}(e) = P_h o\Psi^-_{Q}(e), P_h o\Psi^+_{R^*}(ef) = P_h o\Psi^+_{R}(ef)$, $P_h o\Psi^-_{R^*}(ef) = P_h o\Psi^-_{R}(ef)$, $P_h o\gamma^{*+}(e, ef) = P_h o\gamma^+(e, ef)$ and $P_h o\gamma^{*-}(e, ef) = P_h o\gamma^-(e, ef), \forall e \in N, ef \in E, h = 1, 2, \cdots, m$.

The consequences of deleting a node or an edge from a m-BPFIG are started in the following assertions.

Proposition 3.5: An m-BPFISG of a m-BPFIG $\breve{\xi}$ must be a m-BPFIPSG of $\breve{\xi}$.

Proof: Let $\breve{H}$ be an m-BPFISG of an m-BPFIG $\breve{\xi}$. By the definitions, we get $\breve{H}$ holds all the properties to be an m-BPFIPSG of the m-BPFIG $\breve{\xi}$. Thus, $\breve{H}$ is an m-BPFISG of the m-BPFIG $\breve{\xi}$.

Proposition 3.6: If $\breve{H}$ be an m-BPFISG of an m-BPFIG $\breve{\xi}$, then an m-BPFG H is an m-BPFSG of an m-BPFG ξ.

Proof: Let $\breve{H}$ be an m-BPFISG of an m-BPFIG $\breve{\xi}$. By the definitions, we get the corresponding H holds all the properties to be an m-BPFSG of an m-BPFG . Thus, H is an m-BPFSG of an m-BPFG ξ.

Definition 3.7: An m-BPFIG $\breve{\xi} = (N, Q, R, \gamma)$ is called to be complete m-BPFIG if

$$P_h o\gamma^+(q, qr) = \min\{P_h o\Psi^+_{Q}(q), P_h o\Psi^+_{R}(qr)\},$$

$$P_h o\gamma^-(q, qr) = \max\{P_h o\Psi^-_{Q}(q), P_h o\Psi^-_{R}(qr)\},$$

for all $q \in N$, $qr \in E, h = 1, 2, \cdots, m$.

Definition 3.8: An m-BPFIG $\breve{\xi} = (N, Q, R, \gamma)$ is called to be strong m-BPFIG if

$$P_h o\gamma^+(q, qr) = \min\{P_h o\Psi^+_{Q}(q), P_h o\Psi^+_{R}(qr)\},$$

$$P_h o\gamma^-(q,qr) = \max\{P_h o\Psi_Q^-(q), P_h o\Psi_R^-(qr)\},$$

for all pairs (q,qr) in $\breve{\xi}, h = 1,2,\cdots,m$.

If $\breve{\xi} = (N,Q,R,\gamma)$ is a complete m-BPFIG and the nodes q,r are adjacent to the edge qr, then $P_h o\gamma^+(q,qr) = \min\{P_h o\Psi_Q^+(q), P_h o\Psi_R^+(qr)\} = P_h o\Psi_R^+(qr) = \min\{P_h o\Psi_Q^+(r), P_h o\Psi_R^+(qr)\} = P_h o\gamma^+(r,qr)$ and $P_h o\gamma^-(q,qr) = \max\{P_h o\Psi_Q^-(q), P_h o\Psi_R^-(qr)\} = P_h o\Psi_R^-(qr) = \max\{P_h o\Psi_Q^-(r), P_h o\Psi_R^-(qr)\} = P_h o\gamma^-(r,qr)$.

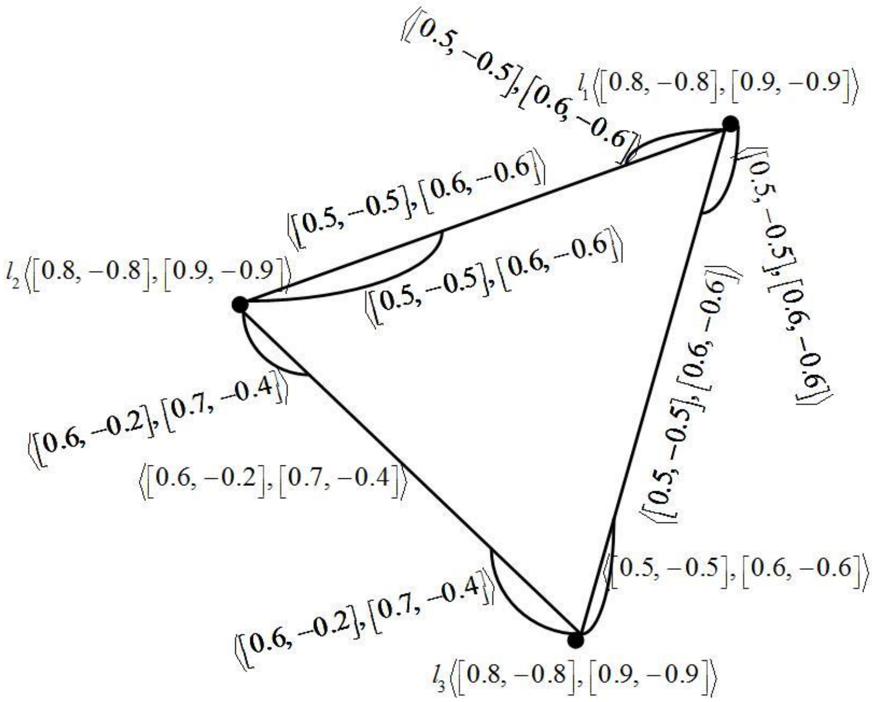

Figure 3: Complete m-BPFIG $\breve{\xi}$ and also a strong m-BPFIG

Theorem 3.9: A complete m-BPFIG is a strong m-BPFIG.

Proof: Let $\breve{\xi} = (N,Q,R,\gamma)$ be a complete m-BPFIG and (q,qr) be a pair in $\breve{\xi}$. Then $P_h o\gamma^+(q,qr) = \min\{P_h o\Psi_Q^+(q), P_h o\Psi_R^+(qr)\}$ and

$$P_h o\gamma^-(q,qr) = \max\{P_h o\Psi_Q^-(q), P_h o\Psi_R^-(qr)\},$$

for all $q \in N,\ qr \in E, h = 1,2,\cdots,m$. Hence, $P_h o\gamma^+(q,qr) = \min\{P_h o\Psi_Q^+(q), P_h o\Psi_R^+(qr)\}$ and

$P_h o\gamma^-(q,qr) = \max\{P_h o\Psi_Q^-(q), P_h o\Psi_R^-(qr)\}$, for all pairs (q,qr) $in\ \breve{\xi}, h = 1,2,\cdots,m$.

Therefore $\breve{\xi}$ is a strong m-BPFIG.

4. Paths and Connectivities

Here, we used examples to describe various incidence paths in m-BPFIGs. A few theorems are presented.

Definition 4.1: Let $\boldsymbol{x = l_1, l_2, \cdots, l_{k-1} = y, l_k = z}$ are the $\boldsymbol{k}$ nodes in an m-BPFIG $\breve{\boldsymbol{\xi}}$. Then

$\boldsymbol{l_1, (l_1, l_1l_2), l_1l_2, (l_2, l_1l_2), l_2, \cdots, y, (y, yz), yz, (z, yz)}$ is said to be an incidence path in $\breve{\boldsymbol{\xi}}$. The incidence energy of the path is represented as $[\boldsymbol{P_h o\gamma^+(x, yz), P_h o\gamma^-(x, yz)}]_{\boldsymbol{h=1}}^{\boldsymbol{m}}$ and is defined as $\boldsymbol{P_h o\gamma^+(x, yz) = P_h o\gamma^+(l_1, l_1l_2) \wedge P_h o\gamma^+(l_2, l_1l_2) \wedge \cdots \wedge P_h o\gamma^+(y, yz) \wedge P_h o\gamma^+(z, yz)}$ and $\boldsymbol{P_h o\gamma^-(x, yz) = P_h o\gamma^-(l_1, l_1l_2) \vee P_h o\gamma^-(l_2, l_1l_2) \vee \cdots \vee P_h o\gamma^-(y, yz) \vee P_h o\gamma^-(z, yz)}$.

The incidence energy of connected between $\boldsymbol{x}$ and $\boldsymbol{yz}$ in $\breve{\boldsymbol{\xi}}$ is $\boldsymbol{IECON\,M_{\breve{\xi}}(x, yz) = \left[\left(P_h oIECON\,M_{\breve{\xi}}^+(x, yz), P_h oIECON\,M_{\breve{\xi}}^-(x, yz)\right)\right]_{h=1}^{m}}$, where $\boldsymbol{P_h oIECON\,M_{\breve{\xi}}^+(x, yz) = max\{P_h o\gamma^+(x, yz)\}}$ and $\boldsymbol{P_h oIECON\,M_{\breve{\xi}}^-(x, yz) = min\{P_h o\gamma^-(x, yz)\}}$.

Example 4.2: Take the connected m-BPFIG $\breve{\boldsymbol{\xi}}$ shown in the Figure 4.

Here $\boldsymbol{\left[P_h oIECON\,M_{\breve{\xi}}^+(l_1, l_1l_2)\right]_{h=1}^{m} = (0.3, 0.2), \left[P_h oIECON\,M_{\breve{\xi}}^-(l_1, l_1l_2)\right]_{h=1}^{m} = (-0.3, -0.2)}$.

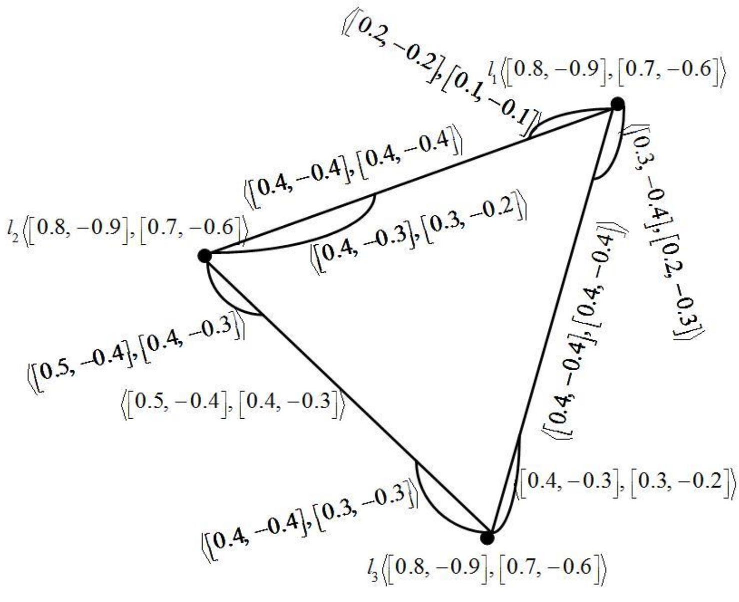

Figure 4: An m-BPFIG $\breve{\boldsymbol{\xi}}$

Definition 4.3: Let $\boldsymbol{xy}$ be an edge of an m-BPFIG $\breve{\boldsymbol{\xi}} = \boldsymbol{(N, Q, R, \gamma)}$. If $\boldsymbol{P_h o\gamma^+(q, qr) > 0}$, $\boldsymbol{P_h o\gamma^+(r, qr) > 0}$, $\boldsymbol{P_h o\gamma^-(q, qr) < 0}$ and $\boldsymbol{P_h o\gamma^-(r, qr) < 0, h = 1, 2, \cdots, m}$, then $\boldsymbol{(q, qr)}$ and $\boldsymbol{(r, qr)}$ are called pairs. If there is an incidence path connecting each pair, then $\breve{\boldsymbol{\xi}}$ is said to be paired.

Theorem 4.4: Let $\breve{\xi} = (N, Q, R, \gamma)$ be an m-BPFIG and $\breve{H} = (N^*, Q^*, R^*, \gamma^*)$ be an m-BPFISG of $\breve{\xi}$. Then for any pair (q, qr) in $\breve{\xi}$. $P_h oIECON\, M_{\xi}^{+}(q, qr) \geq P_h oIECON\, M_{H}^{+}(q, qr)$ and $P_h oIECON\, M_{\xi}^{-}(q, qr) \leq P_h oIECON\, M_{H}^{-}(q, qr)$, $h = 1, 2, \cdots, m$.

Proof: Take $\breve{H}$ is an m-BPFISG of $\breve{\xi}$. By the definition of an m-BPFISG, we get $P_h o\gamma^{*+}(q, qr) = P_h o\gamma^{+}(q, qr)$ and $P_h o\gamma^{*-}(q, qr) = P_h o\gamma^{-}(q, qr)$, for all pair (q, qr) in $\breve{H}$. But,

$P_h oIECON\, M_{\xi}^{+}(q, qr), P_h oIECON\, M_{H}^{+}(q, qr)$ and $P_h oIECON\, M_{\xi}^{-}(q, qr), P_h oIECON\, M_{H}^{-}(q, qr)$, $h = 1, 2, \cdots, m$ lies on same incidence pair of $\breve{H}$ and $\breve{H}$ or lies on distinct pairs of $\breve{H}$ and $\breve{H}$.

Here, two cases are appeared.

Case i. Assume $P_h oIECON\, M_{\xi}^{+}(q, qr), P_h oIECON\, M_{H}^{+}(q, qr)$ and $P_h oIECON\, M_{\xi}^{-}(q, qr)$, $P_h oIECON\, M_{H}^{-}(q, qr)$, $h = 1, 2, \cdots, m$ lies on same pair (q, qr) of $\breve{H}$ and $\breve{\xi}$. Then by the definition of an m-BPFISG, we get $P_h o\gamma^{*+}(q, qr) = P_h o\gamma^{+}(q, qr)$ and $P_h o\gamma^{*-}(q, qr) = P_h o\gamma^{-}(q, qr)$. Then $P_h oIECON\, M_{\xi}^{+}(q, qr), P_h oIECON\, M_{H}^{+}(q, qr)$ and $P_h oIECON\, M_{\xi}^{-}(q, qr), P_h oIECON\, M_{H}^{-}(q, qr)$, $h = 1, 2, \cdots, m$.

Case ii. Assume $P_h oIECON\, M_{\xi}^{+}(q, qr), P_h oIECON\, M_{H}^{+}(q, qr)$ and $P_h oIECON\, M_{\xi}^{-}(q, qr)$, $P_h oIECON\, M_{H}^{-}(q, qr)$, $h = 1, 2, \cdots, m$ lies on the pairs $(q_1, q_1 r_1)$ in $\breve{\xi}$ and $(q_2, q_2 r_2)$ in $\breve{H}$. This means both the pairs $(q_1, q_1 r_1)$ and $(q_2, q_2 r_2)$ are the pairs of $\breve{\xi}$. If $P_h o\gamma^{+}(q_1, q_1 r_1)$ $= P_h o\gamma^{+}(q_2, q_2 r_2)$ and $P_h o\gamma^{-}(q_1, q_1 r_1)$ $= P_h o\gamma^{-}(q_2, q_2 r_2)$, then $P_h oIECON\, M_{\xi}^{+}(q, qr) = P_h oIECON\, M_{H}^{+}(q, qr)$ and $P_h oIECON\, M_{\xi}^{-}(q, qr) = P_h oIECON\, M_{H}^{-}(q, qr)$. If $P_h o\gamma^{+}(q_1, q_1 r_1) \neq P_h o\gamma^{+}(q_2, q_2 r_2)$ or $P_h o\gamma^{-}(q_1, q_1 r_1) \neq P_h o\gamma^{-}(q_2, q_2 r_2)$ or both, then $P_h oIECON\, M_{\xi}^{+}(q, qr) > P_h oIECON\, M_{H}^{+}(q, qr)$ or $P_h oIECON\, M_{\xi}^{-}(q, qr) < P_h oIECON\, M_{H}^{-}(q, qr)$, $h = 1, 2, \cdots, m$ or both.

Thus in any case $P_h oIECON\, M_{\xi}^{+}(q, qr) \geq P_h oIECON\, M_{H}^{+}(q, qr)$ or $P_h oIECON\, M_{\xi}^{-}(q, qr) \leq P_h oIECON\, M_{H}^{-}(q, qr)$, $h = 1, 2, \cdots, m$.

5. Strong Pair and Cut Pair in an m-BPFIG

Here are definitions and examples for Strong pair and Cut pair in m-BPFIG. Additionally, several associated theorems are provided.

Definition 5.1: A pair (q, qr) in $\breve{\xi} = (N, Q, R, \gamma)$ is strong pair if $P_h o\gamma^{+}(q, qr) \geq P_h oIECON\, M_{\xi}^{+}(q, qr)$ and $P_h o\gamma^{+}(q, qr) \leq P_h oIECON\, M_{\xi}^{+}(q, qr)$.

Theorem 5.2: Let $\breve{\xi} = (N, Q, R, \gamma)$ be an m-BPFIG and a pair (q, qr) in $\breve{\xi}$ such that $([P_h o\gamma^{+}(q, qr), P_h o\gamma^{+}(q, qr)]_{h=1}^{m}) = (\vee \{P_h o\gamma^{+}(a, ab)\}, \wedge \{P_h o\gamma^{-}(a, ab)\})$, for all pairs (a, ab) in $\breve{\xi}$. Then the pair (q, qr) is a strong pair.

Proof: Let $(\boldsymbol{q},\boldsymbol{qr})$ be a pair in an m-BPFIG $\breve{\boldsymbol{\xi}}$. If there is a unique pair $(\boldsymbol{q},\boldsymbol{qr})$ in $\breve{\boldsymbol{\xi}}$ such that $\boldsymbol{P_h o\gamma^+(q,qr)} = \vee\{\boldsymbol{P_h o\gamma^+(a,ab)}\}$ and $\boldsymbol{P_h o\gamma^-(q,qr)} = \wedge\{\boldsymbol{P_h o\gamma^-(a,ab)}\}$, for all $(\boldsymbol{a},\boldsymbol{ab})$in $\breve{\boldsymbol{\xi}}$,

thus, the energy of all the pathways without $(\boldsymbol{q},\boldsymbol{qr})$ must have a positive portion that is smaller than $\boldsymbol{P_h o\gamma^+(q,qr)}$ and a negative part that is bigger than $\boldsymbol{P_h o\gamma^+(q,qr)}$. Then, a strong pair is $(\boldsymbol{q},\boldsymbol{qr})$.

Again, if there are more than one pair $(\boldsymbol{q},\boldsymbol{qr})$ in $\breve{\boldsymbol{\xi}}$ such that $\boldsymbol{P_h o\gamma^+(q,qr)} = \vee\{\boldsymbol{P_h o\gamma^+(a,ab)}\}$ and $\boldsymbol{P_h o\gamma^-(q,qr)} = \wedge\{\boldsymbol{P_h o\gamma^-(a,ab)}\}$, for all $(\boldsymbol{a},\boldsymbol{ab})$in $\breve{\boldsymbol{\xi}}$, then $\boldsymbol{P_h o\gamma^+}(\mathbf{q},\mathbf{qr}) = \mathbf{P_h o\gamma^+}(\mathbf{a},\mathbf{ab})$ and $\mathbf{P_h o\gamma^-}(\mathbf{q},\mathbf{qr}) = \mathbf{P_h o\gamma^-}(\mathrm{a},\mathrm{ab})$ for all pair (a,ab)in $\breve{\xi} - \{(\mathrm{q},\mathrm{qr})\}$. Now by the definition, we get $\mathrm{P_h o\gamma^+(q,qr)} \geq \mathrm{P_h oIECON\ M_{\breve{\xi}}^+(a,ab)}$ and $\mathrm{P_h o\gamma^-(r,qr)} \leq \mathrm{P_h oIECON\ M_{\breve{\xi}}^-(a,ab)}$ for all the pair (a,ab) in $\breve{\xi}$. So (q,qr) is a strong pair.

Definition 5.3: Let $\breve{\xi} = (\mathrm{N},\mathrm{Q},\mathrm{R},\gamma)$ be an m-BPFIG and $\breve{\mathrm{H}} = (\mathrm{N}^*,\mathrm{Q}^*,\mathrm{R}^*,\gamma^*)$ be an m-BPFISG of $\breve{\xi}$ such that for a pair (q,qr) in $\breve{\xi}$, $\breve{\mathrm{H}} = \breve{\xi} - \{(\mathrm{q},\mathrm{qr})\}$. If $\mathrm{P_h oIECON\ M_{\breve{\xi}}^+(q,qr)} > \mathrm{P_h oIECON\ M_{\breve{H}}^+(q,qr)}$ and $\mathrm{P_h oIECON\ M_{\breve{\xi}}^-(q,qr)} < \mathrm{P_h oIECON\ M_{\breve{H}}^-(q,qr)}$, $\mathrm{h} = 1,2,\cdots,\mathrm{m}$ for some pair (q,qr) in $\breve{\xi}$, then the pair (q,qr) is said to be an incidence cut pair of $\breve{\xi}$.

Theorem 5.4: Let (q,qr) be a pair in an m-BPFIG $\breve{\xi} = (\mathrm{N},\mathrm{Q},\mathrm{R},\gamma)$ such that

$$\mathrm{P_h o\gamma^+(q,qr)} = \min\{\mathrm{P_h o\Psi_Q^+(q)}, \mathrm{P_h o\Psi_R^+(qr)}\}, \mathrm{P_h o\gamma^-(q,qr)} = \max\{\mathrm{P_h o\Psi_Q^-(q)}, \mathrm{P_h o\Psi_R^-(qr)}\}.$$

Then the pair (q,qr) is a strong pair of $\breve{\xi}$.

Proof: Let $\breve{\mathrm{H}} = (\mathrm{N}^*,\mathrm{Q}^*,\mathrm{R}^*,\gamma^*)$ be an m-BPFISG of an m-BPFIG $\breve{\xi} = (\mathrm{N},\mathrm{Q},\mathrm{R},\gamma)$ such that $\breve{\mathrm{H}} = \breve{\xi} - \{(\mathrm{q},\mathrm{qr})\}$, where $(\mathrm{q},\mathrm{qr})$is a pair of $\breve{\xi}$. If $\breve{\mathrm{H}}$ is disconnected, then (q,qr)must be a cut pair of $\breve{\xi}$. Then $\mathrm{P_h oIECON\ M_{\breve{\xi}}^+(q,qr)} > \mathrm{P_h oIECON\ M_{\breve{H}}^+(q,qr)}$ and $\mathrm{P_h oIECON\ M_{\breve{\xi}}^-(q,qr)} < \mathrm{P_h oIECON\ M_{\breve{H}}^-(q,qr)}$, $\mathrm{h} = 1,2,\cdots,\mathrm{m}$.

Then from the definition (q,qr) is a strong pair. If $\breve{\mathrm{H}}$ is disconnected, then there exists a pairs (q,qs) for some $\mathrm{q} \neq \mathrm{s}$, such that (q,qs) and (r,qr) lies in a incidence path from q to qr in $\breve{\mathrm{H}}$. Then

$\mathrm{P_h oIECON\ M_{\breve{H}}^+(q,qr)} \leq \min\{\mathrm{P_h o\gamma^+(q,qs)}, \mathrm{P_h o\gamma^+(r,qr)}\} \leq \min\{\mathrm{P_h o\Psi_Q^+(q)}, \mathrm{P_h o\Psi_R^+(qs)}, \mathrm{P_h o\Psi_Q^+(r)}, \mathrm{P_h o\Psi_R^+(qr)}\} \leq \min\{\mathrm{P_h o\Psi_Q^+(q)}, \mathrm{P_h o\Psi_R^+(qr)}\} = \mathrm{P_h o\gamma^+(q,qr)}$ and

$\mathrm{P_h oIECON\ M_{\breve{H}}^-(q,qr)} \geq \min\{\mathrm{P_h o\gamma^-(q,qs)}, \mathrm{P_h o\gamma^-(r,qr)}\} \geq \min\{\mathrm{P_h o\Psi_Q^-(q)}, \mathrm{P_h o\Psi_R^-(qs)}, \mathrm{P_h o\Psi_Q^-(r)}, \mathrm{P_h o\Psi_R^-(qr)}\} \geq \min\{\mathrm{P_h o\Psi_Q^-(q)}, \mathrm{P_h o\Psi_R^-(qr)}\} = \mathrm{P_h o\gamma^-(q,qr)}$.

Thus $\mathrm{P_h o\gamma^+(q,qr)} \geq \mathrm{P_h oIECON\ M_{\breve{H}}^+(q,qr)}$ and $\mathrm{P_h o\gamma^-(q,qr)} \geq \mathrm{P_h oIECON\ M_{\breve{H}}^-(q,qr)}$. Therefore (q,qr) is a strong pair of $\breve{\xi}$.

Theorem 5.5: Every pair of a complete m-BPFIG is a strong pair.

Proof: Let (q, qr) be a pair in complete m-BPFIG $\breve{\xi} = (N, Q, R, \gamma)$. Then $P_h o\gamma^+(q, qr) = \min\{P_h o\Psi_Q^+(q), P_h o\Psi_R^+(qr)\}, P_h o\gamma^-(q, qr) = \max\{P_h o\Psi_Q^-(q), P_h o\Psi_R^-(qr)\}$. Now from Theorem 5.4, it can be showed that (q, qr) is a strong pair of $\breve{\xi}$. Since the pair (q, qr) is arbitrary in the complete m-BPFIG $\breve{\xi}$. Then all the pairs of $\breve{\xi}$ are strong pair.

Example 5.6: Take the connected m-BPFISG $\breve{H}$ of $\breve{\xi}$shown in the Figure 5.

Here $[P_h oIECON\ M_H^+(l_1, l_1 l_2)]_{h=1}^m = (0.2, 0.1), [P_h oIECON\ M_H^-(l_1, l_1 l_2)]_{h=1}^m = (-0.2, -0.1)$.

Clearly, $P_h oIECON\ M_{\breve{\xi}}^+(q, qr) \geq P_h oIECON\ M_H^+(q, qr)$ or $P_h oIECON\ M_{\breve{\xi}}^-(q, qr) \leq P_h oIECON\ M_H^-(q, qr), h = 1, 2, \cdots, m$.

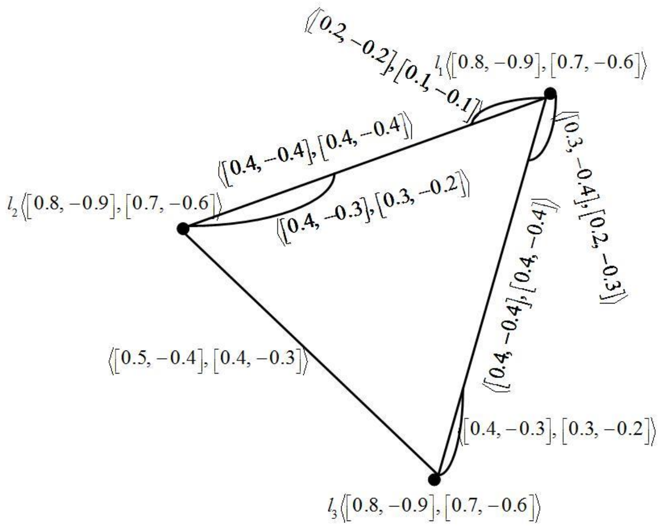

Figure 5: An m-BPFISG $\breve{H}$of an m-BPFIG $\breve{\xi}$

6. Conclusions:

An m-BPFG has made use of the incidence connection between nodes and edges. Firstly, the definition and attributes of m-BPFISGs and m-BPFIG are described by examples. Again, various occurrence routes along with their energies, connections, and characteristics. We examine the degree of connectivity between pairs in the m-BPFISGs and m-BPFIG. In the last section the idea of strong and cut pairs with their attributes had been explored utilizing incidence routes and the energy of connectivity of pairs.

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