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A Statistical Model for the Evaluation on Extended Exponential Dagum Distribution with Quantile Regression: Properties and Evaluation of Size of Brain Tumor

Vidya P, Parthiban S

Department of Mathematics and Statistics, School of Applied Sciences and Humanities, Vignan's Foundation for Science, Technology, and Research, Vadlamudi, Guntur District, Andhra Pradesh, India
Email: selvam.parthiban1979@gmail.com

The current study set out to investigate the consequences of age and race on the size of main brain tumors have been analyzed by using Weibull and Dagum distributions for the better observation of the malicious tumor sizes. In this study some suitable model has been applied to compute the connection between dependent variable and independent variable. In this proposed work, Weibull-G exponential distribution model and Dagum with more parameters has been applied to analyze some medical data and found more accuracy of results connected with brain tumor malignant tumor size etc. Finally, it has been concluded with valid proofs that EED distribution is the better model to evaluate and analyze the medical data and relevant observations. Quantile regression model for EED were used to analyze the effects of different tumor sizes on gender, age, race we acquire a better grasp of the size of brain tumor. The size of brain tumors and their epidemiological measures have been clearly extended in the literature. The Results have established effective Statistical model for heterogenous and high measurements data with high degree of quality.

Keywords: Mortality, Uncertainity, Likelihood Estimation, Exponential Distribution, Dagum distribution, Quantile regression.

1. Introduction

In the recent literature we investigate some articles [28][10][7]. Now-a-days cancer related deaths of male ages 20-40 are the second leading causes and female ages 20-40 are the fifth leading causes registered [28][10][7]. Secondary brain tumors are those that have supper to the brain from another part of the body (metastasize) [28][26]. The malignancies that are the most likely to influence the major dependent parts of human body such as kidney, breast, lungs and skin. The CNS tumor and prime brain tumor have both examined to the suggested technique [20][26]. We present the data set with tumor size it is detected that it has a well fit with Weibull and Dagum distribution [21]. In order to understand in what way, the maximum likelihood estimators and their corresponding results distribution function with moments, generating functions, Quantiles for finite tumor sample sizes, probability density function, cumulative distribution function, hazard function, Survival function, cumulative hazard function, we talked about some numerical calculations that are performed. Weibull and Dagum with two parameters are the most common distributions for analyzing real life data and these distributions signifies the scale and shape parameters. It is very flexible to analyze the positive real data [26]. In this paper WPDF (Weibull Probability Density Function) and WCDF (Cumulative Distribution Function of Weibull distribution) of two parameters are assumed by the equations (4), (5) respectively. The DPDF (Probability Density Function of Dagum distribution) and DCDF (Dagum Cumulative Distribution Function) of four parameters are certain by the equations (10), (11) respectively. Further in section-3 the histology based average tumor sizes (figure 1 and 2) has been analyzed and the probability density curve which can be characterized the tumor sizes for six sub population are identified with corresponding parameters.

Let X be a continuous RV (random variable) has a Weibull distribution with three parameters a> 0, $b > 0$ and c. [12] In the event that the RV (random variable) $Y =$

$(\frac{X-c}{b})^{a}$ has an exponential distribution with PDF

$P_r(y) = e^{-y}, y > 0$ (1)

A RV (random variable) X has the PDF of a general Weibull distribution with parameters a> 0, $b > 0$ and c, (WPDF) is provided by [3]

v

$f(x;t,u,v) = f(x) = \frac{t}{u}(\frac{x-v}{u})^{t-1} e^{-(\frac{x-v}{u})^{t}}$ $x \geq v, u < 0$ (2)

Where t>0 and u>0 and is the location parameter. The general WCDF is provided by [29]

$F(x) = 1 - e^{-(\frac{x}{u})^{t}}$ $x \geq 0, u > 0$ (3)

We put u = 1and v = 1 in equation (1) we get PDF of standard Weibull distribution which depends with single parameter 't' is given by [12]

$P_X(x) = t\, x^{t-1}. exp(-x^{t})\ ;\ x > 0, t > 0$

$\mu'_r = E(X^r) = \int_0^{\infty} e^{-y} y^{r/t}\, dy$

$\mu'_r = E(X^r) = \Gamma(\frac{r}{t} + 1)$

$$\text{Mean} = E(X^r) = \Gamma\left(\frac{r}{t} + 1\right), \text{Var}(X) = \Gamma\left(\frac{1}{t} + 1\right) - \left[\Gamma\left(\frac{1}{t} + 1\right)\right]^2$$

2. Statistical Model and Assumptions Weibull distribution model

The Weibull distribution model is applicable to the study of survival analysis. The WPDF and WCDF in its simplest form is given by the following equations respectively. The WCDF of generated Weibull distribution is provided by [14],[16],[17]

F(x:P_α,Q_β,R_γ) =$[1 - exp(-Q_\beta)[G_w(x)/(1 - G_w(x))]^{R\gamma}]^{P\alpha}$ where x > 0 P_α, Q_β > 0 (4) The PDF that corresponds (2) is provided by [14],[16],[17]

$$f(x: P_\alpha, Q_\beta, R_\gamma) = P_\alpha Q_\beta R_\gamma \frac{g(x) - g(x)G_w(x) + g(x)G_w(x)}{(1 - G_w(x))^2} \quad [18],[1],[3]$$

$$= [1 - exp(-Q_\beta)[G_w(x)/(1 - G_w(x))]^{R\gamma}]^{P\alpha - 1} \quad (5)$$

$$g(x: P_\alpha, Q_\beta) = (P_\alpha/Q_\beta)(x/Q\beta)^{P\alpha - 1} \exp(-(x/Q_\beta)^{P\alpha} \quad (6)$$

$$G(x: P_\alpha, Q_\beta) = 1 - \exp\left(-\left(\frac{x}{Q_\beta}\right)\right)^{P\alpha} \quad \text{where } P_\alpha, Q_\beta > 0 \quad (7)$$

The mean of Weibull distribution with two - parameter is derived by

$$E(X) = Q_\beta \Gamma\left(\frac{1}{P_\alpha} + 1\right) \quad (8)$$

The variance of Weibull distribution with two - parameter is derived by [14],[16],[17]

$$\boldsymbol{V(X) = Q_\beta{}^2\left(\left(\Gamma\frac{1}{P_\alpha} + 1\right) - \left(\Gamma\frac{1}{P_\alpha} + 1\right)^2\right)} \quad (9)$$

3. Dagum distribution model

The Dagum distribution was originally proposed by Dagum (1977), and it is defined bycdf and pdf given (for x > 0)

The generalized exponential distribution defined by

$$\boldsymbol{F(x) = \{1 - [1 - G(x)]^{D_\alpha}\}^{D_\theta}, x \in R} \quad (10)$$

Where G(x) is a baseline continuous cumulative distribution function $\boldsymbol{a, b > 0}$

The probability density function corresponding (1) is defined by

$$f(x) = D_\alpha\, D_\theta\, g(x)\, [1 - G(x)]^{D_\alpha - 1}\{1 - [1 - G(x)]^{D_\alpha}\}^{D_\theta - 1}, x \in R \tag{11}$$

where $g(x) = dG(x)\,/dx$

The Dagum distribution was originally proposed by Dagum (1977), and it is defined by the cdf and pdf given (for x > 0) by

$$G(x\!: D_\theta\,, D_\gamma, D_\delta) = [1 - D_\gamma x^{-D_\delta}]^{D_\theta} \tag{12}$$

and $g(x\!: D_\theta, D_\gamma, D_\delta\,) = D_\theta\, D_\gamma\, D_\delta\, x^{-(1+D_\delta)}\, (1 + D_\gamma x^{-D_\delta})^{(1+D_\theta)}$ (13)

where D_θ, D_γ, D_δ> 0. The parameter D_γ is a scale parameter, whereas D_θ and D_δ are shape parameters. $D_\theta\, D_\delta$ > 1 and $D_\theta\, D_\delta \leq 1$. Moreover, the τ^{th} quantile is

x(τ) = ${D_\gamma}^{-1/D_\delta}(\tau^{-1/D_\theta} - 1)^{-1/D_\delta}$and the t^{th} ordinary moment is given by

E $(X^t) = D_\theta {D_\gamma}^{t/D_\delta}\, B\, (D_\theta + t/D_\delta\,, 1\; - t/D_\delta)$ where $t < D_\delta, B(.)$ is the beta function.

We investigate some mathematical properties of a new five parameter Dagum distribution with $(0, \infty)$ called the extended exponential Dagum distribution ("EED") defined from equation (11). The (DCDF) cdf of X having EED distribution $X \sim EED(D_\alpha, D_\theta, D_\beta, D_\gamma, D_\delta)$ say, can be obtained from (10) and (11) as

$$F(x) = \{\, 1 - [1 - (1 + D_\gamma x^{-D_\delta})^{-D_\beta}]^{D_\alpha}\}^{D_\theta} \qquad x \in (0, \infty) \tag{14}$$

Whereas (DPDF) pdf of EED distribution as

$$f(x) = D_\alpha\, D_\theta\, D_\beta\, D_\gamma\, D_\delta\, x^{-(1+D_\delta)}\, ((1 + D_\gamma\, x^{-D_\delta})^{-(1+D_\beta)}[1 - (1 + D_\gamma\, x^{-D_\delta})^{-D_\beta}]^{(D_\alpha - 1)}\{1 - [1 - (1 + D_\gamma\, x^{-D_\delta})^{-D_\beta}]^{D_\alpha}\}^{(D_\theta - 1)} \tag{15}$$

Maximum Likelihood Estimation

Let $x_1, \ldots, x_n$ be a random sample of size n from the EED distribution given by (15). The log-likelihood function for the vector of parameters

We consider the estimation of the unknown parameter of the EED model by the method of maximum likelihood. Let $x_1, \ldots, x_n$ be a random sample of size n from the EED distribution given by (15). The log-likelihood function for the vector of parameters

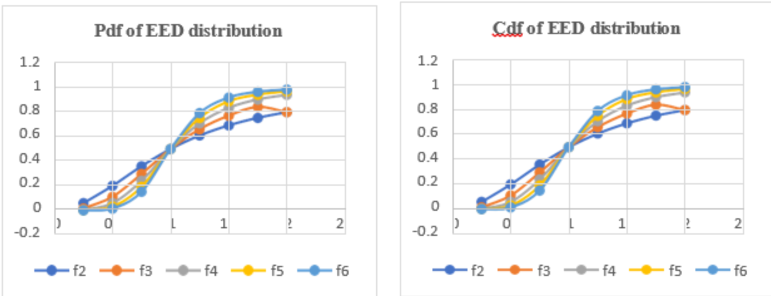
$\psi = (D_\alpha\, D_\theta\, D_\beta\, D_\gamma\, D_\delta)^T$ can be stated as

$$l(\psi) = n \log\, (D_\alpha\, D_\theta\, D_\beta\, D_\gamma\, D_\delta) - (1 + D_\delta) \sum_{i=1}^{n} \log\; x_i - (1 + D_\beta) \sum_{i=1}^{n} \log\, (1 + D_\gamma\, {x_i}^{-D_\delta}) + (D_\alpha - 1) \sum_{i=1}^{n} \log\, [1 - (1 + D_\gamma\, {x_i}^{-D_\delta})^{-D_\beta}] + (D_\theta - 1) \sum_{i=1}^{n} \log\, \{1 - [1 - (1 + D_\gamma\, {x_i}^{-D_\delta})^{-D_\beta}]^{D_\alpha}\} \tag{16}$$

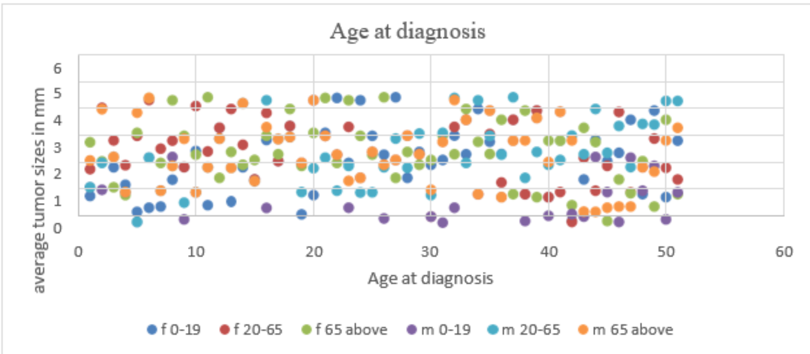
Then Differentiate

$\frac{d\psi}{dD_\alpha} = 0, we\ get\ \overline{D_\alpha}$, $\frac{d\psi}{dD_\theta} = 0, we\ get\ \overline{D_\theta}$, $\frac{d\psi}{dD_\beta} = 0, we\ get\ \overline{D_\beta}$, $\frac{d\psi}{dD_\delta} = 0, we\ get\ \overline{D_\delta}$, $\frac{d\psi}{dD_\gamma} = 0, we\ get\ \overline{D_\gamma}\ respectively.$

4. Applications



a



b

Figure 1. (a) PDFand CDF of EED distribution (b) Tumor size on average versus age at diagnosis

5. Dagum Distribution with corresponding parameters (Male):

Table 1: The Dagum distributions and associated parameters with malignant tumor best fit size data.

D_δ=0.36	D_α=5.46	D_β=55.76	D_γ=0.07
D_δ=0.35	D_α5.64	D_β=56.44	D_γ=0.012
D_δ= 0.47	D_α3.68	D_β=45.43	D_γ=0.412
D_δ=0.36	D_α=5.19	D_β=57.56	D_γ=0.376

Weibull Distribution with corresponding parameters (Female):

Table 2: Malignant tumor best fit size data by the Weibull distribution and the related parameters.

D_α=2.02	D_β=43.513
D_α=2.09	D_β=43.978
D_α=1.89	D_β=44.131

The probability distributions are selected, Kolmogorov Smirnov goodness of fit test was used to detect the distribution that best fits the aforementioned data. [19]. Plotted probability density functions of tumor best fit size data for six sub population. The fundamental features of tumor size for those subgroups derived figure 2(a), 2(b) and 3(a),

3(b). We examined the distributions and distinguished between tumor sizes in men and women, and the methods of the specific tumor size distributions were clearly discovered. Females' probability distributions were more skewed than any other recognized distributions. MLE was applied to the recognized distributions to find corresponding parameters. Estimated parameters are registered in tables 1 and 2 respectively, and the identified corresponding PDF and CDF, along with estimates, are plotted below.

6. Probability density functions

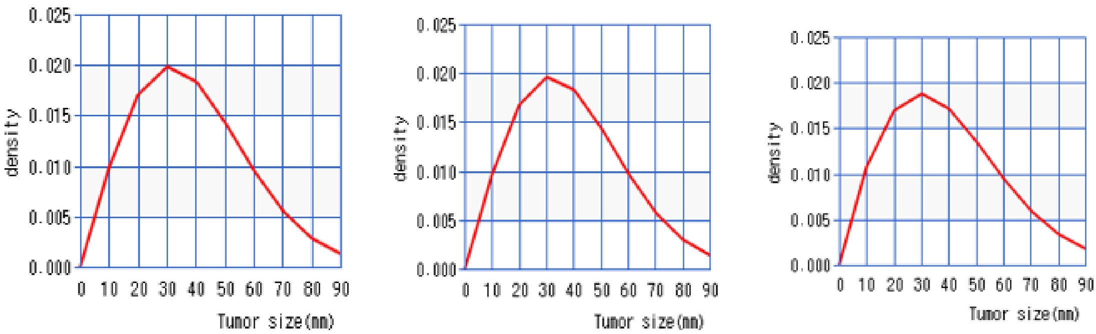


Figure 2 (a): WPDF sizes of tumors categorized for gender (female)

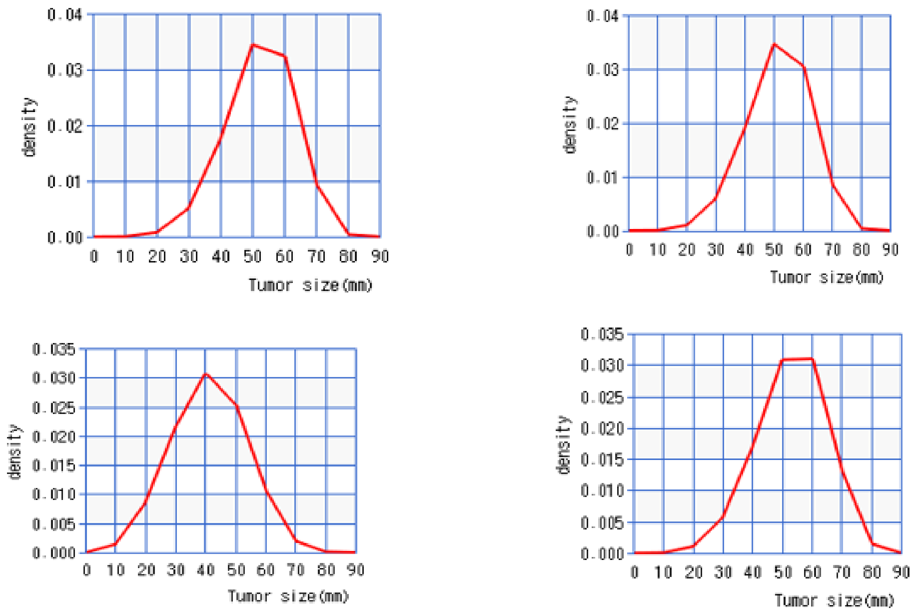


Figure 2 (b): WPDF size of tumors categorized for gender (male)

7. Cumulative Distribution Functions

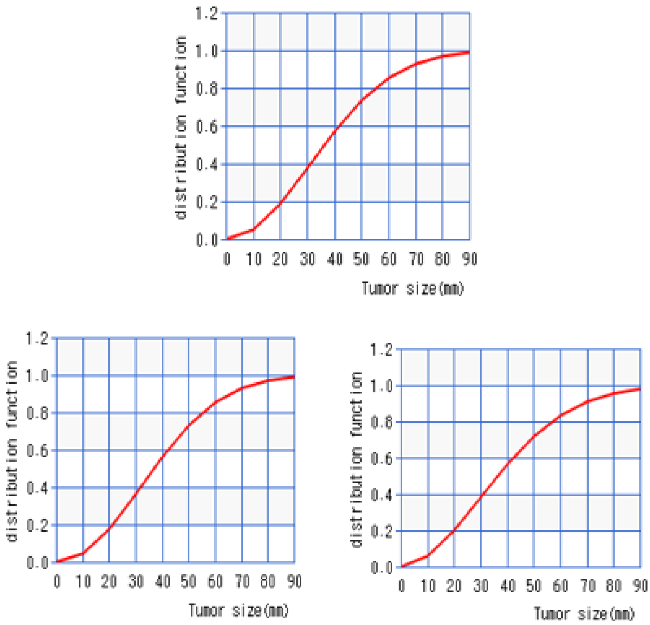


Figure 3 (a): DCDF size of tumors categorized according to gender (female)

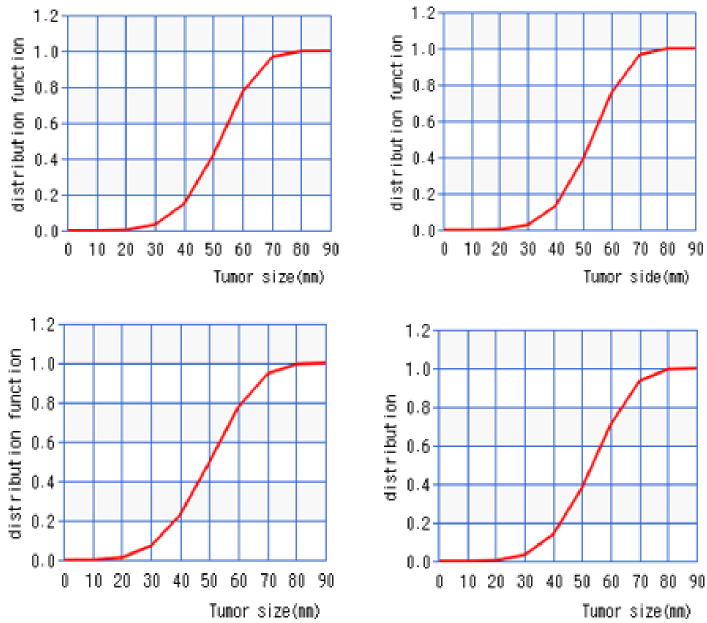


Figure 3 (b): DCDF size of tumors categorized according to gender (Male)

The above figures shown the DCDF for the sizes of CNS and brain tumors female and male. The DCDF is a complete explanation of the random tumor size variable. We Estimated the values parameters of Weibull distribution to liken the basic features of the size of tumors for the inhabitants of curiosity. We found there has been identical related

to the outcome of the race and gender on the above probability distributions. Discussed about the WPDF of the size of tumors and heavy tailed Weibull and Dagum distributions are found to fit both male and female population segments. Since the brain tumor sizes exhibit a highly skewed behaviour, we model these data using methods other than ordinary linear regression.

8. Survival rate function

Suppose that $FX(x)$ is the cdf of EED distribution supported on the interval (0, ∞) as proposed in equation then, the survival function of EED is given by

$$S_X(x) = P(X > x) = 1 - F(x) = \{ 1 - [1 - (1 + D_\gamma x^{-D\delta})^{-D\beta}]^{D\alpha}\}^{D\theta} , x \in (0, \infty) \quad (17)$$

Rate of survival after diagnosis for the population we measured. The results could play an important role in planning individual treatment strategies after diagnosed with cancer. The outcomes show uniformity of the observed survival improvement between sexes. Finally, the implementation part well fit with the probability distribution and the outcomes are well related with the medical report. Future medical fields will find this paper to be of great use.

9. Hazard function

Suppose that $FX(x)$ is the cdf of EED distribution in (0, ∞) as intended in equation then, the hazard function of EED is specified by

$$h_X(x) = \frac{f_X(x)}{1 - F(x)}$$

$$h_X(x) = D_\alpha D_\theta D_\beta D_\gamma D_\delta x^{-(1+D\delta)} ((1 + D_\gamma x^{-D\delta})^{-(1+D\beta)}[1 - (1 + D_\gamma x^{-D\delta})^{-D\beta}]^{(D\alpha-1)} \quad (18)$$

10. Cumulative Hazard function

Suppose that $SX(x)$ is the survival of EED distribution in (0, ∞) as intended in equation then, the hazard function of EED is specified by

$$H_X(x) = -ln\, S_X(x)$$

$$H_X(x) = -ln \{ 1 - [1 - (1 + D_\gamma x^{-D\delta})^{-D\beta}]^{D\alpha}\}^{D\theta} \quad (19)$$

11. Quantile Regression model

We investigated these novel methods in because of the growth of quantile regression models, epidemiological studies of brain and central nervous system data (Koenker and Hallock 2001).

In general, we believe the following: LMM

$y_{ij} = X^T \beta + {}_{ij} \; Z_{ij} \; b_i + \epsilon_{ij}$, i= 1, 2…., n, j= 1, 2…….n_i[5]

(20) We define p[th] quantile function of yi j as

$Qp(y_{ij} | X_{ij}, \; b_i) \; \underline{=}^{ij} \; X^T \; \beta_p \; + \; Z_{ij} b_i$ [5]

(21)

Let R be a real valued random variable then CPDF of R can be given by,

$F(r) = P(R \leq r).$ [17]

(22)

The q^{th} Quantile of R can be derived by the inverse function (Koenker 2005),

$F^{-1}(t) = Q(t) = in \; f(r{:} \; FI \geq t), \; 0 < t < 1 \; and \; median \; Q(1/2)$ [17]

(23) Quantile function linear conditional model $Q(t | X = x) = x^T Qnt(t)$, [17], Qnt (t)= min ($y_i - x^T Q(t)$) , $t \in (0,1)$ and Qnt(t) called t^{th} regression quantile.

(24) Let R= ($r_1, r_2, r_3 I. . . r_n$) be a random variable and $y = (y_1, y_1 I. . y_n)$ be n detected responses. The quantile regression linear model is given by $y = X^T \; Qnt(t) + \in_t$

The q^{th} regression quantile can be defined as

$$min_{t \in R} \sum_{\{y_i \geq x_i^T \; Qnt(t)\}} t \,|\, y_i - x_i^T Q(t) \,| + \sum_{\{y_i \geq x_i^T \; Qnt(t)\}} (1-t) \,|\, y_i - x_i^T Q(t) \,| \quad (25)$$

A random variable, uniformly distributed on the interval [0, 1], the quantile function of the EED distribution for is given by

$$Qnt_x \quad \tau = exp \left[\frac{1}{D\gamma \, D\delta} \left(- ln\{1 - (1-\tau^{D_\theta})^{1/D_\gamma}\}^{-1/D_\beta} - 1 \right) \right] \quad (26)$$

To found the Quantile regression coefficients using simplex method that we have literature to use LPP [5] to obtain the median regression coefficients. At different quantiles of the CPDF (conditional probability distribution function) of tumor sizes, we are calculating the coefficient estimates for the regression model derived from the given equation.

Qnt (t) = $D_\alpha + D_\beta \, Sex + D_\theta \, Age + D_\gamma \, Age * Sex + D_\delta Age * Race + D_\alpha Sex * Race + \epsilon$ (27)

12. Regression Statistics

Table 3

R	0.45264	R-Squared	0.2048	Adjusted R squared	0.1651
MSE	1.3859	S	1.1772	MAPE	92.6257
Durbin- Watson	2.0622				
AIC	3.2507	AICc	3.5298		
Schwarz criterion (BIC)	3.3499	Hannan-Quinn criterion (HQC)	3.2741		
PRESS	33.4169	PRESS RMSE	1.2324	Predicted R Squared	0.04146

R-Square is useful to investigate multivariate models and fractional variations in dependent variable that is the result of all independent variable. R-Squared = 0.2048, values at or closed to zero, which means the correlation is closed at zero, there is no measurable relations. We observed RMSE error value 1.3859, it is small the model selected is a good fit at predicting, because most of the points on regression line.

Table 4

Test	Test Statistic	p-value	H0(5%)
Kolmogorov Smirnov (Lilliefors)	0.2037	0.0181	Rejected

Kolmogorov Smirnov test a very effectual way to regulate if two samples are significantly different from each other. The p-value is small and reject the null hypothesis at (5%) level of significance, So the mean of medulloblastoma on tumors does not differ.

Table 5

	coefficients	Std Err	Lower Confidence Level	Upper Confidence Level	t stat	p- value	VIF	Tol	Beta
intercept	2.7134	0.5841	1.4948	3.9319	4.6449	0.00015			
	-0.0269	0.01189	-0.0517	-0.0021	-2.2701	0.0344	1	1	-0.4526
T (5%)	2.0859								

LCL - Lower limit at confidence interval 95%, UCL - Upper limit at confidence interval 95%

Scatter Plot

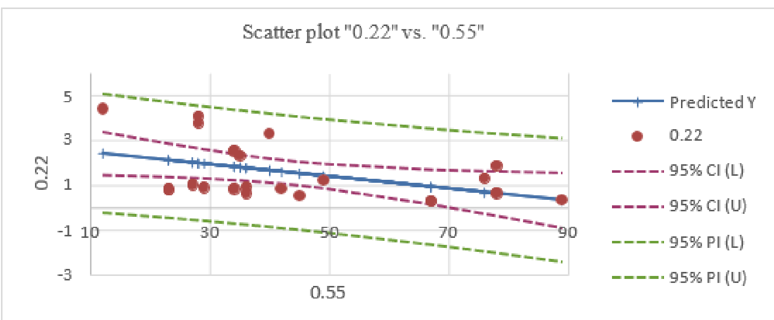


Figure 4 Confidence Interval CI - 95%, Prediction Interval PI - 95%

Normal Q-Q Plot – Residuals

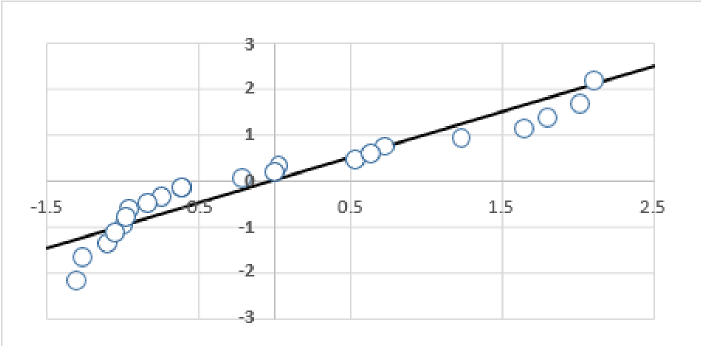


Figure 5

Tested mean of the tumor sizes(mm) for all the mentioned histology among male and female. For all the evaluations were made by Kolmogorov Smirnov (Lilliefors) test. In this experiment, we were unable to reject the null hypothesis in the case of medulloblastoma, so that mean of the tumor sizes of the other medulloblastoma does not differ. We can apply the same for rest of groups we reject in all cases.

13. Results and Discussion

Results showed that the average size of tumor by major histological type for primary brain tumor diagonally diverse age group. The mean of medulloblastoma on tumors does not differ when the Kolmogorov Smirnov (Lilliefors) test is used. We conversed about the tumor size probability density function. The Dagum and Weibull distributions with heavy tails were found to fit male and female population subgroups. Estimated PDF and CDF for identified probability distributions, as well as stated best-fit probability distributions The presented graphical representations of PDF and CDF Estimated parameters and quantile regression.

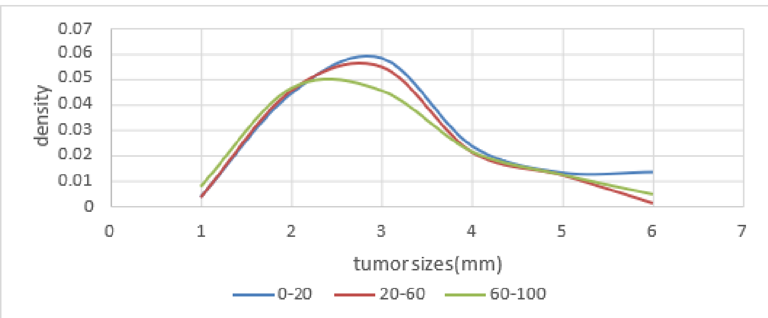


Figure 6

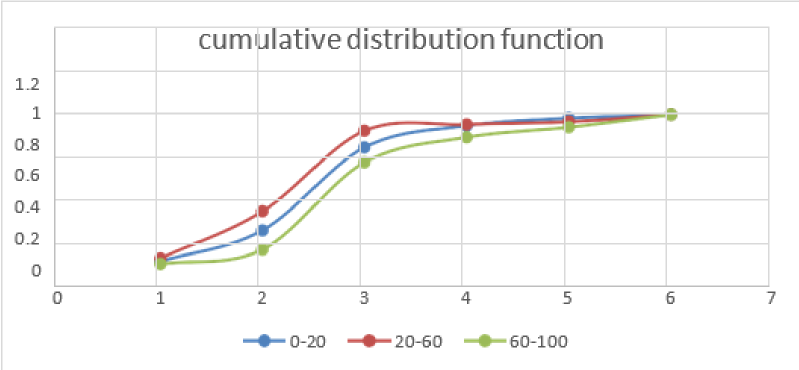


Figure 7

Figure 6 and figure 7 shows PDF and CDF of EED distribution with size tumor

Quantiles computed based on age and tumor size revealed a significant interaction between upper and lower quantiles. Coefficients that are significant at the 5% level of identification Tables 3to 6 can be used to draw the following conclusions. A median regression of tumor sizes was identified, the effect on tumor size is designated by the regression coefficients for 19 different quantiles. Estimated $5th$, $10th$, $15th$, $20th$, $25th$, $30th$, $35th$ quantiles of the tumor sizes from Quantile regression model equation (17), 0.061, 0.093,0.08, -0.014,0.09, 0.016,0.072 respectively. Higher quantiles did not significantly differ from the CPDF of tumor sizes as on age. The variables on rest of sex and age were significant at the level of 95%.

14. Contributions

According to the specific subject matter, the PDF for various subgroups of brain tumor sizes serve as more significant pillars in order to advance statistical inference. Weibull and Dagum distributions were used to discover the best characteristic distribution of brain size of tumor in both men and women, from which we could derive predicted tumor size and confidence intervals. The PDF and CDF of the Weibull and Dagum probability distributions are the crucial instrument for the treatment process. These findings will be of broad use to the scientific and biomedical communities.

15. Conclusions

In this paper we suggest literature-based five parameter Exponential Dagum distributions called EED model. The ability to examine data from real-world situations is a clear benefit of generalizing a standard distribution. In the case of the female population, the disparity between the mean median is roughly twice as great as it is in the case of the other group. Also, compared to other subgroups, females have a bigger skewness, or the percentage of tumor sizes that are farther from the median. When kurtosis size in the data sets is compared, male individuals are three times larger than the second largest. When compared to all other data sets, the number of exciting observations from the mean in the tumor fit size data for females were the lowest. The extremely skewed behaviour of size of brain tumor obligated us to model these data in ways other than ordinary linear regression. Finally, we explained the utility of the model showing that this one provides reliably better fit model.

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References

1. Alzaatreh A, Lee C and Famoye. F, "A new method for generating families of continuous distributions," Metron, vol. 71, no. 1, pp. 63 -79, 2013.
2. Aslam, M., Shoaib, M., Khan, H. (2011). Improved group acceptance sampling plan for Dagum distribution under percentiles lifetime. Communications of the Korean Statistical Society 18:403-411.
3. Bourguignon M, Silva R.B, and Cordeiro G.M, "The Weibull-G family of probability distributions, "Journal of data science, vol. 12, no. 1, pp.53-68, 2013.
4. Barrodale and Roberts F.D.K, "An improved algorithm for discrete $l1$ linear approximation," SIAM Journal of Numerical Analysis, Volume 35, pp. 1019-1030,1964.
5. Christian E. Galarzaa Dipankar Bandyopadhyayb ∗ Victor H. Lachosa. "Quantile Regression for Linear Mixed Models: A Stochastic Approximation EM approach" Department of de Estatística, Universidade Estadual de Campinas, Campinas, Brazil Division of Biostatistics, University of Minnesota, Minneapolis, MN, USA. August13, 2015.
6. Chin-Diew Lai Massey University "Weibull Distributions and Their Applications "Article · February 2006 DOI: 10.1007/978-1-84628-288-1_3 ·
7. Cancer Statistics, 2020: Report from National Cancer Registry program, India Prashant Mathur, DNB, Ph. D; Krishnan Sathish Kumar, M. Sc; Meesha Chaturvedi, MBBS; Priyanka Das, B-Level; Kondalli Lakshmi Narayana Sudarshan, MSc; Stephen Santhappan, MSc, MPhil; Vinod Nallasamy, MSc, MPhil; Anish John, BE; Sandeep Narasimhan; and Francis Selvaraj Rosalind, MSc; on behalf of ICMR-NCDIR-NCRP Investigator Group.
8. Dagum, C. (1977). A new model of personal income distribution: specification and estimation. Economic appliqué's 30:413-437.
9. Domma, F., Giordano S., Zenga, M. (2011a). Maximum likelihood estimation in Dagum distribution from censored samples. Journal of Applied Statistics 38:2971-2985.
10. Ferlay, J.et al. Estimating the global cancer incidence and mortality in (2018): Globocan sources and methods, Int. J. Cancer 1941-1953 (2019).
11. Frank Gomes- Silva, Ronaldo V. da Silva, Ana Percontini, Manoel Wallace A. Ramos and Gauss M. Cordeiro, "An extended Dagum Distribution: Properties and Applications" Int. J. Appl. Math. Stat.: Vol. 56; Issue No. 1; Year 2017, ISSN 0973-1377.
12. Gupta S G., Kapoor V K: Twelfth Edition: Fundamentals of Mathematical Statistics, July 2020, ISBN:978-93-173-8&(TC-499)
13. Gupta R.D., Kundu D.: Exponentiated Exponential Family: An Alternative to Gamma and Weibull Distributions, Biometrical journal, (2001) 117-130.
14. Gupta R. D., Kundu D.: Discriminating between Weibull and generalized exponential distributions, Computational Statistics and Data Analysis, (2003), 1798-196.
15. Hassan A. and Elgarhy M., "A new family of exponential Weibull- generated distributions, "International journal of Mathematics and its Application, vol.1, pp.135- 148,2016.
16. Kleiber, C., Kotz, S. (2003). Statistical Size Distributions in Economics and Actuarial Sciences, New York: Wiley.

17. Koenker. R, Quantile Regression. Cambridge university Press, 2005.
18. Majdah Mohammed Badr, Amal T Badawai, Alya S Alzubidi (2022). "A New Extension of the Exponential Weibull Model Mathematical properties and modelling," Saudi Arabia and Insurance department, faculty of Commerce, Mansoura university, Egypt. https://doi.org/10.1155/2022/4669412.
19. Massey. F. J, "the Kolmogorov – Smirnov Test for goodness of fit," Journal of the American Statistical Association, volume 46, no. 253, pp. 66-78, 1951.
20. Noman Rasheed, Department of Statistics, Panjab College Hasilpur, Pakistan, "Topp- Leone Dagum Distribution: Properties and its Applications" Research Journal of Mathematical and Statistical Sciences; Vol.8(1), 16-30, January (2020).
21. Oguntude P., Balogun O., Okagbue H., and Bishop S., "The Weibull-exponential distribution: its properties and applications," Journal of Applied Sciences, vol.15, no. 11, pp. 1305-1311, 2015. York: Wiley.
22. Parthiban S, Gajivaradhan P. One-factor ANOVA model using trapezoidal fuzzy numbers through alpha cut interval method: Annals of Pure and Applied Mathematics, Vol. 11, No. 1, (2016), pp. 45-61. http://www.researchmathsci.org/apamart/apam- v11n1-6.pdf.
23. Parthiban S, Gajivaradhan P. Statistical hypothesis test in three factor ANOVA model under fuzzy environments using trapezoidal fuzzy numbers: Bulletin of Mathematical Sciences and Applications, Vol. 14, (2016), pp. 23-42. https://www.scipress.com/BMSA.14.23.
24. Parthiban S, Gajivaradhan P. A comparative study of chi-square goodness-of-fit under fuzzy environments: Mathematical Theory and Modelling, Vol. 6, No. 2, pp. 82-97. http://www.iiste.org/Journals/index.php/MTM/article/view/28576/29338
25. Rameshwar D. Gupta*, Exponentiated Exponential Family: An Alternative to Gamma and Weibull Distributions"; Department of Mathematics, Statistics and Computer Sciences, The University of New Brunswick Saint John Canada, Biometrical Journal 43 (2001) 1, 117–130.
26. Soumaya Ghnimi 1 Soufiane Gasmi2 July (2014). "Parameter Estimations for Some Modifications of the Weibull Distribution" by authors and Scientific Research Publishing Inc.
27. Stewart B. and Wild C.P., World cancer report, WHO 2013.
28. Weibull W.: Wide applicability, Journal of applied mechanics, (1951), 203-210