Impact of thin nanoengineered coatings on the stress–strain state of cylindrical cladding made from Zr-based alloys for nuclear fuel elements**

Yu.V. Romashov,[1,2,*] E.V. Povolotskii[2] and A.G. Mamalis[3,*]

[1] *V.N. Karazin Kharkiv National University, Ukraine*
[2] *National Technical University "Kharkiv Polytechnic Institute", Ukraine*
[3] *Project Centre for Nanotechnology and Advanced Engineering, NCSR "Demokritos", Greece, Athens*

A current problem concerning the use of thin nanoengineered coatings for increasing operability of nuclear fuel cladding made from widely used zirconium-based alloys is considered from the point of view of the mechanical elastic interaction between the coating and the cladding. A mathematical model of thin coatings on a cylindrical cladding is presented in the form of special boundary conditions of the equations of elasticity theory that define the stress–strain state of the cladding. It is shown that thin coatings can noticeably decrease stresses in the cladding.

Keywords: elasticity, thick solids, thin shells, strength

1. Introduction

Using thin nanoengineered coatings is one of the ways widely discussed to increase the operability and the safety of cladding for the fuel elements of nuclear power reactors.[1–3]

---

* Corresponding authors. E-mails: yu.v.romashov@gmail.com; a.mamalis@inn.demokritos.gr
** This paper was first presented at the eleventh Japanese–Mediterranean workshop (JAPMED'11), 15–19 July 2019 in Batumi, Georgia.

1 Bischoff, J., Delafoy, C., Vauglin, C., Barberis, P., Roubeyrie, C., Perche, D., Duthoo, D., Schuster, F., Brachet, J.-C., Schweitzer, E.W., Nimishakavi, K. AREVA NP's enhanced accident-tolerant fuel developments: Focus on Cr-coated M5 cladding. *Nucl. Engng Technol.* **50** (2018) 223–228.

2 Alat, E., Motta, A.T., Comstock, R.J., Partezana, J.M., Wolfe, D.E. Ceramic coating for corrosion (c3) resistance of nuclear fuel cladding. *Surf. Coatings Technol.* **281** (2015) 133–143.

3 Maier, B.R., Garcia-Diaz, B.L., Hauch, B., Olson, L.C., Sindelar, R.L., Sridharana, K. Cold spray deposition of Ti_2AlC coatings for improved nuclear fuel cladding. *J. Nucl. Mater.* **466** (2015) 712–717.

Nanotechnology Perceptions **16** (2020) 202–208
Received 24 September 2019; *accepted* 26 May 2020
© 2020 Collegium Basilea
doi: 10.4024/N20RO19A.ntp.16.02

The main effects of the application of the coatings are connected both to the properties of the coating materials and to mechanical interaction between thick- and thin-walled structures, representing the cladding and its coating, respectively. The purpose of this paper is to propose a mathematical model of mechanical interaction between the widely used cylindrical cladding made from Zr-based alloys and its thin coating made from stainless steel and to use the model to estimate the impact of the thin coatings on the stress–strain state of the cladding. This is highly topical due to the trend of using thin protective coatings in future realizations of nuclear fuel cladding, which requires clarification of the strength and lifetime of structural elements with thin coatings as well as of the coatings themselves.

2. Mathematical model of cylindrical cladding with a thin coating

The cladding is represented as a cylinder with internal radius a, external radius b and length L (Fig. 1). It is assumed that the various external factors impacting on the cladding are reduced to the internal pressure p_a of the gases in the gap between the nuclear fuel pellets and the cladding and to the external pressure p_b from the moving heat carrier. The cylinder must be considered as thick-walled, since the cylinder's average radius $\frac{1}{2}(a+b)$ is comparable with the thickness $b-a$ of the wall in most designs of fuel elements. Hence, the cladding is schematized as a long thick-walled axially symmetric cylinder, as in the well known problem of plane strain in the theory of elasticity.[4] Due to the axial symmetry of the shape and pressures, the stress–strain state of this elastically deforming pressurized cylinder is reduced to the radial displacement $u = u(r)$, the radial stress $\sigma_r = \sigma_r(r)$, and the circumferential stress $\sigma_\theta = \sigma_\theta(r)$, all depending on the radial coördinate r only as shown in Fig. 1. It is known that the stresses of such an axially symmetric cylinder can be represented via the displacement:[4]

$$\sigma_r = \frac{E}{1-\nu^2}\left(\frac{\mathrm{d}u}{\mathrm{d}r} + \nu\frac{u}{r}\right), \quad \sigma_\theta = \frac{E}{1-\nu^2}\left(\nu\frac{\mathrm{d}u}{\mathrm{d}r} + \frac{u}{r}\right), \tag{1}$$

where E and ν are the effective Young's moduli and Poisson's ratio of the material of the cylinder under the plane strain state; these quantities are defined by the Young's modulus E' and Poisson's ratio ν' of the material of the cylinder by the relations $E = E'/1-\nu'^2$ and $\nu = \nu'/1-\nu'$.

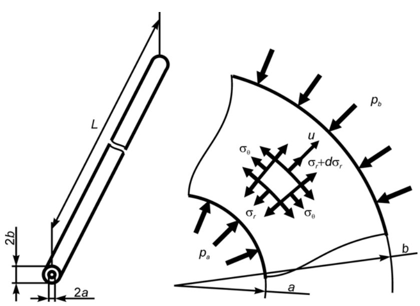

Figure 1. The cladding and a fragment of its cross-section.

[4] Timoshenko, S.P., Goodier, J.N. *Theory of Elasticity*. New York: McGraw-Hill (1951).

Due to the relations (1), the strain state of this axially symmetric cylinder is defined by one differential equation for unknown displacement:

$$\frac{d^2u}{dr^2}+\frac{1}{r}\frac{du}{dr}-\frac{u}{r^2}=0,\ a<r<b\,, \tag{2}$$

which must be considered with the boundary conditions on the inner and outer surfaces $r = a$ and $r = b$. In the inner surface without the coating the boundary condition is defined by the relation $\sigma_r(a) = -p_a$, which considering the stresses (1) has the form:[4]

$$\frac{E}{1-\nu^2}\left(\frac{du}{dr}+\nu\frac{u}{r}\right)=-p_a,\ r=a\,. \tag{3}$$

If the cladding has no external thin coating, the boundary condition on the external radius can be defined by the relation $\sigma_r(b) = -p_b$:

$$\frac{E}{1-\nu^2}\left(\frac{du}{dr}+\nu\frac{u}{r}\right)=-p_b,\ r=b\,. \tag{4}$$

To take into account the thin coating on the external radius of the cylinder (Fig. 2), it is necessary to consider the interaction between the cylinder and the thin coating. This interaction is significantly defined by the properties of the thin coating, which can be represented as a bending moment-free thin cylindrical shell due to the axial symmetry and the loadings.[5] The interaction is this defined by the equilibrium conditions between the external pressure p_b, acting on the coating, the radial stress $\sigma_r(b)$, acting on the coating from the side of the cladding, and the internal circumferential forces N_θ in the coating, represented by the thin bending moment-free cylindrical shell (Fig. 2). The condition of equilibrium along the radial direction of an elementary element of the coating, defined by the infinitesimal angle $d\theta$ (Fig. 2), can be written as:

$$-\sigma_r(b)\left(R_c-\frac{h_c}{2}\right)d\theta-p_b\left(R_c+\frac{h_c}{2}\right)d\theta-2N_\theta\sin\frac{d\theta}{2}=0\,, \tag{5}$$

where h_c and $R_c = b + h_c/2$ are the thickness and radius of the central surface of the thin coating (cf. Fig. 2).

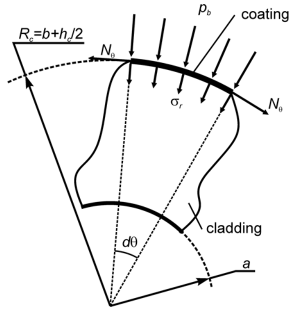

Figure 2. Fragment of cladding with a thin coating on the external radius.

[5] Timoshenko, S.P., Woinowsky-Krieger, S. *Theory of Plates and Shells*. New York: McGraw-Hill (1959).

It is possible to neglect the thickness h_c compared with the radius R_c of the midsurface of the thin coating. Besides, it is necessary to take into account the well known relation $\sin\frac{\mathrm{d}\theta}{2}\approx\frac{\mathrm{d}\theta}{2}$ for the small angle $\mathrm{d}\theta$.[6] Considering all these circumstances, the equilibrium condition (5) can be reduced to:

$$\sigma_r\left(b\right)=-p_b-N_\theta/R_c\ . \tag{6}$$

It is known that the circumferential force N_θ of a bending-free thin cylindrical shell can be related to the deflexion:[5]

$$N_\theta=E_c h_c w/R_c\ , \tag{7}$$

where E_c is the Young's modulus of the coating and w is the deflexion of the thin cylindrical shell, representing the thin coating.

It is obvious that the deflexion w of the shell is equal to the displacement $u(b)$ of the external boundary of the cladding, considering which and relation (6), the radial stress (5) can be written as:

$$\sigma_r\left(b\right)=-p_b-\frac{E_c h_c}{R_c^2}u\left(b\right). \tag{8}$$

Along with eqn (1), defining the radial stress, eqn (8) allows formulating the boundary condition of the external surface of the cladding with the external thin coating as:

$$\frac{E}{1-\nu^2}\left(\frac{\mathrm{d}u}{\mathrm{d}r}+\nu\frac{u}{r}\right)+\frac{E_c h_c}{R_c^2}u=-p_b,\ r=b\ . \tag{9}$$

The well known boundary condition (4) for the cladding without a thin coating is the particular case of the boundary condition (9) for the cladding with the thin coating, corresponding to the zero-thickness cladding $h_c = 0$. Thus, the mathematical model of the stress–strain state of the cylindrical cladding with the thin coating is proposed in the form of differential equation (2) with boundary conditions (3) and (9), as well as the relations (1), which allow the stresses to be defined from a known displacement. This model can be substantiated by going to the limit.

3. The stress–strain state of the cylindrical cladding with a thin coating

Solving the ordinary differential equation (2) with boundary conditions (3) and (9) allows the stress–strain state of the cladding with the external thin coating to be found. The general solution of this linear homogeneous differential equation (2) and the stresses corresponding to this solution can be written as:

$$u\left(r\right)=C_1 r+\frac{C_2}{r}\ , \tag{10}$$

$$\sigma_r\left(r\right)=\frac{E}{1-\nu^2}\left(\left(1+\nu\right)C_1+\frac{\nu-1}{r^2}C_2\right),\ \sigma_\theta\left(r\right)=\frac{E}{1-\nu^2}\left(\left(1+\nu\right)C_1+\frac{1-\nu}{r^2}C_2\right), \tag{11}$$

where C_1 and C_2 are the integrating constants, which must be found from the boundary conditions, considering which the radial stress (11) can be represented as:

---

[6] Dwight, H.B. *Tables of Integrals and Other Mathematical Data*. New York: Macmillan (1957).

$$(1+\nu)C_1 + \frac{\nu-1}{a^2}C_2 = -p_a\frac{1-\nu^2}{E}, \tag{12}$$

$$\left(1+\nu+\frac{1-\nu^2}{E}\frac{E_c h_c b}{R_c^2}\right)C_1 + \left(\frac{\nu-1}{b^2}+\frac{1-\nu^2}{E}\frac{E_c h_c}{R_c^2 b}\right)C_2 = -p_b\frac{1-\nu^2}{E}, \tag{13}$$

from which the integrating constants C_1 and C_2 can be found, although their analytical expressions are very unwieldy; they can be represented in compact form by using Kramer's rule for a system of two linear equations:

$$C_1 = \frac{\begin{vmatrix} -p_a\dfrac{1-\nu^2}{E} & \dfrac{\nu-1}{a^2} \\ -p_b\dfrac{1-\nu^2}{E} & \dfrac{\nu-1}{b^2}+\dfrac{1-\nu^2}{E}\dfrac{E_c h_c}{R_c^2 b} \end{vmatrix}}{\begin{vmatrix} 1+\nu & \dfrac{\nu-1}{a^2} \\ 1+\nu+\dfrac{1-\nu^2}{E}\dfrac{E_c h_c b}{R_c^2} & \dfrac{\nu-1}{b^2}+\dfrac{1-\nu^2}{E}\dfrac{E_c h_c}{R_c^2 b} \end{vmatrix}}, \tag{14}$$

$$C_2 = \frac{\begin{vmatrix} 1+\nu & -p_a\dfrac{1-\nu^2}{E} \\ 1+\nu+\dfrac{1-\nu^2}{E}\dfrac{E_c h_c b}{R_c^2} & -p_b\dfrac{1-\nu^2}{E} \end{vmatrix}}{\begin{vmatrix} 1+\nu & \dfrac{\nu-1}{a^2} \\ 1+\nu+\dfrac{1-\nu^2}{E}\dfrac{E_c h_c b}{R_c^2} & \dfrac{\nu-1}{b^2}+\dfrac{1-\nu^2}{E}\dfrac{E_c h_c}{R_c^2 b} \end{vmatrix}}. \tag{15}$$

Thus, the stress–strain state of the cladding with the external thin coating is represented by eqns (10), (11), (14) and (15).

4. Quantitative estimations of the impact of the coating on the stress–strain state of the cylindrical cladding

It will be considered that the cladding is made from 99%Zr–1%Nb alloy with different thicknesses of coatings made from stainless steel type 1–10.[7] The internal and external

[7] Stoev, P.I., Belous, V.A., Voyevodin, V.N., Kuprin, A.S., Leonov, S.A., Ovcharenko, V.D., Tikhonovsky, M.A., Khoroshih, V.M. Mechanical properties and acoustic parameters tubes of zirconium alloy Zr1%Nb with a protective coating. *Voprosy Atomnoj Nauki i Tekhniki* **5(99)** (2015) 87–97.

diameters of the cladding, the internal and external pressures, and the Young's moduli of the materials of the cladding and the coating are:

$$a = 3{,}885 \ \text{mm}, \ b = 4{,}55 \ \text{mm}, \tag{16}$$

$$p_a = 10 \ \text{MPa}, \ p_b = 16 \ \text{MPa}, \tag{17}$$

$$E' = 96 \ \text{GPa}, \ \nu' = 0{,}33, \ E_c = 210 \ \text{GPa}. \tag{18}$$

These data correspond to the cladding of fuel elements of the VVER-1000 type nuclear reactors, widely used in the eastern European countries.[8]

To obtain quantitative estimations of impact for fuel elements of the VVER-1000 type nuclear reactor, we will compare the stress–strain states of claddings with different thickness h_c of the coating. These states can be obtained for the data (16–18) using the formulae (10), (11), (14) and (15). The results for the radial and circumferential stresses and the radial displacements in the claddings obtained for the different thicknesses of the coating are presented in Fig. 3. They show that the thin coatings have no noticeable impact on the radial stresses (Fig. 3a), but can noticeably decrease the circumferential stresses and the radial displacements (Fig. 3b) in the cladding. This is not contradicted by known data on increasing yield strength of cladding with coatings versus without coatings.[7] Thus, the thin nanoengineered coatings can have a favourable impact on the strength of the cladding at least due to decreasing the circumferential stresses, by about 4%.

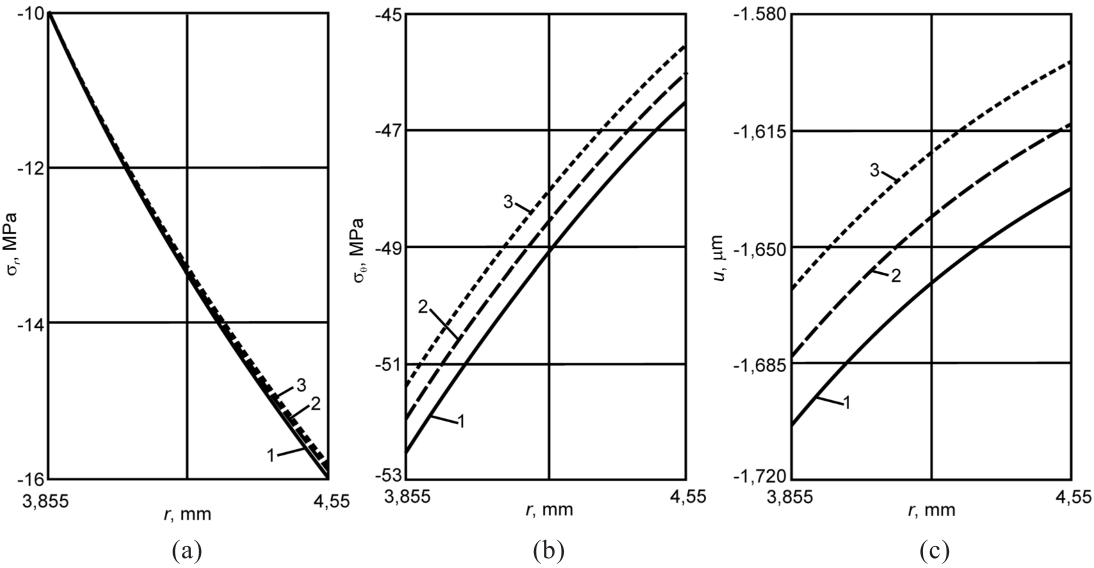

Figure 3. Radial (a) and circumferential (b) stresses and radial displacements (c) in cladding made from Zr-based alloy without coating (curves 1) and with coatings made from stainless steel with thickness h_C = 5 μm (curves 2) and h_C = 10 μm (curves 3).

[8] Király, M., Hózer, Z., Horváth, M., Novotny, T., Perez-Feró, E., Vér, N. Impact of thermal and chemical treatment on the mechanical properties of E110 and E110G cladding tubes. *Nucl. Engng Technol.* **51** (2019) 518–525.

5. Conclusions

A mathematical model of the cylindrical cladding of nuclear reactor fuel elements with a thin coating is proposed in the form of the well known differential equation of the plane problem of the theory of elasticity with a modified boundary condition for considering the effect of the thin coating. The reliability of this proposed modified boundary condition is substantiated by its limit transition, which is reduced to the well known boundary condition of the theory of elasticity in the case of a coating with zero thickness;

- the thin coatings have no noticeable impact on the radial stresses, but noticeably decrease the circumferential stresses and the radial displacements in the cladding;
- the modified boundary condition for considering the effect of the thin coating allows it to be asserted that the impact of the thin coating on the stress–strain state of the cladding is approximately proportional to the Young's modulus of the material and the thickness of the coating. Due to this circumstance, increasing the Young's modulus of the material allows the thickness of coating to be decreased while retaining the effect of the coating. Thus, to improve the operational properties of the cladding of fuel elements of nuclear reactors it seems very promising to make ultrathin coatings from ultrarigid materials using nanotechnology;
- it is recommended to further consider the bending of the cladding of fuel elements, which will require consideration of the bending forces in the thin shells representing the coatings; it is also recommended to consider the temperature and creep deformations of cladding with a thin nanoengineered coating to assess possible exfoliation of the coating under operating conditions.

Acknowledgment

We would like to thank Jan Zdebor, Vice-Dean for Industrial Coöperation of the Faculty of Mechanical Engineering of the University of West Bohemia in Pilsen (Czech Republic), for his interest in our previous results about operability of the cladding of fuel elements of nuclear reactors, which encouraged us to pursue the work reported in the present paper.