Nanotechnology Perceptions
ISSN 1660-6795
www.nano-ntp.com

Wavelet Transform and Thresholding Techniques in Medical Ultrasound Image Speckle Reduction

Meera Nair, Bijukumar S P

NSS College Rajakumari, Idukki, Kerala, India

Speckle is a paculiar type of multiplicative noise affecting the quality of medical ultrasound image and SAR(Synthetic Aperture Radar) image making the interpretation and processing of the image difficult. Among many despeckling strategies wavelet domain offers promising results since it follows a multi-resolution analysis of image and helps in identifying the noise from other important details of the image. In this review, we bring together various wavelet-based despeckling techniques used in medical ultrasound imaging, along with different thresholding methods, and assess their effectiveness in different imaging situations.

Keywords: SAR(Synthetic Aperture Radar), image speckle.

1. Introduction

Wavelet transform is a mathematical tool which helps in the multi resolution analysis of the image. This helps in the decomposition of the image into high frequency and low frequecy components allowing the examination of various features of the image which is helpful in various image processing tasks like compression, feature extraction, classification, denoising etc. The wavelets are extensively used diverse research areas. Wavelet based methodologies can be used to enhance the quality of the image in denoising, optimise the compression, improve accuracy in feature extraction in segmentation and classification.

Speckle noise is a granular salt-and-pepper-like noise that appears in ultrasound imaging. This paculiar type of noise cannot be prevented since it occurs due to peculiar characteristics of acoustic radiations used in ultrasound imaging. It affects the diagnostic efficiency of the images which is major issue. It affects the clarity of the images and also can introduce artifacts in the images which can lead to misdiagnosis. It may lead to difficulties in distinguishing between different tissues, edges or boundaries of various structures etc. This degradation in image quality hampers the effectiveness of diagnostic procedures.

The economic advantage, portability and non-invasive nature gives ultrasound imaging an important place in medical diagnosis. But the presence of speckle noise is an important issue in ultrasound imaging, greatly affecting its diagnostic efficiency. Despeckling is done in spatial or transformed domain. In spatial despeckling, the statistical properties of the pixels are explored for maximum speckle reduction. But many of them alters the texture, blur the fine details and edges and also introduces artifacts to the images. Despeckling in the transform domain was a major turning point in the despeckling studies. Initial studies using Fourier transform consider only the frequency domain of the image losing all the spatial domain information including where particular frequencies occur. It does not give promising results in despeckling since it cannot distinguish the noise information from that of the image. Wavelet-based despeckling techniques have been widely studied to reduce the negative impact of speckle noise. The wavelet transform's multi-resolution analysis enables the separation of noise from significant image features. Thresholding methods in the wavelet domain distinguishes true high frequency components like edges from unwanted high frequency noise component. The effectiveness of wavelet-based despeckling methods lies in their ability to analyze images at multiple scales while selectively reducing noise without compromising the fine details, textures, and edges which is important in applications like medical imaging, where preserving image quality is crucial. This study focuses on the recent advancements in wavelet-based despeckling techniques, comparing various methods and their effectiveness in different imaging scenarios.

2. Theoretical Background

The despeckling process using wavelet transform can be summarized in the following steps:[33].

- Apply a logarithmic transformation to the ultrasound image to convert the multiplicative speckle noise into additive noise. Many denoising methods are available for removing additive noises which can be used after this conversion.

- Perform a wavelet transform on the log-transformed image. This decomposes the image into approximate low frequency components representing large-scale features and smooth variations in the image and detailed high frequency components representing fine details, edges and noise in the image.

- Apply a thresholding technique to the detailed wavelet coefficients, where speckle noise is more prominent. Thresholding techniques are applied to remove the noise in the image.

- Apply the inverse wavelet transform to the thresholded coefficients which reconstructs the denoised image in the logarithmic domain .

- Exponentiate the resultant denoised log-transformed image producing the final despeckled ultrasound image.

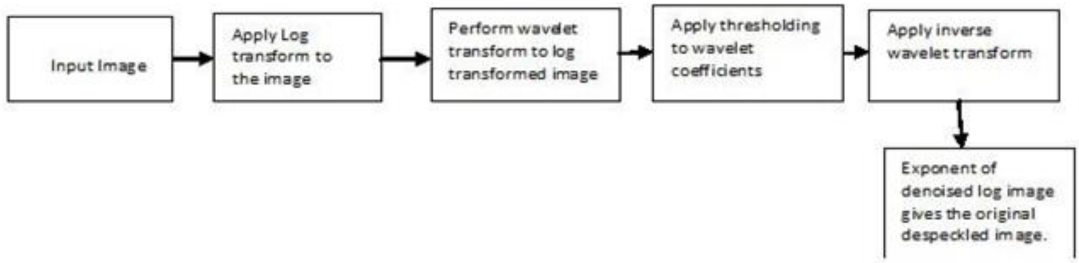

Fig 1. Wavelet based despeckling

2.1 Types of Wavelets Used in Despeckling

Exploiting the wavelet theory, many different wavelets are available. The choice of the wavelets depends on the purpose of the despeckling study and area of interest in the image. We need maximum speckle removal while preserving fine details in the image. Here we discuss some of the wavelets used in despeckling and their effects on the quality of output image.

The scaling function $\phi(t)$, called father wavelet is a fundamental building block in wavelet theory. They are the approximate coefficients in a wavelet transform. The scaling function captures the coarse details of the signal represented by low frequency components. The wavelet function, denoted by $\psi(t)$ that captures the high-frequency components of signal which represents the fine details, edges, noise etc in the image. It is also called the mother wavelet. The scaling function and wavelet function allows the image to be decomposed at various levels into its components and thus providing a powerful tool for analyzing signals.

2.1.1 Haar Wavelets

It is the simplest type. It is commonly used for simple signal processing tasks. It is used in fast denoising since computation is fast. They are not effective in preserving smooth regions in the image. They can be used in initial despeckling studies since they are fast and also they can identify sharp changes in the image like edges.

Scaling Function (Father Wavelet) $\phi(t)$:

$$\phi(t) = \left\{\begin{matrix} 1 & 0 \le t < 1 \\ 0 & \text{otherwise} \end{matrix}\right) \tag{1}$$

Wavelet Function (Mother Wavelet) $\psi(t)$:

$$\psi(t) = \left\{\begin{matrix} 1 & 0 \le t < \frac{1}{2} \\ -1 & \frac{1}{2} \le t < 1 \\ 0 & \text{otherwise} \end{matrix}\right) \tag{2}$$

2.1.2 Daubechies Wavelets

They are characterised by a number of vanishing moments. As the vanising momemts increases it can represent the smooth structures more effectively. The family is represented as dbN which varies from db1 to db10. db1 represents haar wavelets since the number of vanishing moments is 1. db2 having two vanishing moments and can represent linear features in the images. For representing the polynomials we need to use db4. It can capture more smooth edges and so ideal for image denoising tasks. db6 represents cubic polynomials and will be ideal for preserving fine details in the images while denoising.

Scaling Function is,

$$\phi\ (t) = \sqrt{2}\sum_{k=0}^{2N\ \ 1} h_k\ \ (2t - k) \tag{3}$$

[1] $\phi(t)$: The scaling function also known as the father wavelet. It represents the low frequency approximate coefficients.

[2] h_k : These are the filter coefficients that represents specific mathematical properties such as orthogonality and compact support.

[3] $\sqrt{2}$: This factor ensures that the scaling function is properly normalized.

[4] $(2t-k)$: This term represents a scaling by 2 and shifting by k.The scaling function is shifted and scaled to generate different versions .

Wavelet Function is ,

$$\psi(t) = \sqrt{2}\sum_{k=0}^{2N\ \ 1}(-1)^k h_{2N\text{-}1\text{-}k}\ \ (2t - k) \tag{4}$$

1. $\psi(t)$: The wavelet function known as the mother wavelet. It represents the detailed coefficients.

2. $(-1)^k$: This factor reverses the sign, which helps the wavelet function to capture the high-frequency details of the signal.

3. h_{2N-1-k} : This is the same filter coefficients used in the scaling function, but they are reversed . N determines the number of vanishing moments wich is a measure of the wavelet's ability to represent polynomials.

Daubechies Wavelets are used widely in deseckling studies. In a study they proposed a complex Daubechies transform. In this wavelet transform is taken in the original image without taking log transform of the image. It has the advantage of shift invariance and adaptive thresholding is used which helps in preserving the edge information while despeckling. They claimed superior performance. In another study[5] an adaptive threshoding techniques is divised and observed that Daubechies wavelets were giving better results as compared to other state of art despeckling methods. In[4] comparative analysis various despeckling filers is done and observed better performance by daubechies.

2.1.3 Symlets

Symlets are a modified version of Daubechies wavelets. Unlike Daubechies wavelets, which are not symmetric because they focus on maximizing vanishing moments and having compact support, Symlets are designed to be nearly symmetric.

The scaling function is

$$(t) = \sqrt{2}\sum_{k=0}^{2N-1} g_k \;(2t - k) \tag{5}$$

The wavelet function is

$$\psi(t) = \sqrt{2}\sum_{k=0}^{2N-1} (-1)^k g_{2N-1-k} \;(2t - k) \tag{6}$$

The function is similar to daubechies, only the filter coefficient is different. The filter coefficients are designed mathematically to capture the special symmetric property offered by the symlets.

Symlets are used in despeckling tasks where preserving the symmetry is more important for perfect and accurate reconstruction of the image like in medical imaging where the structural integrity of the reconstructed image matters a lot. There are many studies in literature using Symlets. In [14] Symlet based despeckling is used and proposed a new method for calculating threshold exploiting the statistical properties of the image using weighted window. In another study [25] a new adaptive thresholding technique is proposed and they get better results for Symlet8.

2.1.4 Coiflets

Coiflets is symmetric and have 2N vanishing moments[23]. While Daubechies wavelets are excellent for detecting sharp features and discontinuities in the image, Coiflets offer better performance in applications where smoothness and edge preservation are critical. This makes Coiflets more suitable for ultrasound image despeckling where maintaining edges, texture and other fine details of anatomical structures is important.

2.1.5 Biorthogonal Wavelets

All the waveletes discussed above are orthogonal ie, they use same functions for the decomposition of the image into wavelets and reconstruction of wavelets back to the image which ensures energy conservation and perfect reconstruction. In Biorthogonal wavelets, different functions can be used for decomposition and reconstruction. This provides more flexibility in selecting the filters which can balance smoothness and preserve fine details. These wavelets are complex and are particularly useful in applications where quality of the image is crucial. [9],[22]

2.2 Thresholding Techniques in Wavelet-Based Despeckling

The effectiveness of the wavelet-based techniques greatly depends on the choice of the thresholding. Varioius thresholding techniques that are used in wavelet based despeckling are discussed here [13]

2.2.1.Hard Thresholding

Hard thresholding is one of the simplest wavelet thresholding techniques. It involves setting all wavelet coefficients below a certain value to zero and keeping those above unchanged. This method is easier to implement but can introduce artifacts, which makes it unsuitable for medical imaging applications where a small change in the image significantly affects diagnosis. Hard thresholding[16] is defined as:

$$\hat{w}_i = \begin{cases} w_i, & \text{if } |(w_i)| > \lambda, \\ 0, & \text{if } |(w_i)| \leq \lambda. \end{cases} \tag{7}$$

w_i - The original wavelet coefficient. Wavelet coefficients represent both noise and signal information. ώ- The thresholded wavelet coefficient after applying hard thresholding. λ - The threshold value.

It assumes that small coefficients are likely to be noise and sets them to zero. Coefficients above the threshold are considered significant and are kept unchanged.

2.2.2 Soft Thresholding

Soft thresholding reduces artifacts and produces smoother images compared to hard thresholding. But along with the noise components it may shrink the image coefficients also which can result in the loss of important signal features.

For a wavelet coefficient w_i and a threshold λ soft thresholding[16] is defined as:

$$w_i = \begin{cases} \text{sign}(w_i)((|w_i|) - \lambda), & \text{if } |(w_i)| > \lambda, \\ 0, & \text{if } |(w_i)| \leq \lambda. \end{cases} \tag{8}$$

sin(w_i)- checks if the wavelet is positive or negative and thus preserves the direction of the wavelet coefficient. If the value of the coefficient is greater than the threshold, the new coefficient is obtained by reducing the threshold value from the value of the coefficient and if it is less than the threshold coefficient will be set to zero. This method removes any abrupt changes which was a problem in hard thresholding. This helps in smoother transitions, thus reducing the artifacts and helping with better noise removal.

Soft thresholding is widely used in ultrasound image despeckling since it preserves the image's smoothness, which is crucial for clinical interpretation.[17]

2.2.3 Universal Thresholding(VisuShrink)

This method selects a threshold based on the noise level. It is a simple and effective technique for balancing noise reduction and feature preservation. Universal thresholding is effective in high-noise environments but can lead to over-smoothing in images with low noise, potentially obscuring important details[16].

$$\lambda = \sigma\sqrt{2\log N} \tag{9}$$

σ is the standard deviation of the noisy image. N is the number of wavelet coefficients. λ is the estimated threshold.

It is effective in despeckling images with high noise. But it can lead to over smoothening of the image. This will result in loosing fine details of the image which greatly affects the diagnostic power of medical ultrasound images [30][35].

2.2.4 BayesShrink

It's an adaptive thresholding. It considers noise as Gaussian and selects the threshold such that it minimizes Bayesian risk. It tries to balance noise reduction and detail preservation like edges and other fine details.[28].

$$\lambda = \frac{\sigma^2}{\hat{\sigma}_X} \tag{10}$$

$$\lambda = arg \min_{\lambda} \mathrm{SURE}(\lambda)$$

$$\mathrm{SURE}(\lambda) = N \quad 2|\{i:)w_i|\lambda\})+\sum_{i=1}^{N} min\left(w_i^2, \lambda^2\right)$$

In a study a new hybrid model is proposed which is a combinatiom of SRAD and Bayeshrink thresholding. Proposed method produces output which is cleaner, smoother and also it preserves edges and other fine details[12].

2.2.5 Adaptive Thresholding

Adaptive thresholding selects threshold values based on local image characteristics, allowing for more precise noise reduction in regions with varying noise levels. Adaptive thresholding is particularly useful in ultrasound imaging, where speckle noise can vary across different regions, requiring a more balanced approach to noise reduction [25].

$$\lambda(i) = f\left(w_i, \sigma\right) \tag{11}$$

In the study[7] the threshold value changes with each coefficient and they have proved that adaptive thresholds provide better despeckling compared to uniform thresholds. In another study [20] using adaptive thresholding, cuckoo search algorithm is used which they claimed to provide better despeckling results for clinical use.

2.3 Performance Evaluation Metrics

Performance metrics provides a quantifiable measurement to the results rather than providing a qualitative value. Multiple matrices are used since a single metric cannot effectively capture different aspects of performance of various algorithms [19][34][24]. The common performance matrices used in despeckling studies, which compares the original and despeckled images to analyze the performance of despeckling algorithms are discussed here.

2.3.1. Mean Square Error (MSE):

MSE measures the average squared difference between the original and despeckled images. A lower MSE indicates better similarity between the two images, as it represents smaller differences in pixel values. The Mean Squared Error (MSE) is given by:

$$MSE = \frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\left[I_{\text{original}}(i,j) - I_{\text{reconstructed}}(i,j)\right)^2 \qquad (12)$$

2.3.2 Universal Quality Index (UQI):

UQI is a measure of similarity between two images which considers the luminance and contrast of images. It takes mean, cross-correlation and standard deviation of the pixels of the original and despeckled images. A higher UQI indicates better similarity between the two images.

$$\text{UQI} = \frac{4.\ \mu_{I_{\text{original}}}.\ \mu_{I_{\text{reconstructed}}}.\ \sigma_{I_{\text{original}}.I_{\text{reconstructed}}}}{\left(\mu^2_{I_{\text{original}}} + \mu^2_{I_{\text{reconstructed}}}\right)\ \left(\sigma^2_{I_{\text{original}}} + \sigma^2_{I_{\text{reconstructed}}}\right)} \qquad (13)$$

$\mu_{I_{\text{original}}}$ and $\mu_{I_{\text{reconstructed}}}$ are the means of the original and reconstructed images, respectively.

$\sigma_{I_{\text{original}}}$ and $\sigma_{I_{\text{reconstructed}}}$ are the standard deviations.

$\sigma_{I_{\text{original}}}, I_{\text{reconstructed}}$ is the covariance between the original and reconstructed images.

2.3.3 Peak Signal-to-Noise Ratio (PSNR)

It is a measure that is used in assessing the quality of despeckled images . It is derived from the Mean Squared Error (MSE) value. A higher PSNR value indicates better quality of the methodology used and it shows that the despeckled image is closer to the original.

$$\text{PSNR} = 10.\log_{10}\left(\frac{\text{MAX}^2}{\text{MSE}}\right) \qquad (14)$$

where

MAX is the maximum pixel value of the image and MSE is the Mean Squared Error

2.3.4 Coefficient of Contrast (CoC):

CoC measures the contrast of the despeckled image relative to its mean intensity. It shows the linear correlation between the original and reconstructed images. It is calculated as the standard deviation of the despeckled image divided by its mean intensity. Higher CoC values indicate higher contrast.

The Correlation Coefficient (COC) is defined as:

$$\text{COC} = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\left(I_{\text{original}}(i,j) - \mu_{I_{\text{original}}}\right)\left(I_{\text{reconstructed}}(i,j) - \mu_{I_{\text{reconstructed}}}\right)}{\sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n}\left(I_{\text{original}}(i,j) - \mu_{I_{\text{original}}}\right)^2 \cdot \sum_{i=1}^{m}\sum_{j=1}^{n}\left(I_{\text{reconstructed}}(i,j) - \mu_{I_{\text{reconstructed}}}\right)^2}} \quad (15)$$

2.3.5 Edge Preservation Index (EPI):

EPI evaluates how well the edges in the original image are preserved in the reconstructed image. Higher EPI values indicate better preservation of edges.

The Edge Preservation Index (EPI) is given by:

$$\text{EPI} = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\nabla I_{\text{original}}(i,j) \cdot \nabla I_{\text{reconstructed}}(i,j)}{\sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n}\left(\nabla I_{\text{original}}(i,j)\right)^2} \cdot \sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n}\left(\nabla I_{\text{reconstructed}}(i,j)\right)^2}} \quad (16)$$

∇ represents the gradient of the image I

2.3.6 Structure Similarity Index (SSI):

SSI considers luminance, contrast and structure calculated to compare the patterns between the original and despeckled images.

The Structural Similarity Index (SSI) is calculated using the equation:

$$\text{SSI} = \frac{\left(2\mu_{I_{\text{original}}}\mu_{I_{\text{reconstructed}}} + C_1\right)\left(2\sigma_{I_{\text{original}},I_{\text{reconstructed}}} + C_2\right)}{\left(\mu^2_{I_{\text{original}}} + \mu^2_{I_{\text{reconstructed}}} + C_1\right)\left(\sigma^2_{I_{\text{original}}} + \sigma^2_{I_{\text{reconstructed}}} + C_2\right)}$$

(17)

Where C_1 and C_2 are constants used to stabilize the division.

2.3.7 Contrast-to-Noise Ratio (CNR):

CNR measures the difference in mean intensity between the original and despeckled images relative to the standard deviation of their pixel values.

The Contrast-to-Noise Ratio (CNR) is defined as:

$$\text{CNR} = \frac{\left(\left|\mu_{\text{signal}} - \mu_{\text{background}}\right|\right)}{\sigma_{\text{background}}} \quad (18)$$

where:

1. μ_{signal} is the mean signal intensity.

2. $\mu_{\text{background}}$ is the mean background intensity.

3. $\sigma_{\text{background}}$ is the standard deviation of the background noise.

2.3.8 Figure of Merit (FOM):

FOM provides a single numerical value representing the overall quality of the despeckled image. It is calculated as a function of MSE and UQI, with a higher FOM indicating better image quality.

The Figure of Merit (FOM) is given by

$$\text{FOM} = \frac{1}{max(N_1, N_2)} \sum_{i=1}^{N_2} \frac{1}{1 + d_i^2} \tag{19}$$

7. N_1 is the number of true edge points in the original image.

8. N_2 is the number of detected edge points in the reconstructed image.

9. d_i is the distance between the i-th detected edge point and the nearest true edge point.

3 Some of the despeckling studies using wavelets

By leveraging to the theory of wavelets in despeckling numerous innovative methods have been developed there by significantly advancing in the precision and effectiveness of despeckling techniques in ultrasound imaging. In a study [13] they proposed wavelet packet transform(WPT) and compare it with discrete wavelet transform(DWT). In DWT further decomposition is done in approximate coefficients only while in the new WPT based despeckling, detailed coefficients are also decomposed in each level. This leads to better results across various wavelet types, especially with hard thresholding. In another study [17] a new orthogonal family of wavelets called USI was designed. They compared the performance with other wavelet families of filters and also BM3D filter. The new filter outperforms other filters in terms of KMSE, PSNR, and β, the first two matrices are used for showing noise reduction and the third one shows edge preservation for low and median noise but for high noise levels BM3D shows better results.

In [36] a new direction-sensitive wavelet is designed, having better edge preservation as it follows the anisotropic scaling law of curvelets. In the paper [37] a novel multiscale nonlinear wavelet diffusion (MNWD) method for ultrasound speckle suppression and edge enhancement is introduced. The results are validated using synthetic and real time ECG images. The despeckled images are also tested in segmentation algorithm and observed better results. In [31] they proposed novel despeckling method for medical ultrasound images called bishrink which uses Hyperanalytic Wavelet Transform (HWT) with a Maximum a Posteriori (MAP) filter. They compared the results of the proposed algorithm with seven well known filters in despeckling and observed that the proposed filter gives the best values in terms of MSE, PSNR and SSIM.

In wavelet-based despeckling methodologies, innovation extends beyond the development of new wavelet strategies to include the creation of advanced thresholding techniques. The effectiveness of noise removal is heavily influenced by the choice of threshold, making it a critical factor in achieving optimal results. Consequently, significant research has been devoted to refining thresholding methods, leading to a range of sophisticated approaches designed to enhance despeckling performance. In [28] a new thresholding technique called BayesShrink is introduced which also does compression along with denoising. It performs better than SureShrink thresholding, which was a widely method accepted in terms of MSE value. In [1] a variant of the BayesShrink, of the previously discussed paper is developed. BayesShrink is a widely accepted thresholding methodology giving promising results. Several wavelet-based despeckling studies [26], [18], [12], [6] have successfully employed the BayesShrink method, underscoring its reliability. Building on this foundation, researchers have continued to explore adaptive thresholding techniques to further enhance optimal noise removal. A new adaptive threshold based on weighted variance is proposed in [32]. In [27] also weighted value is calculated as an adaptive thresholding technique. In [38] they designed a new improved threshold function to compensate for the deficiencies in hard and soft thresholding. In this method based on the frequencies of the wavelet coefficients, an adjustment factor will be calculated and the threshold is calculated based on it.

Usually in wavelet-based despeckling, thresholding is applied only to the detailed coefficient as suming of all the noise components are present in the detailed coefficients. But in real scenarios, the noise can also be present in the approximate coefficients. In the study[10] they proposed a hybrid filter where thresholding is applied to the detailed coefficients to remove the noise in the high-frequency components and an advanced Kuan filter is used to remove the noise in the approximate coefficient.

In another study [8] they proposed a double filter bank method where discrete wavelet transform is applied to the noisy image to convert the multiplicative speckle noise to additive noise. Out of 64 subbands produced by three level decomposition of wavelets, only few are selected using SVD technique. In this selected subbands different fuzzy based clustering algorithms like FCM, PCM, and PFCM are applied to remove noise from detailed components. Bilateral filter is applied to low-frequency components to remove noise in the approximate components. Then detailed and approximate coefficients are rejoined and inverse log transform is taken to obtain the despeckled image.

In the study [2] non-local means filter is applied in the wavelet domain for noise removal and observed better results. A hybrid filter [11], Daubechies-Weiner provides better despeckling results than the individual wavelet filters.

Comparative analysis of various despeckling algorithms done in [3],[29],[21]

3. Conclusion

Wavelet-based despeckling has emerged as a powerful approach for despeckling. The comparative studies reviewed here highlight the significant progress made in both the design of new wavelet families, such as WPT and USI, and the refinement of thresholding methods like BayesShrink and adaptive thresholds. These advancements have led to improvements in noise reduction while preserving edge and other fine details in the image and thus preserving overall image quality. The integration of hybrid and double filter bank approaches, as well as the exploration of non-local means filtering, represent promising directions for further enhancing despeckling performance.

However, wavelets have limitations, particularly in capturing anisotropic features and complex structures within images. To address these drawbacks, other transformed domain techniques such as contourlet, shearlet, curvelet, and directionlet transforms can be used. These methods offer a more flexible and efficient representation of multidimensional data, capturing directional and geometric features more effectively than traditional wavelets. As a future study, we plan to concentrate on curvelet transforms in despeckling due to their superior ability to represent edges and other geometrical structures in images.

References

1. S. Gupta I R.C. Chauhan I S.C. Sexana 2. Wavelet-based statistical approach for speckle reduction in medical ultrasound images. Medical and Biological Engineering and Computing, 42, 2004.
2. Adib Akl and Charles Yaacoub. A hybrid filter for image despeckling with wavelet-based denoising and spatial filtering. In2013 Third International Conference on Communications and Information Technology (ICCIT), pages 325–329, 2013.
3. K V Archana and R. Vanithamani. A comparison on speckle noise suppression algorithm for intravascular ultrasound (ivus) images. Indian Journal of Computer Science and Engineering (IJCSE), 12:634–640, 2021.
4. Nishtha Attlas and Dr. Sheifali Gupta. Wavelet based techniques for speckle noise reduction in ultrasound images. Journal of Engineering Research and Applications, 4, 2014.
5. Anterpreet Kaur Bedi and Ramesh Kumar Sunkaria. Ultrasound speckle reduction using adaptive wavelet thresholding. Multidim Syst Sign Process, 33:275–300, 2021.
6. Mantosh Biswas and Hari Om. An image denoising threshold estimation method. Advances in Computer Science and its Applications, 2:0975 – 8887, 2013.
7. S Grace Chan, Bin Y, and Martin Vetterl. Adaptive wavelet thresholding for image denoising and compression. IEEE TRANSACTIONS ON IMAGE PROCESSING, VOL. 9, NO. 9, SEPTEM-BER 2000l, 9(9), 2000.
8. Vimalraj Chinnathambi1, Esakkirajan Sankaralingam, Veerakumar Thangaraj, and Sreevidya Padma4. Despeckling of ultrasound images using directionally decimated wavelet packets with adaptive clustering. IET Image Processing, 2018.
9. Sonia Dahdouh, Mathieu Dubois, Emmanuelle Frenoux, and Angel Osorio. A 1D wavelet filtering for ultrasound images despeckling. In Jan D'hooge and Stephen A. McAleavey, editors, Med-ical Imaging 2010: Ultrasonic Imaging, Tomography, and Therapy, volume 7629, page 762907. International Society for Optics and Photonics, SPIE, 2010.
10. China Debarghya, Mitra Pabitra, Chakraborty Chandan, and K. M. Mandana. Wavelet based non local means filter for despeckling of intravascular ultrasound image. In 2015 International Conference on Advances in Computing, Communications and Informatics (ICACCI), pages 1361– 1365, 2015.
11. Irraivan Elamvazuthi, Muhammad Luqman Bin Muhd Zain, and K.M. Begam. Despeckling of ultrasound images of bone fracture using multiple filtering algorithms. Mathematical and Com-puter Modelling, 57(1):152–168, 2013. Mathematical and Computer Modelling in Power Control and Optimization.
12. Karthikeyan K and Chandrasekar C. Speckle noise reduction of medical ultrasound images using bayesshrink wavelet threshold. International Journal of Computer Application, 22(9):8–14,

2011.
13. P. Kalavathi, S. Boopathiraja, , and M. Abinaya. Despeckling of ultrasound medical images using dw and wp transform techniques. International Journal of Engineering and Technology (IJET), 9, 2017.
14. Parvinder Kaur and Baljit Singh. A multiresolution technique to despeckle ultrasound images. In High Performance Architecture and Grid Computing, pages 391–396, Berlin, Heidelberg, 2011. Springer Berlin Heidelberg.
15. Ashish Khare, Manish Khare, Yongyeon Jeong, Hongkook Kim, and Moongu Jeon. Despeckling of medical ultrasound images using daubechies complex wavelet transform. Signal Processing, 90(2):428–439, 2010.
16. Donoho David L. and Johnstone Iain M. Ideal spatial adaptation by wavelet shrinkage. Biometrika, 81(3):425–455, 1994.
17. Andr´eia Seixas Leal and Henrique Mohallem Paiva. A new wavelet family for speckle noise reduction in medical ultrasound images. Measurement, 140:572–581, 2019.
18. Ikeda M, Makino R., and Imai K. A new evaluation method for image noise reduction and usefulness of the spatially adaptive wavelet thresholding method for ct images. Australas Phys Eng Sci Med, 35:475–483, 2012.
19. Sameera V. Mohd Sagheer and Sudhish N. George. A review on medical image denoising algo-rithms. Biomedical Signal Processing and Control, 61:102036, 2020.
20. WangNo N, Chiewchanwattana S, and Sunat K. An efficient adaptive thresholding function optimized by a cuckoo search algorithm for a despeckling filter of medical ultrasound images. Journal of Ambient Intelligence and Humanized Computing, 2020.
21. Sima Navabian, Fereshteh Yousefi Rizi, and Zahra Alizadeh Sani. A comparative study on de-speckling techniques in intravascular ultrasound images. Signal Processing and Renewable Energy, 2019.
22. Om Prakash, Manish Khare, and Ashish Khare. Image denoising technique based on soft thresh-olding of biorthogonal wavelet coefficients.National Conference on Advances in Mobile Commu-nications, Networking and Computing, pages 27–28, 2013.
23. Ayu Jati Puspitasari, Ika Cismila Ningsih, Muhammad Sulthonur Ridwan, and Halim Hamadi. Planar scintigraphy image de-noising using coiflet wavelet. Pusat Sains dan Teknologi Akselerator (PSTA) - Badan Tenaga Nuklir Nasional (BATAN), 24, 2021.
24. Anterpreet Kaur Bedi Ramesh Kumar Sunkaria. A review on various ultrasound image despeckling techniques. Conference Paper ·January 2021, 2021.
25. Simarjot Kaur Randhawa, Ramesh Kumar Sunkaria, and Emjee Puthooran. Despeckling of ul-trasound images using novel adaptive wavelet thresholding function. Multidimensional Systems and Signal Processing, 30:1545–1561, 2019.
26. Hashemi SayedMasoud and Beheshti Soosan. Adaptive image denoising by rigorous bayesshrink thresholding. In 2011 IEEE Statistical Signal Processing Workshop (SSP), pages 713–716, 2011.
27. Ms. Alka Vishwa Ms. Shilpa Sharma. Speckle noise reduction in ultrasound images by wavelet thresholding. Computer Science and Software Engineering, 2, 2012.
28. Chang Shih-Chang, Yu Bin, and Vetterli Martin. Adaptive wavelet thresholding for image de-noising and compression. IEEE Transactions on Image Processing, 9(9):1532–1546, 2000.
29. Palwinder Singh. Speckle reduction techniques for ultrasound images. International Journal of Computer Techniques, 3, 2016.
30. SoodvDeepika, Kaur Anureet, and Adhikary Kaushik. A survey on despeckling methods. Inter-national Journal of Emerging Technology and Advanced Engineering, 3(7), 2013.
31. Cristina Stolojescu-Crisan. A hyperanalytic wavelet based denoising technique for ultrasound images. IWBBIO 2015, 2, 2015.
32. S Sudha G R Suresh and R Sukanesh. Speckle noise reduction in ultrasound images by wavelet

thresholding based on weighted variance. International Journal of Computer Theory and Engineering, 1, 2009.
33. Solbo S Eltoft T. Homomorphic wavelet-based statistical despeckling of sar images. IEEE Trans Geosci Remote Sens, 42:711–721, 2004.
34. Dr.Seema Kawathekar Vaishali Kumbhakarna, Vijaya R.Patil. Review on speckle noise reduction techniques for medical ultrasound image processing. International Journal of Computer Tech-niques, 2:102036, 2015.
35. R Vanithamani and G Umamaheswari. Wavelet thresholding techniques in despeckling of medical ultrasound images. Int. J. Eng. Technol.(IJET), 5(6):5098–5106, 2014.
36. C. Vimalraj, S. Esakkirajanand Thangaraj Veerakumar, and P. Sreevidya. Direction sensitive wavelet packet for despeckling of ultrasound images. 2016.
37. Yong Yue, Mihai M Croitoru, Akhil Bidani, Joseph B Zwischenberger, and Jr John W Clark. Ultrasound speckle suppression and edge enhancement. Engineering in Medicine and Biology 27th Annual Conference, 2005.
38. Yang Zhang, Weifeng Ding, and Zhifang Panand Jing Qin. Improved wavelet threshold for image de-noising. Frontiers in Neuroscience, 2019.