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On Ms -Connectedness and One Point Compactification in Minuscule Topological spaces

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The main purpose of this paper to propose the notions of connectedness, compactness and One point Compactification in Ms -topological spaces. There is also an attempt to define M-Hausdroff, M-lindelof space, Minuscule connectedness, Minuscule compactness and M-One point Compactification

Keywords: Minuscule Compactness, Minuscule connectedness, M-Hausdroff space, M-lindelof space, Minuscule One point Compactification..

1. Introduction

In general topology, the concepts of compactness and their characteristics are extensively studied. David A. Rose and Hamlett T.R. [11] introduced the concept of one point I compactification in 1992. The concept of Minuscule topology was first developed in 2023 by R. Alagar et.cl [18]. It was described as the symmetric difference, with respect to an equivalence relation on it, of a subset of the universe, along with approximations. A novel family of functions known as Ms -top.spaces and related characterizations for continuous functions were already investigated . The notions of minuscule compactness and one-point compactification are represented in terms of Minuscule compactness and M-one-point compactification, have been introduced in this study.

2. Preliminary

Let us discuss the definitions that will be helpful in the sequel.

Definition 2.1 (18). *Suppose a non-empty finite set* U *containing components known as the universe. Let* E *be an equivalence relation on* U, *commonly referred to as the indiscernibility relation. After that, the set* U *is divided into disjoint equivalence classes.*

Elements in the same equivalence class are seen as indiscernible from other elements. The pair (U, E) *is often referred to as the approximation space in academic works. Let* H *be a subset of* U.

1. *The lower approximation of the set* H *with respect to the relation* E *refers to the collection of objects that can be unambiguously categorized as belonging to* H *with respect to the conditions defined by* E. *This lower approximation can be expressed as* $L_E(H)$. *That is,*

$$L_E(H) = \bigcup_{x \in U} \{E(H) : E(H) \subseteq H\}$$

where E(H) *denotes the equivalence class determined by H*.

2. *The lower minimal approximation:*

$$L_E^{\Lambda}(H) = \bigcup_{x \in U} \{E(H) : E(H) \subseteq H\} - H = L_E(H) - H$$

3. *The upper approximation of* H *with regard to* E *is the set of all objects that can be classified as* H *with respect to* E. *and it is highlighted by* $U_E(H)$. *That is,*

$$U_E(H) = \bigcup_{x \in U} \{E(H) : E(H) \bigcap H \neq \Phi\}$$

4. *The upper minimal approximation:*

$$U_E^{\Lambda}(H) = \bigcup_{x \in U} \{E(H) : E(H) \bigcap H \neq \Phi\} - H = U_E(H) - H.$$

5. *Let* $L_E(H)$ *and* $UE^{\Lambda}(H)$ *be two sets. The symmetric difference of the sets* $L_E(H)$ *and* $UE^{\Lambda}(H)$ *is* $L_E(H)\Delta UE^{\Lambda}(H)$ *and it is highlighted by,*

$$L_E(H)\Delta U_E^{\Lambda}(H) = (L_E(H) - U_E^{\Lambda}(H)) \bigcup (U_E^{\Lambda}(H) - L_E(H))$$

Definition 2.2.[18] *Let* U *be the universe,* E *be an equivalence relation* U *and* $\urcorner_E(H) = \{U, \Phi, L_E(H), U_E(H), L_E{}^{\Lambda}(H), U_E{}^{\Lambda}(H), L_E(H) \nabla U_E{}^{\Lambda}(H)\}$. *Here* $LE^{\Lambda}(H)$ *is always be* Φ. Φ *is always within the topology. So,* $LE^{\Lambda}(H)$ *and it is ignored. Then the topology* $\natural_E(H) = \{U, \Phi, L_E(H), U_E(H), UE^{\Lambda}(H), L_E(H) \nabla UE^{\Lambda}(H)\}$ *where* $H \subseteq E$. $\natural_E(H)$ *satisfies the subsequent axioms:*

1. U *and* $\Phi \in \natural_E(H)$.

2. *The union of the elements of any sub collection of* $\natural_E(H)$ *is in* $\natural_E(H)$.

3. *The intersection of all elements of any finite sub collection of* $Ɥ_E(H)$ *is in* $Ɥ_E(H)$. *That is,* $Ɥ_E(H)$ *is a topology on* U *called the* Minuscule *topology on* U *with respect to* H. *We call* $(U, Ɥ_E(H))$ *is a* Ms*-top.space it is highlighted by* Ms*-top.space* . *The elements of* $Ɥ_E(H)$ *are called* Minuscule *opensets and it is denoted by* Mso.

Definition 2.3. *If* $(U, Ɥ_E(H))$ *is a* Ms*-top.space ,where* $H \subseteq U$ *and if* $A \subseteq U$, *The* Minuscule *interior of the set* A *is* M*int(A), which is the union of all* M-*open* *subsets of* A. *The concept of the set* M-*closure is defined as the intersection of all* Msc*sets that contain* A. *The* M-*closure is represented by* M*cl*(A).

Properties:[18] If (U, E) is an approximation space and $H, W \subseteq U$, then

1. $L_E(H) \subseteq H \subseteq U_E(H)$.
2. $L_E(\phi) = U_E(\phi) = \phi$ and $L_E(H) = U_E(H) = U$
3. $L\,E^{\Lambda}(H) = \phi$
4. $U_E(H \cap W) \subseteq U_E(H) \cap U_E(W)$
5. $U_E^{\wedge}(H \cup W) \subseteq U_E^{\wedge}(H) \cup U_E^{\wedge}(W)$
6. $L_E(H \cap W) = L_E(H) \cap L_E(W)$
7. $U_E(U_E^{\wedge}(H)) = L_E(U\,E^{\wedge}(H)) = U\,E^{\wedge}(H)$.
8. $U_E(H \cup W) = U_E(H) \cup U_E(W)$
9. $L\,E^{\wedge}(H) \cap U_E(H) = \phi$
10. $L_E(H \cup W) \supseteq L_E(H) \cup L_E(W)$
11. $UE^{\Lambda}(H \cap W) = UE^{\Lambda}(H) \cap UE^{\Lambda}(W)$
12. $L_E(H) \subseteq L_E(W)$ and $U_E(H) \subseteq U_E(W)$ whenever $H \subseteq W$
13. $L\,E^{\wedge}(H)$ ⊽ $U_E(H) = U_E(H)$.

Example 2.4. *Let* $U = \{\varpi_{ax}, \varpi_{bx}, \varpi_{cx}, \varpi_{dx}\}$ *with* $U/E = \{\{\varpi_{ax}\}, \{\varpi_{bx}, \varpi_{cx}\}, \{\varpi_{dx}\}\}$

Let $H = \{\varpi_{ax}, \varpi_{cx}\} \subseteq U$ *Then,* $Ɥ_E(H) = \{U, \phi, \{\varpi_{ax}\}, \{\varpi_{ax}, \varpi_{bx}, \varpi_{cx}\}, \{\varpi_{bx}\}, \{\varpi_{ax}, \varpi_{bx}\}\}$

and the M−*closed sets in* U *are* $U, \phi, \{\varpi_{bx}, \varpi_{cx}, \varpi_{dx}\}, \{\varpi_{dx}\}, \{\varpi_{ax}, \varpi_{cx}, \varpi_{dx}\}$, *and* $\{\varpi_{cx}, \varpi_{dx}\}$.

3. Minuscule connectedness

Definition 3.1. *A* Ms*-top.space* $(U, Ɥ_E(H))$ *is stated to be minuscule connected if* $(U, Ɥ_E(H))$ *cannot be expressed as a disjoint union of two* $\neq \phi$ Mso. *A subset of* $(U, Ɥ_E(H))$ *is minuscule connected as a subspace and it is highlighted as* Ms-*contd.*

A subset is said to be minuscule discontd iff it is not Ms-*contd.*

Example 3.2. *Let* $U = \{\varpi_a, \varpi_b, \varpi_c, \varpi_d\}, X = \{\varpi_a, \varpi_d\} \subset U$ *and* $U/R = \{\{\varpi_a\}, \{\varpi_b\}, \{\varpi_c\}, \{\varpi_d\}\}$ *with* Ms-*top.space* ɥ$_E$(H) = $\{U, \phi, \{\varpi_a, \varpi_d\}\}$ *then it is* Ms-*contd.*

Theorem 3.3. *For a* Ms-*top.space* (U, ɥ$_E$(H)) *the subsequent are equivalent*

(i) (U, ɥ$_E$(H)) *is* Ms-*contd*

(ii) (U, ɥ$_E$(H)) *and* ϕ *are the only subsets of* U *which are both* Mso *and minuscule closed it is marked as* Msc.

(iii) *Every map that is minuscule continuous it is highlighted as* Ms-*conts* (U, ɥ$_E$(H)) *and has two points or more in discrete space* (V, ɥ′$_E$(I)) *is a constant map.*

Proof. (1) $\Rightarrow$(2) Let G be a Mso and Msc subset of (U, ɥ$_E$(H)). Then Z − G is also both Mso and Msc. Then Z = G $\cup$(Z − G) a disjoint union of two ≠ ϕ Mso which contradicts the fact that (U, ɥ$_E$(H)) is Ms-contd. Hence G= ϕ or Z.

(2) $\Rightarrow$(1) suppose that Z = $J \cup K$ where J and K are disjoint ≠ ϕ Mso subsets of (U, ɥ$_E$(H)). Since J = Z − K, then J is both Mso and Msc.

By assumption $J = \phi$ or Z, which is a contradiction. Hence (U, ɥ$_E$(H)) is Ms-contd.

(2) $\Rightarrow$(3) Let φ: (U, ɥ$_E$(H)) → (V, ɥ′$_E$(I)) be a Ms-conts map where (V, ɥ′$_E$(I)) is discrete space with atleast two points. Then $\varphi(\{\acute{y}\})$ is Msc and Mso for each $\acute{y} \in Y$. That is, (U, ɥ$_E$(H)) is covered by Msc and Mso covering $\{\varphi(\{\acute{y}\}): \acute{y} \in Y\}$. By assumption, $\varphi(\acute{y}) = \phi$ *or*Z for each $\acute{y} \in Y$. If $\varphi^{-1}(\acute{y}) = \phi$ for each $\acute{y} \in Y$. Then φ fails to be map. Therefore $\exists$ atleast one point $\varphi^{-1}(\{\acute{y}\}) = \phi, \acute{y} \in Y$ such that, $\varphi^{-1}(\{\acute{y}\})$ = Z. It is evident from this φ is a constant map.

(3) $\Rightarrow$(2) Let G be both Mso and Msc in (U, ɥ$_E$(H)). Suppose G ≠ ϕ. Let φ: (U, ɥ$_E$(H)) → (V, ɥ′$_E$(I)) be a Ms-conts map defined by φ(G) = $\{a\}$ and φ(Z − G) = $\{b\}$ where $a \neq b$ and $a, b \in Y$. By assumption, φ is constant so G = Z. □

Theorem 3.4. *If* φ: (U, ɥ$_E$(H)) → (V, ɥ′$_E$(I)) *is* Ms-*conts surjection and* Z *is* Ms-*contd, then* Y *is* Ms-*contd.*

Proof. Suppose that Y is not Ms-contd. Let $Y = J \cup K$ where J and K are disjoint ≠ ϕ open sets in (V, ɥ′$_E$(I)). Since φ' is Ms-conts and onto. Z = $\varphi^{-1}(J) \cup \varphi^{-1}(K)$ where $\varphi^{-1}(J)$ and $\varphi^{-1}(K)$ are disjoint ≠ ϕ Mso subsets in (U, ɥ$_E$(H)). In contrary to it, this (U, ɥ$_E$(H)) is Ms-contd. Hence (V, ɥ′$_R$(I)) is Ms-contd. □

Theorem 3.5. *If* φ *is* Ms-*conts mappings of a* Ms-*contd space* (U, ɥ$_E$(H)) *onto an arbitrary* *top.space* (V, ɥ′$_E$(I)) *is* Ms-*contd.*

Proof. Let (V, ɥ′$_E$(I)) be a Ms-contd. Then $\exists$ a $\neq \phi$ proper subset G of (V, ɥ′$_E$(I)) which is both Mso and Msc in (V, ɥ′$_E$(I)). Since φ is Ms-conts and onto (V, ɥ′$_E$(I)), φ^{-1} (G) is $\neq \phi$ proper subset of (U, ɥ$_E$(H)) which is both Mso and Msc in (U, ɥ$_E$(H)) and therefore, (U, ɥ$_E$(H)) is discontd. which is a contradiction. Hence (V, ɥ′$_E$(I)) must be contd. □

Theorem 3.6. *A* Ms-*top.space* (U, ɥ$_E$(H)) *is* Ms-*contd iff every* $\neq \phi$ *proper subset of U has a* $\neq \phi$ *frontier.*

Proof. Let every $\neq \phi$ proper subset of (U, ɥ$_E$(H)) have a $\neq \phi$ frontier. To show that U is Ms-contd. If, U is Ms-contd. Then $\exists \neq \phi$ disjoint sets I and K both are Mso and Msc in U such that $U = I \cup K$. Therefore $I' = I^0 = \bar{I}$ but $Fr(I) = \bar{I} - I^0$. Hence $Fr(I) = \phi$ this contradicts our hypothesis. Hence U must be Ms -contd. Conversely, Let U be contd, if $\exists$ a$\neq \phi$ proper subset D of U such that $Fr(D) = \phi$. Now $\underline{D} = D^0 \cup Fr(D) = D \cup Fr(\underline{D})$. Thus, every proper subset of U must have a $\neq \phi$ frontier. □

4. Minuscule Compactness

Definition 4.1. *A collection* $\{Q_j : j \in \mathrm{J}\}$ *of* Mso *sets in a* Ms-*top.space* (U, ɥ$_E$(H)) *is called a* Mso *cover of subset* B *of* U *if* B $\subset \{Q_j : j \in \mathrm{J}\}$ *holds.*

Definition 4.2. *A subset* B *of a* Ms -*top.space* (U, ɥ$_E$(H)) *is stated to be* Ms *compact relative to* (U, ɥ$_E$(H)), *if for every collections* $\{Q_j : j \in \mathrm{J}\}$ *of* Mso *subsets of* (U, ɥ$_E$(H)) *such that* B $\subset \{Q_j : j \in \mathrm{J}\}$ $\exists$ *a finite subset* I_0 *of* **I** *such that* B $\subset \{Q_j : j \in \mathrm{J}_0\}$.

Definition 4.3. *A subset* B *of a* Ms -*top.space* (U, ɥ$_E$(H)) *is said to be minuscule compact and it is highlighted as* Ms -*compt. if* B *is* Ms -*compt. as a subspace of* (U, ɥ$_E$(H)) .

Theorem 4.4. *A* Msc *subset of* Ms -*compt. space* (U, ɥ$_E$(H)) *is* Ms -*compt. relative to* (U, ɥ$_E$(H)).

Proof. Let Q be a Ms -compt. subset of a Ms -top.space (U, ɥ$_E$(H)). Then Q^c is Mso in (U, ɥ$_E$(H)). let $S = \{Q_j : j \in \mathrm{J}\}$ be an Mso cover of Q by Mso subsets in (U, ɥ$_E$(H)). Then $S^* = S \cup Q^c$ is a Mso cover of (U, ɥ$_E$(H)). That is U = $(\cup_{j \in \mathrm{J}} Q_j) \cup Q^c$. By hypothesis (U, ɥ$_E$(H)) is Ms -compt. and hence S^* is reducible to a finite sub cover of (U, ɥ$_E$(H)) say U = $Q_{j1} \cup Q_{j2} \cup \ldots \ldots \cup Q_{jn} \cup Q^c, Q_{jk} \in S^*$. Thus, a Mso cover S of Q contains a finite sub cover. Hence Q is Ms-compt. relative to (U, ɥ$_E$(H)).

Theorem 4.5. *A* Ms-*top.space* (U, ɥ$_E$(H)) *is* Ms-*compt. iff every family of* Msc *sets of* (U, ɥ$_E$(H)) *having finite intersection property has a* $\neq \phi$ *intersection.* □

Theorem 4.6. *The image of a* Ms-*compt. space under a* Ms-*conts map is* Ms- *compt.*

Proof. Let ϕ: (U, ɥ$_E$(H)) $\Rightarrow$ (V, ɥ$_E$(Q)) be a Ms -conts map from a Ms -compt. space (U,

$ɥ_E(H))$ onto a Ms -top.space $(V, ɥ_E(Q))$. Let $\{Q_j: j \in J\}$ be an Mso cover of $(V, ɥ_E(Q))$. Then $\{h^{-1}(Q_j): j \in j\}$ is a Mso cover of $(U, ɥ_E(H))$.

Since h is Ms -conts. As $(U, ɥ_E(H))$ is Ms -compt., the Mso cover $\{h^{-1}(Q_j): j \in J\}$ of $(U, ɥ_E(H))$ has a finite sub cover $\{h^{-1}(Q_j): j = 1, 2, 3 ... n\}$. There- fore $U = \cup_{j \in J} h^{-1}(Q_j)$. Then $h(X) = \cup_{j \in J} Q_j$, that is $V = \cup_{j \in J}(Q_j)$. Thus, $\{Q_1, Q_2, ... Q_n\}$ is a finite sub cover of $\{Q_j: j \in J\}$ for $(V, ɥ_E(Q))$. Hence, $(V, ɥ_E(Q))$ is Ms -compt. □

Definition 4.7. *A* Ms -*top.space* $(U, ɥ_E(H))$ *is countably* Ms -*compt. if every countable* Mso *cover of* $(U, ɥ_E(H))$ *has a finite sub cover.*

Theorem 4.8. *Let* $(U, ɥ_E(H))$ *be a* Ms -*top.space and* $(V, ɥ_E(Q))$ *be a* M − Hausdroff. *If* $h: (U, ɥ_E(H)) \rightarrow (V, ɥ_E(Q))$ *is* Ms -*conts injective, then* $(U, ɥ_E(H))$ *is* M − Hausdroff.

Proof. Let H and Y be any two distinct points of $(U, ɥ_E(H))$. Then $h(H)$ and $h(y)$ are distinct points of $(V, ɥ_E(Q))$, because h is injective. Since $(V, ɥ_E(Q))$ is M − Hausdroff, there are disjoint Mso sets J and K in $(V, ɥ_E(Q))$ containing $h(H)$ and $h(Y)$ resp. Since h is Ms -conts and $J \cap K = \phi$, we have $h^{-1}(J)$ and $h^{-1}(K)$ are disjoint Mso sets in $(U, ɥ_E(H))$ such that $x \in h^{-1}(J)$ and $y \in h^{-1}(K)$.

Hence $(U, ɥ_E(H))$ is M − Hausdroff. □

Theorem 4.9. *If* $h(U, ɥ_E(H)) \rightarrow (V, ɥ_E(Q))$ *is* Ms-*conts and bijective and if* U

is Ms-*compt. and* V *is* Hausdroff, *then h is a* M-*homeomorphism.*

Proof. It is obvious from the theorem 4.7&4.8. K is Ms-compt. Since V is

M − Hausdroff space implies that h(Q) is Msc in $(V, ɥ_R{}'(Y))$. □

Definition 4.10. *A* Ms -*top.space* $(U, ɥ_E(H))$ *is stated to be* Ms −*lindelof space if every* Mso *cover of* $(U, ɥ_E(H))$ *has a countable sub cover.*

Theorem 4.11. *Every* Ms-*compt. space is a* Ms − *lindelof space.*

Proof. Let $(U, ɥ_E(H))$ be Ms -compt. Let $\{Q_j: j \in J\}$ be Mso cover of $(U, ɥ_E(H))$. Then $\{Q_j: j \in J\}$ has a finite sub cover $\{Q_j: j = 1, 2, ... n\}$, since $(U, ɥ_E(H))$ is Ms -compt. Since every finite sub cover is always a countable sub cover and therefore, $\{Q_j: j = 1, 2, ... n\}$, is countable sub cover of $\{Q_j: j \in J\}$ for $(U, ɥ_E(H))$. Hence $(U, ɥ_E(H))$ is Ms -lindelof space. □

Theorem 4.12. *The image of* Ms − *lindelof space under a* Ms-*conts map is*

Ms -*compt.*

Proof. $h: (U, ɥ_E(H)) \rightarrow (V, ɥ_E(Q))$ be a Ms -conts map from a Ms − *lindelof* space $(U, ɥ_E(H))$ onto a Ms -top.space $(V, ɥ_E(Q))$. Let $\{Q_j: j \in J\}$ be an Mso cover of $(V, ɥ_E(Q))$, then $\{h^{-1}(Q_j): j \in J\}$ be an Mso cover of $(U, ɥ_E(H))$, since h is Ms -conts. As $(U, ɥ_E(H))$ is Ms−*lindelof*, the Mso cover $\{h^{-1}(Q_j): j \in J\}$ of $(U, ɥ_E(H))$ has a countable sub cover $\{h^{-1}(Q_j,) j = 1, 2, ... n\}$. Therefore $H = \cup_{j \in J} h^{-1}(Q_j)$ which implies $f(U) = V = \cup_{j \in J} Q_j$, that is $\{Q_1, Q_2, Q_3, \dots \dots \dots Q_n\}$ is a countable sub family of $\{Q_j: j \in J\}$ for $(V, ɥ_E(Q))$. Hence $(V, ɥ_E(Q))$ is Ms − *lindelof* space. □

Theorem 4.13. *If* (U, ɥ$_E$(H)) *is* Ms − *lindelof space and Countably* Ms -*compt. space, then* (*V*, ɥ$_E$) *is* Ms -*compt.*

Proof. Suppose, (U, ɥ$_E$(H)) is Ms − *lindelof* and countably Ms -compt. space. Let $\{Q_j: j \in I\}$ be an Mso cover of (U, ɥ$_E$(H)). Since (U, ɥ$_E$(H)) is Ms − *lindelof* $\{Q_j: j \in J\}$ has a countable sub cover $\{Q_{in}: n \in N\}$. Therefore

$\{Q_{in}: n \in N\}$ is a countable sub cover of (U, ɥ$_E$(H)) and $\{Q_{in}: n \in N\}$ is subfamily of $\{Q_j: j \in J\}$ and so $\{Q_{in}: n \in N\}$ is a countable Mso cover of (U, ɥ$_E$(H)). Again since (U, ɥ$_E$(H)) is countably Ms -compt., $\{Q_{in}: n \in N\}$ has a finite sub cover $\{Q_{jk}: k = 1, 2, ...n\}$. Therefore $\{Q_{jk}: k = 1, 2, ...n\}$ is a finite sub cover of $\{Q_j: j \in J\}$ for (U, ɥ$_E$(H)). Hence (U, ɥ$_E$(H)) is Ms -compt. space.

□

Theorem 4.14. *A* Ms -*top.space* (U, ɥ$_E$(H)) *is* Ms -*compt. iff every basic* Mso *cover of* (U, ɥ$_E$(H)) *has a finite sub cover.*

Proof. Let (U, ɥ$_E$(H)) be Ms -compt. then every Mso cover of (U, ɥ$_E$(H)) have a finite sub cover. Conversely, Suppose that every basic Mso cover of (U, ɥ$_E$(H)) has a finite sub cover and let $C = \{G_\delta: \delta \in \Psi\}$ be any Mso cover of (U, ɥ$_E$(H)). If $K = \{D_\gamma: \gamma \in \Delta\}$ be any Mso base for (U, ɥ$_E$(H)), then, every G_δ represents the union of a subset of *K* members, and the total of all these members of *K* is clearly

a basic Mso cover of (U, ɥ$_E$(H)) By hypothesis this collection of *K* members has a finite sub cover, $\{D_{\delta i}: i = 1, 2...n\}$ for each $D_{\delta i}$ in this finite sub cover, we can select a G_δ from *C*. Such that $D_{\gamma i} \subset G_{\delta i}$. It follows that the finite sub collection

$\{G_{\delta i}: i = 1, 2, 3...n\}$. which arises in this way is a sub cover of *C*. Hence (U, ɥ$_E$(H)) is Ms -compt.

□

5. Minuscule One-point Compactification

Definition 5.1. *A* Ms -*top.space* (U, ɥ$_E$(H)) , x ∈ H *we denote it by* ɥ$_E$ = {*v* ∈ ɥ$_E$: x ∈ U}. *A space* J ⊆ H *is called a neighbourhood of* x *if* ∃ U ∈ ɥ$_E$ *such that* x ∈ U ⊆ *A*.

Definition 5.2. *A* M - Hausdroff *space* (U, ɥ$_E$(H)) *is stated to be locally* Ms - *compt. iff* (U, ɥ$_E$(H)) *is locally* M- H *closed abbreviated as* MHC.

Definition 5.3. *A* Ms -*top.space* (U, ɥ$_E$(H)) *is stated to be* M − H *closed iff it is* M − H *and quasi* M − H *closed* (QMHC).

Definition 5.4. *A* Ms -*top.space* (U, ɥ$_E$(H)) *is claimed as strongly locally* Ms - *compt. if each point in* H *has a* Ms -*compt. neighbourhood.*

Definition 5.5. *A* Ms -*top.space* (U, ɥ$_E$(H)) *is stated to be quasi* M - H *closed abbreviated as* QMHC *iff A finite subcollection of each open cover of* H *covers a dense subset of* H.

Definition 5.6. *A* M - Hausdroff *space* (U, ɥ$_E$(H)) *is considered to be locally* M - *H closed if each point in* H *has a neighbourhood which is* M - *H closed on a subspace of* (U,

$ɥ_E(H))$.

Definition 5.7. *A* Ms-*top.space* (G, Ω) *is considered to be a compactification of* $(H, ɥ)$ *iff*

1. $H \subseteq G$,
2. $ɥ = \Omega|H = \{W \cap H: W \in \Omega\}$, *and*
3. (G, Ω) *is* Ms-*compt.*

If, in addition, we have

4. $MCl_{\Omega}(H) = G$,

Then (G, Ω) *is said to be a* Ms-*compt. extension of* $(H, ɥ)$. *Furthermore, if* $G - H = \{r\}$, *then the* M- *top.space* (G, Ω) *is said to be a one-point compactification (or* Ms-*compt. extension) of* $(H, ɥ)$.

Example 5.8. *Let* $Y = \{\varpi_{a1}, \varpi_{a2}, \varpi_{a3}, \varpi_{a4}\}$, $X = \{\varpi_{a1}, \varpi_{a2}, \varpi_{a3}\}$ *with* $R = \{\{\varpi_{a1}\}, \{\varpi_{a2}, \varpi_{a3}, \varpi_{a4}\}\}$. *and* $U = \{\varpi_{a1}, \varpi_{a2}\}$, *Then* $ɥ_{(H)} = \{X, \phi, \{\varpi_{a1}\}, \{\varpi_{a1}, \varpi_{a3}\}, \{\varpi_{a3}\}\}$ *and* $\Omega_{(Y)} = \{Y, \phi\{\varpi_{a1}\}, \{\varpi_{a1}, \varpi_{a3}, \varpi_{a4}\}, \{\varpi_{a3}, \varpi_{a4}\}$. *hence the* M- *closed sets in* Y *are* $Y, \phi, \{\varpi_{a2}, \varpi_{a3}, \varpi_{a4}\}, \{\varpi_{a2}\}, \{\varpi_{a1}, \varpi_{a2}\}$. $MCl_{\Omega(H)} = G$. *Furthermore,* $G - H = \{\varpi_{a4}\}$, *then the* M- *top.space* (G, Ω) *is said to be a one-point compactification of* $(H, ɥ)$.

Example 5.9. *Let* $Y = \{\varpi_{a1}, \varpi_{a2}, \varpi_{a3}, \varpi_{a4}, \varpi_{a5}\}$, $X = \{\varpi_{a1}, \varpi_{a2}, \varpi_{a3}\}$ *with* $R = \{\{\varpi_{a1}\}, \{\varpi_{a2}, \varpi_{a3}, \varpi_{a4}, \varpi_{a5}\}\}$. *and* $U = \{\varpi_{a1}, \varpi_{a2}\}$,

Then $ɥ_{(H)} = \{X, \phi, \{\varpi_{a1}\}, \{\varpi_{a1}, \varpi_{a3}\}, \{\varpi_{a3}\}\}$ *and* $\Omega_{(Y)} = \{Y, \phi\{\varpi_{a1}\}, \{\varpi_{a1}, \varpi_{a3}, \varpi_{a4}, \varpi_{a5}\}, \{\varpi_{a3}, \varpi_{a4}, \varpi_{a5}\}$.*hence the* M- *closed sets in* Y *are* $Y, \phi, \{\varpi_{a2}, \varpi_{a3}, \varpi_{a4}, a_5\}, \{\varpi_{a2}\}, \{\varpi_{a1}, \varpi_{a2}\}$.$MCl_{\Omega(H)} = G$. *but,* $G - H = \{\varpi_{a4}, a_5\}$, *Therefore, Clearly it is not a one-point compactification of* $(H, ɥ)$.

Theorem 5.10. *If* (G, Ω) *is a* Hausdroff *one-point compactification of* $(H, ɥ)$, *then we have the following:*

1. $ɥ \subseteq \Omega$,
2. $(H, ɥ)$ *is Hausdorff and strongly locally* Ms-*compt., and*
3. *if* $G - H = \{r\} \in \Omega$, *then* $(H, ɥ)$ *is* Ms-*compt.*

Proof. (1) Since points are closed in (G, Ω), $H \in \Omega$ and hence $\Omega|H = ɥ \subseteq \Omega$.

(2) Clearly $(H, ɥ)$ is Hausdorff. If $x \in H$ and $G - H = \{r\}$, then $x \neq r$ and there are disjoint Ω-opensets U and V with $x \in U$, $r \in V$. Then $U \subseteq Cl_{\Omega}(U) = Cl_r(U) \subseteq G - V \subseteq H$, so that $(H, ɥ)$ is strongly locally Ms-compt. since closed subsets of Ms-compt. spaces are Ms-compt.

(3) If $G - H = \{r\} \in \Omega$, then H is Ms-compt. since it is a closed subset of

Ms-compt. space (G, Ω). Thus, $(H, \Omega|H) = (H, ɰ)$ is Ms-compt. □

Theorem 5.11. *For any space* $(H, ɰ)$, $ɰ^{\Psi}$ *is a* M- *topology on* H^{Ψ} *and* (H^{Ψ}, r^{Ψ}) *is a one-point compactification of* $(H, ɰ)$.

Proof. Clearly, $(W \cap H|W \in ɰ^{\Psi}) = ɰ$, so that if $ɰ^{\Psi}$ is a topology, $ɰ^{\Psi}|H = ɰ$. Since finite unions of Ms-compt. sets are Ms-compt. and ɰ is closed under finite intersection, then $ɰ^{\Psi}$ is closed under finite intersection. Now, if $\emptyset \neq V_{\gamma} \in A$ with each $H - V_{\gamma}$ compt., then $\cup(\{r\} \cup V_{\gamma}) = r \cup (\cup_{\gamma} V_{\gamma}) \in ɰ^{\Psi}$. since $\cup_{\gamma} V_{\gamma} \in ɰ$ and $H - (\cup_{\gamma} V_{\gamma})$ is Ms-compt., being a closed subset of an Ms-compt. set. Similarly, $U \cup (r) \cup V \in r^{\Psi}$ if U, $V \in ɰ$ and $H - V$ is Ms-compt. Therefore, $ɰ^{\Psi}$ is closed under arbitrary union and it forms a topology.

To see that $(H^{\Psi}, ɰ^{\Psi})$ is Ms-compt., let W be an r^{Ψ}-open cover of H^{Ψ}. If $r \in W_0 \in$ W, then $W_0 = \{r\} \cup V$ for some V with $V \in ɰ$ and $H - V$ is Ms-compt. Since $ɰ^{\Psi}|H = ɰ$, $\{W \cap H|W \in W$, and $W \neq W_0\}$ is a r-open cover of $H - V$. Hence, there is a finite subset $(W_1, W_2, \ldots\ldots, W_n \subseteq W)$ such that $W_1 \cap H, \ldots \ldots \ldots, W_n \cap H$ is a finite M-cover of $H - V$. Thus, $W_0, W_1, \ldots\ldots\ldots, W_n$ is a finite M-sub cover of W for H^{Ψ}.

We note that $(H^{\Psi}, ɰ^{\Psi})$ is a Ms-compt. extension of $(H, ɰ)$ iff $(H, ɰ)$ is not Ms-compt. In any case, $(H^{\Psi}, ɰ^{\Psi})$ is T_1 iff $(H, ɰ)$ is T_1, since for every ideal M, finite and hence singleton subsets of H are always Ms-compt. At the remainder point r, the smallest T_1 topology that can be generated for any one-point compactification of a T_1 space $(H, ɰ)$ is locally cofinite. □

Corollary 5.12. *If* $(H, ɰ)$ *has a* M - *Hausdorff one-point compactification iff* $(H, ɰ)$ *is a strongly locally* M *-compt. Hausdorff space.*

Proof. Theorem 4.1, part (2), contains the necessity. It is sufficient to demonstrate $(H^{\Psi}, ɰ^{\Psi})$ is Hausdroff. Since $(H, ɰ)$ is Hausdorff, The only thing left to check is whether disjoint $ɰ^{\Psi}$-open sets can distinguish each $x \in H$ from $r \in H^{\Psi}$ - H. Let K be a ɰ- closed M - compt. neighbourhood of $x \in H$. Then $x \in Int_r K \in ɰ^{\Psi}$ since $ɰ \subseteq ɰ^{\Psi}$, and $r \in H^{\Psi} - K \in ɰ^{\Psi}$.

□

6. Conclusion

This paper explains the concepts of M-Hausdroff space, strongly locally Ms com- pact M-lindelof space, Minuscule compactness and M-One-point Compactifica- tion. It is planned to define a weaker version of open sets in the future, as well as in Ms-top.spaces.

References

1. Mark Anthony Armstrong. Basic topology. Springer Science & Business Media, 2013.

2. Sakkraiveeranan Chandrasekar. On micro topological spaces. Journal of New Theory, (26):23–31, 2019.
3. S Gunavathy and R Alagar. Nano compactness with respect to nano ideal in nano topological space.
4. S Krishnaprakash, R Ramesh, and R Suresh. Nano-compactness and nano- connectedness in nano topological spaces. International Journal of Pure and Applied Mathematics, 119(13):107–115, 2018.
5. Kazimierz Kuratowski. Topology: Volume I, volume 1. Elsevier, 2014.
6. AA Nasef, AI Aggour, and SM Darwesh. On some classes of nearly open sets in nano topological spaces. Journal of the Egyptian Mathematical Society, 24(4):585–589, 2016.
7. Jack Porter. On locally h-closed spaces. Proceedings of the London Mathe- matical Society, 3(2):193–204, 1970.
8. A Pushpalatha and S Sridevi. Nano semi cˆ{ ast} generalized closed sets in nano topological spaces. Malaya Journal of Matematik (MJM), (1, 2019):361– 366, 2019.
9. Ilangovan Rajasekaran and Ochanan Nethaji. Simple forms of nano open sets in an ideal nano topological spaces. Journal of New theory, (24):35–43, 2018.
10. Carmel Richard. Studies on nano topological spaces. 2013.
11. David A Rose and ; T R Hamlett. Mathematica slovaca on one-point i-compactification and local i-compactness, 1992.
12. P Sathishmohan, V Rajendran, S Brindha, and PK Dhanasekaran. Between nano-closed and nano semi-closed. Nonlinear Studies, 25(4), 2018.
13. M Lellis Thivagar, J Kavitha, and D Evangeline Christina Lily. Minimal and maximal open sets in nano topological space.
14. M Lellis Thivagar and Carmel Richard. On nano continuity. Mathematical theory and modeling, 3(7):32–37, 2013.
15. M Lellis Thivagar and Carmel Richard. on nano continuity in a strong form.
16. International Journal of Pure and Applied Mathematics, 101(5):893–904, 2015.
17. R Vaidyanathaswamy. The localisation theory in set-topology. In Proceedings of the Indian Academy of Sciences-Section A, volume 20, pages 51–61. Springer India, 1944.
18. R Vaidyanathaswamy. Set topology. Courier Corporation, 1960.
19. R.Alagar, S.Purushoth Kumar, and R.Venkatesan. On Ms -top.spaces.Journal of chemical health risks, Vol.13 No.6, Pages 3550-3558— ISSN:2251-6727.
20. S. Purushoth Kumar, Dr. R. Alagar. A Novel Forms of MSL − Closed and MSW –Open Sets in Minuscule Topological Spaces, Communicated.