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Analytical Modeling And Validation Of Residual Stresses In The Coiling And Uncoiling Process Of Advanced High Strength Steel

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Residual stresses generated during the roll forming process play a critical role in defect formation, as well as influencing the strength and behavior of the material. Traditionally, these residual stresses are measured through experimental methods, which are often costly. This study introduces a novel approach by providing an analytical solution for the coiling and uncoiling process of metal sheets, particularly focusing on Advanced High Strength Steel (AHSS), which serves as input for the roll forming process. An analytical model has been developed for the coiling and uncoiling processes, considering the elastic-plastic plane strain pure bending of AHSS. The model employs the von Mises yield criterion and the Prandtl-Reuss flow rule to derive the analytical solution. The accuracy of the analytical predictions is validated by comparing them with experimental results obtained through the X-ray diffraction method. The experimental results show a close match with the analytical predictions. Consequently, the findings present a rapid and efficient method for measuring residual stresses, marking a significant advancement in the field.

Keywords: Residual Stress, Coiling of metal sheets, Longitudinal strain, X-ray diffraction.

Nomenclature

σ_{xc}	Coiling stress in transverse direction (x direction)
σ_{zc}	Coiling stress in longitudinal direction (z direction)
σ_{yc}	Coiling stress in radial direction (y direction)
ε_{xc}	Strain in x direction
ε_{zc}	Strain in z direction
ε_{yc}	Strain in y direction
R_c	Coiling curvature
y	Arbitrary distance of a coiling stress acting from neutral axis
Y_L	Limiting value of Y for yielding.
ω_c	Stress ratio
$d\varepsilon^E$	Elastic strain increment

$d\varepsilon^P$	Plastic strain increment
v	Poisson's ratio
E	Modulus of Elasticity
R_{uy}	Uncoiling curvature limit
$\sigma_{z,r}$	Final stress in Z direction
t	Thickness of metal sheet
$d\varepsilon_{zpc}$	Longitudinal plastic strain increment
$d\varepsilon_{xpc}$	Transverse plastic strain increment
σ_y	Yield stress
$d\overline{\epsilon_{pc}}$	Equivalent plastic strain increment

1. Introduction:

Residual stresses in roll forming products play an important role in determining their strength. Different destructive and non-destructive techniques for residual stress measurements are available and each has its own advantages and limitations (Withers, 2001). The distribution patterns of the residual stresses in a roll formed product are generally very complex. And it will be difficult and time consuming to predict residual stresses through simulations due to complexity of various influencing factors and conditions (Sun et al., 2015). Experimental measurements of residual stress are time consuming, difficult, with limited accuracy (Sun et al., 2017). Only surface residual stresses are measured (Livia et al., 2017).

The amount of the residual stresses depends on the external circumstances during the cold forming (Ingvarsson, 1975). Accurate theoretical predictions of residual stresses in roll forming process are required which are currently not available. A limited study of modelling of residual stress due to bending (Kato and Aoki, 1978) is done. Plastic bending of a wide plate was modelled by incremental numerical analysis process (Rondal, 1987). He assumed plane strain condition and used Prandtl-Reuss theory for his study and projected an imprecise approach of developing residual stresses in channel sections based on the results from his pure bending analysis. All these three studies were based on numerical analysis and they showed residual stresses have variations across the plate thickness.

Kato and Aoki (1978) analysed residual stresses on the basis of mathematical plasticity. Rondal, (1987) proposed a theoretical method for predicting residual stresses due to cold-rolling. He presented distribution and size of the residual stresses with respect to the dimensions of the profiles. Pourboghrat and Chu (1995) described a unique method for predicting springback in wall curls by using moment-curvature relationships which is derived for sheets undergoing plane strain stretching, bending and unbending deformations. Zhang and Hu (1998) briefly reviewed residual stress calculation methods in plane strain bending. He examined the influence of deformation theory and incremental theory on residual stress distributions. Johnson and Yu (1981) used Von Mises and Tresca criteria for evaluation of elastic springback of rectangular plate subjected to biaxial elastic-plastic pure bending. Jun and Korsunsky (2010) developed novel approach Eigenstrain Reconstruction Method (ERM) for residual stress analysis. Residual Stresses were computed by using an analytical elasto-plastic model and a relaxation procedure by Ulutan et al. (2007). With the developed analytical model, substantial computational time is reduced for predictions of residual stresses

Above researchers studied pure bending of plates into plastic range. Among all these existing studies, few are based on deformation theory (Quach et al., 2004, 2006). All these researchers ignored the effect of deformation history on residual stress. But this effect was important and it was shown by Zhang and Hu (1998), which concluded that flow theory should be used for accurate predictions of residual stresses in wide plates. All these studies based on the flow theory were shown numerical results and few theories (Ingvarsson, 1975) of them neglected elastic deformation which is very important for coiling process of steel metal sheets. This paper shows precise analytical solution for the coiling and uncoiling process modelled as a plane strain pure bending problem. All important factors are considered and added while deriving equation. Residual stress measurement is done subsequently using X-ray diffraction method to facilitate its application and to validate analytical results.

2. Analytical modelling

2.1 Coiling of metal sheet

2.1.1 Assumptions made

i. Flat sheet is free from residual stress.

ii. Strains induced during coiling and uncoiling process are relatively small. Therefore, strain hardening will not take place. This shows steel can be assumed to have elastic-perfectly plastic stress-strain curve.

2.1.2 Terminology

σ_{xc} = stress in x directions (width direction)

σ_{yc} = stress along y direction (thickness direction)

σ_{zc} = stress in z direction (length direction)

Bending direction is longitudinal direction along Z- axis. Width direction is transversal direction along X-axis and direction normal to curved plate surface is radial direction along Y-axis.

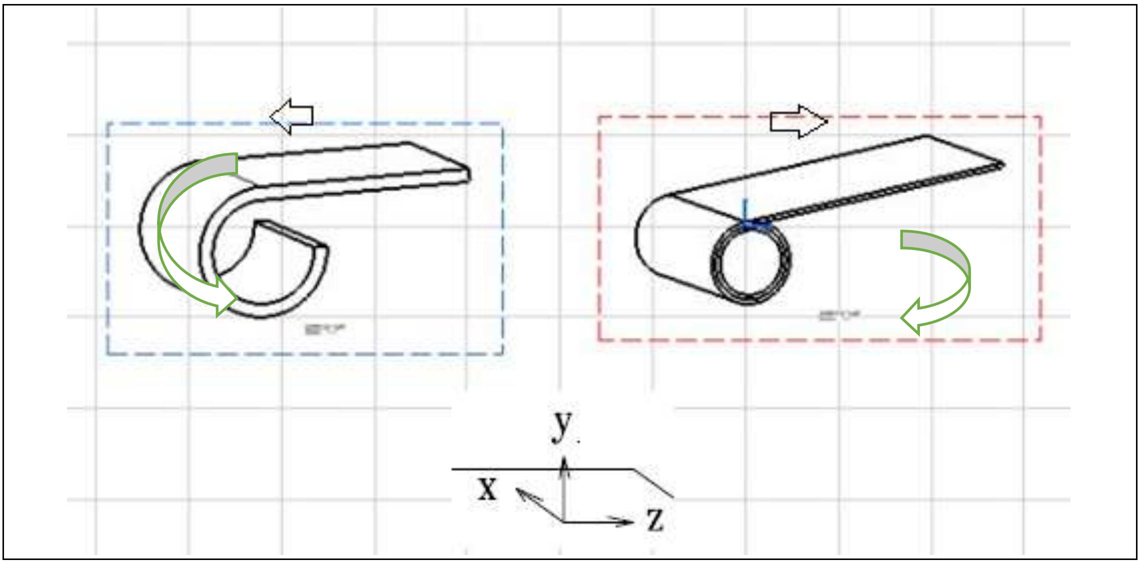

Figure 1: Coiling and uncoiling Process

Assuming σ_{yc} is zero as it is acting along thickness.
Thus, the Von Mises yield criteria for bi-axial is given by

$$\sigma_y{}^2 = \sigma_{xc}{}^2 - \sigma_{xc}\sigma_{zc} + \sigma_{zc}{}^2 \quad (1)$$

Superposition of the components of strain in x, y, z direction

$$\varepsilon_{xc} = \frac{1}{E}\left[\sigma_{xc} - v(\sigma_{yc} + \sigma_{zc})\right] \quad (2)$$

$$\varepsilon_{yc} = \frac{1}{E}\left[\sigma_{yc} - v(\sigma_{zc} + \sigma_{xc})\right] \quad (3)$$

$$\varepsilon_{zc} = \frac{1}{E}\left[\sigma_{zc} - v(\sigma_{xc} + \sigma_{yc})\right] \quad (4)$$

By considering strain along longitudinal and transverse direction, above equation 2, 3, 4 are simplified as

$$\varepsilon_{xc} = \frac{1}{E}[\sigma_{xc} - v(\sigma_{zc})] \quad (5)$$

$$\varepsilon_{zc} = \frac{1}{E}[\sigma_{zc} - v(\sigma_{xc})] \quad (6)$$

Considering plane strain condition along x-direction i.e. width direction and plane stress condition along y-direction i.e. thickness direction.
For a case of plane stress ($\sigma_y = 0$), two simple and useful equations relating stress to strain are obtained by solving equation number 5, 6 simultaneously

$$\left.\begin{aligned} \sigma_{zc} &= \frac{E}{1-v^2}(\varepsilon_{zc}) \\ \sigma_{xc} &= \frac{E}{1-v^2}(\varepsilon_{xc}) \\ \epsilon_{z,c} &= R_c\, y \end{aligned}\right\} \textit{these equations are valid upto elastic straining} \quad (7)$$

$$\varepsilon_{zc} = \frac{(1-v^2).\sigma_y}{E\sqrt{v^2 - v + 1}} \quad (8)$$

$$\sigma_{z,c} = \pm \frac{\sigma_y}{\sqrt{(\omega_c{}^2 - \omega_c + 1)}} \quad (9)$$

$$\sigma_{x,c} = \pm \frac{\sigma_y\,.\omega_c}{\sqrt{(\omega_c{}^2 - \omega_c + 1)}} \quad (10)$$

If $y > 0$, then value obtained from equation 9 and 10 of $\sigma_{x,c}$ and $\sigma_{z,c} \geq 0$

To determine coiling stress at any ‘y’ value for coiling curvature R_c, we have to make relation of stress ratio in Prandtl-Reuss Flow rule.

Applying Prandtl-Reuss Equations

Total strain increment is the sum of an elastic strain increment $d\varepsilon^E$ and a plastic strain increment $d\varepsilon^P$[19]

$$d\varepsilon = d\varepsilon^E + d\varepsilon^P \quad 11(a)$$

$$\begin{Bmatrix} d\epsilon_{zpc} \\ d\epsilon_{xpc} \end{Bmatrix} = d\overline{\epsilon_{pc}} \frac{3}{\sqrt{\sigma_{xc}{}^2 + \sigma_{zc}{}^2 - \sigma_{xc}\,\sigma_{zc}}} \begin{Bmatrix} s_{zc} \\ s_{xc} \end{Bmatrix} \quad 11(b)$$

s_{zc}, s_{xc} are deviatoric stresses.

$$s_{zc} = \frac{2\sigma_{zc} - \sigma_{xc}}{3} \ldots\ldots 11(c)$$

$$R_{cy} = \frac{(t_c.E\sqrt{v^2 - v + 1)}}{(1-v^2).2\sigma} \quad (12)$$

The magnitude of residual stresses in coiling is found depending upon yield stress of steel and radius of coiling curvature. If curvature R_c≥R_cy, plastic bending will not be involved during coiling. So, no existence of residual stresses. If curvature R_c<R_cy , yielding of material occurs due to coiling curvature in metal sheets. This higher yielding results residual stress development at the end of coiling.

2.2 Uncoiling of metal sheet.

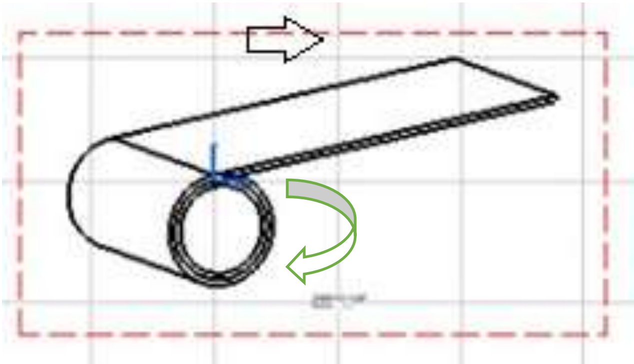

Figure 2: Uncoiling of Metal sheet

Final stresses at any point can be calculated by adding coiling and uncoiling stresses.

$$\left.\begin{aligned} \sigma_{zr} &= \sigma_{zc} + \sigma_{zu} \\ \sigma_{xr} &= \sigma_{xc} + \sigma_{xu} \end{aligned}\right\} \qquad (13\,a)$$

Uncoiling stresses are elastic. They are given by,

$$\left.\begin{aligned} \sigma_{z,u} &= \frac{E}{1-v^2}(R_u.y) \\ \sigma_{x,u} &= \frac{E}{1-v^2}(R_u.y) \end{aligned}\right\} \qquad (14\,b)$$

Due to uncoiling process, reverse yielding may occur. Thus final resultant stresses also obey Von Mises yield criteria.

$$\sigma_{ys}{}^2 = \sigma_{z,r}{}^2 - \sigma_{x,r}\sigma_{z,r} + \sigma_{x,r}{}^2 \qquad (15)$$

Uncoiling curvature limit R_{uy} can be calculated by substituting stress ratio ω_c, equations 10, 14(a), 14(b) into equation 15.

$$\sigma_y{}^2 = \sigma_{z,r}{}^2 - \sigma_{x,r}\sigma_{z,r} + \sigma_{x,r}{}^2$$
$$\sigma_{zr} = \sigma_{zc} + \sigma_{zu}$$

From equation 9 and equation 14(a)

$$\sigma_{zr} = \frac{\sigma_y}{\sqrt{(\omega_c{}^2 - \omega_c + 1)}} + \frac{E}{1-v^2}(R_u.y)$$
$$\sigma_{xr} = \sigma_{xc} + \sigma_{xu}$$

From equation 10 and equation 14(a)

$$\sigma_{xr} = \frac{\omega_c.\sigma_y}{\sqrt{(\omega_c{}^2 - \omega_c + 1)}} + \frac{E}{1-v^2}(R_u.y)$$

Substituting these equations in equation 15

$$\sigma_y{}^2 = \left[\frac{\sigma_y}{\sqrt{(\omega_c{}^2 - \omega_c + 1)}} + \frac{E}{1-v^2}(R_u.y)\right]^2 - \left[\frac{\omega_c.\sigma_y}{\sqrt{(\omega_c{}^2 - \omega_c + 1)}} + \frac{E}{1-v^2}(R_u.y)\right]\left[\frac{\sigma_y}{\sqrt{(\omega_c{}^2 - \omega_c + 1)}} + \frac{E}{1-v^2}(R_u.y)\right] + \left[\frac{\omega_c.\sigma_y}{\sqrt{(\omega_c{}^2 - \omega_c + 1)}} + \frac{E}{1-v^2}(R_u.y)\right]^2$$

Simplifying this equation

$$R_u.y = \left[\frac{\sigma_y(1-v^2)}{(1-v+v^2)}\right]\left[\frac{[2-v+(2v-1)\omega_c}{(E)\sqrt{\omega_c{}^2 - \omega_c + 1}}\right]$$

Uncoiling curvature limit is given by

$$R_{uy} = \frac{\sigma_y(1-v^2)[2-v+(2v-1)\omega_c}{(E|y|(1-v+v^2)\sqrt{\omega_c{}^2 - \omega_c + 1}} \tag{16}$$

The total longitudinal strain due to uncoiling is

$$\epsilon_{z,uy} = (R_c + R_{uy})y \tag{17}$$

Uncoiling Stresses are

$$\left.\begin{aligned} \sigma_{z,uy} &= \frac{E}{1-v^2}(R_{uy}.y) \\ \sigma_{x,uy} &= \frac{vE}{1-v^2}(R_{uy}.y) \end{aligned}\right\} \tag{18}$$

Stress ratio is given by

$$\omega_{uy} = \frac{\sigma_{x,c} + \sigma_{x,uy}}{\sigma_{z,c} + \sigma_{z,uy}} \tag{19}$$

Using equations 8, 9, 10, 16, 18, 19

$$\sigma_{z,c} = \pm\frac{\sigma_y}{\sqrt{\omega_c{}^2 - \omega_c + 1}} \tag{9}$$

$$\sigma_{x,c} = \pm\frac{\omega_c.\sigma_y}{\sqrt{\omega_c{}^2 - \omega_c + 1}} \tag{10}$$

And putting them in stress equation, stress equation becomes,

$$\omega_{uy} = \frac{\sigma_{x,c} + \sigma_{x,uy}}{\sigma_{z,c} + \sigma_{z,uy}}$$

$$\omega_{uy} = \frac{\dfrac{\omega_c . \sigma_y}{\sqrt{{\omega_c}^2 - \omega_c + 1}} + \dfrac{vE}{1-v^2}(R_{uy}.y)}{\dfrac{\sigma_y}{\sqrt{{\omega_c}^2 - \omega_c + 1}} + \dfrac{E}{1-v^2}(R_{uy}.y)}$$

$$\omega_{uy} = \frac{\omega_c . \sigma_y(1-v^2) + vE(\sqrt{{\omega_c}^2 - \omega_c + 1})(R_{uy}.y)}{\sigma_y(1-v^2) + \sigma_{z,uy}E(\sqrt{{\omega_c}^2 - \omega_c + 1})(R_{uy}.y)}$$

Substituting R_{uy} value from equation 19

$$\omega_{uy} = \frac{\omega_c . \sigma_y(1-v^2) + vE(\sqrt{{\omega_c}^2 - \omega_c + 1)(\dfrac{\sigma(1-v^2)[2-v+(2v-1)\omega_c}{(E|y|(v^2-v+1)\sqrt{{\omega_c}^2 - \omega_c + 1}}.y)}}{\sigma_y(1-v^2) + \sigma_{z,uy}E(\sqrt{{\omega_c}^2 - \omega_c + 1)(\dfrac{\sigma(1-v^2)[2-v+(2v-1)\omega_c}{(E|y|(v^2-v+1)\sqrt{{\omega_c}^2 - \omega_c + 1}}.y)}}$$

$$\omega_{uy} = \frac{\omega_c.(v^2-v+1) - v[2-v+(2v-1)\omega_c]}{(v^2-v+1) - v[2-v+(2v-1)\omega_c]}$$

$$\omega_{uy} = \frac{\omega_c - v\omega_c - v[2-v+(2v-1)\omega_c}{(v^2-v+1) - v[2-v+(2v-1)\omega_c]}$$

$$\omega_{uy} = \frac{\omega_c - v\omega_c + v^2\omega_c + 2v - v^2 + 2v^2\omega_c - v\omega_c}{(v^2-v+1) + 2 - v + 2v\omega_c - \omega_c}$$

$$\omega_{uy} = \frac{\omega_c - v^2\omega_c + 2v - v^2}{\omega_c - 2v\omega_c - 1 + v^2}$$

$$\omega_{uy} = \frac{(1-v^2)\omega_c - v(2-v)}{(1-2v)\omega_c - (1-v^2)} \qquad (20)$$

Therefore when $R_c \leq |R_{uy}|$,

$$\left.\begin{aligned} \sigma_{zu} &= \frac{-E}{1-v^2}(R_c.y) \\ \sigma_{xu} &= \frac{-vE}{1-v^2}(R_y.y) \end{aligned}\right\} \qquad (21)$$

When $R_c > |R_{uy}|$, reverse yielding occurs. The final stress will be

$$\left.\begin{aligned} \sigma_{z,r} &= \pm \frac{\sigma_y}{\sqrt{{\omega_u}^2 - \omega_u + 1}} \\ \sigma_{x,r} &= \pm \frac{\omega_c.\sigma_y}{\sqrt{{\omega_u}^2 - \omega_u + 1}} \end{aligned}\right\} \qquad (22)$$

$$\omega_u = \frac{\sigma_{x,r}}{\sigma_{z,r}} = \frac{\sigma_{x,c} + \sigma_{x,u}}{\sigma_{z,c} + \sigma_{z,u}}$$

$\sigma_{x,r}$ and $\sigma_{z,r} \leq 0$ when $y \geq 0$. Hence from equation 8, 16, 22, the uncoiling stresses are calculated as

$$\sigma_{zr} = \sigma_{zc} + \sigma_{zu}$$

$$\sigma_{zu} = \sigma_{zr} - \sigma_{zc}$$

$$\sigma_{zu} = \frac{\sigma_y}{\sqrt{{\omega_u}^2 - \omega_u + 1}} - \frac{\sigma_y}{\sqrt{{\omega_c}^2 - \omega_c + 1}}$$

$$\sigma_{zu} = \pm\left(\frac{\sigma_y}{\sqrt{{\omega_c}^2 - \omega_c + 1}} + \frac{\sigma_y}{\sqrt{{\omega_u}^2 - \omega_u + 1}}\right) \qquad 23\ (a)$$

$$\sigma_{xr} = \sigma_{xc} + \sigma_{xu}$$

$$\sigma_{xu} = \sigma_{xr} - \sigma_{xc}$$

$$\sigma_{xu} = \frac{\omega_u . \sigma_y}{\sqrt{{\omega_u}^2 - \omega_u + 1}} - \frac{\omega_c . \sigma_y}{\sqrt{{\omega_c}^2 - \omega_c + 1}}$$

$$\sigma_{xu} = \pm\left(\frac{\omega_c . \sigma_y}{\sqrt{{\omega_c}^2 - \omega_c + 1}} - \frac{\omega_u . \sigma_y}{\sqrt{{\omega_u}^2 - \omega_u + 1}}\right) \qquad 23\ (b)$$

Applying Prandtl-Reuss Equations

Total strain increment is the sum of an elastic strain increment $d\varepsilon^E$ and a plastic strain increment $d\varepsilon^P$

$$d\varepsilon = d\varepsilon^E + d\varepsilon^P \qquad 11(a)$$

$$\begin{Bmatrix} d\epsilon_{zpu} \\ d\epsilon_{xpu} \end{Bmatrix} = d\overline{\epsilon_{pu}} \frac{3}{\sqrt{{\sigma_{xu}}^2 + {\sigma_{zu}}^2 - \sigma_{xu}\,\sigma_{zu}}} \begin{Bmatrix} s_{zu} \\ s_{xu} \end{Bmatrix} \qquad 23(c)$$

s_{zu}, s_{xu} are deviatoric stresses.

$$s_{zu} = \frac{2\sigma_{zu} - \sigma_{xu}}{3}$$

$$s_{xu} = \frac{2\sigma_{xu} - \sigma_{zu}}{3}$$

From equation 23(c),

$$\frac{d\epsilon_{zpu}}{d\epsilon_{xpu}} = \frac{s_{zu}}{s_{xu}}$$

$$\frac{d\epsilon_{zpu}}{d\epsilon_{xpu}} = \frac{\frac{2\sigma_{zu} - \sigma_{xu}}{3}}{\frac{2\sigma_{xu} - \sigma_{zu}}{3}}$$

$$\frac{d\epsilon_{zpu}}{d\epsilon_{xpu}} = \frac{2\sigma_{zu} - \sigma_{xu}}{2\sigma_{xu} - \sigma_{zu}} \qquad 23(d)$$

Substituting stress equation $\omega_u = \frac{\sigma_{x,u}}{\sigma_{z,u}}$ in equation 23 (d)

$$\frac{d\epsilon_{zpu}}{d\epsilon_{xpu}} = \frac{2-\omega_u}{2\omega_u - 1} \qquad 23(e)$$

The longitudinal strain increment $d\epsilon_{zu}$ consists of elastic strain increment and a plastic strain increment

$$d\epsilon_{zu} = d\epsilon_{zeu} + d\epsilon_{zpu} \qquad 23\,(f)$$

Due to plain strain condition, transverse coiling strain increment is zero.

$$d\epsilon_{xu} = d\epsilon_{xeu} + d\epsilon_{xpu} \qquad 23\,(g)$$

Substituting equation 23(e) and 23(g) in 23(f),

$$d\epsilon_{zu} = d\epsilon_{zeu} - \left(\frac{2-\omega_u}{2\omega_u - 1}\right) d\epsilon_{zeu} \qquad 23(h)$$

The incremental elastic strains are given by

$$\begin{Bmatrix} d\epsilon_{zpu} \\ d\epsilon_{xpu} \end{Bmatrix} = \frac{1}{E}\begin{bmatrix} 1 & -v \\ -v & 1 \end{bmatrix}\begin{Bmatrix} d\sigma_{zu} \\ d\sigma_{xu} \end{Bmatrix} \qquad 26(i)$$

Differentiating σ_{zu}

$$\sigma_{zu} = \frac{\sigma_y}{\sqrt{{\omega_u}^2 - \omega_u + 1}}$$

$$d\sigma_{zu} = \frac{\left(\sqrt{{\omega_u}^2 - \omega_u + 1}\right) X\, 0 - \sigma_y \frac{1}{2}(1 - \omega_u + {\omega_u}^2)^{-\frac{1}{2}}(-1 + 2\omega_u)}{{\omega_u}^2 - \omega_u + 1}$$

$$d\sigma_{zu} = \frac{-\sigma_y \frac{1}{2}(1 - 2\omega_u)}{({\omega_u}^2 - \omega_u + 1)^{\frac{2}{2}}.({\omega_u}^2 - \omega_u + 1)^{\frac{1}{2}}}$$

$$d\sigma_{zu} = \pm \frac{\sigma_y(1 - 2\omega_u)}{2({\omega_u}^2 - \omega_u + 1)^{\frac{3}{2}}} d\omega_u \qquad 23(j)$$

Differentiating equation

$$\omega_u = \frac{\sigma_{x,u}}{\sigma_{z,u}}$$

$$d\omega_u = \omega_u d\sigma_{z,u} + \sigma_{z,u} d\omega_u \qquad 23(k)$$

Substituting $\sigma_{zu} = \frac{\sigma_y}{\sqrt{1-\omega_u+{\omega_u}^2}}$, 23 (j) into 23 (k)

$$d\sigma_{x,u} = \omega_u\left(\frac{\sigma_y(1 - 2\omega_u)}{2({\omega_u}^2 - \omega_u + 1)^{\frac{3}{2}}} d\omega_u\right) + \left(\frac{\sigma_y}{\sqrt{{\omega_u}^2 - \omega_u + 1}}\right) d\omega_u$$

$$d\sigma_{x,u} = \omega_u \left(\frac{\sigma_y (1 - 2\omega_u)}{2({\omega_u}^2 - \omega_u + 1)^{\frac{3}{2}}} d\omega_u \right) + \left(\frac{\sigma_y}{\sqrt{{\omega_u}^2 - \omega_u + 1}} \right) d\omega_u$$

$$d\sigma_{x,u} = \left(\frac{\sigma_y (\omega_u - 2{\omega_u}^2) d\omega_u}{2({\omega_u}^2 - \omega_u + 1)^{\frac{3}{2}}} \right) + \left(\frac{\sigma_y (1 - \omega_u + {\omega_u}^2) X2}{2({\omega_u}^2 - \omega_u + 1)^{\frac{1}{2}} X ({\omega_u}^2 - \omega_u + 1)^{\frac{2}{2}}} \right)$$

$$d\sigma_{x,u} = \left(\frac{\sigma_y (2 - \omega_u)}{2({\omega_u}^2 - \omega_u + 1)^{\frac{3}{2}}} d\omega_u \right) \qquad 23\ (l)$$

Substituting equation 23 (i), 23 (j), 23 (l) into 23 (h)

$$d\epsilon_{zu} = \pm \frac{\sigma_y}{2E({\omega_u}^2 - \omega_u + 1)^{\frac{3}{2}}} [(1 - 2\omega_u) - 2v(2 - \omega_u) - \frac{(2 - \omega_u)^2}{(2 - 1)} d\omega_u \qquad (24)$$

Taking integration of above equation w.r.t. ω_u

$$\int_{\epsilon_{zuy}}^{\epsilon_{zr}} d\epsilon_{zu} = \int_{\omega_{uy}}^{\omega_u} \pm \frac{\sigma_y}{2E({\omega_u}^2 - \omega_u + 1)^{\frac{3}{2}}} [(1 - 2\omega_u) - 2v(2 - \omega_u) - \frac{(2 - \omega_u)^2}{(2 - 1)} d\omega_u$$

$$\epsilon_{zr} - \epsilon_{z,uy} = \pm \frac{\sigma_y}{E} \left[\frac{\omega_u (1 - 2v)}{\sqrt{{\omega_u}^2 - \omega_u + 1}} + \frac{\sqrt{3}}{2} cot\, h^{-1} \left(\sqrt{\frac{4({\omega_u}^2 - \omega_u + 1)}{3}} \right) \right]_{\omega_{uy}}^{\omega_u} \qquad (25)$$

As uncoiling includes flattening which results in $\epsilon_{zr} = 0$.

$$0 - \epsilon_{z,uy} = \pm \frac{\sigma_y}{E} \left[\frac{\omega_u (1 - 2v)}{\sqrt{{\omega_u}^2 - \omega_u + 1}} + \frac{\sqrt{3}}{2} cot\, h^{-1} \left(\sqrt{\frac{4({\omega_u}^2 - \omega_u + 1)}{3}} \right) \right]_{\omega_{uy}}^{\omega_u}$$

$$|y| = \frac{\sigma_y}{E(R_c + R_{uy})} \left[\frac{\omega_u (1 - 2v)}{\sqrt{{\omega_u}^2 - \omega_u + 1}} + \frac{\sqrt{3}}{2} cot\, h^{-1} \left(\sqrt{\frac{4({\omega_u}^2 - \omega_u + 1)}{3}} \right) \right]_{\square}^{\square} \qquad (26)$$

By using equation 26, the stress ratio, ω_u can be calculated at any location y for $R_c > \left| R_{uy} \right|$. The values of R_{uy} and ω_{uy} can be calculated by equation 16 and 20.

$$R_{uy} = \frac{\sigma(1 - v^2)[2 - v + (2v - 1)\omega_c}{(E|y|(v^2 - v + 1)\sqrt{{\omega_c}^2 - \omega_c + 1}} \qquad (16)$$

$$\omega_{uy} = \frac{(1 - v^2)\omega_c - v(2 - v)}{(1 - 2v)\omega_c - (1 - v^2)} \qquad (20)$$

Total residual stresses can be calculated by equations 23(a), 23(b) and 14(a)

$$\left.\begin{aligned}\sigma_{zu} &= \pm\left(\frac{\sigma_y}{\sqrt{{\omega_c}^2-\omega_c+1}}+\frac{\sigma_y}{\sqrt{{\omega_u}^2-\omega_u+1}}\right)\\ \sigma_{xu} &= \pm\left(\frac{\omega_c.\sigma_y}{\sqrt{{\omega_c}^2-\omega_c+1}}-\frac{\omega_u.\sigma_y}{\sqrt{{\omega_u}^2-\omega_u+1}}\right)\end{aligned}\right\} \quad (23)$$

$$\left.\begin{aligned}\sigma_{zr} &= \sigma_{zc} + \sigma_{zu}\\ \sigma_{xr} &= \sigma_{xc} + \sigma_{xu}\end{aligned}\right\} \quad (14\,a)$$

3. Methodology

Table 1: Material properties for Advanced High Strength steel (AHSS)

Sr. No.	Name of Material		Tensile Strength	Modulus of Elasticity	Poisson's Ratio	Thickness of sheet
			(N/mm^2)	E (N/mm^2)	v	t(mm)
1	AHSS	DP-600	671.3	188440	0.295	2
2		DP-780	807.5	188440	0.295	2
3		DP-980	1003	188440	0.295	1, 2, 3

Table 1 shows material properties for Advanced High Strength steel (AHSS) taken for residual stress calculations by analytical method

4. Results and discussion

Different 6 iterations are completed with variations (Table 2) in thickness ranging from 1 mm to 3 mm and three different grades of Advanced High Strength Steel.

Table 2: Analytical and Experimental results for coiling and uncoiling

Iterations	Metal sheet thickness	Material	Analytical Results		Experimental Results	
			Residual stress generated due to coiling	Residual stress generated due to uncoiling	Residual stress generated due to coiling	Residual stress generated due to uncoiling
1	1	DP980	722	294	740	301
2	2	DP980	596	123	622	138
3	3	DP980	325	43	321	59
4	2	DP600	345	52	365	63
5	2	DP780	454	193	493	205
6	2	DP980	596	219	622	227

Table 2 shows Analytical and Experimental results for coiling and uncoiling of metal sheets. Figure 3 shows different experimental methods of residual stress measurement.

Experimental results are obtained by X-ray diffraction method due to its advantages over other experimental methods for coiling and uncoiling of metal sheets.

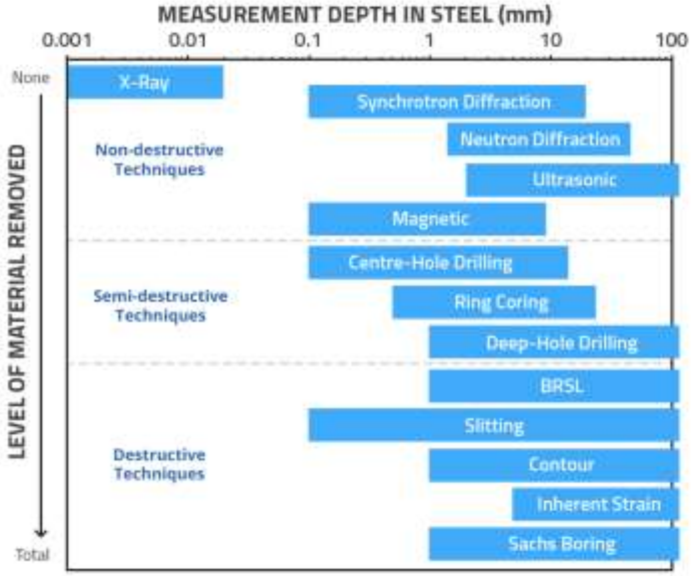

Figure 3: Different experimental methods of residual stress measurement

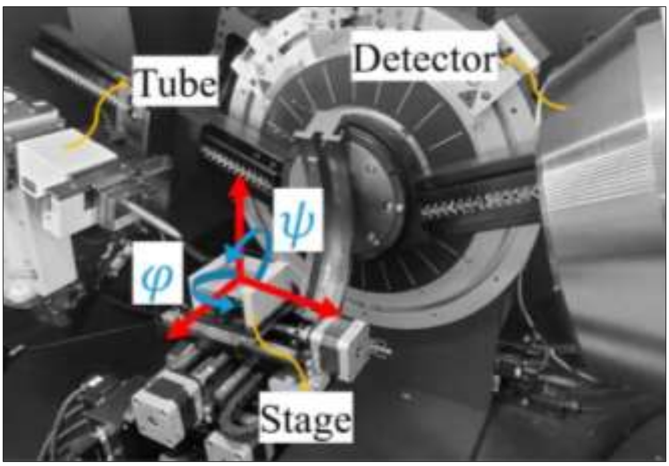

Figure 4: Experimental setup for residual stress measurement due to coiling and uncoiling.

Figure 4 shows experimental setup for residual stress measurement due to coiling and uncoiling. Table 3shows X-ray diffraction Test set up Parameters

Table 3: X-ray diffraction Test set up Parameters

Material analyzed	AHSS		Beta Oscillation	3 Deg
Bragg angle	156.41		Aperture size	2 mm
Source tube	Cr K alpha		Number of exposures	10
Wavelength	2.291 A		Exposure time	1 sec
X-Ray Elastic Constant	5.92 x 10^{-6} MPa		Current/Voltage setting	25 kV, 5.0 mA
Beta angle range	±25 Deg		Peak Location	Pearson VII 85%

Three samples for each iteration are considered and average value of these three samples are displayed at table 2. Figure 5 shows comparison of residual stresses between the analytical and Experimental analysis for variations in metal sheet thickness and grades of AHSS

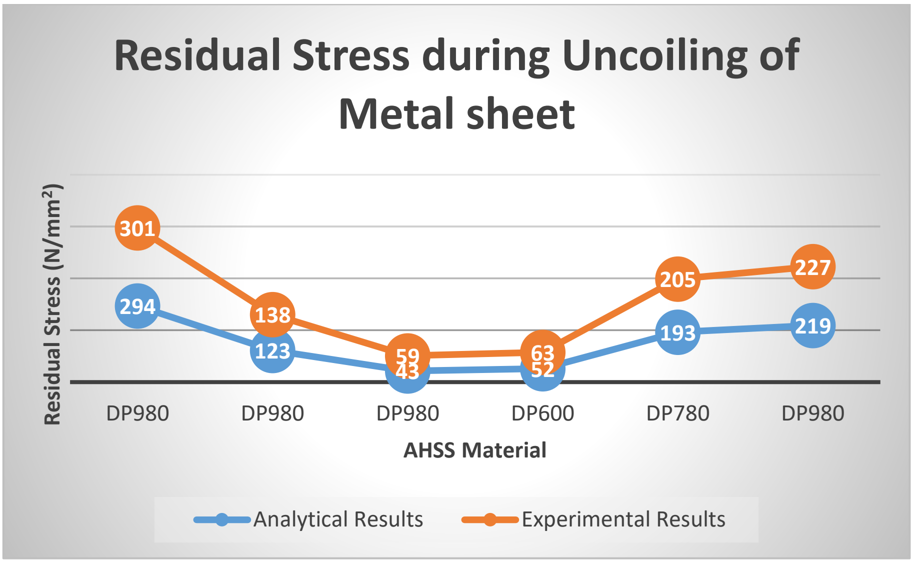

Figure 5: Comparison of residual stresses between the analytical and Experimental analysis for variations in metal sheet thickness and grades of AHSS

5. Conclusions:

Residual stresses in roll forming process is generated due to cold forming. The raw material provided to roll forming process is in the form of metal sheet which are coiled during transportation and uncoiled before roll forming. Residual stresses are generated due to coiling and uncoiling process as a result of plastic bending.

In this research paper analytical modeling for the coiling and uncoiling process of metal sheets is presented which is necessary as a starting point for the evaluation of residual stresses during roll forming process. A plane strain pure elastic– plastic bending model by applying Prandtl-Reuss flow rule and von Mises yield criteria is considered for the development of analytical solutions of a coiling and uncoiling process. The magnitude and extent of residual stresses were found to be sensitive to the coiling and uncoiling radius and the yield stress of AHSS (Advanced High Strength Steel)

The analytical results are validated with experimentation. X-ray diffraction method is used to measure residual stresses in coiling and uncoiling of metal sheets. Experimental results show a closed match with analytical results. Results obtained from analytical and experimentation showed non-linear variations of residual stresses along the thickness of metal sheets.

The analytical solution presented in this paper for coiling and uncoiling gives accurate residual stresses. This results can be specified as initial stresses in a finite element analysis of the roll forming process.

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