Predicting the efficiency of high-speed diamond grinding of superhard materials using 3D models of macro- and micro-levels

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The relevance of solving the title problem is dictated by the high labour intensity and low productivity of the process of diamond grinding of superhard materials (SHM) based on synthetic diamond (SD). The high consumption of expensive diamond grains leads to a high cost of processing. Reliability and quality of the SHM tool must be improved, without which it is impossible to use it in automated production. Creation and use of expert systems is a conceptual stage of technology development. At the heart of problem-solving in a particular subject area lies the principle of reproducing the knowledge of experienced specialists (experts). On the base of experience, experts analyse the situation and recognize the most useful information, optimize decision-making, abandon dead ends. The efficiency of the process of high-speed diamond grinding of SHM is largely determined by the properties of the tool (especially its diamond layer), which are established at the manufacturing (sintering) stage. The necessary properties of the tool, the sintering modes of the diamond layer, and the processing modes were studied on the basis of dynamic 3D models built at macro- and micro-levels. As a result of model experiments carried out using the finite-element method (FEM), it seemed possible to obtain a mathematical model (function) that approximately characterizes the sintering process of the diamond layer of a wheel for high-speed grinding in the expected range of variation of independent factors. At the stage of manufacturing of diamond wheels for high-speed grinding, the output indicator for the function is the reduced stresses in the "diamond grain–coating–metal phase binder" system. When studying the process of high-speed grinding, the output

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indicator for the function is the productivity and specific consumption of diamond grains in the working layer of the diamond wheel, calculated using dynamic 3D modeling at the micro-level. A 3D micro-level extended system "diamond grain–coating–metal phase binder–processed material" is considered.

Keywords: diamond wheel, expert system, finite element method, optimization, simulations

1. Introduction

To solve the title problem, a methodology is proposed based on 3D models from the macro- to the micro-level. The methodology is based on finite-element calculations of the stress–strain state of the sintering (manufacturing) zone of diamond abrasive tools and the grinding zone (SHM processing). At the same time, without lengthy, labour-intensive and expensive experimental studies, a rational structure (grain size, concentration of diamond grains, rational design of a wheel for high-speed grinding) and its physical and mechanical properties (binder properties, grade of diamond grain, maximum allowable and optimal grinding speed) are determined by calculation. However, the industrial implementation of these results requires continual problem-solving, which requires the continuous presence of a team of highly qualified specialists capable of carrying out difficult calculations and processing the results.[1] With this in mind, industrial implementation of the results of dynamic 3D modeling of high-speed diamond grinding processes can be achieved using an expert system that allows solving problem situations.[1,2] Such a system is a specially designed computer program that stores the database as well as the knowledge base of the expert and has a user interface for convenient operation under modern production conditions.

The main difference between expert systems and other software products is that they use not only data, but also knowledge, as well as a special mechanism for deriving solutions (and hence new knowledge) based on existing ones. Knowledge in expert systems is presented in a form that can be easily processed by a computer, namely as mathematical laws and algorithms. As a rule, expert systems are created to solve practical problems in highly specialized areas, where the knowledge of experienced specialists plays an important role. Expert systems were the first developments able to draw significant attention to the results of research in the field of artificial intelligence. Expert systems have, however, one significant difference from other artificial intelligence systems: they are not designed to solve universal problems, unlike, for example, neural networks.[3,4]

1 Mamalis, A.G., Grabchenko, A.I., Fedorovich, V.A., Romashov, D.V. and Fedorenko, D.O. Principles of 3D modelling of the production and application of diamond composite materials. *Nanotechnol. Perceptions* **8** (2012) 132–138.

2 Agrawal, A., Dubey, A.K. and Shrivastava, P.K. Intelligent modeling and multi-objective optimization of powder mixed electrical discharge diamond grinding of MMC. In: Proc. Intl Conf. on Industrial Engineering and Engineering Management (IEEM-2016), Bali, 4–7 December 2016, pp. 1036–1040.

3 Lee, K.B., Cheon, S. and Kim, C.O. A convolutional neural network for fault classification and diagnosis in semiconductor manufacturing processes. *IEEE Trans. Semiconductor Manufacturing* **30** (2017) 135–142.

4 Palsson, F., Sveinsson, J.R. and Ulfarsson, M.O. Multispectral and hyperspectral image fusion using a 3-D-convolutional neural network. *IEEE Geosci. Remote Sensing Lett.* **14** (2017) 639–643.

Expert systems are designed for high-quality problem-solving in a specific area (in rare cases, wide areas). Expert knowledge is a combination of theoretical understanding of the problem and practical skills for solving it, the effectiveness of which has been proven as a result of practical activities. The foundation of the developed expert system for the SHM high-speed diamond grinding process is theoretical (with the help of FEM) and experimental studies, the results of which have led to mathematical models transcribable into computer algorithms. The value of the expert system as a finished product is determined by both the quality of the created knowledge base and the quality of programming. An expert system is not a simple program written by one or more programmers; it is the result of a collaborative effort between subject matter experts, knowledge engineers and programmers. There are cases where programs have been written by experts themselves, as in this study. To implement the idea of creating an expert system for high-speed SHM diamond grinding, we used the tools of the open and free-to-use integrated software development environment LAZARUS, built on the FreePascal compiler. This environment is almost a complete analogue of the well-known commercial system Delphi. With the use of the LAZARUS system, many software projects for various purposes have been developed.[2]

2. Literature review

Currently, the most technologically advanced way of processing SHM is diamond grinding.[5,6] However, the processes used today for diamond grinding with wheels on organic and metal binders do not fully solve the problems of low productivity (which is 10,000 times lower than when processing, for example, corundum ceramics), a significant specific consumption of diamond grains (sometimes reaching 30 carats of grains per 1 carat of the allowance removed) and a significant percentage of rejects due to the appearance of a network of microcracks on the treated SHM surface.[7] Processing products made of superhard and brittle materials has always been associated with high costs.[7,8] In addition, due to the growing demand for materials with optimal properties in terms of wear resistance, as well as thermal and chemical resistance, there is a need for further development and improvement (optimization) of existing processing methods.[8] At this stage of using diamond abrasive tools, one of the most promising areas for improving the efficiency of grinding operations and expanding its technological capabilities is to increase the processing speed.[8,9]

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5 Webster, J. and Tricard, M. Innovations in abrasive products for precision grinding. *Ann. CIRP* **53** (2004) 597–617.

6 Mamalis, A.G., Grabchenko, A.I., Romashov, D.V., Fedorenko, D.O., Lagoudas, D., Fedorovich, V.A. and Kundrak, J. Determination of the diamond wheel structure in high-speed grinding using nanoindentation techniques: experimental and numerical simulation. *Nanotechnol. Perceptions* **9** (2013) 187–197.

7 Żyłka, Ł., Płodzień, M. and Babiarz, R. The influence of grinding speed on the creep-feed grinding process. *J. Mech. Energy Engng* **2** (2018) 285–290.

8 Oliveira, J.F.G., Silva, E.J., Guo, C. and Hashimoto, F. Industrial challenges in grinding. *Ann. CIRP* **58** (2009) 663–680.

9 Zhu, X.-M., Liu, Y., Zhang, J.-H., Wang, K. and Kong, H.-H. Ultrasonic-assisted electrochemical drill-grinding of small holes with high-quality. *J. Adv. Res.* **23** (2020) 151–161.

When developing new diamond tools, it is important to select the right diamond and bonding materials, as well as create the right structure for the grinding wheel.[10] Diamond abrasive machining at the present stage is characterized by the search for ways to improve the productivity and efficiency of the process, and the quality of the workpieces.[11, 12]

The key areas of scientific and technological progress are computerization and electronicification, designed to ensure the development and widespread use in the economy, including scientific research, education and everyday life, of information and computing, especially microprocessor-based, technology, which will revolutionize social production through the use of fundamentally new technological systems, and give new dynamism to intellectual and creative processes.[13] Research in the field of artificial intelligence is directly related to the problem of its effectiveness.[13, 14] Many currently existing methods of working with unconfirmed data (fuzzy logic, Bayesian logic, confidence coefficients, multivalued logic, etc.) have been tested in various expert systems, and most of them have proven to be quite effective.[13] This is most likely due to the fact that the *organization* of knowledge has a greater role than the numerical values associated with it.[14] Many knowledge bases provide for redundancy, allowing an expert system to reach the correct conclusion in several different ways.[13, 14] The most difficult problem in the formation of knowledge bases is that of automating the process of extracting knowledge.[15, 16] Knowledge has a huge social value. The most complete knowledge in a particular area is possessed by people classified as experts. Their opinions and evaluation often dominate in reaching managerial decisions, such as the approval of large projects. Of course, experts are better than others regarding awareness of their subject areas; they have rich practical experience from which to draw in order to arrive at certain conclusions. At the same time, a paradox is often found, consisting in the fact that the experts themselves are not always able to logically, clearly and accessibly—even for specialists—relate how they came to certain conclusions, and their explanations often turn out to be purely external.

Using expert systems is most appropriate for the following cases: diagnostics; where there is no established theory; where there is an insufficient number of specialists; where there is significant informational noise. Diagnostic tasks are understood not only through analogy with medical diagnoses; they include any search in which there are many possible answers, and the difficulty is in choosing one correct one or, at least, discarding incorrect ones. Areas where,

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[11] Wei Li, Mingjia Liu, Yinghui Ren and Qidi Chen. A high-speed precision micro-spindle use for mechanical micro-machining. *Intl J. Adv. Manufacturing Technol.* **102** (2019) 9–12.

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[14] Crawford, K. *Atlas of AI: Power, politics, and the planetary costs of artificial intelligence*. New Haven: Yale University Press (2021).

[15] Botta, A., De Donato, W., Persico, V. and Pescapé, A. Integration of cloud computing and internet of things: a survey. *Future Generation Computer Systems* **56** (2016) 684–700.

[16] Kaur, N. and Kaur, A. Predictive modelling approach to data mining for forecasting electricity consumption. *Proc. IEE Intl Conf. on Cloud Systems Big Data Engineering* (Confluence), 14–15 January 2016, Noida, India, pp. 331–336.

essentially, a complete theory has been worked out include problems such as (short-term) weather forecasting, engine repair, and many medical issues, although their excess of variables make it difficult to create a complete and integral theory; as a result of this the solution of problems largely depends on the experience and intuition of practitioners.[13] In this study, in order to create a knowledge base built on dynamic 3D models of macro- and micro-levels, experiment planning is considered *qua* cybernetics.

In the general case, we can give the following definition: experiment design is the optimal control of an experiment with incomplete knowledge of the mechanism of the phenomena.[17] This approach is based on the cybernetic principle of the "black box".[17, 18] It is a system of connexions inaccessible to observation, since we do not know anything (or know only partially) about the content and mechanism of the process. Only the inputs of the box—the variables involved in the process—and the outputs—the results of the process—are known (Fig. 1). For example, the process of diamond grinding on any machine can be considered as a black box, the content of which—the mechanism of the microcutting process during grinding—is completely unknown to us. The input variables X_i that are known to us and can be controlled might be the parameters of the tool, cutting conditions and some properties of the material being processed. The input variables W and Z that are not controlled, or may be unknown, are some properties of the workpieces being processed, changes in the physical properties of cutting fluids over time, changes in the physical properties of the tool due to its replacement and as a result of wear, fluctuations in processing speed, etc.

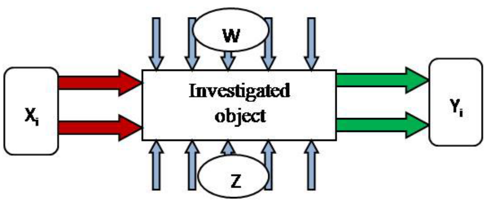

Figure 1. Black box concept. X_i are the adjustable input variables (factors); W, Z the unregulated (unknown) impacts on the system; and Y_i is the output parameter.

The output variables Y_i are the parameters of interest to us: process productivity, specific consumption of diamonds, stresses in the system under study, etc. By changing the input variables, one can observe how the output will change, and at the same time choose from all possible combinations of input variables the one that will provide the minimum specific consumption of diamond grains or, equivalently, the maximum productivity of the process. With such a formulation of the problem, we are not interested in either the mechanism of the cutting process, or the interaction of factors in the process, or the patterns of wear of the cutting tool. The black box is the simplest model of any system, the internal structure of which is not visible. The meaning of the proposed statement of the problem lies in the fact that the mathematical theory of the experiment gives the optimal plan for the study of the black box. This plan provides not only the fastest way to optimize the output variable, but also the means to obtain a mathematical model of the process,

[17] Katsev, P.G. *Statistical Research Methods of Cutting Tools* (2nd edn). Moscow, "Engineering" (1974).
[18] Gayathri Ravichandran. Big data processing with Hadoop. *Intl Res. J. Engng Technol.* **4** (2017) 448–451.

which is an extremely compact and convenient tool for researching and controlling a real process. In many cases, the model also makes it possible to reveal the mechanism of the process; i.e., the internal structure of the black box, using physical analogies and mathematical analysis.

The condition for development of the model is information sufficiency,[16] meaning that the developer must have sufficient understanding of what are the input and output variables in the system under study and what factors influence the process of its functioning. If the level of information sufficiency is low, then it is impossible to create a model that can be used to obtain new knowledge about the original object.[17] If the level of information sufficiency is high, i.e. the system is already well studied, then the question of creating a model loses its meaning, since it will not give new knowledge either. The performance indicator allows one to evaluate (more precisely, describe) the result of an operation obtained using a specific strategy. However, even if such evaluations are obtained for the entire set of admissible strategies, this is still not enough to choose one of them, i.e. the one that will be implemented.

Expert systems (ES) are one of the prominent areas of artificial intelligence (AI) research;[4,19] they are computer applications designed to solve complex problems in a particular area at the level of extraordinary human intelligence and experience. ES components include the database (e.g., properties of system elements), the knowledge base, the inference engine, and a user interface. The ES knowledge base is a repository of factual and heuristic knowledge. The former is widely accepted by engineers and scientists; the latter encompasses practice, accurate judgment and the ability to evaluate and guess.[19] The user interface provides the means of interaction between the user of the ES and the ES itself. This is usually natural language processing, suitable for a user who is well versed in the subject area, but who does not have to be an AI expert.

The creation of a 3D methodology for modeling the behaviour of a tool at all the main stages of its life cycle is one of the least expensive ways to increase its efficiency, and hence the efficiency of processing itself. Based on this, with the available tool materials, it is necessary to create scientific foundations and a methodology for choosing such tool characteristics and processing modes that ensure a stable and rational process of high-speed grinding during processing (sharpening) of SHM tools.

3. Research methodology

The efficiency of using SHM grinding wheels is determined, first of all, by the productivity of the process, the specific consumption of diamond grains, and the quality of parts processing. The decisive factor in improving the efficiency of diamond-abrasive tools is the use of scientifically based grinding modes, which make it possible to significantly increase processing productivity and reduce the specific consumption of diamonds in a wheel (this becomes especially important when sharpening SHM tools). At present, the processing of synthetic diamonds is accompanied by low productivity and high specific consumption of diamond wheels; the utilization rate of diamond grains typically does not exceed 5–10%. Using synthetic diamonds as a blade tool requires their sharpening and finishing.

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[19] Valarmathy, S., Ramani, R., Fahim Akhtar, Selvaraju, S. and Ramachandran, G. Automatic ration material distributions based on GSM and RFID technology. *Intl J. Intelligent Systems Applications* **5** (2013) 47–54.

High-speed grinding is especially important for the processing of SHM, since at significant processing speeds not only productivity increases, but also the accuracy of tool sharpening. This is due to the high impact speeds of the diamond grains on the material being processed that has comparable, or the same, hardness as the grains, and the consequent formation of sharp microedges on the cutting grains.

Our methodology was determined by the already widespread use of 3D modeling[1] of processes such as ours, in conjunction with experimental data to assess the adequacy of the models (Fig. 2). All elements of the studied systems (diamond grains, metal phase, binder, coating of the diamond grains, processed SHM) and processing conditions are considered in interaction. Dynamic 3D modeling is used to determine performance parameters and consider the processes of destruction of diamond grains and binder fate during high-speed grinding (Fig. 2). ANSYS, LS-DYNA, CosmosWorks and Abaqus calculation systems were used to study the stress–strain state in the grinding zone and in the tool manufacturing process, to simulate the quantitative and qualitative characteristics of the grinding process. The construction of 3D models imitating the mentioned processes was used by the SolidWorks system of automation of CAD work in three dimensions (Fig. 2).

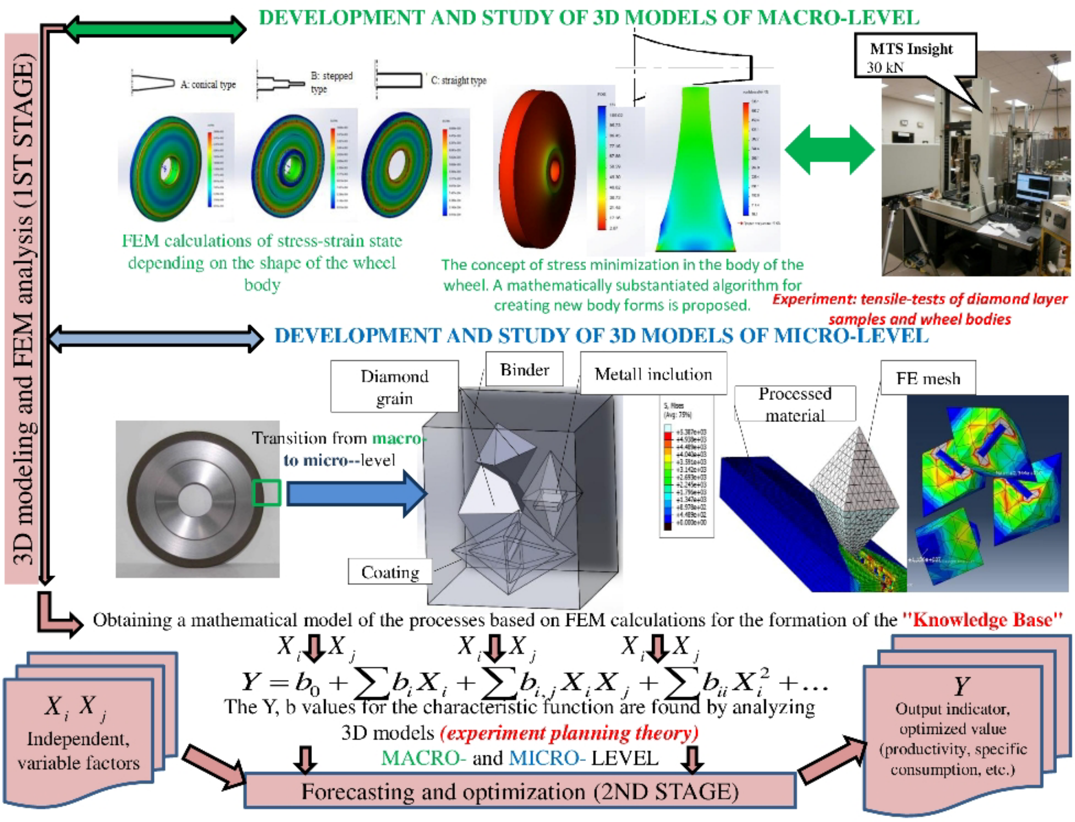

Figure 2. The concept of creating a knowledge base built on mathematical 3D models of macro- and micro-levels for the processes of sintering the diamond layer and of high-speed grinding of the SHM used in the developed expert system.

As can be seen from Fig. 2, the research is generally divided into two stages: 1, creation of 3D models of the macro- and micro-levels, followed by FEM analysis according to the method of planning model experiments and calculation of parameters Y and b for regression equations describing the processes of sintering and high-speed grinding;[1, 6] 2, creation of a software product (expert system) for forecasting and optimizing the processes considered at the first stage.[1, 6] Both stages are closely related. The present paper focuses on the second stage—prediction and optimization of SHM high-speed diamond grinding processes based on 3D macro- and micro-level models. We have considered the first stage in detail in previous papers[1, 6]

To apply the methods of optimization theory, it is necessary to propose a correct statement of the problem and choose the most suitable method for its numerical solution. The correct formulation of the optimization problem[17, 19] necessarily includes: determination of system boundaries; formulation of the characteristic criterion; choice of independent variables; and construction of a system model. The choice of a method for solving the optimization problem is determined taking into account the peculiarities of its mathematical formulation. As applied to high-speed diamond grinding, the above scheme for solving optimization problems takes the following form; the boundary of the object under study separates this object from other objects interacting with it and allows separation of the characteristics (object parameters) into external and internal. When considering the problems of optimization of high-speed grinding, the boundary of the object under study is chosen by the content of the problem under consideration. The boundary of the object under study can be determined by the spatial area of the cutting tool when solving the problems of choosing the optimal characteristics of diamond wheels at the stage of their design and manufacture. When solving problems of optimizing high-speed grinding processes using available cutting tools, taking into account the characteristics of the material being processed, the boundary of the object under study can be determined by the area covering the processing diamond grain with the material being processed around their contact interaction.

The independent variables are a set of parameters characterizing the state of the object under consideration within a selected boundary:

$$\{x\} = \{x_1 \quad x_2 \quad \dots \quad x_n\}^T , \tag{1}$$

where $\{x\}$ is a vector of object-independent variables $x_1, x_2, \dots, x_n$, n is the number of these variables, and T denotes the transpose. When solving optimization problems, each independent variable x_k, $k = 1,2,...,n$ represents some numerical characteristic. In our case they can be: technological modes of sintering the diamond layer; physical and mechanical properties of the diamond layer; processing modes; and physical and mechanical properties of the processed material. Each of the independent variables is expressed in units of the value it represents and naturally can vary within certain limits:

$$x_k^{\min} \le x_k \le x_k^{\max}, k = 1,2,\dots,n , \tag{2}$$

where, $x_k^{\min}$, $x_k^{\max}$ are the maximum and minimum possible values of the independent variable x_k. It is more convenient to work with dimensionless independent variables:

$$-1 \le X_k \le 1, k = 1,2,\dots,n , \tag{3}$$

where the X_k are normalized values of the independent variables, from which we can determine their dimensioned values (2) by:

$$x_k = \alpha_k X_k + \beta_k, \quad k = 1,2,\dots,n , \tag{4}$$

where α_k, β_k are conversion coefficients, the determination of which reduces to solving a system of linear algebraic equations:

$$\begin{Bmatrix} -\alpha_k + \beta_k = x_n^{\min} \\ \alpha_k + \beta_k = x_n^{\max} \end{Bmatrix}, \tag{5}$$

where $k = 1,2,...,n$. It is convenient to write the solution of system (5) using Cramer's rule:

$$\alpha_k = \frac{\begin{vmatrix} x_k^{\min} & 1 \\ x_k^{\max} & 1 \end{vmatrix}}{\begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix}}, \beta_k = \frac{\begin{vmatrix} -1 & x_k^{\min} \\ 1 & x_k^{\max} \end{vmatrix}}{\begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix}}, \tag{6}$$

where $k = 1,2,...,n$. Calculation by eqns (6) allows us to write:

$$\alpha_k = \frac{x_k^{\max} - x_k^{\min}}{2}, \beta_k = \frac{x_k^{\max} + x_k^{\min}}{2}, \tag{7}$$

where $k = 1,2,...,n$.

The inverse transformation (4) is written as:

$$X_k = \frac{1}{\alpha_k} x_n - \frac{\beta_k}{\alpha_k}, \tag{8}$$

where $k = 1,2,...,n$. Application of the normalized variable (8) greatly simplifies the numerical solution of the optimization problem. Further, instead of vector (1), we will consider normalized quantities (8) as independent variables:

$$\{X\} = \{X_1 \quad X_2 \quad \ldots \quad X_n\}^T, \tag{9}$$

where $\{X\}$ is the vector of normalized object-independent variables, and X_1, X_2 … X_n the individual object-independent variables.

The characteristic function is a criterion to evaluate the characteristics of the system in order to ensure the possibility of choosing the best conditions for its functioning. In general, the values of the characteristic function depend on the independent variables:

$$Y = Y(X_1, X_2 \ldots X_n), \tag{10}$$

where $Y(\ldots)$ is some given function (cf. Fig. 2, mathematical model obtained by analysing the results of 3D modeling). In the problems of optimizing the processes of high-speed grinding, the form of the characteristic function (10) is determined by the content of the problem under consideration. The objective function, as a rule, is the stresses in the composition of the diamond layer, as well as the output indicators of the technological process recalculated from them: grinding performance and specific consumption of diamond grains.

The main difficulty of the problem of optimizing the manufacturing processes and the use of wheels for high-speed grinding is that the characteristic function cannot be constructed in an explicit analytical form. This is due to the complexity of the mathematical models describing the manufacturing and application of the diamond wheels; the models are akin to the boundary value problems of the theories of heat conduction and elasticity, symbolically:[20]

$$\mathbf{A} = (\mathbf{u}) = \mathbf{f}, \tag{11}$$

where **A** is the operator of the boundary value problem, **u** the vector of parameters of the state of the studied system, and **f** some given vector that usually characterizes interaction with the environment.

The most general approximate method for solving boundary value problems is FEM. The main idea of the method is that instead of the unknown **u** (eqn 11), which is a function of the coördinates of the points of the area that defines the boundary of the object under study, the values of the function are found at the nodes of finite elements and approximated for the entire object. The solution of such problems can only be obtained using numerical methods and performed using computers due to the large amount of calculations. Currently, there are a large number of systems that allow solving such problems by FEM. The independent variables introduced in the form (1) in our problem are the parameters that determine the initial data. At the same time, the solution of a boundary value problem, even on modern high-speed computers, can take quite a long time; the search for a solution to an optimization problem requires determining the characteristic function at a large number of points. The time for solving the problem of optimizing the high-speed grinding process can be significantly reduced if the values of the characteristic function are calculated at a finite number of points within intervals (2) of independent variables. Using these values instead of the characteristic function (10) one can consider its approximation constructed using its individual calculated values. Of course, this will introduce some error into the calculation; however, such an error will be quite acceptable with a sufficiently large number of approximation nodes within the intervals (2). Instead of dimensioned independent variables (2), dimensionless ones (8) can be used. As an example, some results of constructing the characteristic function of the problem of optimizing the processes of high-speed grinding of SHM are given in Fig. 3.

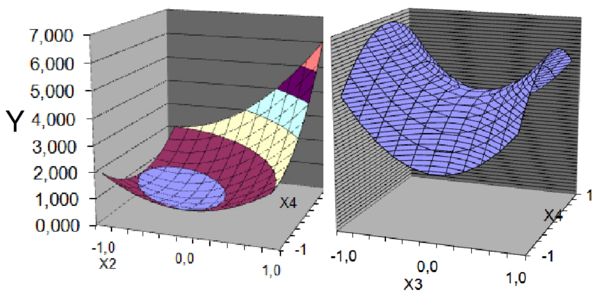

Figure 3. Examples of visualization of the characteristic function of the process of high-speed diamond grinding using FEM: left, the minimum at a stationary point; right, a saddle stationary point.[1, 6]

The results presented in Fig. 3 indicate that the value of the characteristic function can be obtained quite accurately by approximating a relatively small number (24) of its nodal values. These preliminary results (Figs 1 and 3) show that such a function can be quite accurately represented using an approximation in the form of a quadratic function (11), which in the general case of *n* independent variables (factors) is written as:

[20] Kalitkin, N.N. *Numerical Methods*. Moscow: Nauka (1978).

$$Y = b_0 + \sum_{i=1}^{n} b_i X_i + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} b_{ij} X_i X_j + \sum_{i=1}^{n} b_{ii} X_i^2 \tag{12}$$

where Y is the optimized value of the output indicator (e.g., grinding performance; stresses in the sintering zone) at given values of factors X. This function (12) of the independent variables (9) can be represented in matrix vector form as:

$$Y = \frac{1}{2}\{X\}^T [A]\{X\} + \{B\}^T \{X\} + B_0 \tag{13}$$

where $[A]$ is a specified matrix of numerical coefficients, $\{B\}$ a specified vector of numerical coefficients, and B_0 a specified numerical coefficient. They are written as:

$$[A] = \begin{bmatrix} 2b_{11} & b_{12} & b_{13} & \cdots & b_{1n} \\ b_{12} & 2b_{22} & b_{23} & \cdots & b_{2n} \\ b_{13} & b_{23} & 2b_{33} & \cdots & b_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ b_{1n} & b_{2n} & b_{3n} & \cdots & 2b_{nn} \end{bmatrix}, \{B\} = \begin{Bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_n \end{Bmatrix}, B_0 = b_0 \quad . \tag{14}$$

For the characteristic function (13), the stationary condition is written as follows:

$$[A]\{X\} = \{B\} . \tag{15}$$

The minimum and maximum conditions for function (13) take, respectively, the following forms:

$$\{X\}[A][X] \geq 0 , \tag{16}$$

$$\{X\}[A][X] \leq 0 , \tag{17}$$

To establish the maximum or minimum at the stationary point, it is necessary to establish the properties of positive (negative) definiteness of the matrix $[A]$ from (13). A wide variety of criteria can be used for this; the best known is the Sylvester criterion,[20] formulated by first letting the square matrix look like:

$$[A] = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}. \tag{18}$$

By diagonal minors Δi, $i = 1,2,...n$ name the quantities:

$$\Delta_1 = a_{11}, \Delta_2 = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}, \dots, \Delta_i = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1i} \\ a_{21} & a_{22} & \dots & a_{2i} \\ \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & \dots & a_{ii} \end{vmatrix}, \dots \Delta_n = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}. \tag{19}$$

In accordance with the criterion, a symmetric matrix [*A*] is positive definite if all its diagonal minors (19) are positive:[21]

$$\Delta_1 > 0, \Delta_2 > 0, \ldots, \Delta_i > 0, \ldots \Delta_n > 0\,. \tag{20}$$

The calculation of the determinant is carried out using any available numerical method (it is provided in the subroutine for performing LU decomposition).

4. Results

For high-speed grinding processes, the most important output indicators are the specific consumption of diamond grains of the wheel and the productivity of the grinding process. For the first time for the SHM high-speed grinding process, it was possible to theoretically calculate these output characteristics using dynamic 3D modeling. Based on a significant number of theoretical studies, mathematical models have been obtained that approximately characterize the grinding process of SHM at high speed (cf. Fig. 2). Let us consider the example of the mathematical apparatus used for programming optimization calculations in the expert system, using the example of minimizing the specific consumption of diamond grains during SHM grinding. The task of optimization at this stage is to minimize (for the output indicator specific consumption) function (12) of the studied system "diamond grain–metal phase binder–processed material".

To check the correctness of the application of this approach, the values of the coefficients from the regression equation obtained for the specific consumption at the first stage of research[1,6] were substituted into expression (14). With the initial data obtained from the results of 3D modeling (Fig. 2), the matrix [*A*] and the vector {*B*} from expression (14) are:

$$[A] = \begin{bmatrix} 2.56 & 0.01 & 0.006 & 0.23 \\ 0.01 & 3 & -0.03 & 0.14 \\ 0.006 & -0.03 & 2.5 & 0.031 \\ 0.23 & 0.14 & 0.031 & -4.72 \end{bmatrix}, \{B\} = \begin{Bmatrix} 0.04 \\ -0.17 \\ 3.66 \\ 0.56 \end{Bmatrix}, B_0 = 23.12. \tag{21}$$

As a result of solving the system, a stationary point with coördinates X_{stat} was obtained:

$$X_{\text{stat}} = \begin{Bmatrix} 2.21330534540417\text{E}-0002 \\ -3.70032562205910\text{E}-0002 \\ 1.46485496531225\text{E}+0000 \\ -1.09042235095715\text{E}-0001 \end{Bmatrix}, Y_{\text{stat}} = 3.1081222095627346\text{E}+001 \tag{22}$$

where Y_{stat} is the value of the characteristic function at the stationary point. For the determinant of the matrix from (21), det[*A*] = –91.139. The negative value violates the condition of the positive sign of the matrix from the Sylvester criterion. Taking into account this circumstance, the minimum value of the characteristic function takes place at the boundary points of the considered interval.

[21] Berezin, I.S. and Zhidkov, N.P. *Computational Methods, Vol. II*. Moscow: State. Publishing House of Physics and Mathematics (1959).

As a result of the optimization procedure, the optimal point X_{min} was obtained, corresponding to the minimum value of the function written in the form (21) with the initial data (21):

$$X_{min} = \begin{Bmatrix} 1.00000000000000E+0000 \\ 1.00000000000000E+0000 \\ -1.00000000000000E+0000 \\ -1.00000000000000E+0000 \end{Bmatrix}, Y_{min} = 2.01349999999999998E+001. \tag{23}$$

From this we can conclude that checking the behaviour (Sylvester criterion) at a (near) stationary point to describe the process of high-speed grinding is mandatory. From (22) we see that the minimum of the function, i.e. the specific consumption of diamond grains in the "diamond grain–metal phase binder–processed material" system during high-speed grinding, is on the border of the considered range of independent variables (factors) X and corresponds to the coördinate vector X_{min} (23) in coded values.

The optimization problem for the high-speed grinding expert system HSGES is implemented in the form of three specially designed modules called LAPDEF, LAPFILE and LAPEQNS.

The LAPDEF module contains a description of classes representing vectors and matrices. These classes provide a variable to deallocate memory, set sizes, and access elements. The representation of matrices and vectors in the form of classes using the technology of object-oriented programming makes it possible to significantly simplify the use of vectors and matrices in a computer expert system. It should be especially noted that the elements of matrices and vectors in general form have the EXTENDED type, which provides a representation of a real number with 19–20 significant digits in the range $1.9\times10^{-4932}\ldots.1.1\times10^{4932}$. The choice of this type of data is due to the desire to ensure the highest possible accuracy of calculations by diminishing rounding errors to the greatest possible extent.

The LAPFILE module contains a set of subroutines that provide writing to a text file and reading data from matrices and vectors from a text file, as well as saving and loading these data.

The LAPEQNS module contains subroutines that allow solving a system of linear algebraic equations using the LU decomposition method. These subprograms are developed on the basis of programs proposed in a reference book adapted to modern programming languages.[15]

According to the conceived concept, the expert system should: combine and store the accumulated databases and knowledge base of model and experimental calculations; optimize the mathematical models described above; calculate statistical errors; assess the risks of making a decision; and update the databases and mathematical models. At the same time, it is necessary to enable the user to access the obtained knowledge and use it to solve the industrial or scientific problem of interest. To implement this possibility, it is necessary to develop a special computer application that includes a graphical interface with which the user (worker or researcher) could operate all the functions of the system. Such an application (HSGES) has been designed, tested and debugged.

Next, we consider some of the steps in creating the main working modules, functions and control panel. Fig. 4 shows the main window of the application; it shows the process of creating a "Button1Click" object to control the function of accessing the system database, which is responsible for selecting the task that the user needs. In the source code window we can see the procedure responsible for creating the "Button" object called "Select Task", and the types of variables (TStrings, integer) that this object can work with. In a similar way, other objects of the system workspace are created, which are focused on the execution of a specific programmed task.

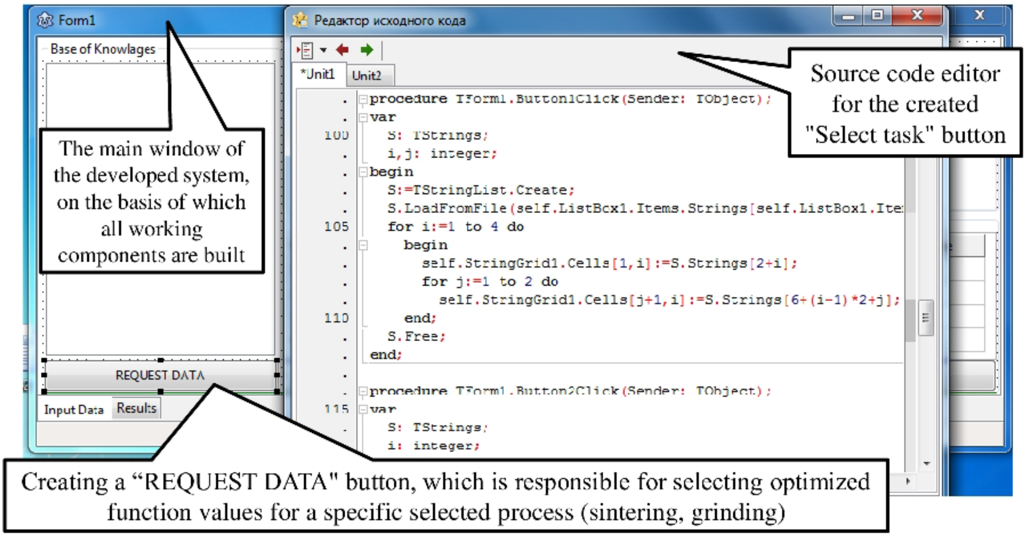

Figure 4. One of the stages of development of the working interface of the HSGES system.

Consider also the process of creating the "ListBox" field (Fig. 5), with the help of which the database with solutions for the obtained mathematical models will be displayed and called upon request, considered in the work of the processes of sintering diamond wheels and SHM high-speed grinding. The "ListBox" component is an array of strings. Using this object, it is possible to load data from disk and save information to a file. Also "ListBox" can sort rows. The user can select a line in the list by clicking on it with the left mouse button and access the description of the selected database object.

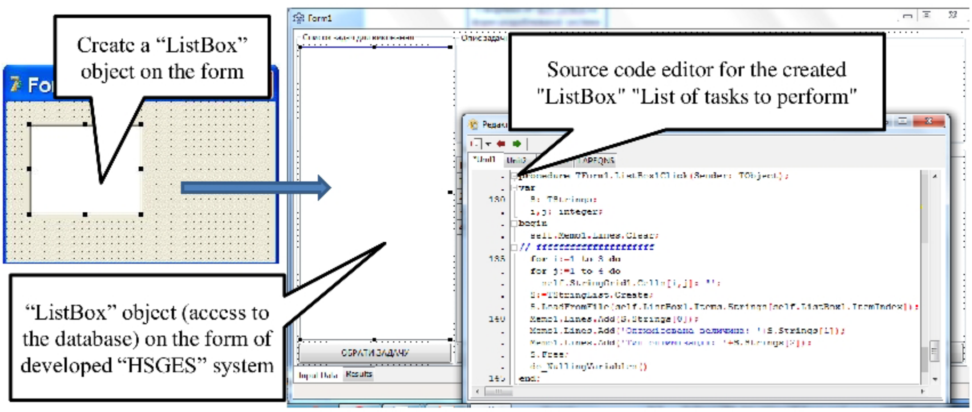

Figure 5. Creation and description of the "ListBox" field for the HSGES expert system which is responsible for working with databases of the system.

The system also comprises working modules that programmatically implement mathematical methods for calculating stationary points of mathematical models of the process, checking solutions for minimum and maximum, as well as assessing the risks of decisions made. Consider the developed program code that implements the search for the stationary value of the characteristic function, which in turn reduces to solving a system of linear algebraic equations. Fig.6 shows the program code created for the LAPEQNS module using the LU decomposition method.

*Unit1 | Unit2 | LAPDEF | *LAPEQNS

```
procedure LU_solve(A: TMatriceExtended; r: TVectorWord; b: TVectorExtended);
    p: extended;
begin
 self.ClearAll;
 self.AddInfo('Linear equations system');
 self.AddInfo('method  = Kraut column main element scheme');
 self.AddInfo('equations count = '+InttoStr(A._CountRows));
 self.NameProcess:='Counter motion';

  self.NameAction:='rows interchanging';
  for i:=1 to A._CountRows do if r._Coef[i]<>i
                             then begin
                                   self.ProgressAction:=round(i/A._CountRows*100);
                                    p:=b._Coef[i];
                                    b._Coef[i]:=b._Coef[r._Coef[i]];
                                    b._Coef[r._Coef[i]]:=p;
                                  end;

  self.NameAction:='Uv=b system solving';
  for i:=1 to A._CountRows
  do begin
       self.ProgressAction:=round(i/A._CountRows*100);
       for j:=1 to i-1 do b._Coef[i]:=b._Coef[i]-A._Coef[i,j]*b._Coef[j];
       b._Coef[i]:=b._Coef[i]/A._Coef[i,i];
       self.ProgressProcess:=round(i/A._CountRows*50)
     end;

  self.NameAction:='Lx=v system solving';
  for i:=A._CountRows downto 1
  do begin
       self.ProgressAction:=round((A._CountRows-i)/(A._CountRows-1)*100);
       for j:=A._CountRows downto i+1 do b._Coef[i]:=b._Coef[i]-A._Coef[i,j]*b._Coef[j];
       self.ProgressProcess:=round((1+(A._CountRows-i)/(A._CountRows-1))*50)
     end
end;
```

Figure 6. Program code that implements the LU decomposition method for solving a system of equations when searching for optimal values for the functions of studied processes.

As shown, the working modules of the system consist of a set of functions and procedures that are automatically called on demand when the user works with the system interface. The user sets the input data from the proposed set of parameters in the range of values covered by the system (Fig. 7). The parameters are those independent variables (factors) that, according to the expert, have the greatest impact on the output parameter (optimized value) of the mathematical model of the selected task; e.g., the SHM grinding process. Consider the optimization of the specific consumption of diamond grains. To do this, we turn to the knowledge base of the system, in the application we select the process of interest to us. The user has the opportunity to enter the input parameters of interest to him and evaluate them for the SHM processing process (the field "Entered parameter values"). In this case, the user is prompted to enter the following parameters (X_i): thermal expansion coefficient (CTE) of the metal phase, elastic modulus of the binder, cross-feed and grinding speed. Next, the system optimizes the output indicator of the function (from the knowledge base) that describes the SHM grinding process and compares the data entered by the user with the optimal ones that minimize the specific consumption of diamond grains.

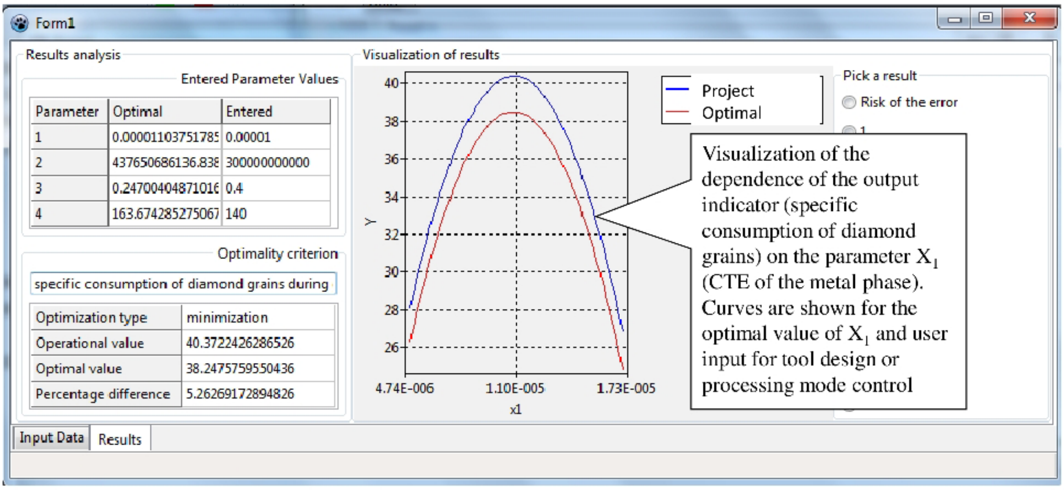

Figure 7. Operation of the HSGES system for minimizing the specific consumption of diamond grains during the SHM grinding depending on the input parameters: CTE of the metal phase, elastic modulus of the binder, transverse feed and processing speed.

The system interface has two languages (Ukrainian and English) and can be used on any computer running any of the popular operating systems. After clicking on the "Evaluate entered values" button, the "Results" tab opens to the user (Fig. 7). Table 1 shows some results of the program.

Table 1. Results from the HSGES expert system in solving the problem of minimizing the specific consumption of diamond grains during SHM processing.

<table>
<tr><td colspan="2">Parameters
(input variable indicators)</td><td>Optimized by the system $X_{1..4}$</td><td>Example of user-entered values $X_{1..4}$, when solving a forecasting technological problem directly in production (closest to the optima of the existing ones)</td></tr>
<tr><td>1</td><td>CTE of metal phase/K^{-1}</td><td>0.0000110375178552597</td><td>0.00001</td></tr>
<tr><td>2</td><td>Elasticity modulus of binder/N m^{-2}</td><td>437650686136.838</td><td>300000000000</td></tr>
<tr><td>3</td><td>Transverse feed/mm</td><td>0.247004048710169</td><td>0.4</td></tr>
<tr><td>4</td><td>Processing speed/m s^{-1}</td><td>163.674285275067</td><td>140</td></tr>
<tr><td colspan="2"></td><td></td><td></td></tr>
<tr><td colspan="4">Calculation of the output indicator (optimization criterion) with optimal input data and user input</td></tr>
<tr><td colspan="4">Specific consumption of diamond grains during SHM grinding (mg/mm^3) depending on:</td></tr>
<tr><td colspan="2">User-entered ($X_{1..4}$)</td><td>Optimized parameters ($X_{1..4}$)</td><td>Percentage difference</td></tr>
<tr><td colspan="2">40.3722426286526</td><td>38.2475759550436</td><td>5.26269172894826</td></tr>
</table>

The obtained optimal values of the four factors have two practical applications:

1. These values are the terms of reference for developers (and the equipment on which these optimal processing modes can be implemented) of the tool that must have the obtained optimal properties;

2. Since it is unlikely that among mass-produced tools and equipment there are those that have optimal properties, the user enters into the system those real properties (values) of all four factors closest in value to the optimal ones (hereinafter referred to as “user-entered”, Table 1) and gets the predicted output of the process, and its difference from the optimum as a percentage.

The developed HSGES system, based on computer processing of the results of a wide range of experimental and modeling studies, allows the user to determine the optimal conditions for the process of designing and manufacturing diamond wheels and for the SHM high-speed diamond grinding process. The system is flexible and “learnable”. The software is able to run on the main most-used operating platforms. Modernization of the HSGES system can be accomplished by developing new models, both based on FEM and other modeling methods available to the expert, which will expand the knowledge base. One direction of development of the system is its adaptation for use with mobile devices.

5. Conclusions

Optimization, storage and analysis of data in modern conditions require additional knowledge and skills—this is a separate issue for the direct practical application of the developed models. The present work is connected with previous studies that took place at the first stage (Fig. 2) and briefly describes their results in order to understand the direct relationship between them. Now we are interested in the second stage—the development of the foundations for the practical application of the results of the analysis of the developed 3D models of the diamond grinding process from macro- to micro-level. The work at the second stage (Fig. 2) is largely devoted to the description of the programming process in the Lazarus environment, as a result of which an expert system was created, capable of storing and processing a large number of research results of 3D models of macro- and micro-levels in a convenient and modern way. (Figs 4–7). The mathematical foundations and algorithms for analysing the results, as well as the creation of a working application with a user interface (HSGES) for working with them (Fig. 7), have been described.

A scientifically substantiated method for designing and manufacturing (sintering) a diamond abrasive tool is proposed, taking into account the peculiarities of its manufacture and processing conditions at high speeds. The technique is based on the wide application of 3D models of macro- and micro-levels, on the basis of which mathematical models of the processes under study are created, acceptable for optimization using programming. The developed software product (HSGES) is flexible and allows for its further improvement, as well as application to other machining processes. Modernization of the HSGES expert system can be carried out by developing new models both based on the finite element method and other modeling methods available to an expert. The subsequent analysis of the simulation results and the formation of mathematical models by various planning methods that describe studying processes will expand the knowledge base of the system and, accordingly, increase its efficiency, as well as the degree of applicability when introduced into production processes and scientific research.