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Complex Fermatean Fuzzy Soft Right Semi Group and Ideal Structures

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In this article, we define a new structure of complex Fermatean fuzzy soft left ideals (resp., right) of a semi group S. Union and Intersection of ideals is again ideal. Also, we prove the set of all idempotent elements of S from a right zero semi group of S, then C(x) = C(y) for all idempotent elements of x and y of S. If C1 and C2 are complex Fermatean fuzzy soft left and right ideals of a semi group S, respectively, then C1 * C2 C1 C2. For every Complex Fermatean fuzzy soft right ideal C1 and every Complex Fermatean fuzzy soft left ideal C2 of a semi group S, if C1 * C2 = C1 C2, then S is regular.

Keywords: fuzzy set, soft set, complex Fermatean fuzzy set, soft ideal, idempotent, right zero semi group.

1. Introduction

Molodtsov [14] initiated the concept of soft set theory as a new approach for modeling uncertainities. Then Maji et.al [[15],[16]] expanded this theory to fuzzy soft set theory. The algebraic structures of soft set theory have been studied increasingly in recent years. Aktas and Cagman [2] defined the notion of soft groups. Feng et.al [2008] initiated the study of soft semi rings and finally soft rings are defined by Acar et.al [3]. But in real life situation, the problems in economics, engineering, environment, social science, medical science etc do not always involve crisp data. Consequently, we cannot successfully by using the traditional classical methods because of various types of uncertainties in this problem. There are several theories, for example, theory of fuzzy sets [20], theory of intuitionistic fuzzy sets [1], vague sets [20], interval mathematics [20] and rough sets [10] which can be considered as mathematical tools for dealing with uncertainties. But all these theories have this inherent difficulty as what where point out by Molodtsov in [14]. The reason for these difficulties is possibly the inadequacy of the parameterization tool of the theories.

Indeed, the applications of complex fuzzy sets span various fields, such as signal processing [2002], physical phenomena [2002] and economics [2002]. Chen et.al developed a neuro-fuzzy architecture using complex fuzzy sets [2011]. Jun et.al successfully applied complex fuzzy sets in multiple periodic factor prediction problems [10]. Additional background on complex fuzzy sets can be found in [2002]. Similarly to the case of an intuitionistic fuzzy set, a complex intuitionistic fuzzy set is characterized by a complex grade of membership and complex grade of non-membership [2012]. The complex intuitionistic fuzzy sets enable defining the values of membership and non-membership for any object in these complex-valued functions.

In this article, we define a new structure of complex Fermatean fuzzy soft left ideals (resp., right) of a semi group S. Union and Intersection of ideals is again ideal. Also, we prove the set of all idempotent elements of S from a right zero semi group of S, then C(x) = C(y) for all idempotent elements of x and y of S. If C_1 and C_2 are complex Fermatean fuzzy soft left and right ideals of a semi group S, respectively, then $C_1 * C_2 \subseteq C_1 \cap C_2$. For every Complex Fermatean fuzzy soft right ideal C_1 and every Complex Fermatean fuzzy soft left ideal C_2 of a semi group S, if $C_1 * C_2 = C_1 \cap C_2$, then S is regular.

2. Basic concepts and preliminaries

In the section, we define some ideals namely complex Fermatean (left,right,interior) ideal in semi group, with help of examples and study some of its related results.

Definition 2.1: A mapping $\mu: X \rightarrow [0, 1]$ where X is an arbitrary non empty set and is called Fuzzy set in X.

Definition 2.2: Let G be a any group. A mapping $\mu : G \rightarrow [0, 1]$ is a Fuzzy Subgroup if

(i) $\mu (x \bullet y) \geq \min \{\mu (x), \mu (y)\}$

(ii) $\mu (x^{-1}) = \mu (x)$, for all $x, y \in G$.

Definition 2.3: A pair (f,A) is called a soft set over the lattice L, if $f : A \rightarrow P(L)$. Here L is the initial universe and E is the set of parameters. Let P(L) denotes the power set of L and I^L denotes the set of all fuzzy sets on L.

Definition 2.4: A pair (f,A) is called a fuzzy soft set over L, where $f: A \rightarrow I^L$, ie. for each $a \in A$, $f_a: L \rightarrow I$ is a fuzzy set in L.

Definition 2.5: Let (f,A) be a non-null soft set over a ring R. Then (f,A) is said to be a soft ring over R if and only if f(a) is sub ring of R for each $a \in A$.

Definition 2.6: Let $\mu: X \rightarrow Y$ and $\upsilon: A \rightarrow B$ be two functions, where A and B are parameter sets for the crisp sets X and Y respectively. Then the pair (μ , υ) is called a fuzzy soft function from X to Y.

Definition 2.7: A complex bi fuzzy soft set C on a semi group S is known as complex bi fuzzy soft left ideal of S if $C (xy) > C (y)$.

Definition (Fuzzy number) 2.8: It is a fuzzy set the following conditions:

(i) convex fuzzy set (ii) normalized fuzzy set (iii) its membership function is piecewise continuous.

(iv) it is defined in the real number.

A complex fuzzy subset A, defined on a universe of discourse X, is characterized by a membership function $\tau_A(x)$ that assigns any element $x \in X$ a complex valued grade of membership in A. The values of $\tau_A(x)$ all lie within the unit circle in the complex plane and thus all of the form $P_A(x)\, e^{j\mu_A(x)}$ where $P_A(x)$ and $e^{j\mu_A(x)}$ are both real valued and $P_A(x) \in [0,1]$. Here $P_A(x)$ is termed as amplitude term and $e^{j\mu_A(x)}$ is termed as phase term.

The complex fuzzy set may be represented in the set form as $A = \{(x, \tau_A(x)) / x \in X\}$. It is denoted by CFS.

The phase term of complex membership function belongs to $(0,2\pi)$. Now we take those forms which Ramot et.al presented in [8] to define the game of winner, neutral and lose.

$$\mu_{A\cup B}(x) = \begin{cases} \mu_A(x) & \text{if } P_A > P_B \\ \mu_B(x) & \text{if } P_A < P_B \end{cases}.$$

This is a novel concept and it is the generalization of the concept " winner take all" introduced by Ramot et.al [8] for the union of phase terms.

Example 2.9: Let $X = \{x_1, x_2, x_3\}$ be a universe of discourse. Let A and B be complex fuzzy sets in X as shown below.

$$A = \{0.6\, e^{i(0.8)}, 0.3\, e^{i\frac{3\pi}{4}}, 0.5\, e^{i(0.3)}\}$$

$$B = \{0.8\, e^{i(0.9)}, 0.1\, e^{i\frac{\pi}{4}}, 0.4\, e^{i(0.5)}\}$$

$$A\cup B = \{0.8\, e^{i(0.9)}, 0.3\, e^{i\frac{3\pi}{4}}, 0.5\, e^{i(0.3)}\}$$

We can easily calculate the phase terms $e^{i\mu_{A\cap B}(x)}$ on the same line by winner, neutral and loser game.

3. Some standard results in complex number

Theorem-3.1: If $\overline{\omega} = \overline{z_1} + \overline{z_2}$, then $\overline{\omega^{\alpha}} = \mu^{\alpha}$ and if $\overline{\omega} = \overline{z_1}\,\overline{z_2}$, then $\overline{\omega^{\alpha}} = p^{\alpha}$

Proof: In this theorem we have two cases arise.

Case (i): $-1 \le \alpha < 0$

Let $\omega \in \overline{\omega^{\alpha}}$. Then we can find $z_1 \in \overline{z_1}$ and $z_2 \in \overline{z_2}$

Because $\overline{\omega} = \overline{z_1} + \overline{z_2}$ such that $z_1 + z_2 = \omega$ and $\pi_k(z_1, z_2) < \alpha$.

Since $\delta_k(\omega) < \alpha$ for $k \in P$. This implies that $\delta_k(z_i) < \alpha$ for $i = 1,2$ and $k \in P$

.

$\Rightarrow z_1 \in \overline{z_1^\alpha}$ and $z_2 \in \overline{z_2^\alpha}$ $\Rightarrow (z_1, z_2) \in \overline{z_1^\alpha} \times \overline{z_2^\alpha}$, where $\overline{z_1^\alpha} \times \overline{z_2^\alpha}$ is the Cartesian product between $\overline{z_1^\alpha}$ and $\overline{z_2^\alpha}$.Thus from the given definition of μ^α, $\omega \in \mu^\alpha$

Conversely, let $\omega \in \mu^\alpha$.

Then $\omega = z_1 + z_2$ with $\delta_k(z_i) < \alpha$ for $i = 1,2$ and $k \in P$

$\Rightarrow \pi_k(z_1, z_2) < \alpha \Rightarrow \max\{\pi_k(z_1, z_2) / z_1 + z_2 = \omega\} < \alpha$

$\Rightarrow \pi_k(\omega) < \alpha$ for all $k \in P$

$\Rightarrow \omega \in \overline{\omega^\alpha}$.Hence $\overline{\omega^\alpha} = p^\alpha$

Case (ii): $\alpha = -1$

Let $\omega \in \mu^{-1}$. Then there exist z_1 and z_2 such that $z_1 + z_2 = \omega$ and $\pi_k(z_1, z_2) = -1$ for all $k \in P$.

Thus, $\delta_k(\omega) = -1$ for all $k \in P \Rightarrow \omega \in \overline{\omega^{-1}}$.

Conversely,

Let $\omega \in \overline{\omega^{-1}}$. Then we can find z in $\sup(\overline{z_1})$ and $(\overline{z_{2n}})$ in $\sup(\overline{z_2})$ for $n = 1,2,3,\ldots\ldots.$ such that $z_{1n} + z_{2n} = \omega$ and $\pi_k(z_{1n}, z_{2n}) < -1 - \frac{1}{n}$ for all $p \in P$.

Since the supremums are compact, we may choose subsequences $z_{1nk} \to z_1$ and $z_{2nk} \to z_2$ with $z_1 + z_2 = \omega$ and $\pi_k(z_1, z_2) < -1 - \frac{1}{n}$ for all $k \in P$, because π_k is continuous.

$\Rightarrow \inf\{\delta_k(z_1), \delta_k(z_2)\} \le -1$ for all $k \in P \Rightarrow \delta_k(z_i) \le -1$ for $i = 1,2$ and for all $k \in P$.

$\Rightarrow z_i \in \overline{z_i^{-1}}$ for $i = 1,2$

$\Rightarrow (z_1, z_2) \in \overline{z_1^{-1}}, \overline{z_2^{-1}}$. Hence, if $\overline{\omega} = \overline{z_1} + \overline{z_2}$, then $\overline{\omega^{\alpha}} = \mu^{\alpha} \Rightarrow \omega \in P^{-1}$

Before proceeding to the mxt theorem, we will discuss some important results which are needed for the upcoming results.

Lemma-3.2: If $\overline{\omega} = \overline{z_1} + \overline{z_2}$ or $\overline{\omega} = \overline{z_1}\,\overline{z_2}$, then $\overline{\omega^{\alpha}}$ is open for $-1 \le \alpha < 0$

Proof: Let $\overline{\omega} = \overline{z_1} + \overline{z_2}$ and $\omega \in \overline{\omega^{\alpha}}$ for $-1 \le \alpha < 0$.

Then by theorem (3.1), $\Im(z_1, z_2) \in \overline{z_1} \times \overline{z_2}$ such that $z_1 + z_2 = \omega$.

Since $\overline{z_2^{\alpha}}$ is open so, choose on disk $O(z_2, \varepsilon)$ with radius $\varepsilon > 0$, centered at z_2 such that $O(z_2, \varepsilon)$ contained in $\overline{z_2^{\alpha}}$, thus $z_1 + O(z_2, \varepsilon)$ is an open set containing $\omega = z_1 + z_2$ and contained in $\mu^{\alpha} = \overline{\omega^{\alpha}}$. Therefore $\overline{\omega^{\alpha}}$ is open. If $\overline{\omega} = \overline{z_1}\,\overline{z_2}$, then by similar argument by choosing $\omega = z_1 z_2$ and $z_1.O(z_2, \varepsilon)$ in place of $\omega = z_1 + z_2$ and $z_1 + O(z_2, \varepsilon)$ respectively. Thus $\overline{\omega^{\alpha}}$ is open.

Lemma-3.3: Let $\overline{\omega} = \overline{z_1} + \overline{z_2}$ or $\overline{\omega} = \overline{z_1}\,\overline{z_2}$. Assuming that $\omega_n = \overline{\omega^{0}}$ for $n \in P$ converges to ω and $\delta_k(\omega_n)$ converges to λ_k in [-1,0] for $k \in P$, then $\delta_k(\omega) \le \lambda_k$ for all $k \in P$.

Proof: Let $\overline{\omega} = \overline{z_1} + \overline{z_2}$. Then for every $\varepsilon > 0$, there is $z_{1n} \in \overline{z_1^{0}}$ and $z_{2n} \in \overline{z_2^{0}}$ such that $z_{1n} + z_{2n} = \omega_n$ and $\delta_k(\omega_n) - \varepsilon < \pi_k(z_{1n}, z_{2n}) \le \delta_k(\omega_n)$ for every $k \in P$. Hence all z_{1n}, z_{2n} and ω_n for $n \in P$ belongs to compact sets. Thus, we can choose subsequences such that $z_{1nk} \to z_1$, $z_{2nk} \to z_2, \omega_{nk} \to \omega$ and $z_1 + z_2 = \omega$ with $\lambda_k - \varepsilon < \pi_k(z_1, z_2) \le \lambda_k$ for $k \in P$, because π_k is continuous. Since ε is arbitrary, so $\pi_k(z_1, z_2) = \lambda_k$ for $k \in P$, thus $\delta_k(\omega) \le \lambda_k$ for $k \in P$. By similar argument, we can prove that if $\overline{\omega} = \overline{z_1}\,\overline{z_2}$, then $\delta_k(\omega) \le -k$ for $k \in P$.

Theorem-3.4: If $\overline{z_1}$ and $\overline{z_2}$ are fuzzy complex numbers, then so $\overline{z_1}+\overline{z_2}$, $\overline{z_1}\,\overline{z_2}$, $\overline{z_1}-\overline{z_2}$ and $\frac{\overline{z_1}}{\overline{z_2}}$.

Proof: (i) Let us assume that $\overline{\omega}=\overline{z_1}+\overline{z_2}$. We have to show $\overline{\omega}$ is a fuzzy complex number, our first target is to show $\delta_k(\omega)$ is continuous for all $k \in P$ by arguing that $\omega_n \to \omega$ implies $\delta_k(\omega_n) \to \delta_k(\omega)$ for $k \in P$.

It is sufficient to choose ω_n in $\overline{\omega^0}$. Since $\delta_k(\omega_n) \in [-1,0]$, so, there exists a subsequence $\delta_k(\omega_n) \to \lambda_k \in [-1,0]$ for $k \in P$.

We have, $\lim\min \delta_k(\omega_n) \le \lambda_k \le \lim\max$ for $k \in P$.

Also by lemma-3.2, $\{\omega / \delta_k(\omega) \ge t\}$ is a closed set for all real t. Therefore, $\delta_k(\omega)$ for $k \in P$ is a lower semi continuous and we have, $\lim\min \delta_k(\omega_n) \ge \delta_k(\omega)$ for $k \in P$.

Again by lemma-3.3, $\delta_k(\omega) \le \lambda_k$ for $k \in P$. Thus $\lim\min \delta_k(\omega_n) = \lambda_k = \delta_k(\omega)$ for $k \in P$.

Hence there is a subsequence $\delta_k(\omega_{nj}) \to \lim\min \delta_k(\omega_n)$.

Also by lemma-3.3 implies, $\delta_k(\omega) = \lim\max \delta_k(\omega_n)$,

Therefore, $\lim\min \delta_k(\omega_n) = \delta_k(\omega) = \lim\max \delta_k(\omega_n)$ for all $k \in P$.

$\Rightarrow \lim \delta_k(\omega_n) = \delta_k(\omega)$ for $k \in P$ $\Rightarrow \delta_k(\omega)$ is continuous for all $k \in P$.

Since $\overline{\omega^\alpha}$ for $-1 \le \alpha < 1$ is the sum of two bounded sets by theorem-3.1,

So $\overline{\omega^\alpha}$ for $-1 \le \alpha \le 0$ is bounded. Hence, $\overline{\omega}$ satisfies all the conditions of fuzzy complex number.

(ii) By replacing $\overline{\omega}=\overline{z_1}+\overline{z_2}$ by $\overline{\omega}=\overline{z_1}\,\overline{z_2}$ in (i), we can prove that $\overline{\omega}=\overline{z_1}\,\overline{z_2}$ is a fuzzy complex number.

(iii) Since $\overline{z_2}$ is a fuzzy complex number, then so is $-\overline{z_2}$ and therefore $\overline{z_1} - \overline{z_2}$ is a fuzzy complex number.

(iv) Since the mapping $z \to z^{-1}, z \neq 0$ is continuous and $\left(z^{-1}\right)^{\alpha} = \left(\overline{z^{\alpha}}\right)^{-1}$.

Thus $z^{-1}$is a fuzzy complex number. Hence $\dfrac{\overline{z_1}}{\overline{\overline{z_2}}}$ is a fuzzy complex number.

By using theorem 3.1, lemma 3.2 and lemma 3.3, we can prove the following result:

Lemma-3.5: $\left|\overline{z}\right|^{\alpha} = \left|\overline{z^{\alpha}}\right|$ for $-1 \leq \alpha \leq 0$

Theorem-3.6: Let $\overline{z_1}, \overline{z_2}$ be fuzzy complex numbers. Then

i. $\left|\overline{z_1} + \overline{z_2}\right| \leq \left|\overline{z_1}\right| + \left|\overline{z_2}\right|$

ii. $\left|\overline{z_1}\,\overline{z_2}\right| = \left|\overline{z_1}\right|\left|\overline{z_2}\right|$

iii. $\left|\dfrac{\overline{z_1}}{\overline{\overline{z_2}}}\right| = \dfrac{\left|\overline{z_1}\right|}{\overline{\overline{\left|\overline{z_2}\right|}}}$.

Proof: (i) By theorem 3.1 and lemma-3.3, we have

$$\left|\overline{z_1} + \overline{z_2}\right|^{\alpha} = \left|\left(\overline{z_1} + \overline{z_2}\right)^{\alpha}\right| = \left|\overline{z_1^{\alpha}} + \overline{z_2^{\alpha}}\right| = \left\{ \left|\overline{z_1} + \overline{z_2}\right| / \overline{z_1} \in \overline{z_1^{\alpha}}, \overline{z_2} \in \overline{z_2^{\alpha}} \right\}$$

$\to (1)$ Also, by lemma-3.3, we have

$$\left(\left|\overline{z_1}\right| + \left|\overline{z_2}\right|\right)^{\alpha} = \left|\overline{z_1}\right|^{\alpha} + \left|\overline{z_2}\right|^{\alpha} = \left|\overline{z_1{}^{\alpha}}\right| + \left|\overline{z_2{}^{\alpha}}\right|$$

$$= \left\{ \left|\overline{z_1}\right| + \left|\overline{z_2}\right| / \overline{z_1} \in \overline{z_1^{\alpha}}, \overline{z_2} \in \overline{z_2^{\alpha}} \right\} \to (2)$$

Hence from (1) and (2),

$$\left|\overline{z_1} + \overline{z_2}\right| \leq \left|\overline{z_1}\right| + \left|\overline{z_2}\right| \text{ for all } z_1, z_2 \in C .$$

(ii) We have $\left|\overline{z_1}\,\overline{z_2}\right|^{\alpha} = \left|\left(\overline{z_1}\,\overline{z_2}\right)^{\alpha}\right| = \left|\overline{z_1^{\alpha}}\,\overline{z_2^{\alpha}}\right|$

$$= \left\{ \left|\overline{z_1}\,\overline{z_2}\right| / \overline{z_1} \in \overline{z_1^{\alpha}}, \overline{z_2} \in \overline{z_2^{\alpha}} \right\} \rightarrow (3)$$

Also

$$\left(\left|\overline{z_1}\right|\left|\overline{z_2}\right|\right)^{\alpha} = \left|\overline{z_1}\right|^{\alpha}\left|\overline{z_2}\right|^{\alpha} = \left|\overline{z_1^{\alpha}}\right|\left|\overline{z_2^{\alpha}}\right| = \left\{ \left|\overline{z_1}\right|\left|\overline{z_2}\right| / \overline{z_1} \in \overline{z_1^{\alpha}}, \overline{z_2} \in \overline{z_2^{\alpha}} \right\} \rightarrow (4)$$

Hence from (3) and (4), $\left|\overline{z_1}\,\overline{z_2}\right| = \left|\overline{z_1}\right|\left|\overline{z_2}\right|$.

(iii) We see that

$$\left|\frac{\overline{z_1}}{\overline{z_2}}\right|^{\alpha} = \left|\overline{z_1}\,\overline{z_2^{-1}}\right|^{\alpha} = \left|\overline{z_1^{\alpha}}\left(\overline{z_2^{-1}}\right)^{\alpha}\right| = \left|\overline{z_1^{\alpha}}\left(\overline{z_2^{\alpha}}\right)^{-1}\right|$$

$$= \left\{ \left|\frac{\overline{z_1}}{\overline{z_2}}\right| / \overline{z_1} \in \overline{z_1^{\alpha}}, \overline{z_2} \in \overline{z_2^{\alpha}} \right\} \rightarrow (5)$$

Also

$$\left(\frac{\left|\overline{z_1}\right|}{\left|\overline{z_2}\right|}\right)^{\alpha} = \left(\left|\overline{z_1}\right|\left|\overline{z_2}\right|^{-1}\right)^{\alpha} = \left|\overline{z_1}\right|^{\alpha}\left(\left|\overline{z_2^{-1}}\right|\right)^{\alpha} = \left|\overline{z_1}\right|^{\alpha}\left(\left|\overline{z_2}\right|^{\alpha}\right)^{-1}$$

$$= \left\{ \left|\frac{\overline{z_1}}{\overline{z_2}}\right| / \overline{z_1} \in \overline{z_1^{\alpha}}, \overline{z_2} \in \overline{z_2^{\alpha}} \right\} \rightarrow (6)$$

From (5) and (6), α-cuts are equal and hence $\left|\frac{\overline{z_1}}{\overline{z_2}}\right| = \frac{\left|\overline{z_1}\right|}{\left|\overline{z_2}\right|}$ is proved.

4. Some characterisations of Complex Fermatean fuzzy soft ideals

In this section, we deal with main theorems based on complex Fermatean fuzzy soft left ideals (resp., right).

Theorem 4.1: Let C be a complex Fermatean fuzzy soft left ideal of a semi group S. If the set of all idempotent elements of S form a left Zero Semi group of S, then C(x) = C(y) for all idempotent elements of x and y of S.

Proof: Let us assume that Idm(s) be the set of all idempotent elements of S and is a left Zero Semi group of S.

For any x,y ∈ Idm (S), we have xy = x and yx = y. Thus

$P^3c(x)\bullet e^{i\,\delta c(xy)} = P^3c(xy)\bullet e^{i\,\delta c(xy)} \geq P^3c(y)\bullet e^{i\,\delta c(y)} = P^3c(y)\bullet e^{i\,\delta c(yx)} \geq P^3c(x)\bullet e^{i\,\delta c(x)}$

$= P^3c(x)\bullet e^{i\,\delta c(y)}$ and

$\mathfrak{I}^3_C(x) \bullet e^{i\,\Delta_{c(xy)}} = \mathfrak{I}^3_C(xy) \bullet e^{i\,\Delta_{c(xy)}}$

$\leq \mathfrak{I}^3_C(y) \bullet e^{i\,\Delta_{c(y)}} = \mathfrak{I}^3_C(yx) \bullet e^{i\,\Delta_{c(yx)}} \leq \mathfrak{I}^3_C(x) \bullet e^{i\,\Delta_{c(x)}}$

$= \mathfrak{I}^3_C(x) \bullet e^{i\,\Delta_{c(y)}}$

Thus for C(x) = C(y) for all x,y ∈ Idm (S).

Theorem 4.2 : Let C be a complex Fermatean fuzzy soft right ideal of a semi group S. If the set of all idempotent elements of S form a right zero semi group of S, then C(x) = C(y) for all idempotent elements of x and y of S.

Proof: Proof is similar to the theorem 4.1.

Proposition 4.3: If S is a Semi group, then the following properties are hold.

(i) The intersection of two complex Fermatean fuzzy soft semi groups of S is a complex Fermatean fuzzy soft semi group of S.

(ii) The intersection of two complex Fermatean fuzzy soft left (respectively right) ideals of S is a complex Fermatean fuzzy soft left (respectively right) ideal of S.

Proof: Let

$C_1 = \{ C_{1T} = P^3c_1 \bullet e^{i\,\delta c1}, C_{1F} = r^3c1 \bullet e^{i\,\Delta c1} \}$ and

$C_2 = \{ C_{2T} = P^3c_2 \bullet e^{i\,\delta c2}, C_{2F} = r^3c2 \bullet e^{i\,\Delta c2} \}$ be any two complex Fermatean fuzzy soft semi groups of S

Let x, y ∈ S. Then

$(P^3c_1 \bullet e^{i\,\delta c_1} \cap P^3c_2 \bullet e^{i\,\delta c_2})(xy)$

$= \inf \{ P^3c_1(xy)\bullet e^{i\,\delta c_1(xy)}, P^3c_2(xy)\bullet e^{i\,\delta c_2(xy)} \}.$

$\geq \inf \{\{\inf P^3c_1(x)\bullet e^{i\,\delta c_1(x)}, P^3c_1(y)\bullet e^{i\,\delta c1(y)} \},$

$\{ \inf P^3c_2 (x) \bullet e^{i\,\delta c_2(x)}, P^3c_2 (y) \bullet e^{i\,\delta c_2(y)} \}\}.$

$= \inf \{\{\inf P^3c_1 (x) \bullet e^{i\,\delta c_1(x)}, P^3c_2 (x) \bullet e^{i\,\delta c_2(x)} \}$,

$\{\inf P^3c_1 (y) \bullet e^{i\,\delta c1(y)}, P^3c_2 (y) \bullet e^{i\,\delta c_2(y)} \}\}.$

$= \inf \{ (P^3c_1 \bullet e^{i\,\delta c_1} \cap P^3c_2 \bullet e^{i\,\delta c2})(x),$

$(P^3c_1 \bullet e^{i\,\delta c_1} \cap P^3c_2 \bullet e^{i\,\delta c_2})(y) \}.$

$(r^3c_1 \bullet e^{i\,\Delta c_1} \cup r^3c_2 \bullet e^{i\,\Delta c_2}) (xy)$

$= \text{Sup} \{ r^3c_1 (xy) \bullet e^{i\,\Delta c_1(xy)}, r^3c_2 (xy) \bullet e^{i\,\Delta c_2(xy)} \}.$

$\leq \text{Sup} \{\{\text{Sup } r^3c_1 (x) \bullet e^{i\,\Delta c_1(x)}, r^3c_1 (y) \bullet e^{i\,\Delta c_1(y)} \},$

$\{\text{Sup } r^3c_2 (x) \bullet e^{i\,\Delta c_2 (x)}, c_2 (y) \bullet e^{i\,\Delta c_2(y)} \}\}.$

$= \text{Sup} \{\{\text{Sup } r^3c_1 (x) \bullet e^{i\,\Delta c_1(x)}, r^3c_2 (x) \bullet e^{i\,\Delta c_2 (x)}\}$,

$\{\text{Sup } r^3c_1 (y) \bullet e^{i\,\Delta c_1(y)}, r^3c_2 (y) \bullet e^{i\,\Delta c_2(y)} \}\}.$

$= \text{Sup} \{(r^3c_1 \bullet e^{i\,\Delta c_1} \cup, r^3c_2 \bullet e^{i\,\Delta c_2})(x),$

$(r^3c_1 \bullet e^{i\,\Delta c_1} \cup r^3c_2 \bullet e^{i\,\Delta c_2})(y) \}.$

Thus for $C_1 \cap C_2$ is a Complex Fermatean fuzzy soft semi group of S.

(ii) Let C_1 and C_2 be any two Complex Fermatean fuzzy soft left ideals of semi group S and x, y ∈ S. Then

$(P^3c_1 \bullet e^{i\,\delta c_1} \cap P^3c_2 \bullet e^{i\,\delta c_2}) (xy)$

$= \inf \{P^3c_1 (xy) \bullet e^{i\,\delta c_1(xy)}, P^3c_2 (xy) \bullet e^{i\,\delta c_2(xy)} \}$

$\geq \inf \{P^3c_1 (y) \bullet e^{i\,\delta c_1(y)}, P^3c_2 (y) \bullet e^{i\,\delta c_2(y)} \}$

$= \{ P^3c_1 \bullet e^{i\,\delta c_1} \cap P^3c_2 \bullet e^{i\,\delta c_2} \} (y)$

and

$r^3c_1 \bullet e^{i\,\Delta c_1} \cup r^3c_2 \bullet e^{i\,\Delta c_2}) (xy)$

$=\text{Sup} \{r^3c_1 (y) \bullet e^{i\,\Delta c_1(y)}, r^3c_2 (y) \bullet e^{i\,\Delta c_2(y)} \}$

$= \{ r^3c_1 \bullet e^{i\,\Delta c_1} \cup r^3c_2 \bullet e^{i\,\Delta c_2} \} (y)$

Thus for $C_1 \cap C_2$ is a Complex Fermatean fuzzy soft semi left ideal of S. The intersection of Complex Fermatean fuzzy soft right ideals can be proved in a similar manner.

Proposition 4.4: If S is a semi group. Then the following properties are hold.

(i) The Union of two Complex Fermatean fuzzy soft semi groups of S is a complex Fermatean fuzzy soft semi group of S.

(ii) The Union of two Complex Fermatean fuzzy soft left (respectively right) ideals of S is a Complex Fermatean fuzzy soft left (respectively right) ideal of S.

Theorem 4.5: If C_1 and C_2 are complex Fermatean fuzzy soft left and right ideals of a semi group S, respectively, then $C_1 * C_2 \subseteq C_1 \cap C_2$.

Proof: If C_1 is Complex Fermatean fuzzy soft right ideal and C_2 is any Complex left bi fuzzy soft ideal of S.

We have $C_1 * C_2 \subseteq C_1 * S \subseteq C_1$ and $C_1 * C_2 \subseteq S * C_2 \subseteq C_2$.

Hence $C_1 * C_2 \subseteq C_1 \cap C_2$.

Theorem 4.6: If S is regular semi group, then $C_1 * C_2 \subseteq C_1 \cap C_2$ for every Complex Fermatean fuzzy soft right ideal of C_1 and C_2 of S.

Proof: Let α be any element of S. Since S is regular, there exist an element x ∈ S such that α x α = α.

Hence we have

$$(P^3c_1 \bullet e^{i\delta c_1} \cap P^3c_2 \bullet e^{i\delta c_2})(\alpha)$$

$$=(\max)_{\alpha=y\rho} \{ \inf \{P^3c_1(y) \bullet e^{i\delta c_1(y)}, P^3c_2(\rho) \bullet e^{i\delta c_2(\rho)}\}$$

$$=(\max)_{\alpha x\alpha=y\rho} \{ \inf \{P^3c_1(y) \bullet e^{i\delta c_1(y)}, P^3c_2(\rho) \bullet e^{i\delta c_2(\rho)}\}$$

$$\geq \inf \{P^3c_1(\alpha) \bullet e^{i\delta c_1(\alpha)}, P^3c_2(\alpha) \bullet e^{i\delta c_2(\alpha)}\}$$

$$= \{P^3c_1 \bullet e^{i\delta c_1} \cap P^3c_2 \bullet e^{i\delta c_2}\}(\alpha)$$

And

$$(r^3c_1 \bullet e^{i\Delta c_1} \cup r^3c_2 \, e^{\Delta c_2})(xy)$$

$$=(\min)_{\alpha=y\rho} \{ \sup \{r^3c_1(y) \bullet e^{i\Delta c_1(y)}, r^3c_2(\rho) \bullet e^{i\Delta c_2(\rho)}\}$$

$$=(\min)_{\alpha x\alpha=y\rho} \{ \sup \{r^3c_1(y) \bullet e^{i\Delta c_1(y)}, r^3c_2(\rho) \bullet e^{i\Delta c_2(\rho)}\}$$

$$\leq \sup \{r^3c_1(\alpha) \bullet e^{i\Delta c_1(\alpha)}, r^3c_2(\alpha) \bullet e^{i\Delta c_2(\alpha)}\}$$

$$= \{r^3c_1 \bullet e^{i\Delta c_1} \cup r^3c_2 \bullet e^{i\Delta c_2}\}(\alpha)$$

We have $C_1 * C_2 \supseteq C_1 \cap C_2$ and $C_1 * C_2 \subseteq C_1 \cup C_2$ is true from theorem 3.6.

Hence $C_1 * C_2 = C_1 \cap C_2$.

Theorem 4.7: For any non-empty subset H of a semi group S, We have

(i) H is a Semi group of S if and only if the characteristic complex Fermatean fuzzy soft set C_H of H in S is a complex Fermatean fuzzy soft semi group of S.

(ii) H is a left (respectively right) ideal of S if and only if the characteristic complex bi fuzzy soft set C_H of H in S is a complex Fermatean fuzzy soft left (respectively right) ideal of S.

Proof: The Proof is straight forward.

Theorem 4.8: For every complex Fermatean fuzzy soft right ideal C_1 and every complex Fermatean fuzzy soft left ideal C_2 of a semi group S, if $C_1 * C_2 = C_1 \cap C_2$, then S is regular.

Proof: Assume that $C_1 * C_2 = C_1 \cap C_2$ for every complex Fermatean fuzzy soft right ideal C_1 and every complex Fermatean fuzzy soft left ideal C_2 of a semi group S. Let M and N be any right and left ideals of S, respectively.

In order to see that $M \cap N \subseteq MN$ holds. Let α be only element of $M \cap N$. Then the characteristic complex Fermatean fuzzy soft set C_M and C_N on complex Fermatean fuzzy soft right ideal and a complex Fermatean fuzzy left ideal of S, respectively by theorem 4.7.

At follows from the hypothesis,

$$T_{CMN}(\alpha) = (T_{CM} \bullet T_{CN})(\alpha) = (T_{CM} \cap T_{CN})(\alpha) = 1.e^{i2\pi}$$

$$F_{CMN}(\alpha) = (F_{CM} \bullet F_{CN})(\alpha) = (F_{CM} \cup F_{CN})(\alpha) = 0$$

So that $\alpha \in MN$. Thus $M \cap N \subseteq MN$. Since the inclusion in the other direction always holds. We obtain that $R \cap L \subseteq RL$. It follows that S is regular.

Theorem 4.9: If C_1 and C_2 are two complex Fermatean fuzzy soft sets of a semi group S, then

(i) $(C_1 \cap C_2)_x = C_{1x} \cap C_{2x}$

(ii) $(C_1 \cup C_2)_x = C_{1x} \cup C_{2x}$

(iii) $C_{1x} + C_{2x} = (C1 + C2)x$

Proof:

Let $f(x) = \sum_{i=0}^{n} a_i x^i$ be any element of S. Then

$$\text{(i) } \delta(C_1 \cap C_2)_x (f(x)) = \delta_{(C1 \cap C2)x}$$

$$= (\inf)_i \{ \delta_{(C1 \cap C2)}(ai) \}$$

$$= (\inf)_i \{\inf\{ \delta_{C1}(ai), \delta_{C2}(ai) \}\}$$

$$= (\inf) \{ \delta_{C1x}(f(x)), \delta_{C2X}(f(x)) \}$$

$$= \delta_{(C1x \cap C2x)}(f(x)).$$

Similarly, we can show that $\Delta\ (C_1 \cap C_2)_x\ f(x) = \Delta_{(C1x \cap C2x)}(f(x))$

Hence $(C_1 \cap C_2)_x = C_{1x} \cap C_{2x}$

(ii) $\delta\ (C_1 \cup C_2)_x\ (f(x)) = \delta_{(C1 \cup C2)x}$

$= (\inf)_i \{ \delta_{(C1 \cup C2)}(ai)\}$

$= (\inf)_i \{\mathrm{Sup}\{ \delta_{C1}(ai), \delta_{C2}(ai)\}\}$

$\geq (\mathrm{Sup}) \{(\inf)i \{ \delta_{C1}(ai), \delta_{C2}(ai)\}\}$

$= (\mathrm{Sup})\{(\inf)i\ \delta_{C1}(ai), (\inf)i\ \delta_{C2}(ai)\}$

$= (\mathrm{Sup}) \{ \delta_{C1x}(f(x)), \delta_{C2x}(f(x))\}$

$= \delta_{(C1x \cup C2x)}(f(x)).$

Similarly, we can show that $\Delta\ (C_1 \cup C_2)_x\ f(x) = \Delta_{(C1x \cup C2x)}(f(x))$

Hence $(C_1 \cup C_2)_x = C_{1x} \cup C_{2x}$

$$\text{Now } \delta_{C1x+C2x}(f(x)) = \left\{ \underset{f(x)=g(x)+h(x)}{\mathrm{Sup}} \{ \inf \{ \delta Ax(g(x), \delta Bx(h(x)\} g(x)\} \right\}$$

$$= \sum_{i=0}^{p} b_i x^i \quad h(x) = \sum_{i=0}^{p} C_i x^i$$

$$= \left\{ \underset{f(x)=g(x)+h(x)}{\mathrm{Sup}} \{ \inf \{\inf(\delta c1(bi), \delta c2(ci))\}\} \right\}$$

$$= \left\{ \underset{a_i=b_i+c_i}{\mathrm{Sup}} \{\inf \{\inf(\delta c_1(bi), \delta c_2(ci))\}\} \right\}$$

$$= \left\{ \underset{a_i=b_i+c_i}{\inf} \{\max \{\{\inf(\delta c_1(bi), \delta c_2(ci))\}\} \right\}$$

$$= \delta\ (c_1+c_2)\ x\ (f(x))$$

Similarly, we can show that $\Delta_{c1x+c2x}(f(x)) = \Delta_{(c1+c2)x}(f(x))$.

5. Conclusion:

we proved the set of all idempotent elements of S from a right zero semi group of S, then C(x) = C(y) for all idempotent elements of x and y of S. If C_1 and C_2 are complex Fermatean fuzzy soft left and right ideals of a semi group S, respectively, then $C_1 * C_2 \subseteq C_1 \cap C_2$. For every complex Fermatean fuzzy soft right ideal C_1 and every complex Fermatean fuzzy soft left ideal C_2 of a semi group, if $C_1 * C_2 = C_1 \cap C_2$, then S is regular.

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