Evaluation of the Altshul Friction Factor Equation for Engineering Calculations

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In the teaching of Fluid Mechanics, the calculations of the coefficient of friction for turbulent flow are complex due to the numerous operations that are necessary to perform for problem-solving with the use of the expressions traditionally used. Consequently, the objectives were to define the commonly used ranges for the Reynolds number and relative roughness and to evaluate the ac-curacy of the Altshul equation for the calculation of the coefficient of friction in turbulent flow within the established range. The study of the information allowed the identification of the level of complexity and precision of the available calculation expressions and the determination of the commonly used ranges of the calculation parameters based on the values used by the different authors consulted. The effectiveness of simple Altshul versus Colebrook equation for the practical ranges of relative roughness and Reynolds numbers was evaluated, with the help, in their order, of Matlab functions and Newton-Raphson method. The results showed that the practical ranges are in range: relative roughness: 0.00008 and 0.003; Reynolds number: 1.5E+03 and 1E+06 and that for these values the Altshul expression shows very good coincidence with those obtained with the help of the CW equation, with a maximum relative error of approximately 8 %. It concludes the con-venience of using the Altshul simple equation for practical ranges both in teaching and in field engineering work.
Keywords: fluid flow friction; turbulent flow regime; explicit models; pipe flow.

1. Introduction

Fluid Mechanics is a subject included in the curriculum of most engineering careers and their different specialties. Its purpose is to study the fundamentals of the operation of liquids and gases, especially the former. Fluid Mechanics provides essential support to other subjects such as pneumatic and fluid power systems, thermodynamics, and industrial installations, among others. It is usually a complicated subject for most students due to the numerous calculations they must perform to solve piping problems for whose solution, they must use complex formulas to calculate the friction coefficients in turbulent flow regime. Professionals in this specialty must face similar situations during engineering practice when it requires an abbreviated or preliminary calculation to determine pressure loss or pipe diameter.

Translated with www.DeepL.com/Translator (free version)The calculations associated to fluids pressure drops in pipes can be the following types [1–4], namely:

I. Pressure drops calculation when flow speed and pipe diameter are known.

II. Speed calculation (and flow) when pressure drop, and pipe diameter are known.

III. Calculation of the required pipe diameter for pressure drop and flow given.

The key function of the hydraulic calculation of pipes is to determine the head or pressure loss by friction (Δh o Δp). The basic equation for its calculation is the classic Darcy-Weisbach formula, which until now remains the basis of most hydraulic calculations of pipes (1).

$$\Delta p = \frac{\lambda l \rho v^2}{2d} \quad (1)$$

In this formula, λ is the hydraulic friction coefficient (a dimensionless quantity); l: etimated pipe length; ρ: flow density; v: speed of the average flow; d: pipe design diameter.

To determine the value of the hydraulic friction coefficient λ in formula (1) is difficult because it depends on the liquid flow regime and the pipe material properties. The fluids transport systems generally work in a turbulent flow regime. For this reason, a lot of formulas to calculate the hydraulic friction coefficient are proposed.

One of the first and most used methods to determine the coefficient λ in a turbulent flow regime is Colebrook–White formula, which is given by equation (2). The relation ε/d represents the relative roughness of the pipe.

$$\frac{1}{\sqrt{\lambda}} = -2\log_{10}\left(\frac{\frac{\varepsilon}{d}}{3.7} + \frac{2.51}{Re\sqrt{\lambda}}\right) \quad (2)$$

The main disadvantage of this formula is that the model is implicitly defined in the friction factor and some repetitions are needed to calculate this factor. Therefore, several investigations have been conducted to define explicit models to calculate the Darcy friction factor [5–8]. For a manual calculation, it is recommended to have an expression as simple and compact as possible because it reduces the number of steps in the calculation and is easier to remember [9].

Small approximation errors can be accepted in practical problems, especially if these are less important than other inevitable errors involved in the calculations. Likewise, Colebrook-White (CW) formula and the Moody diagram are not completely correct for all conditions [10–12]. It is estimated that these formulas are only effective up to 15% [13, 14]. Moreover, in these kinds of calculations there is great uncertainty not only in basic quantities like pipe roughness (which is hard to define), but also in design quantities such as the design flow rate [15].

The explicit equations are used to avoid an inconvenient repetitive method when determining the friction factor. However, the values obtained by using these equations lead to errors [7, 16]. To apply the explicit friction factor on field practice and education, the equation should not be complicated or have too many variables [17].

Throughout history, countless expressions have been developed to solve these problems. These expressions have been analyzed by different authors[1, 6, 25, 26, 12, 18–24]. The development of new expressions is based on the principle of finding more efficient methods to solve the Colebrook–White equation (or approximations) for Reynolds numbers between 4000 and 108, and for the relative roughness of an inner pipe surface between 0 and 0.05 [26] or similar ranges [27]. Most of these equations are explicit expressions approximated for this purpose which have differences (relative error) in the calculation result regarding the universally used pattern (C-W formula) that goes from 0,1 to 46 % ([6, 21]). The analysis of the recent bibliography [2–4, 14, 28, 29] shows that the most diffused expressions in academia are Haaland's ones, [30] and Swamee-Jain [31].

The Altshul equation [32] is characterized by its simplicity and acceptable accuracy, but in practice its main constraint is that this equation has a higher relative error than the aforementioned expressions. Mainly when dealing with extreme relative roughness values and large Reynolds numbers. Nevertheless, in engineering calculations and in the study of Fluid Mechanics, the range in which these calculations are developed is much more limited, large Reynolds numbers (over 106) are used in special technologies and constructions [17]. The results with this equation in these intervals are accurate and less laborious.. Consequently, this expression is a viable, accurate and less complex option in field engineering calculations and in the engineering teaching- learning process for the values and more frequent cases of the engineering practice.

During the literature review, no adequate foundations were found to establish practical ranges of Reynolds numbers common usage. Similarly, no relative roughness for calculating the friction coefficient in turbulent flow regime and evaluations outcomes of simple expressions under these conditions were noticed. This represents a problem that should be addressed.

In the light of the abovementioned information, the objectives were to define the commonly used ranges for Reynolds number and relative roughness and to evaluate the accuracy of the Altshul equation for friction coefficient calculations in turbulent flow within the practical range determined in this research.

2. Case and Methodology

For the development of the research, a documentary analysis of 51 works published in Spanish, Russian and English from 1963 to date was carried out, with a predominance of works from the last decade, which was complemented with the use of Newton Raphson's numerical method and the expressions of Fluid Mechanics with the help of Matlab functions, according to the following logic:

1. Analysis of the complexity and level of accuracy of the friction coefficient calculation expressions for turbulent flow, based on the analysis of the current state of the art, in the following order:

- Accuracy of the most used formulas for the calculation of the friction factor where this depends on both Re and relative roughness (ε/d), the most frequent case in engi-neering practice.

- Study of the accuracy results and convenience of using the available explicit alge-braic expressions and graphical methods, as well as the potential of the less wide-spread and little studied Altshul equation, as a viable tool for the fast calculation of λ.

2. Determination of the commonly used ranges of the calculation parameters from the results provided by the main scientific publications from 2011 to date, which in their order are the practical ranges: a) of the effective roughness for different materials of the most used pipes, b) of relative roughness considering the most frequently used pipe diameters and c) of Reynold's Number.

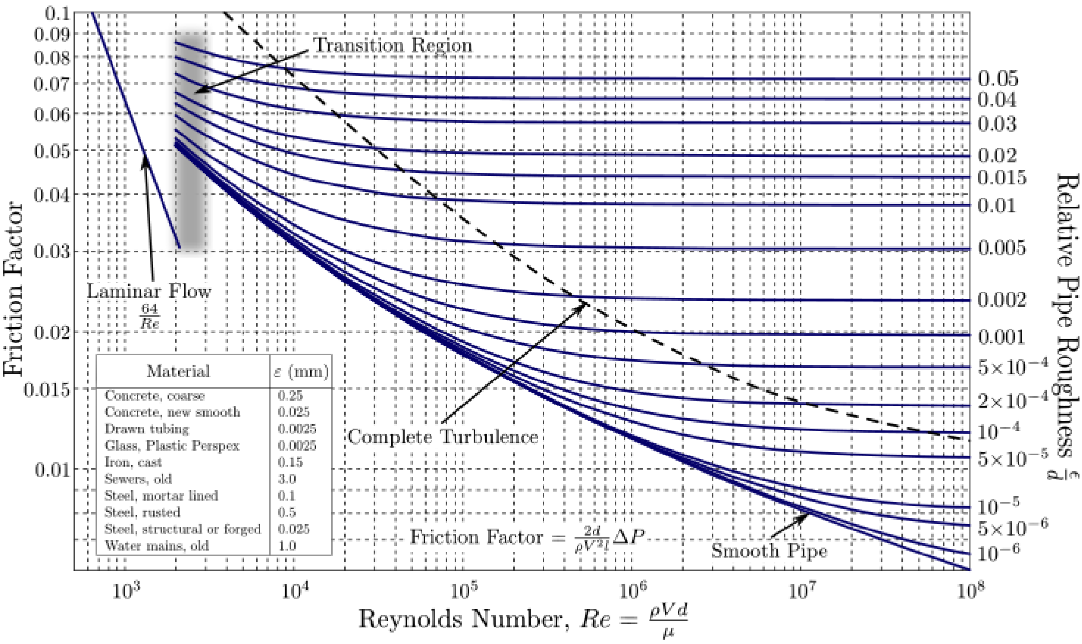
3. The evaluation of the effectiveness of the simple Altshul versus CW equation for prac-tical ranges of the determined λ parameters and Reynolds numbers, using the New-ton-Raphson numerical method for the CW equation and Matlab functions to perform an analysis of the behavior of the exponential Altshul relation.

3. Results

Several experiments with technical pipes, mainly G. A. Murin, Colebrook and White, have shown that for a turbulent flow regime, λ not only varies with a change of Re, but al-so the technical condition of the pipe affects λ (Figure 1).

In this diagram, three resistance regions under turbulent conditions are clearly dis-tinguished:

Figure 1 Moody chart.



Region I- corresponds to hydraulically smooth pipes, where λ coefficient only depends on the Reynolds number and not on the pipe material. Also, region III is a quadratic resistance of rough conducts area where the value of λ does not depend on Re, but on the relation ε/d (complete turbulence). There is a consensus among scholars about which formulae should be used in those regions. Those formulas are not complex for calculating the friction factor. Moreover, those formulae are the most common ones when solving practical problems in fluid Mechanics, especially if that fluid is water. In region I for Re values of (4000<Re< 10 d/ε), it is recommended to use formula (3) developed by Blasius. In region III, where Re>560 d/ε , it is advisable to use formula (4) by Shifrinson.

$$\lambda = \frac{0.3164}{\mathrm{Re}^{0.25}} \qquad (3)$$

$$\lambda = 0.11\left(\frac{\varepsilon}{d}\right)^{0.25} \qquad (4)$$

In region II (transition region), the friction factor depends not only on Re (105 < Re ≤ 9,2 x 105), but also on the relative roughness (ε/d). Considering both, the smooth and rough pipe laws, the researchers have not reached into an agreement about which is the most accurate formula to calculate the friction factor. Even though, Colebrook-White equation is the most appropriate method, according to Gregory & Fogarasi [33] and Schroeder [34], this is a complex equation to be used in fluids Mechanics due to the multiple interactions necessary [35].

It has been proved that the models with more logarithmic functions are more precise [25]. However, the more precise results are in comparison with CW equation, the more complex the equation becomes [36, 37]. That is why the equations generally referred to as the favorite ones regarding precision, are indeed, the least suitable for routine calculations in engineering fluid Mechanics courses and everyday practical applications.

To find the friction factor implicitly given in Colebrook, it is necessary to use numeric algorithms that are not as fast as approximations. In those situations, reliable explicit approximations are preferred to a faster estimation of λ.

The use of the Moody diagram as an alternative of Colebrook equation eliminates the iteration requirement. However, this graphic tool is not convenient for computer simulations, nor for students to repeatedly solve type II and III problems. Thus, students usually read the diagram in a wrong way.

The two most frequently used equations, both in Fluid Mechanics studies in engineering careers, and by field engineers are those of Haaland (5) and Swamee – Jain (6). Both equations are cited by the most relevant authors of fluid Mechanics manuals [2–4, 14]. Most of the latest inves-tigations about the explicit approximations to calculate λ agreed with the abovementioned au-thors. These approximations have an adequate precision and not a high level of complexity [38]. However, these logarithmic equations have a lot of arithmetic operations which frequently lead students to make some calculation mistakes.

$$\lambda = \left[-1.8\log\left(\left(\frac{\varepsilon/\mathrm{d}}{3.7}\right)^{1.11} + \frac{6.9}{\mathrm{Re}}\right)\right]^{-2} \qquad (5)$$

$$\lambda = \frac{0.25}{\left[log_{10}\left(\frac{\varepsilon/d}{3.7}+\frac{5.74}{Re^{0.9}}\right)\right]^2} \qquad (6)$$

It would be advantageous to have a λ explicit equation, easy to memorize and that could cover a wide range of Reynolds numbers and relative roughness. Besides, this equation could be used to solve most of the problems, and it would be particularly useful for piping network problems [39]. This way, it could be avoided to use friction factor graphics like the Moody diagram or complex mistake- provoking equations.

After analyzing the explicit logarithmic equation (7) stated by Altshul [40] and some modi-fications proposed by other authors [39], it was verified that all of them produce similar results to Colebrook within the limits of Reynolds numbers from 3x103 to 1x106 [16, 40–42]. This allows to directly calculate (without consecutive approximations) the λ coefficient value [40, 43].

$$\frac{1}{\sqrt{\lambda}} = 1.8log\frac{Re}{Re\frac{\varepsilon}{d}+7} \qquad (7)$$

Nevertheless, for engineering calculations it is more convenient to use a simplified version of this equation, the equation (8), proposed by Altshul [41, 42]. This can be said owing to the same values of ε/d , the curves of this formula extend somewhat below the Colebrook curves, but higher than the experimentally obtained Murin graphic [42].

$$\lambda = 0.11\left(\frac{\varepsilon}{d} + \frac{68}{Re}\right)^{0.25} \qquad (8)$$

Moreover, formula (8) is valid for all homogeneous Newtonian fluids and the divergence between formula (8) and the logarithmic equation proposed by Altshul (7) in a wide range of ε/d and for any Reynolds number, does not go beyond 3- 4 % [44]. Besides, Altshul formula (8) is more convenient for calculations because those are reduced to elementary algebraic operations, and it is easy to learn by heart.

Some experiments for conditions commonly encountered in practice have been conducted to determine the error percentage that occurs when the Altshul approximated formula (8) is applied (Table 1). In those conditions, the mean square error is 3 % of the estimate [45]. Other authors state that the error is 7,32 % [46], or between -1 and 9 % [47].

Table 1. Mean square error when calculating λ, based on Altshul formula (8), according to some authors in the mixed friction area of the turbulent flow regime for different experimental se-quences.

N°	Experiment conditions	Relative error [%]
1	Water (d = 40.2 mm)	1.92
2	Water (d = 40.5…50.6 mm)	1.94
3	Water, kerosene (d = 205 mm)	1.72
4	Air, water, kerosene (d = 74.5…359 mm)	2.83
5	Water (d = 143 mm)	3.28
6	Air (d = 205 mm)	2.19
7	Air (d = 205…206 mm)	2.44
8	Water (d = 26 mm)	1.81
9	Water (d = 25…100 mm)	9.83 (For flow in a rough pipe)

Asker et. al. [50] stated that relative error of the Altshul correlation decreases for ε/d=1e-4, ε/d =1e-6 and ε/d =1e-8 with the increase of the Reynolds number. However, for ε/d=1e-2, the relative error value stays constant with increasing Reynolds number. According to the analysis carried out by Brkic [19] the error percentage of Altshul formula only exceeds 10% for extreme values of relative roughness (ε/d) and high Reynolds numbers.

In this regard, in a recent study [20], the friction coefficient was calculated by different explicit formulas and it was substituted in the CW formula. Then, an absolute mean deviation was obtained using the Clamond method to solve the Colebrook equation (Table 2). It can be observed in this Table, that for a certain range of relative roughness Altshul equation (8) presents deviations of less than 8.2 % regarding Colebrook equation.

Table 2. Absolute mean deviation of explicit relations with respect to CW.

Equation	Relative roughness	Absolute mean deviation	Mean square deviation
Altshul	0.05	1.03E-03	27.3
Swamee–Jain		1.33E-02	0.8
Haaland		2.31E-03	0.3
Altshul	0.01	3.15E-03	8.2
Swamee–Jain		2.07E-02	1.0
Haaland		1.45E-03	0.2
Altshul	0.001	1.34E-02	0.6
Swamee–Jain		1.21E-02	0.6
Haaland		1.39E-02	0.6
Altshul	0.0001	1.52E-01	5.8
Swamee–Jain		9.05E-03	0.5

Haaland		2.03E-02	0.8
Altshul	0.0000001	1.17	20.1
Swamee–Jain		1.93E-02	0.7
Haaland		1.30E-02	0.6

4. Discussion

After analyzing the abovementioned information, it can be concluded that Altshul equation (8) is an option. However, there is not an evaluation of the Reynolds numbers ranges and relative roughness most found in daily practice.

A problem that has been addressed, but not solved yet, is that there is not a consensus about which are the most common values of relative roughness. Almost every practical case of water flow in commercial pipes is between the two extreme conditions: completely smooth and completely rough. Under those conditions, the friction coefficient varies not only with the Reynolds numbers, but also with the roughness [51].

Table 3 depicts the effective roughness of the most frequently used materials in pipes according to different authors. Also, the roughness values for Fluid Mechanics calcula-tions are defined based on the most recommended ones by the consulted authors. These data are used to determine the relative roughness ε/d (Table 4) and the Reynolds numbers which are frequently encountered in daily practice and in the Fluid Mechanics study (Ta-ble 5).

In Table 4, it can be observed that the relative roughness varies from 0,000005 to 0,006 globally. However, in practical problems the ranges to be considered must be between 0,00008 and 0,003, and the Reynolds numbers must be between 1,5E+03 and 1E+06 according to Table 5.

Summarizing, it can be concluded from the bibliographic analysis, that the common ranges to be considered in daily practice are relative roughness: between 0,00008 and 0,003; Reynolds numbers: between 1.5E+03 and 1E+06 (the subsequent analysis is carried out for turbulent flow with Reynolds numbers starting at 4000).

Those λ ranges and Reynolds numbers were classified as the most common during the Fluid Mechanics study. The Newton – Raphson method was used for the CW equation and Matlab functions to conduct an analysis of Altshul (8) exponential relationship performance (Figure 2). A good level of coincidence with the values obtained through CW equation was observed, the curves of this formula extend somewhat below the Colebrook curves, as stated in [42]. Moreover, there was a maximum relative error of approximately 8 %, which occurs for Reynolds numbers close to 1E+06 and small relative roughness val-ues of 0.00008 (Figure 3).

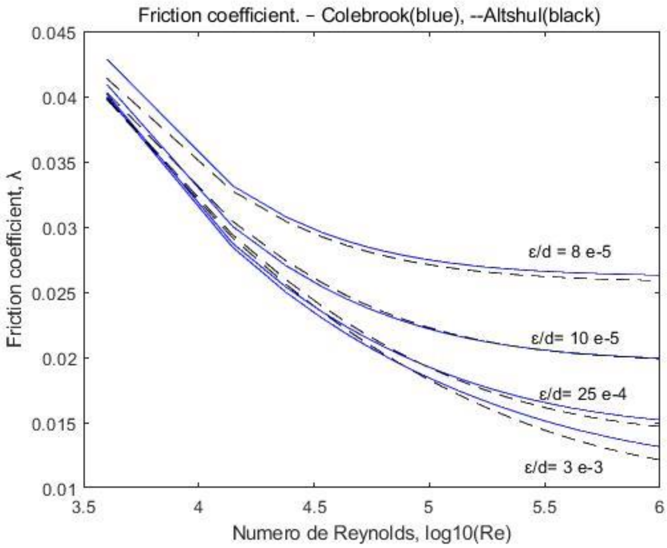


Figure 2. Coefficient of friction calculated by the CW and simplified Altshul equations for the common range of relative roughness and Reynolds number values.

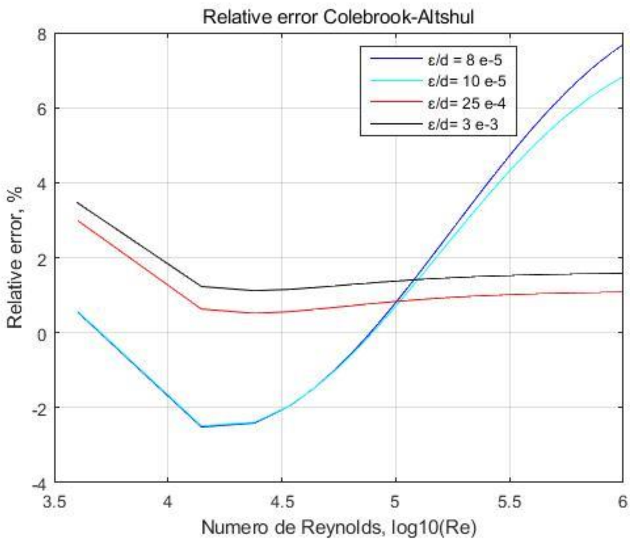


Figure 3. Relative error of the Altshul equation for the common range of relative roughness and Reynolds number val-ues.

The results obtained agree quite well with those obtained by Brkic [19] and Asker [50] in that for small values of relative roughness the relative error of the Altshul correlation increases with increasing Reynolds number, on the other hand for larger values of relative roughness the relative error remains approximately constant with increasing Reynolds number.

Considering what has been stated previously about Colebrook equation accuracy and the uncertainty with respect to the parameters that are part of the λ calculation, it is advisable to use Altshul equation in the teaching- learning process of Fluid Mechanics and engineering calculations for the evaluated range of λ and Reynolds numbers.

5. Conclusions

1. The expressions of greater diffusion and precision in the scientific field, according to the state of the art, is the Colebrook equation, based on implicit method, for the calcula-tion of the friction factor in the range of Re (105 < Re ≤ 9.2 x 105), but it is complex and have a high level of difficulty which affects his use in teaching and engineering practice. Meanwhile the explicit expressions of Haaland and Swamee – Jain have adequate preci-sion and are less complex and laborious than that of implicit type but being of the loga-rithmic type, they require several computational operations, their use by students and en-gineers in engineering practice can lead to mistakes.

2. In the absence of consensus on the range of commonly used values of relative roughness and the Reynolds number for the calculation of coefficient of friction for turbu-lent flow, it was established, based on values used in scientific publications, that the ranges to be considered as common in practice are: for relative roughness between 0.00008 and 0.003; and for the Reynolds number between 1.5E+03 and 1E+06, which facilitates the calculations and evaluations of results by establishing the tuning fork most accepted by the scholars of the subject.

3. The analysis of the behavior of the exponential relation of Altshul, for the values determined as the most common during the study of fluid mechanics, shows a very good coincidence with those obtained with the help of the CW equation, with a maximum rela-tive error of approximately 8 %, which occurs for Reynolds numbers close to 1E+06 and small relative roughness values (0.00008), so it can be said that it is advisable to use it both in the teaching-learning process of the subject Fluid Mechanics and for field engineering calculations.

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