Nanotechnology Perceptions
ISSN 1660-6795
www.nano-ntp.com

Soft R_c-Operators On Soft Topological Space

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In this paper some basic properties of soft regular open set is introduced. We define an operator called soft r_o- operator on soft topological space. The properties of the operator are also discussed.2010 Mathematics Subject Classification: 54A05.

Key words and phrases: soft r_o- operator, soft closure, soft interior, soft regular open.

1 Introduction

The classical mathematical approaches are often insufficient for modelling problems with uncertain data. There are several theories such as fuzzy sets, theory of intuitionistic fuzzy sets, theory of vague sets and theory of rough sets which can be considered as tools for dealing with uncertainties. But these theories have their own difficulties due to the inadequacy of the parametrization tool of the theories as pointed out by Molodstov. In 1999, Molodtsov[5] introduced soft set theory as a mathematical tool for solving complex problems dealing with uncertainties. Recently research works on soft set theory and its applications in various fields are advancing rapidly.

In the year 2011, Shabir and Naz[1] introduced soft topological spaces which are defined over an initial universal set with a fixed set of parameters. Sabir Huzzain and Bashir Ahmad [2] discussed properties of soft interior, soft closure, soft exterior and soft boundary in 2011. Sabir Hussain[4] defined soft regular open and soft regular closed set in soft topological space. In viewpoint of this, here we made an attempt to define an operator on soft topological space for whcich soft regular open sets are fixed points.

This paper is arranged in the following sequence. Section 2 provides some basic definitions and results which are inevitable for the present work. Section 3 defines and discusses some properties of an operator called soft r_c- operator for which soft regular closed sets are fixed points.

The articles by Shabir and Naz[1], Sabir Hussain and Bashir Ahmad[2] are used as resourceful background material for basic definitions and some properties of soft topological space. We begin with some basic definitions and results which are essential for our study. In the entire article, X and A will denote the universal set and fixed parameter set respectively.

2 Preliminaries

Definition 2.1. [1] Let τ be collection of soft sets (F,A) over X. Then τ is said to be a soft topology on X if

1 $\phi_A, \tilde{X}_A \in \tau$
2 Union of any number of soft sets in τ belongs to τ
3 Intersection of two soft sets in τ belongs to τ

Then (X,τ,A) is called soft a topological space over X. The members of τ are called soft open sets.

Definition 2.2. [1] A soft set (F,A) over X is said to be a soft closed set if its relative complement $(F,A)' \in \tau$.

Proposition 2.1. [1] Let (X,τ,A) be a soft topological space over X. Then

1. $\phi_A, \tilde{X}_A$ are soft closed sets over X
2. The intersection of any number of soft closed sets is a soft closed set
3. The union of two soft closed set is a soft closed set.

Proposition 2.2. [1] Let (X,τ,A) be soft a topological space over X. Then $\tau_\alpha = \{F(\alpha)/(F,A) \in \tau\}, \forall \alpha \in A$ is a topology on X.

Definition 2.3. [1] Let (X,τ,A) be a soft topological space over X and (F,A)be a soft set over X. Then soft closure of (F,A), denoted by (F,A), is the intersection of all soft closed sets containing (F,A).

Theorem 2.4.[1] Let (X,τ,A) be a soft topological space over X and let
(F,A) and (G,A) are soft sets over X

1 $\overline{\phi_A} = \phi_A$ and $\overline{\tilde{X}_A} = \tilde{X}_A$

2 $(F,A) \subseteq \overline{(F,A)}$

3 (F,A) *is a soft closed set if and only if* $\overline{(F,A)} = (F,A)$

4 $\overline{\overline{(F,A)}} = (F,A)$

5 $(F,A) \subseteq (G,A) \Rightarrow \overline{(F,A)} \subseteq \overline{(G,A)}$

6 $\overline{(F,A) \cup (G,A)} = \overline{(F,A)} \cup \overline{(G,A)}$

Definition 2.5.[1] Let Let (X,τ,A) be a soft topological space over X .Let (F,A) be a soft set over X. Then $\overline{(F,A)}$ is a soft set over X defined by $\overline{F}(\alpha) = \overline{F(\alpha)}$,∀α ∈A, where $\overline{F}(\alpha)$ is the closure of F (α) in τ_α.
Proposition 2.3. [1] Let Let (X,τ,A) be a soft topological space over X and let (F,A) be a soft set overX. Then $\overline{(F,A)} \subseteq (\overline{F},A)$.
Corollary 2.6. [1] $(\overline{F},A) = \overline{(F,A)}$ if and only if $(\overline{F},A)' \in \tau$.

Definition 2.7. [1] Let Let (X,τ,A) be a soft topological space over X,(F,A) be a soft set over X and x ∈X.Thenx is said to be a soft interior point of (F,A) if there exist a soft open set (G,A) such that x ∈ (G,A) ⊆ (F,A).

Definition 2.8. [1] Let (X,τ,A) be a soft topological space over X,(F,A) be a soft set over X and x ∈X. Then (F,A) is said to be soft neighbourhood of x if there exist a soft open set (G,A) such that x ∈ (G,A) ⊆ (F,A).

Proposition 2.4. [1] Let (X,τ,A) be a soft topological space over X, let (F,A) be a soft set over X and x ∈X. Then if x is a soft interior point of (F,A) then x is an interior point of F (α) in (X,τ_α),∀α ∈A.

Definition 2.9. [2] Let (X,τ,A) be a soft topological space over X and let (F,A) be a soft set overX. Then soft interior of (F,A) is the union of all soft open set contained in (F,A). Soft interior of (F,A) is denoted by $(F,A)^o$. Clearly, $(F,A)^o$ is the largest soft open set contained in (F,A).

Theorem 2.10.[2] Let (X,τ,A) be a soft topological space over X and let
(F,A) and (G,A) are soft sets over X. Then
1 $\phi_A^o = \phi_A, \tilde{X}_A{}^o = \tilde{X}_A$
2 $(F,A)^o \subseteq (F,A)$
3 $[(F,A)^o]^o = (F,A)^o$
4 (F,A) is soft open set if and only if $(F,A)^o = (F,A)$.
5 $(F,A)^o \cap (G,A)^o = [(F,A) \cap (G,A)]^o$
6 $(F,A)^o \cup (G,A)^o = [(F,A) \cup (G,A)]^0$
7 $(F,A) \subseteq (G,A) \Rightarrow (F,A)^o \subseteq (G,A)^o$.

Definition 2.11. [2] Let (X,τ,A) be a soft topological space over X and let (F,A) be a soft set overX. Then (F^o,A) is a soft set defined by $F^o(\alpha) = (F(\alpha))^o$, where $(F(\alpha))^o$ is the interior of F (α) in τ_α,∀α ∈A.

Proposition 2.5. Let (X,τ,A) be a soft topological space over X and let (F,A) be a soft set over X. Then $(F,A)^o \subseteq (F^o,A)$.

Corollary 2.12. $(F,A)^o = (F^o,A)$ if and only if $(F^o,A) \in \tau$.

Definition 2.13. [4] Let (X,τ,A)be a soft topological space over X. Then a soft set (F,A) is a soft regular open set if $(F,A) = \left[\overline{(F,A)}\right]^{o}$.

Definition 2.14. Let (X,τ,A) be a soft topological space over X. Then a soft set (F,A) is soft regular closed set if $[(F,A)o]$ = (F,A).

Definition 2.15. Let (X,τ,A) be a soft topological space over X. Then the $r_o : \tilde{\mathcal{P}}\left(\tilde{X}_A\right) \mapsto \tilde{\mathcal{P}}\left(\tilde{X}_A\right)$ h ______i^{o} operator,defined by $r_o(F,A)$ = (F,A) is called soft $r_o$operator.

3 Soft r_c-Operator

Definition 3.1. Let (X,τ,E) be a soft topological space. Then the operator r_c:$S_E(X)$ 7→ $S_E(X)$ defined by $r_c(F,E) = \overline{[(F,E)^{o}]}$ is called soft r_c-operator.

roposition 3.1. Let (X,τ,E) be a soft topological space and let (F,E) be any soft set over X. Then,

1. $(F,E)^{o} \subseteq r_c(F,E)$.
2. $\overline{(F,E)} \supseteq r_c(F,E)$.
3. For any open soft set (F,E), soft r_c- operator is an expansion operator. i.e., (F,E) $\subseteq r_c(F,E)$.
4. For any closed soft set (F,E), soft r_c- operator is a shrinking operator. i.e., (F,E) $\supseteq r_c(F,E)$.
5. For any soft regular closed set (F,E), soft r_c-operator is an idempotent operator.

Proof.1. By definition of soft closure,

$(F,E)^{o} \subseteq \overline{[(F,E)^{o}]} = r_c(F,E)$.

.

2. By definition of soft interior,

$r_c(F,E) = \overline{[(F,E)^{o}]} \subseteq \overline{(F,E)} = (F,E)$.

3. Since (F,E) is soft open set,

$(F,E) = (F,E)^{o} \Rightarrow (F,E) \subseteq r_c(F,E)$ By [1.]

.

4. Since (F,E) is a soft closed set,

$r_c(F,E) \subseteq (F,E) = (F,E)$

.

5. It is clear from the definition of soft regular closed set.

.□

Proposition 3.2. Let (X,τ,E) be a soft topological space and let (F,E) and

(G,E) be any two soft sets over X. Then,

1. If (F,E) ⊆ (G,E), r_c(F,E) ⊆ r_c (G,E).
2. r_c[(F,E) ∩ (G,E)] ⊆r_c(F,E) ∩ r_c(G,E).
3. r_c[(F,E) ∪ (G,E)] ⊇r_c(F,E) ∪ r_c(G,E).

Proof. 1.

$$(F,E)^o \subseteq (G,E)^o \Rightarrow \overline{[(F,E)^o]} \subseteq \overline{[(F,E)^o]}$$
$$\Rightarrow r_c(F,E) \subseteq r_c(G,E).$$

2. By [1.],
r_c[(F,E) ∩ (G,E)] ⊆r_c(F,E).
r_c[(F,E) ∩ (G,E)] ⊆r_c(G,E).
Hence, r_c[(F,E) ∩ (G,E)] ⊆r_c(F,E) ∩ r_c(G,E)
3. By [1.],
r_c[(F,E) ∪ (G,E)] ⊇r_c(F,E).
r_c[(F,E) ∪ (G,E)] ⊇r_c(G,E).
Hence, r_c[(F,E) ∪ (G,E)] ⊇r_c(F,E) ∪r_c(G,E).

.□

Proposition 3.3. Let (X,τ,E) be a soft topological space and let (F,E) be any soft sets over X. Then,

1. $r_c(F,E)^o = r_c(F,E).$
2. $r_c\big(r_c(F,E)\big) \subseteq r_c(F,E)$
3. For any soft open set (F,E), $r_c\big(r_c(F,E)\big) = r_c(F,E).$

Proof.1. $r_c(F,E)^o = \overline{[((F,E)^o)^o]} = \overline{((F,E)^o)} = r_c(F,E).$
2. Since (F,E) is a soft open set,
(F,E) ⊆r_c(F,E), by proposition 3.1[3]
⇒r_c(F,E) ⊆r_c[r_c(F,E)] By proposition 3.2[1].

3. Since [(F,E)^o] is a soft closed set,
$r_c\left(\overline{[(F,E)^o]}\right) \subseteq \overline{[(F,E)^o}$
⇒$r_c\big(r_c(F,E)\big) \subseteq r_c(F,E)$.] By proposition 3.1[4].
Hence, $r_c\big(r_c(F,E)\big) \subseteq r_c(F,E)$. So that $r_c\big(r_c(F,E)\big) = r_c(F,E)$, by [2.].

□

Proposition 3.4. Let (X,τ,E) be a soft topological space and let (F,E) be any soft sets over X. Then,

1. For any open soft set (F,E), $r_o\big(r_c(F,E)\big) = r_o(F,E).$
2. For any closed soft set (F,E), $r_c\big(r_o(F,E)\big) = r_c(F,E).$

Proof. 1. $r_o\big(r_c(F,E)\big) = r_o\overline{[(F,E)]^o} = r_o\big(\overline{(F,E)}\big) = \overline{\overline{((F,E))}}^o = \big(\overline{(F,E)}\big)^o =$ $r_c(F,E)$, since (F,E) is a soft open set.

2. $r_c\big(r_o(F,E)\big) = r_c[\overline{(F,E)}]^o = r_c(F,E)^o = \overline{((F,E)^o)}^o = \overline{(F,E)^o} = r_c(F,E)$, since (F,E) is soft closed set

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