Analysis Of Sinusoidal Loads On A Simply Supported Beam Using Hyperbolic Shear Deformation Theory

Renu P Pathak[1], Mahesh V. Ghotkar[2*], Satish G. Gajbhiv[3], Durgesh H. Tupe[4]

[1]*Professor, Department of Mathematics, School of Science, Sandip University, Nashik. (M.S), India.*
[2*]*Research Scholar, Department of Mathematics, School of Science, Sandip University, Nashik. (M.S), India.*
[3]*Department of Mathematics, School of Humanities and Engineering Sciences, MIT Academy of Engineering, Alandi, Pune, Maharashtra, India.*
[4]*Assistant Professor, Department of Civil Engineering, Deogiri Institute of Engineering and Management Studies, Aurangabad (M.S), India.*
Email: [1]renu.pathak@sandipuniversity.edu.in, [3]sggajbhiv@mitaoe.ac.in
[4]durgeshtupe@dietms.org
Corresponding Email: [2]maheshghotkar@gmail.com*

Background: The overlaid bar speculation has been approved in a few fields, including sports, flight, medication, and marine developments. Cantilever radiates, fixed radiates, and basically upheld radiates were dissected under different stacking situations. Arrangements were tracked down as relocations and stresses for fixed radiates, just upheld radiates, and different layered cross-utilize covered composite cantilever radiates exposed to variable, uniform, and sinusoidal burdens. Contrasting angle proportions consider fluctuating results concerning relocation, twisting pressure, and shear pressure. The constitutive relationship and balance condition are utilized to process the shear stresses. The virtual dislodging guideline is utilized to decide the covered composite beams exaggerated capability (HYSDT) and relocation field.

Keywords: Stress, Hyperbolic Shear Deformation Theory, Equilibrium Equation.

Introduction

The composite materials are one of the most generally utilized building materials in common and mechanical structures because of its high quality and moderately light weight. The composite materials are made of framework and fortification materials. Numerous blends of materials named as composite materials, for example, solid, mortar, fiber fortified plastics, fiber strengthened metals and comparable fiber impregnated materials. The unidirectional fiber-strengthened composites are generally utilized in shaping composite overlays.

Unidirectional layers are stacked with various fiber direction to accomplish desired stiffness, strength, and thermal characteristics.

Literature Review

Boay and Wee [1] presented a shut structure articulation to decide the effective flexural modulus of an overlaid composite beam. This effective flexural modulus is applied to the bowing, clasping and free vibration reaction of for the most part overlaid composite beams with different limit underpins. Pagano [2] presented definite versatility answers for composite overlays in tube shaped bowing. A unidirectional overlay and two- and three-layered cross-handle covers exposed to sinusoidal load are thought of. The laminated plate theories based on the Kirchhoff's theory have been developed by Reddy [3] and Timoshenko [5]. Khdeir and Reddy [4] presented the analysis of symmetric and antisymmetric cross-ply laminated beams using classical, first-order, second order and third-order theories. Arya [6] et al. presented a zigzag model for laminated composite beams. The theory satisfies the shear stress free condition at top and bottom and continuity condition at interface of the layer. Krishna Murty [7] detailed a third request beam theory including the transverse shear strain and nontraditional (nonlinear) pivotal pressure. In this theory the explanatory transverse shear pressure dispersion over the profundity of the beam can be acquired utilizing constitutive relations. Maiti and Sinha [8] introduced the limited component investigation of the symmetric and unsymmetric thick covered beams dependent on the higher request theory. Li and Hongxing [9] presented the specific powerful firmness grid of a uniform covered composite beam dependent on trigonometric shear distortion theory. A refined overlaid beam constitutive conditions is determined that considers the broadness bearing strains. Swift and Heller [10] studied overlaid beams by accepting layerwise consistent shear strains and a nonstop transverse dislodging through the thickness. This is a layerwise use of Timoshenko beam theory. The consequences of a sine stacked two layered, unsymmetrically stacked graphite epoxy beam with basic backings are introduced. Ozutok and Madenci [11] studied contemplated blended limited component conditions which depend on a practical are acquired by utilizing Gateaux differential for overlaid composite beams and higher request shear disfigurement theory including nonlinear circulation of shear worry through thickness of covered beam is introduced. Hasim [12] investigate an endeavor has been made here for the isogeometric static investigation of the covered composite plane beams by utilizing refined crisscross theory. Fereidooni et al. [13] studied the essential type of conditions of movement of composite beams exposed to shifting time loads are discretized utilizing a created limited component model. Tahani Masoud [14] presented inside the removal field of a layerwise theory, two covered beam speculations for beams with general overlay are created. In the main theory, a current layerwise covered plate theory is adjusted to overlaid beams. Sayyad et al. [15] studied the trigonometric beam theory (TBT) created for the bowing examination of covered composite and sandwich beams thinking about the impact of transverse shear theory. Shimpi and Ghugal [16] built up another dislodging based layerwise trigonometric shear distortion theory containing just two factors. Theory fulfills the shear free condition at top and base surfaces of the beam. Shimpi and Ghugal [17] introduced another layerwise trigonometric shear deformation theory for the examination of two-layered cross-utilize covered beams is introduced. Pawar et al. [18] a novel Normal and Shear Deformation Theory (NSDT) for investigation of covered composite and sandwich beams, considering shear disfigurement just as should be expected distortion, is created. Sayyad et al. [19] the refined beam theory (RBT) is inspected for the bowing of just upheld isotropic, overlaid composite and sandwich beams.Bassiouni et al. [20] presented the hypothetical and trial examination of the dynamic conduct for overlaid composite beams.

The Displacement Field for HYSDT

Based on the before mentioned assumptions, the displacement field of the present composite laminated beam theory can be expressed as follows,

$$u(x,z) = u(x) - z\frac{dw}{dx} + \left[h\sinh\left(\frac{z}{h}\right) - \frac{4}{3}\frac{z^3}{h^2}\cosh\left(\frac{1}{2}\right)\right]\phi(x)$$

$$w(x,z) = w(x) \tag{1}$$

Where u is the displacement in the *x* direction and w is transverse displacement in the *y* direction of a point on the beam in mid plane. The strain-displacement relations between strain-displacement corresponding to the displacement field are given by,

Along edges $x = 0$ *and* $x = L$

$$\bar{B}_{11}\frac{d^2u}{dx^2} - \bar{D}_{11}\frac{d^3w}{dx^3} + \bar{F}_{11}\frac{d^2\phi}{dx^2} = 0 \text{ or w is prescribed,} \tag{2}$$

$$\bar{B}_{11}\frac{du}{dx} - \bar{D}_{11}\frac{d^2w}{dx^2} + \bar{F}_{11}\frac{d\phi}{dx} - R_{11}T_0 = 0 \text{ or } \frac{dw}{dx} \text{ is prescribed} \tag{3}$$

$$\bar{B}_{11}\frac{du}{dx} - \bar{D}_{11}\frac{d^2w}{dx^2} + \bar{F}_{11}\frac{d\phi}{dx} - R_{11}T_0 = 0 \text{ or } \frac{dw}{dx} \text{ is prescribed,} \tag{4}$$

$$\bar{E}_{11}\frac{du}{dx} - \bar{F}_{11}\frac{d^2w}{dx^2} + \bar{H}_{11}\frac{d\phi}{dx} - S_{11}T_0 = 0 \text{ or } \phi \text{ is prescribed} \tag{5}$$

$$\bar{E}_{11}\frac{du}{dx} - \bar{F}_{11}\frac{d^2w}{dx^2} + \bar{H}_{11}\frac{d\phi}{dx} - S_{11}T_0 = 0 \text{ or } \phi \text{ is prescribed,} \tag{6}$$

$$\bar{A}_{11}\frac{du}{dx} - \bar{B}_{11}\frac{d^2w}{dx^2} + \bar{E}_{11}\frac{d\phi}{dx} - O_{11}T_0 = 0 \text{ or } u \text{ is prescribed} \tag{7}$$

Navier solution

Following are the boundary conditions used for simply supported laminated composite beam along the edges $x = a$ and $x = l$, $w = 0$, $M_x = 0$, $N_x = 0$, $P_x = 0$

Navier's solution procedure is adopted to compute displacement variables. The following is the solution forms for $u_0(x)$, $w(x)$ and $\varphi(x)$ that satisfies the boundary conditions exactly.

$$u_0(x) = \sum_{m=1,3,5}^{\infty} u_m \cos\left(\frac{m\pi x}{l}\right) \tag{8}$$

$$w(\mathrm{x}) = \sum_{m=1,3,5}^{\infty} w_m \sin\left(\frac{m\pi x}{l}\right) \quad (9)$$

$$\varphi(\mathrm{x}) = \sum_{m=1,3,5}^{\infty} \varphi_m \sin\left(\frac{m\pi x}{l}\right) \quad (10)$$

Where u_m, w_m and φ_m are the unknown coefficients to be determined. The thermal and transverse mechanical loads are expanded in single Fourier sine series as given below.

$$T_0(\mathrm{x}) = \sum_{m=1}^{\infty} T_{0m} \sin\left(\frac{m\pi x}{l}\right) \quad (11)$$

$$q(\mathrm{x}) = \sum_{m=1}^{\infty} q_m \sin\left(\frac{m\pi x}{l}\right)$$

Where m is the positive integer and T_{0m} and q_m are the coefficients of Fourier series expansions, respectively for thermal and transverse mechanical loads as follows:

In which T_0 and q_0 are the intensities of thermal and mechanical load respectively.

$$\begin{bmatrix} \bar{A}_{11}\frac{m^2\pi^2}{l^2} & -\bar{B}_{11}\frac{m^3\pi^3}{l^3} & \bar{E}_{11}\frac{m^3\pi^3}{l^3} \\ -\bar{B}_{11}\frac{m^3\pi^3}{l^3} & \bar{D}_{11}\frac{m^4\pi^4}{l^4} & -\bar{F}_{11}\frac{m^3\pi^3}{l^3} \\ \bar{E}_{11}\frac{m^2\pi^2}{l^2} & -\bar{F}_{11}\frac{m^3\pi^3}{l^3} & \bar{H}_{11}\frac{m^2\pi^2}{l^2}+G_{55} \end{bmatrix} \begin{Bmatrix} u_{0m} \\ w_m \\ \varphi_m \end{Bmatrix} = \begin{Bmatrix} T_{0m}\mathrm{O}_{11}\frac{2m\pi}{hl} \\ T_{0m}\mathrm{R}_{11}\frac{2m^2\pi^2}{hl^2}+q_m \\ T_{0m}S_{11}\frac{2m\pi}{hl} \end{Bmatrix} \quad (12)$$

Solving the above set of algebraic equations, the values of u_{0m}, w_m and φ_m can be obtained. Having obtained the values of u_{0m}, w_m and φ_m one can then calculate all the thermal displacements and stresses within the beam .Transverse shear stresses are obtained by integrating equilibrium equations τ_{zx}^{EE} of theory of elasticity with respect to the thickness coordinate, satisfying shear stress free conditions at the top and bottom surface of the laminated beam and which ascertains the continuity of transverse shear stress at the layer interface. This relation can be expressed as given below,

$$\tau_{zx}^{EE} = \int_{-h/2}^{z_k} \frac{\partial \sigma_x}{\partial x} dz + \mathrm{C}_1$$

The constant of integration can be obtained from appropriate boundary conditions. It is expected that this relation will produce accurate transverse shear stresses.

Illustrative Examples

Example : A simply supported beam with sinusoidally distributed load,

$$q(x) = \sin\left(\frac{3\pi x}{L}\right)$$

The simply supported beam is having its origin at left support and is simply supported at *x=0 and x=L*. The beam is subjected to sinusoidally distributed load, $q(x)=\sin\left(\frac{3\pi x}{L}\right)$

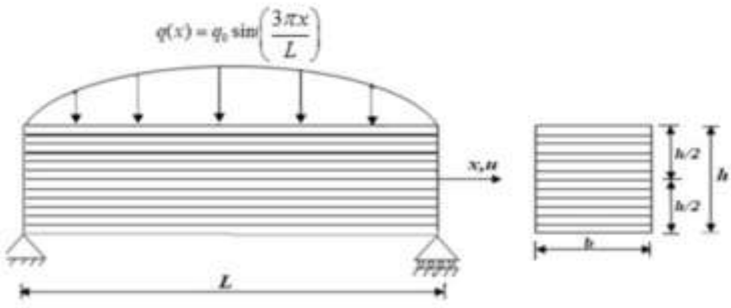

Fig. Simply supported beam with sinusoidally distributed load of single layer beam (0^0).

Non dimensional transverse displacement $\bar{w}$,

For 0^0

$$\bar{w}=\left[\frac{\left(1.6701\times10^5\alpha_L L^2Th^2\pi\right)\left(2003L^2+2.6950\times10^4h^2\pi^2\right)}{\left(4.1728\times10^7L^2h^3\pi^4+7.0043\times10^6h^5\pi^6\right)}\right]\left[\frac{h}{\alpha_L L^2T}\sin\left(\frac{3\pi x}{L}\right)\right]$$

For 90^0

$$\bar{w}=\left[\frac{\left(6680.6599\alpha_L L^2Th^2\pi\right)\left(2003L^2+2695h^2\pi^2\right)}{\left(1.6684\times10^6L^2h^3\pi^4+2.0490\times10^6h^5\pi^6\right)}\right]\left[\frac{h}{\alpha_L L^2T}\sin\left(\frac{3\pi x}{L}\right)\right]$$

For $0^0/90^0/0^0$

$$\bar{w}=\left[\frac{\left(3.4071\times10^5\alpha_L L^2Th^2\pi\right)\left(6.2192\times10^{16}L^2+7.2006\times10^{18}h^2\pi^2\right)}{\left(1.1299\times10^{21}L^2h^3\pi^4+1.7980\times10^{20}h^5\pi^6\right)}\right]\left[\frac{h}{\alpha_L L^2T}\sin\left(\frac{3\pi x}{L}\right)\right]$$

Non dimensional axial displacement $\bar{u}$,

For 0^0

$$\bar{u}=\left[\frac{1}{\alpha_L TL}\right]\left\{\begin{aligned}&\left[-z\cos\left(\frac{3\pi x}{L}\right)\left(\frac{3\pi}{L}\right)\right]\left[\frac{\left(1.6701\times10^{5}\alpha_L L^2Th^2\pi^2\right)\left(2003L^2+2.6950h^2\pi^2\right)}{\left(4.1728\times10^{7}L^2h^3\pi^4+7.0043\times10^{6}h^5\pi^6\right)}\right]+\\&\left[-\cos\left(\frac{3\pi x}{L}\right)\left(z\sinh\frac{z}{h}-\frac{4}{3}\frac{z^3}{h^2}\cosh\frac{1}{2}\right)\left(\frac{5.5616\times10^{9}\alpha_L LTh^2\pi^2}{4.1728\times10^{7}L^2h\pi+7.0043\times10^{6}h^3\pi^3}\right)\right]\end{aligned}\right\}$$

For 90^0

$$\bar{u}=\left[\frac{1}{\alpha_L TL}\right]\left\{\begin{aligned}&\left[-z\cos\left(\frac{3\pi x}{L}\right)\left(\frac{3\pi}{L}\right)\right]\left[\frac{\left(6680.6599\alpha_L L^2Th^2\pi^2\right)\left(2003L^2+26950h^2\pi^2\right)}{\left(1.6684\times10^{6}L^2h^3\pi^4+2.0490\times10^{4}h^5\pi^6\right)}\right]+\\&\left[-\cos\left(\frac{3\pi x}{L}\right)\left(z\sinh\frac{z}{h}-\frac{4}{3}\frac{z^3}{h^2}\cosh\frac{1}{2}\right)\left(\frac{2.2280\times10^{7}\alpha_L LTh^2\pi^2}{1.6684\times10^{6}L^2h\pi+7.0043\times10^{6}h^3\pi^3}\right)\right]\end{aligned}\right\}$$

For $0^0/90^0/0^0$

$$\bar{u}=\left[\frac{1}{\alpha_L TL}\right]\left\{\begin{aligned}&\left[-z\cos\left(\frac{3\pi x}{L}\right)\left(\frac{3\pi}{L}\right)\right]\left[\frac{\left(3.4071\times10^{5}\alpha_L L^2Th^2\pi^2\right)\left(5.6219\times10^{17}L^2+7.2006\times10^{18}h^2\pi^2\right)}{\left(1.1299\times10^{22}L^2h^3\pi^4+1.7980\times10^{20}h^5\pi^6\right)}\right]+\\&\left[-\cos\left(\frac{3\pi x}{L}\right)\left(z\sinh\frac{z}{h}-\frac{4}{3}\frac{z^3}{h^2}\cosh\frac{1}{2}\right)\left(\frac{1.8495\times10^{25}\alpha_L LTh^2\pi^2}{1.1299\times10^{22}L^2h\pi+1.7980\times10^{20}h^3\pi^3}\right)\right]\end{aligned}\right\}$$

Non dimensional axial stresses $\overline{\sigma_x}$,

For 0^0

$$\overline{\sigma_x}=\left[\frac{1}{\alpha_L TE_2}\right]\left\{\begin{aligned}&\left[z\sin\left(\frac{3\pi x}{L}\right)\left(\frac{9\pi^2}{L^2}\right)\right]\left[\frac{\left(1.6701\times10^{5}\alpha_L L^2Th^2\pi^2\right)\left(2003L^2+2.6950h^2\pi^2\right)}{\left(4.1728\times10^{7}L^2h^3\pi^4+7.0043\times10^{6}h^5\pi^6\right)}\right]+\\&\left[\sin\left(\frac{3\pi x}{L}\right)\frac{3\pi}{L}\left(z\sinh\frac{z}{h}-\frac{4}{3}\frac{z^3}{h^2}\cosh\frac{1}{2}\right)\left(\frac{5.5616\times10^{9}\alpha_L LTh^2\pi^2}{4.1728\times10^{7}L^2h\pi+7.0043\times10^{6}h^3\pi^3}\right)\right]\end{aligned}\right\}$$

For 90^0

$$\overline{\sigma_x}=\left[\frac{1}{\alpha_L TE_2}\right]\left\{\begin{array}{l}\left[z\sin\left(\frac{3\pi x}{L}\right)\left(\frac{9\pi^2}{L^2}\right)\right]\left[\frac{\left(6680.6599\alpha_L L^2Th^2\pi^2\right)\left(2003L^2+2695h^2\pi^2\right)}{\left(1.6684\times10^6 L^2h^3\pi^4+2.0490\times10^4 h^5\pi^6\right)}\right]+\\ \left[\sin\left(\frac{3\pi x}{L}\right)\frac{3\pi}{L}\left(z\sinh\frac{z}{h}-\frac{4}{3}\frac{z^3}{h^2}\cosh\frac{1}{2}\right)\left(\frac{2.2280\times10^7\alpha_L LTh^2\pi^2}{1.6684\times10^6 L^2h\pi+2.0490\times10^4 h^3\pi^3}\right)\right]\end{array}\right\}$$

For $0^0/90^0/0^0$

$$\overline{\sigma_x}=\left[\frac{1}{\alpha_L TE_2}\right]\left\{\begin{array}{l}\left[z\sin\left(\frac{3\pi x}{L}\right)\left(\frac{9\pi^2}{L^2}\right)\right]\left[\frac{\left(3.4071\times10^5\alpha_L L^2Th^2\pi\right)\left(5.6219\times10^{17}L^2+7.2006\times10^{18}h^2\pi^2\right)}{\left(1.1299\times10^{22}L^2h^3\pi^4+1.7980\times10^{20}h^5\pi^6\right)}\right]+\\ \left[\sin\left(\frac{3\pi x}{L}\right)\frac{3\pi}{L}\left(z\sinh\frac{z}{h}-\frac{4}{3}\frac{z^3}{h^2}\cosh\frac{1}{2}\right)\left(\frac{3.0562\times10^{24}\alpha_L LTh^2\pi^2}{1.1299\times10^{22}L^2h\pi+1.7980\times10^{20}h^3\pi^3}\right)\right]\end{array}\right\}$$

Non dimensional transverse shear stresses $\tau_{zx}^{\overline{EE}}$ using equilibrium equation,

For 0^0

$$\tau_{zx}^{\overline{EE}}=\left[\frac{1}{\alpha_L TE_2}\right]\left\{\begin{array}{l}\left[\left(\frac{h^2}{8}\left(1-\frac{4z^2}{h^2}\right)\right)\cos\left(\frac{3\pi x}{L}\right)\left(\frac{27\pi^3}{L^3}\right)\right]\left[\frac{\left(1.6701\times10^5\alpha_L L^2Th^2\pi^2\right)\left(2003L^2+2.6950\times10^4h^2\pi^2\right)}{\left(4.1728\times10^7L^2h^3\pi^4+7.0043\times10^6h^5\pi^6\right)}\right]+\\ \left[\begin{array}{l}\left(\left(\left(\cosh\frac{1}{2}-\cosh\frac{z}{h}\right)\frac{9\pi^2h^2}{L^2}\cos\frac{3\pi x}{L}\right)+\left(\left(\frac{1}{3}\frac{z^4}{h^2}\right)-\left(\frac{h^2}{48}\right)\right)\frac{9\pi^2}{L^2}\cosh\frac{1}{2}\cos\left(\frac{3\pi x}{L}\right)\right)\\ \left(\frac{5.5616\times10^9\alpha_L LTh^2\pi}{4.1728\times10^7L^2h\pi+7.0043\times10^6h^3\pi^3}\right)\end{array}\right]\end{array}\right\}$$

For 90^0

$$\tau_{zx}^{\overline{EE}}=\left[\frac{1}{\alpha_L TE_2}\right]\left\{\begin{array}{l}\left[\left(\frac{h^2}{8}\left(1-\frac{4z^2}{h^2}\right)\right)\cos\left(\frac{3\pi x}{L}\right)\left(\frac{27\pi^3}{L^3}\right)\right]\left[\frac{\left(6680.6599\alpha_L L^2Th^2\pi^2\right)\left(2003L^2+2695h^2\pi^2\right)}{\left(1.6684\times10^6L^2h^3\pi^4+2.0490\times10^4h^5\pi^6\right)}\right]+\\ \left[\begin{array}{l}\left(\left(\left(\cosh\frac{1}{2}-\cosh\frac{z}{h}\right)\frac{9\pi^2h^2}{L^2}\cos\frac{3\pi x}{L}\right)+\left(\left(\frac{1}{3}\frac{z^4}{h^2}\right)-\left(\frac{h^2}{48}\right)\right)\frac{9\pi^2}{L^2}\cosh\frac{1}{2}\cos\left(\frac{3\pi x}{L}\right)\right)\\ \left(\frac{2.2280\times10^7\alpha_L LTh^2\pi}{1.6684\times10^6\times10^6L^2h\pi+2.0490\times10^4h^3\pi^3}\right)\end{array}\right]\end{array}\right\}$$

For 0^0 / 90^0 / 0^0

$$\tau_{zx}^{\overline{EE}}=\left[\frac{1}{\alpha_L T E_2}\right]\left\{\begin{array}{l}\left[\left(\frac{h^2}{8}\left(1-\frac{4z^2}{h^2}\right)\right)\cos\left(\frac{3\pi x}{L}\right)\left(\frac{27\pi^3}{L^3}\right)\right]\left[\frac{\left(3.4071\times10^{5}\alpha_L L^2 T h^2\pi^2\right)\left(5.6219\times10^{17}L^2+7.2006\times10^{18}h^2\pi^2\right)}{\left(1.1299\times10^{22}L^2h^3\pi^4+1.7980\times10^{20}h^5\pi^6\right)}\right]+\\ \left[\left(\left(\left(\cosh\frac{1}{2}-\cosh\frac{z}{h}\right)\frac{9\pi^2h^2}{L^2}\cos\frac{3\pi x}{L}\right)+\left(\left(\frac{1}{3}\frac{z^4}{h^2}\right)-\left(\frac{h^2}{48}\right)\right)\frac{9\pi^2}{L^2}\cosh\frac{1}{2}\cos\left(\frac{3\pi x}{L}\right)\right)\right.\\ \left.\left(\frac{3.0562\times10^{24}\alpha_L L T h^2\pi}{1.1299\times10^{22}\times10^{6}L^2h\pi+1.7980\times10^{20}h^3\pi^3}\right)\right]\end{array}\right\}$$

Non dimensional Transverse shear stresses $\tau_{zx}^{\overline{CR}}$ using constitutive relationship

For 0^0

$$\tau_{zx}^{\overline{CR}}=G_{55}\left\{\left[\cosh\frac{z}{h}-\frac{4z^2}{h^2}\cosh\frac{1}{2}\right]\cos\left(\frac{3\pi x}{L}\right)\left[\frac{\left(5.5616\times10^{9}\alpha_L L^2 T h^2\pi\right)}{\left(4.1728\times10^{7}L^2h^3\pi^4+7.0043\times10^{6}h^3\pi^3\right)}\right]\right\}$$

For 90^0

$$\tau_{zx}^{\overline{CR}}=G_{55}\left\{\left[\cosh\frac{z}{h}-\frac{4z^2}{h^2}\cosh\frac{1}{2}\right]\cos\left(\frac{3\pi x}{L}\right)\left[\frac{\left(2.2280\times10^{7}\alpha_L L^2 T h^2\pi\right)}{\left(1.6684\times10^{6}L^2h^3\pi^4+7.0043\times10^{6}h^3\pi^3\right)}\right]\right\}$$

For 0^0 / 90^0 / 0^0

$$\tau_{zx}^{\overline{CR}}=G_{55}\left\{\left[\cosh\frac{z}{h}-\frac{4z^2}{h^2}\cosh\frac{1}{2}\right]\cos\left(\frac{3\pi x}{L}\right)\left[\frac{\left(3.0562\times10^{24}\alpha_L L^2 T h^2\pi\right)}{\left(1.2299\times10^{22}L^2h\pi+1.7980\times10^{20}h^3\pi^3\right)}\right]\right\}$$

AS (Aspect Ratio)	**Ply Angle**	**HYSDT**				
		$\overline{w}$	$\overline{u}$	$\overline{\sigma}_x$	$\tau_{zx}^{\overline{EE}}$	$\tau_{zx}^{\overline{CR}}$
4	0^0	1.711	4.661	14.796	3.211	0.593
	90^0	0.369	0.796	2.500	0.524	0.026
	$0^0/90^0/0^0$	3.468	9.491	29.803	0.258	1.206

Table 2: Non-Dimensional Transverse Deflection $\overline{w}$ at (x = 0.5L, z = 0.0), Axial Displacement $\overline{u}$ at (x = 0.5L, z = $h/2$), Axial Stress ($\overline{\sigma}_x$) at (x = 0.5L, z = $h/2$), Maximum Transverse Shear Stresses ($\tau_{zx}^{\overline{EE}}$ and $\tau_{zx}^{\overline{CR}}$) at ($x$ = 0:0, z = $h/2$) for Single Layer, Three

Layers and Four Layers of Laminated Simply Supported Beam Subjected to Sinusoidally Distributed Load $q(x) = \sin\left(\frac{3\pi x}{L}\right)$ for Aspect Ratio 10.

AS (Aspect Ratio)	Ply Angle	HYSDT				
		$\bar{w}$	$\bar{u}$	$\bar{\sigma}_x$	$\tau_{zx}^{\overline{EE}}$	$\tau_{zx}^{\overline{CR}}$
10	0^0	0.465	1.073	3.458	0.286	0.103
	90^0	0.230	0.396	1.243	0.099	0.004
	$0^0/90^0/0^0$	0.965	2.199	6.907	0.023	0.208

Conclusion

a) It is a displacement based, refined shear disfigurement hypothesis.
b) The shear deformation in the beam is appropriately represented.
c) The hypothesis deters the essential of shear rectification factor.
d) Fluctuation predictable are the governing differential conditions and the associated boundary condition

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