Magnetic Field Between Polar Hemispheres: Remarks on the Dislocation of Zones of a Constant Gradient and Force Factor

Anna A. Sandulyak[1], Maria N. Polismakova[1], Darya A. Sandulyak[1], Anand P. Dwivedi[2*], Charalampos C. Doumanidis[2], Alexander V. Sandulyak[1], Vera A. Ershova[1]

[1]MIREA — Russian Technological University, Moscow, Russia
[2]Guangdong Technion-Israel Institute of Technology, Shantou, China
Email: anand.dwivedi@gtiit.edu.cn

To create an inhomogeneous magnetic field, in which there would be zones with a practically constant value of the magnetic gradient (MG) and / or magnetic force factor (MFF), an original solution was indicated: the use of an electromagnetic system with spherical poles. In this case, the coordinate dependencies of MG and MFF in the interpolar region are extreme and the zones localized in the domain of the extrema are practically stable, referred to as the MGConst and MFFConst zones, with individual coordinates xextr of their conditional centers and length Δx (with an allowable error changes in MG and MFF data within these zones). The data following from experimental studies is given on the dislocation of the indicated zones between the poles-hemispheres of diameter D, separated by different distances b; the values of xextr/D and Δx/D depend on b/D. It is shown that the dependencies xextr/D on b/D obtained separately for the MGConst and MFFConst zones, demonstrating, like the dependencies Δx/D on b/D, mutual similarity (up to a constant multiplier), obey power functions with exponents of 0.5 and 0.4, respectively. A closer (to the center-to-center line of pole-hemispheres) and more compact (in length) dislocation of the MFFConst zones compared to the MGConst zones was revealed: almost 1.3 times both in xextr/D and in Δx/D.

Keywords: Spherical pole pieces, magnetic parameters, gradient, force factor, zones of constancy of parameters, inhomogeneous magnetic field.

I. INTRODUCTION

Solving the amount of scientific and practical problems, often becomes necessary to use an inhomogeneous magnetic field and one that would have certain specific characteristics. In particular, it is suggested to create a field, in fact, a certain zone in this field, with relatively constant values of the magnetic gradient (MG), i.e. when gradH Const or gradB Const: by strength H or induction B = μ0H (μ0 = 4π·10-7 H/m – magnetic constant); aforementioned zone can be characterized as an MGConst zone. In addition, talking about creating a field, a certain zone of which is characterized by relatively constant values of the magnetic force factor (MFF), which is the product of H·gradH or B·gradB: where H·gradH Const or B·gradB Const; this zone can be characterized as the MFFConst zone.

For example, this kind of problem is relevant for magnetometers operating on the Faraday ponderomotive principle (Faraday magnetometer, Faraday balance). One of the features of these magnetometers is that the object of study is samples of small spaces in order to measure the ponderomotive (magnetic) force Fm acting in an inhomogeneous magnetic field on the sample under study for the subsequent determination of its magnetic susceptibility χ. This feature becomes an important advantage when one has to deal with a very limited amount of material being studied; such an advantage can become decisive, in particular, in the study of the magnetic properties of media obtained as a result of fine magnetic separation, as well as other composite, powder media [1–8]. Using such magnetometers the known identical expressions for calculating χ are:

$$F_m = \mu_0 V\chi HgradH \text{ or } F_m = V\chi BgradB/\mu_0, \quad (1)$$

indicate that in order to increase the reliability of the results, the studied sample should be placed in the zone of constant field inhomogeneity. The authors [9–11] assume that this is a zone of constant magnetic gradient, i.e. MGConst, and the authors of [12-14] consider that this is a zone of a constant magnetic force factor, i.e. MFFConst.

However, it should be stated that currently, the required amount of reasoned data on the presence, location and size of these zones (and more informatively, on their variable dislocation) in the available cases of technological execute on and functioning of the corresponding magnetic systems is extremely limited. It results in the absence of information about the coordinate characteristics of MG and MFF in the used magnetometers. There is no possibility to judge the presence and the features of the dislocation as well, of the MGConst and/or MFFConst zones (therefore, in [10] it is proposed to give preference to relative measurements, performing only sample's calibration for with known magnetic susceptibility values).

II. MAGNETIC SYSTEM WITH POLAR HEMISPHERES. MGCONST AND MFFCONST ZONES, VARIABLE COORDINATES OF THEIR CONDITIONAL CENTERS

The problem of obtaining zones of practically constant magnetic gradient MGConst and/or magnetic force factor MFFConst was originally solved in [15–18] - in an electromagnetic system containing a magnetic circuit with spheric pole pieces (polar hemispheres, Fig. 1). Due to this shape of the poles, a specific, acceptable within the framework of the problem being solved, x-characteristic of the induction B (or intensity $H = B/\mu 0$) is obtained – in the direction x (Fig. 1), where x comes from the central point of the region between the poles and rushes in the radial direction of the transverse plane of symmetry of this region.

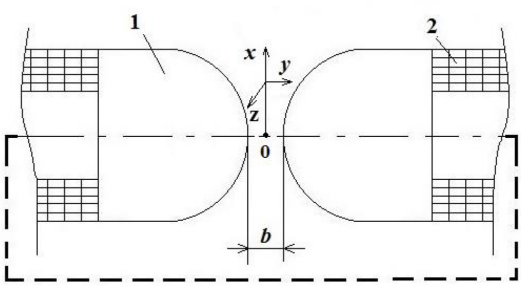

Fig. 1. Illustration of the use of spherical pole pieces in an electromagnetic system, which allow creating local zones of almost constant values of the magnetic gradient and magnetic force factor; 1 - pole piece, 2 - winding of the electromagnetic system

This easy for diagnosing characteristic, for example, by the parameter B when the Hall sensor of the milliteslameter is moved step by step using the coordinate table [15–18]), is tortuous, having a remarkable inflection. Numerous experimental dependencies of B on x obtained in [15, 16, 19] using hemispherical poles (diameters are D = 100 and 135 mm) confirm this fact.

Figure 2a demonstrates one of the families of dependencies B on x – at a distance b = 13 mm between the polar hemispheres with a diameter D = 100 mm for different values of the magnetizing force of the system Iω = 3000...22500 A (I is the value of the power supply current of its winding with the number turns ω). With such a tortuous trend of one or another dependence of B on x (Fig. 2a), its derivative $\partial B/\partial x$ acquires the expected form, which has an extremum (extreme form, Fig. 2b). At the same time $\partial B/\partial x$ dB/dx gradB – taking into account the fact that in the directions y and z (Fig. 1), emanating from any point on the x axis, including that belonging to the MGConst zone, near it: $\partial B/\partial y \rightarrow 0$ and $\partial B/\partial z \rightarrow 0$ [20].

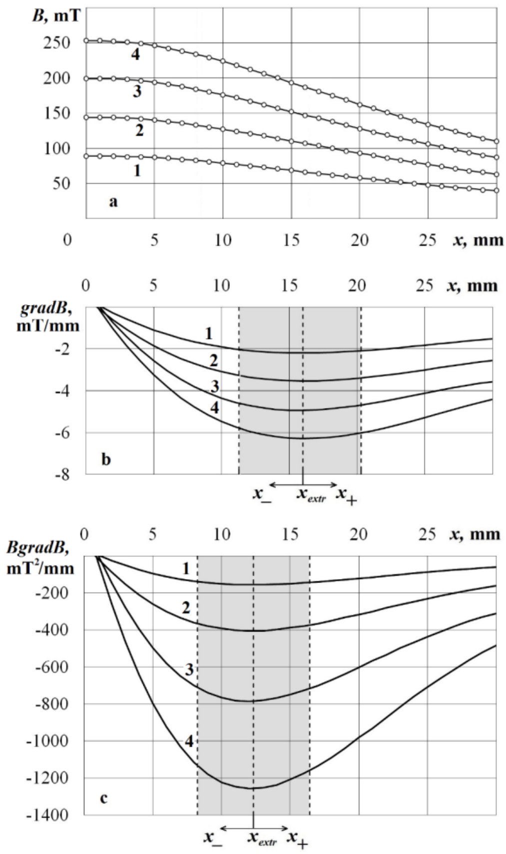

Fig. 2. An example of coordinate magnetic characteristics between hemispherical poles with a diameter D = 100 mm and a distance b = 13 mm: a) winding for induction (points - experiment, line - approximation by a polynomial of the fourth degree), b) extreme for MG, c) extreme for MFF; 1 - Iω = 3000 A, 2 - 6000, 3 - 12000, 4 - 22500; the MG_{Const} and MFF_{Const} zones are dimmed.

This means that the region in the surrounding of the extremum of the magnetic gradient is the zone of an almost constant gradient (MGConst or, what is the same, the zone (gradB)Const) – with an easily identifiable (both visually and by the results of data processing in the program) coordinate of this extremum x = xextr.

The specific value of the position coordinate xextr of the conditional center of the desired zone MGConst can be found not only in the program, but also by the calculation formula [15], if the fourth-degree polynomial, preferably used to approximate the initial data B from x, is analytically investigated for an extremum. For the illustrated particular case (Fig. 2b) at b = 13 mm, the xextr coordinate of the conditional zone center MGConst is xextr 16 mm. This value is practically independent of the magnetizing force Iω [15, 21] in the specified range of Iω.

The values of xextr are determined similarly for other values of b [15] (for the range b =

3.5…15.3 mm) and for the same values of Iω (which also do not affect xextr), which leads to one of the principal dependencies, namely xextr from b. Also, based on the similarity of the trends of related coordinate dependencies confirmed in [19] (by induction, by the magnetic gradient, by the magnetic force factor) for polar hemispheres of different diameters D, one can see the identity of the coordinates of the extrema of the xextr gradient – in the event that was operated with relative values: xextr /D and b/D. Therefore, the obtained dependence of xextr on b should be presented in a more universal form as xextr /D on b/D (Fig. 3a) in the corresponding range b/D = 0.035…0.153.

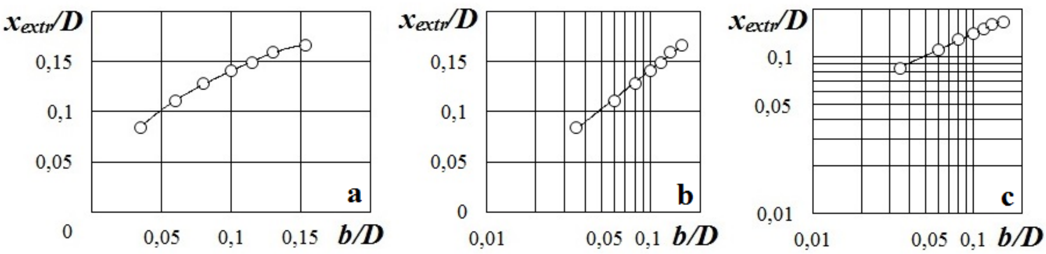

Fig.3. Influence of the relative distance *b*/D between the polar hemispheres on the relative coordinate x_{extr} /D of the conditional center of the zone MG_{Const}; a) in conventional coordinates, b and c) respectively in semi-logarithmic and logarithmic coordinates.

Having the data B and gradB (Fig. 2a,b), one can obtain the data and the corresponding coordinate characteristic of the magnetic force factor (MFF), i.e. data of the product B·gradB. It is obvious that using the same example (b = 13 mm), the obtained dependence of the parameter BgradB on x has (as well as the dependence of the parameter gradB on x in Fig. 2b) a form with extremum (Fig. 2c). Thus, one can also testify to the presence of the MFFConst zone (or, which is the same: the (BgradB)Const zone) – with the identifiable coordinate of this extremum x = xextr, both visually and according to the results of data processing in the program.

For the illustrated particular case at b = 13 mm (Fig. 2c), as well as at other values of b = 3.5…15.3 mm, the coordinate of the extremum xextr as the coordinate of the conditional center of the zone MFFConst practically does not depend on the magnetizing force Iω [15] (within the specified range of Iω). The obtained values of xextr at different b give the dependence of xextr on b [15], which is presented as the dependence of xextr /D on b/D (Fig. 4a) – in the range b/D = 0.035…0.153.

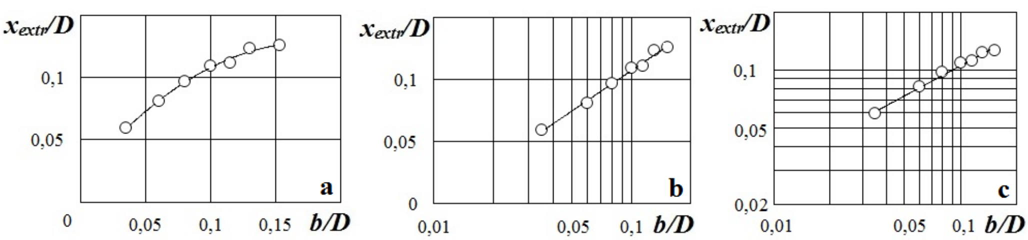

Fig. 4. The same as in Fig.3, but for the conditional center of the zone MFF_{Const}.

It should be noted that both dependencies of the relative coordinates xextr /D of the conditional centers of the MGConst zone (Fig. 3a) and the MFFConst zone (Fig. 4a), obtained on the basis of experimental data, can be approximated by phenomenological expressions.

These dependencies (Fig. 3b and Fig. 4b) are well quasi-linearized in the indicated range of b, which means that they can be described by logarithmic expressions:

$$\frac{x_{extr}}{D} = Q_G \ln\frac{b/D}{K_G}, \quad \frac{x_{extr}}{D} = Q_F \ln\frac{b/D}{K_F}, \qquad (2)$$

where the phenomenological parameters are: Q_G = 0.063…0.064 for the MG_{Const} zone and Q_c = 0.047…0.048 – for the MFF_{Const} zone; KG ≅ KF = 0.009…0.01 – for MG_{Const} and MFF_{Const} zones, i.e. this setting is the same for both zones.

The latter suggests that both variants of expression (2), which can be used to determine relative coordinates xextr /D of the conditional zone center MG_{Const} and the conditional zone center MFF_{Const} are functionally very close: they differ only in the values of the multiplier parameters QG and QF. Hence in addition to the important information received by the user about the dislocation of conditional centers (x_{extr} /D) of each of the zones, namely MG_{Const} and MFF_{Const}, in accordance with (2), i.e. depending on the specific value of the relative distance b/D between the polar hemispheres, the following is just as important. At least in the specified range b/D = 0.035…0.153, the mutual dislocation of these centers remains practically unchanged: the ratio of the coordinate values x_{extr} /D of the conditional centers of the zones MG_{Const} and MFF_{Const} (as the ratio of the corresponding values of the multiplier parameters QG and QF) always the same (average 1.33 with fluctuations from 1.31 to 1.36).

The dependencies in Fig. 3a and Fig. 4a in the specified range of b/D are quasi-linearized quite well in logarithmic coordinates (Fig. 3c and Fig. 4c), additionally. This indicates the possibility of an alternative use for describing these dependencies (and obtaining an additional possibility for calculating the relative coordinates x_{extr} /D of the conditional centers of the MG_{Const} and MFF_{Const} zones) phenomenological expressions of a power type. In addition, here the exponent with the argument b/D in both cases (Fig. 3c and Fig. 4c) is close to 0.5, which allows to write these expressions as:

$$\frac{x_{extr}}{D} = A_G\left(\frac{b}{D}\right)^{0,5}, \quad \frac{x_{extr}}{D} = A_F\left(\frac{b}{D}\right)^{0,5}, \qquad (3)$$

where the phenomenological parameters in calculations according to (3) are: A_G = 0.44…0.45 for the MG_{Const} zone and A_F = 0.33…0.34 for the MFF_{Const} zone.

As before with the variants of expression (2), here both variants of expression (3), by which it can be determined the relative coordinate x_{extr}/D of the conditional center of the MG_{Const} zone and the MFF_{Const} zone, are also very close, they differ only in the values of the multiplier parameters A_G and A_F. Consequently, along with the information obtained in accordance with (3) about the dislocation of each of the conditional centers, i.e. x_{extr} /D data for each of the desired zones, depending on one or another value of *b/D*, information about the mutual dislocation of these two centers of the zones MG_{Const} and MFF_{Const} becomes simply available. Therefore, estimating by the unchanged ratio of the indicated individual values of the parameters A_G and A_F in (3) and, moreover, with the previous estimated result

(1.33), such as mutual dislocation with a specified change in *b/D* remains, as established earlier, nearly unchanged.

III. VARIABLE LENGTH OF MGCONST AND MFFCONST ZONES

The extent of the desired zones MG_{Const} and MFF_{Const} in the surrounding of the magnetic gradient extremum (MG) and the magnetic force factor extremum (MFF) can be estimated with a given error, allowing one or another of its limited values. Such an estimate, but only for the MG_{Const} zones, is given in [21]: using the example of a family of dependencies obtained at a distance of b = 13 mm between the polar hemispheres (Fig. 2b) and then using similar families of dependencies, obtained for different values of b.

At the primary stage of such estimates, a convenient index adopted in [21], which then allows one to estimate the error of the hypothesis being made, can be the comparative deviation between the current (on the x coordinate) value of the parameter of interest (gradient and/or force factor) and its value in the extremum. In this case, the mentioned value of the parameter in the extremum is known in advance and, therefore, is still a control here. Therefore, to estimate the length of the MG_{Const} zone, such an indicator is a relative change in the gradient:

$$E = [(gradB)_{extr} – gradB] / (gradB)_{extr} \times 100\%, \quad (4)$$

and to estimate the length of the zone, MFF_{Const} is the relative change in the force factor:

$$E = [(BgradB)_{extr} – BgradB] / (BgradB)_{extr} \times 100\%, \quad (5)$$

The following comments are essential for implementation of the approach (further comments are given with the involvement of information in Fig. 2b, c). The values of E are determined according to (4) and (5) using the fixed data $(gradB)_{extr}$ and $(BgradB)_{extr}$ from the extreme characteristics MG (Fig. 2b) and MFF (Fig. 2c), as well as the current data *gradB* and *BgradB*, which it is expedient to read with a step-by-step shift, starting from the corresponding extrema. In addition, it should be noted that both for the MG_{Const} zone and for the MFF_{Const} zone, their adjacent half-zones (before and after the extremum) are not strictly symmetrical. Therefore, it is necessary to make a decision about the shift (for reading the specified data) to the left half-zone from the extremum. In Fig. 2b,c, the direction of displacement is indicated by an arrow at the bottom, the displacement value (from the extremum coordinate x_{extr}) is denoted as $x\_$. The choice of just this direction of displacement (to the left) is justified in view of the steeper (than to the right) trend of the *gradB* dependence and the dependence of *BgradB* on x. Then, with a conditional shift to the right, i.e. along the x axis in Fig. 2b,c, where the trend of the dependencies of *gradB* and *BgradB* on x is less steep (in this case, the shift value is denoted as x_+), at values $x_+ = x\_$ results are more suitable parameters for established restrictions.

The family of extreme dependencies of the *gradB* on x (Fig. 2b) can be used to demonstrate a specific quantitative estimate of the length of one of the zones of interest, the MG_{Const} zone. This family has a common extremum coordinate x_{extr} (here $x_{extr} \cong 16$ mm), regardless of the system' value of $I\omega$ and the gradient-value at the extremum, i.e. $(gradB)_{extr}$, of course, depends on $I\omega$.

If, as already mentioned, we start from the extremum of the gradient (Fig. 2b) and increase the step-by-step offset $x\_$ to read the current gradient data, thereby making an attempt to allow an increase in the length $\Delta x \cong 2x\_$ of the desired zone MG_{Const}, then with an increase in $x\_$ and, accordingly, Δx the indicator E determined by (4) increases (Fig. 5). It is noteworthy that this growth is the same for all values of $I\omega$, reaching, in particular, the value of $E \cong 10\%$ at $x\_ = 5.4$ mm. The length of the desired zone is in this case $\Delta x \cong 2x\_ \cong 10.8$ mm (depicted in Fig. 5 by the additional abscissa axis Δx).

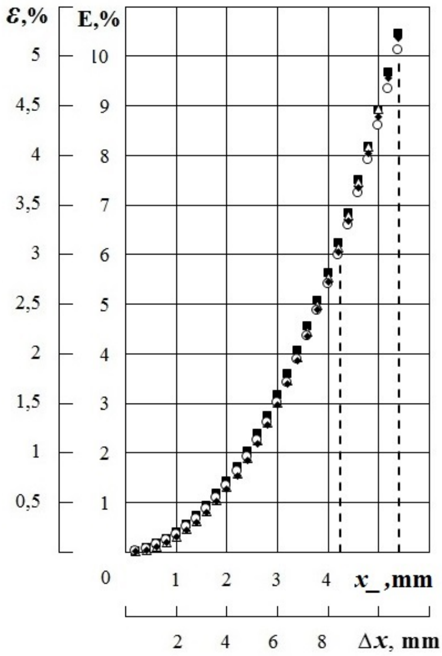

Fig.5. Obtained from the data of Fig. 2b and relations (4), (6) dependence of the indices E and ε on the step-by-step displacement $x\_$ in the half-zone MG_{Const} and the length $\Delta x \cong 2x\_$ of the zone; ♦ - $I\omega$ = 3000 A, ○ - 6000, Δ - 12000, ■ - 22500.

Now, using the data of Fig. 2b, it is possible, using the data of Fig. 2b, to fix the gradient-value at the border of this half-zone, i.e. $(gradB)_Z$ value. This allows to obtain the average gradient-value of the for the half-zone (and the zone as a whole) – as $\langle gradB \rangle = [(gradB)_Z + (gradB)_{extr}]/2$.

Then there are all reasons for evaluating the deviations of the current values of *gradB* with respect to the mean value of $\langle gradB \rangle$ (the values of *gradB* and $\langle gradB \rangle$ modulo). Thus, it is possible to move from the primary indicator E, due to which it is still possible to preliminarily quantify the extent of the MG_{Const} zone in the region close to the extremum, to the following indicator:

$$\varepsilon = (\langle gradB \rangle - gradB) / \langle gradB \rangle, \text{ x } 100\%. \quad (6)$$

It is almost equals $E/2$ and reflects the actual error of the assumption being made. This means that the real zone (which is nevertheless characterized by a certain decrease in the values of *gradB* in it as $x\_$ and Δx increase) can be considered equivalent to a zone with a constant value of the magnetic gradient, i.e. zone MG_{Const}.

According to the example (Fig. 2b), based on the data in Fig. 5, the above is rephrased. So, with an error of $\varepsilon \cong 5\%$ (Fig. 5 additionally has an ordinate axis ε), the MG_{Const} zone is a zone with a length of $\Delta x \cong 2x\_ \cong 10.8$ mm ($\Delta x/D = 0.108$), deployed in the vicinity of the extremum, which occurs here as already mentioned above, to the coordinate $x_{extr} \cong 16$ mm ($x_{extr}/D = 0.16$).

If the condition for the MG_{Const} zone is tighten to such a level that the gradient changes in the vicinity of its extremum do not exceed, for example, the values of $\varepsilon \cong 3\%$ with respect to the average value of the gradient (or $E \cong 6\%$ – with respect to its maximum value), then the length of the MG_{Const} zone will decrease. Thus, from the data in Fig. 5 and Fig. 2b, when with the same value of x_{extr} = 16 mm (x_{extr}/D = 0.16), but with changed values of $(gradB)_Z$ and $\langle gradB \rangle$, this is a reduced value $x\_$ = 4.25 mm and, accordingly, a narrowed zone with a length of $\Delta x \cong 2x\_ \cong 8.5$ mm ($\Delta x / D = 0.085$).

Using the tested approach for families of dependencies obtained at different relative distances *b/D* [15] leads to the results shown in Fig. 6a. These results characterize the change in the relative length $\Delta x/D$ of the MG_{Const} zones (with a change in *b/D*) - despite the fact that within these zones the current values of the gradient differ from its average values by no more than $\varepsilon \cong 5\%$ (line 1) and a value not exceeding $\varepsilon \cong 3\%$ (line 2).

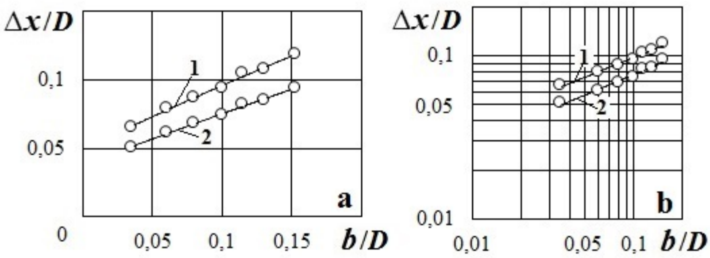

Fig.6. Influence of the relative distance *b/D* between the polar hemispheres on the relative length $\Delta x/D$ of the zone MG_{Const}: when the current data deviates from the average values up to 5% (line 1) and up to 3% (line 2); *a*) and *b*) in ordinary and logarithmic coordinates.

In the specified *b/D* range the data shown in Fig. 6a is fairly quasi-linearized in logarithmic coordinates (Fig. 6b). Thus, it becomes possible to approximate them by a phenomenological dependence of a power-law type and with an exponent at the argument *b/D* close to 0.4 – both for the restrictive condition $\varepsilon \cong 5\%$ and for the restrictive condition $\varepsilon \cong 3\%$. The corresponding calculation expression for calculating the relative length Δx/D of the zone MG_{Const} for one or another value of the relative distance *b/D* between the polar hemispheres looks like this:

$$\frac{\Delta x}{D} = T_G \left(\frac{b}{D} \right)^{0,4}, \qquad (7)$$

where the phenomenological parameter in calculations according to (7) is: T_G = 0.24…0.25 and T_G = 0.19…0.2 – under restrictive conditions, respectively, $\varepsilon \cong 5\%$ and $\varepsilon \cong 3\%$.

As for the quantitative estimate of the length of another zone of interest, namely the MFF_{Const} zone, this estimate is in many respects similar to the estimate of the length of the MG_{Const} zone described above. For demonstration it is convenient to use a similar illustrative example, i.e. a family of dependencies of the force factor *BgradB* on *x* with extremum (Fig. 2c). As well as the family of dependencies of the gradient *gradB* on *x* (Fig. 2b), this family has a common coordinate of the extremum x_{extr}, moreover, regardless of the magnetizing force $I\omega$ of the winding of the magnetic system and the value of the force factor at the extremum, i.e. $(BgradB)_{extr}$, depends on $I\omega$.

Following the recommendation to start from the extremum (Fig. 2c) and increase the step-by-step displacement of $x\_$ to obtain force factor data when calculating according to (5), an attempt is made to increase the length $\Delta x \cong 2x\_$ of the desired zone MFF_{Const} to an acceptable extent. The indicator *E* with an increase in *x_* and, accordingly, Δ*x* increases (Fig. 7): it is also the same for all values of *Iω*. It reaches a value, in particular $E \cong 10\%$ at $x\_ = 4.05$ mm. Therefore, in this case, the length of the proposed zone MFF_{Const} according to the correlation $\Delta x \cong 2x\_$ (in addition, there is an abscissa axis Δ*x* in Fig. 7) is $\Delta x \cong 8.1$ mm, i.e. is less than the value of Δ*x* for a zone similar in *E* to MG_{Const} (Fig. 5).

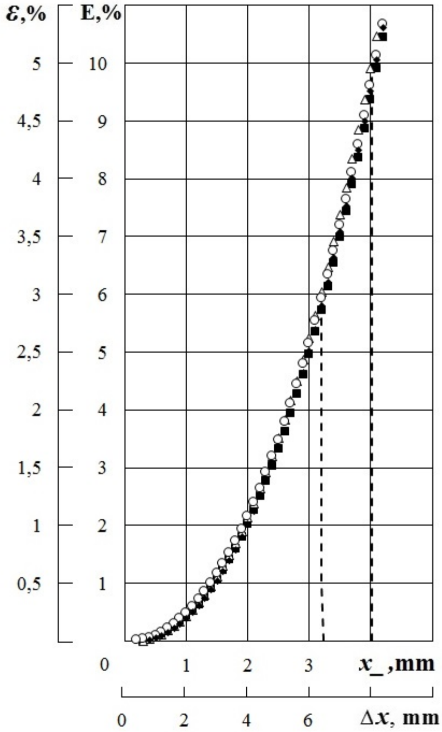

Fig.7. Obtained from the data in Fig. 2c and relations (5), (8) dependence of the indices *E* and *ε* on the step-by-step displacement *x_* in the MFF_{Const} half zone and the length $\Delta x \cong 2x\_$ of the zone; ♦ – Iω = 3000 A, ○ – 6000, Δ – 12000, ■ – 22500.

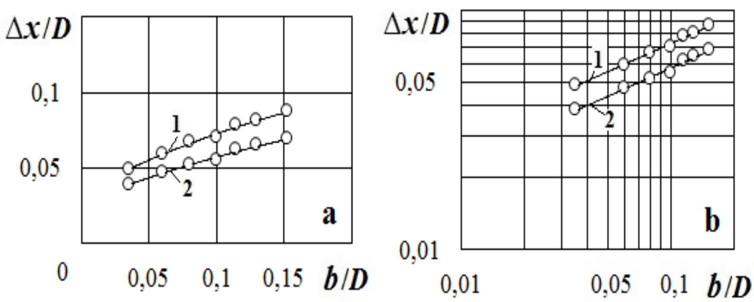

Fig.8. The same as in Fig.6, but for the MFF_{Const} zone.

Further, proceeding from the length of the half-zone *x_* (Fig. 7) established (by such a value of *E*), information about the value of the force factor at its boundary becomes available, i.e. $(BgradB)_Z$, which makes it possible to find the average value of the force factor for the half-zone (and the desired zone as a whole) as $\langle BgradB \rangle$ = [$(BgradB)_Z$ + $(BgradB)_{extr}$] / 2.

This means that the deviations of the current values of *BgradB* can now be estimated with respect to the average value $\langle BgradB \rangle$ (the values of *BgradB* and $\langle BgradB \rangle$ – modulo). Moving on from the indicator *E* (which played an important role in the initial quantitative criterion for the length of the zone MFF_{Const}) to an indicator that reflects the actual error of the assumption being made:

$$\varepsilon = [\langle BgradB \rangle - BgradB] / \langle BgradB \rangle, \times 100\%. \qquad (8)$$

Then the following refined estimate, according to the indicator $\varepsilon \cong E/2$, is: with an error $\varepsilon \cong E/2 \cong 5\%$ (in addition, there is an ordinate axis ε in Fig. 7), the MFF_{Const} zone, judging by the data in Fig. 2c, is a zone with a length of $\Delta x \cong 2x\_ \cong 8.1$ mm ($\Delta x/D = 0.081$).
The condition for the MFF_{Const} zone can be tightened, for example, by limiting the changes in the force factor in the vicinity of its extremum to a value of up to $\varepsilon \cong 3\%$ (in relation to the average value of the force factor or $E \cong 6\%$ in relation to its maximum value). Then, using the data in Fig.7 and Fig.2b, with the same x_{extr} value, but changed values of $(BgradB)_Z$ and $\langle BgradB \rangle$, the reduced length of the half-zone MFF_{Const} will be: $x\_ = 3.23$ mm and the corresponding (reduced) length of the zone: $\Delta x \cong 2x\_ \cong 6.46$ mm (and $\Delta x/D = 0.065$).
Such an approach to estimating the length Δx of the MFF_{Const} zone was also implemented for related families of dependencies obtained in [15, 21] for various *b*. Figure 8a shows a pair of established dependencies $\Delta x/D$ on b/D – for two restrictive conditions: when, within the MFF_{Const} zones, the current values of the force factor should not differ from its average values by no more than $\varepsilon \cong 5\%$ (line 1) and not more than $\varepsilon \cong 3\%$ (line 2).
The data shown in Fig. 8a, presented in logarithmic coordinates, is amenable to quasi-linearization in the specified range of b/D, which means that they are approximated by a phenomenological dependence of a power-law form. The exponent at the argument b/D turned out to be, as before when obtaining the phenomenological expression (7), close to 0.4, both for the restrictive condition $\varepsilon \cong 5\%$ and for the restrictive condition $\varepsilon \cong 3\%$. This allows to express here the correlation between $\Delta x/D$ and b/D as similar to (7):

$$\frac{\Delta x}{D} = T_F \left(\frac{b}{D} \right)^{0,4}, \qquad (9)$$

where the phenomenological parameter in calculations according to (9) is: $T_F = 0.18…0.19$ and $T_F = 0.14…0.15$ – under restrictive conditions, respectively, $\varepsilon \cong 5\%$ and $\varepsilon \cong 3\%$.
The mutual similarity of expressions (9) and (7), which can be used to determine the relative length $\Delta x/D$ of the MG_{Const} zone and the MFF_{Const} zone is indicative: the difference is only in the values of the multiplier parameters T_G and T_F. Hence, in addition to the important information received by the user about the length ($\Delta x/D$) of each of the desired zones, namely the MG_{Const} and MFF_{Const} zones, in accordance with (7) and (9), i.e. depending on the specific value of the relative distance b/D between the polar hemispheres, the following is just as important. At least in the indicated range $b/D = 0.035…0.153$, the mutual ratio of the data on the length of these zones remains practically unchanged, as the ratio of the corresponding values of the multiplier parameters T_G and T_F: it is always the same (average is 1.33).This value also correlates with the value of the mutual dislocation of the conditional centers of the zones MG_{Const} and MFF_{Const} established above, remarkably.

IV. Conclusion

The issue of creating such an inhomogeneous magnetic field, in which there would be zones with specific characteristics, namely, an almost constant value of the magnetic gradient (MG) or magnetic force factor (MFF), which would be confirmed by the appropriate diagnostics of such zones, is usually regarded as complex.

An original solution is indicated, which consists in using a well-proven electromagnetic system with spherical poles for this. In this case, the coordinate dependencies of MG and MFF (in the radial direction of the transverse plane of symmetry of the interpolar region) are extreme. This makes it possible to qualify the zones in the vicinity of extrema as zones of practically constant (within the limits of the defined length) values: MG_{Const} and MFF_{Const} - with individual coordinates $x = x_{extr}$ of their conditional centers and length Δx.

Using the example of an electromagnetic system with hemispherical poles with a diameter of D = 100mm located at one or another mutual distance b (from 3.5 to 15.3 mm), information is given about the dislocation of these specific zones - as values of x_{extr}/D and $\Delta x/D$ dependent on b/D (in the range of 3000 ...22500 ampere turns of the magnetizing force of the winding of the system). It is shown that the x_{extr}/D dependencies on b/D obtained separately for the MG_{Const} and MFF_{Const} zones, demonstrating mutual similarity (up to a constant multiplier coefficient), obey a power-law type with an exponent of 0.5. The dependencies of $\Delta x/D$ on b/D, also demonstrating mutual similarity (up to a constant multiplier coefficient), obey a power-law type with an exponent of 0.4. The data obtained and the dependencies describing them indicate a closer (to the intercenter line of the polar hemispheres) and more compact (in length) dislocation of the MFF_{Const} zones compared to the MG_{Const} zones: almost 1.3 times - both for x_{extr}/D and $\Delta x/D$.

V. Acknowledgements

This research was supported by the Ministry of Science and Higher Education of the Russian Federation, project 0706-2020-0024.

References

[1] A.A. Sandulyak, D.A. Sandulyak, M.N. Polismakova, A.V. Sandulyak, D.O. Kiselev, V.A. Ershova, "Analysis of Concentration Dependences of Magnetic Susceptibilities of Disperse Magnetite-Containing Media", Journal of Engineering Physics and Thermophysics, vol. 90 (4), pp. 845–850, 2017.

[2] A.V. Sandulyak, Magnetic filtration cleaning of liquids and gases. Khimiya, Moscow, 1988. (in Russian). https://dlib.rsl.ru/viewer/01001440011#?page=136

[3] R. Bjork, Z. Zhou, "The demagnetization factor for randomly packed spheroidal particles", Journal of Magnetism and Magnetic Materials, vol. 476, pp. 417–422, 2019.

[4] M. Anhalt, "Systematic investigation of particle size dependence of magnetic properties in soft magnetic composites", Journal of Magnetism and Magnetic Materials, vol. 320, e366–e369, 2008.

[5] K. Bai, J. Casara, A. Nair-Kanneganti, A. Wahl, F. Carle, E. Brown, "Effective magnetic susceptibility of suspensions of ferromagnetic particles", Journal of Applied Physics, vol. 124, 123901, 2018.

[6] A.N. Yurasov, M.M. Yashin, I.V. Gladyshev, et al., "Influence of size effects and granule

distribution by size on optical and magneto-optical properties of nanocomposites", Russian Technological Journal, vol. 3, pp. 49–57, 2021 (in Russian).

[7] Z. Birčáková, P. Kollár, J. Füzer, R. Bureš, M. Fáberová, "Analytical expression for initial magnetization curve of Fe-based soft magnetic composite material", Journal of Magnetism and Magnetic Materials, vol. 423, pp. 140-144, 2017.

[8] R.L. Moore, "Development of a volume fraction scaling function for demagnetization factors in effective media theories of magnetic composites", AIP Advances, vol. 9, 035107, 2019.

[9] B.G. Hosu, K. Jakab, P. Bánki, et al., "Magnetic tweezers for intracellular applications", Review of Scientific Instruments, vol. 74 (9), pp. 4158–4163, 2003.

[10] Kazin P.E., Kulbakin I.V., Methods of researching of material's magnetic properties, Moscow: MGU, 2011. 34 p. (in Russian).

[11] J.A. Cape, R.A. Young, Canted Helmholtz coils for constant-gradient Faraday balance magnetometry, Review of Scientific Instruments, vol. 42 (7), pp. 1061–1063, 1971.

[12] R.K. Quinn, R.C. Knauer, "Low temperature Faraday susceptibility apparatus", Rev Sci Instrum. 43 (10) (1972) 1543–1544.

[13] L. Petersson, A. Ehrenberg, "Highly sensitive Faraday balance for magnetic susceptibility studies of dilute protein solutions", Review of Scientific Instruments, vol. 56 (4), pp. 575–580, 1985.

[14] P. Marcon, K. Ostanina, "Overview of methods for magnetic susceptibility measurement", PIERS Proceedings, Malaysia, Kuala Lumpur. March 27-30, 2012, pp. 420–424.

[15] A.A. Sandulyak, A.V. Sandulyak, M.N. Polismakova, D.O. Kiselev, D.A. Sandulyak, "An approach for choosing positioning of small volume sample at instantiation ponderomotive faraday method in determining its magnetic susceptibility", Russian Technological Journal, vol. 5(2), pp. 57-69, 2017 (in Russian) https://doi.org/10.32362/2500-316X-2017-5-2-57-69

[16] A.A. Sandulyak, A.V. Sandulyak, M.N. Polismakova, D.O. Kiselev, V.A. Ershova, D.A. Sandulyak, "The Use of Spherical Pole Pieces For Performing the Faraday Balance Method", Instruments and Experimental Techniques, vol. 61 (1), pp. 123–126, 2018.

[17] A.A. Sandulyak, A.V. Sandulyak, M. Polismakova, V. Ershova V., D. Sandulyak, D. Kiselev, "On the Issue of Choosing the Measuring Zones in a Faraday Balance When Studying Magnetic Susceptibility of Small Samples", Applied Physics, System Science and Computers (APSAC 2017). Lecture Notes in Electrical Engineering, vol. 428. Springer.

[18] A.V. Sandulyak, A.A. Sandulyak, M.N. Polismakova, D.O. Kiselev, D.A. Sandulyak, V.A. Ershova, "The working zone in the interpolar area of the Faraday balance: an approach to testing the magnetic force factor stability criterion", MATEC Web of Conferences, Tokyo, (ICMAA 2017), vol. 108, pp. 01007. DOI: 10.1051/matecconf/201710801007.

[19] A.A. Sandulyak, D.O. Kiselev, M.N. Polismakova, A.V. Sandulyak, D.A. Sandulyak, About "On the mutual agreement of the coordinates of the interpolar zones of stability of the gradient and the force factor for tips-hemispheres of different diameters". Collection of materials of the International scientific-practical conference "Actual directions of scientific research: development prospects". Chelyabinsk. 2017. p. 137–144. (in Russian).

[20] A.A. Sandulyak, D.O. Kiselev, A.V. Sandulyak, M.N. Polismakova, D.A. Sandulyak, "Faraday magnetometer with spheric pole pieces: 3D analysis of operating zones", Pribory, No. 10, pp. 4–8, 2017 (in Russian).

[21] A.A. Sandulyak, M.N. Polismakova, D.A. Sandulyak, A.V. Sandulyak, V.A. Ershova, "Diagnostics of the Working Zone of Gradient Stability in a Magnetometer with Particle Susceptibility Control (For Magnetic Separation Problems)", Glass and Ceramics, vol. 78(3), pp. 104–110 DOI 10.1007/s10717-021-00356-2