Algorithmic Foundations of Optimization using Finite Element Modeling of High-Speed Grinding Technology in Application to 3D Micro-Level Models

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Optimization consists in choosing the best of all possible options for implementing high-speed grinding technology. A complete enumeration of all options may turn out to be inefficient or practically impossible. Therefore, to solve such a problem, we should apply fundamental mathematical results and numerical methods of optimization theory, which allow choosing the best option without directly checking all possible solutions. Such a choice is realized by means of calculations carried out using special algorithms and is practically impossible without the use of computer technology.

Keywords: 3D modelling, methodology, diamond wheel, high-speed grinding, finite element method, simulations, micro-scale, optimization.

1. Introduction

The results of dynamic 3D modeling of high-speed diamond grinding processes allow to solve the following problems: at the design stage - calculation of the tool design for certain processing modes; at the manufacturing stage - determination of rational conditions for sintering the diamond-bearing layer of the wheel; at the stage of application - the theoretical determination of processing productivity, specific consumption of diamond grains. The solution of these problems allows significantly increase the efficiency of processing during high-speed grinding. (Fig.1)

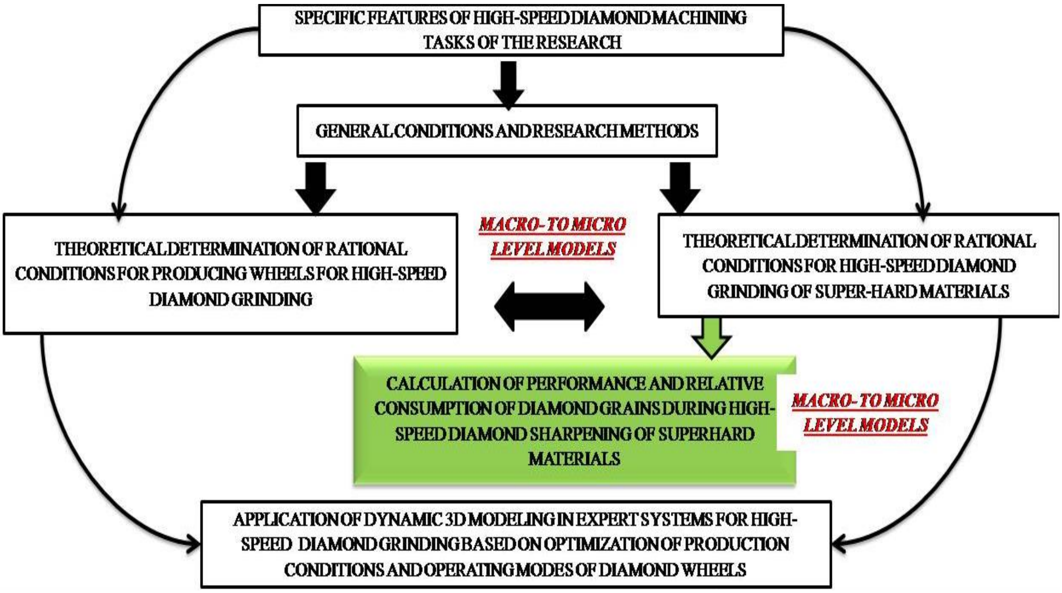


Fig. 1 Structural diagram of the study

However, the industrial implementation of these results requires constant problem solving, which requires the presence of a team of highly qualified experts capable of performing complex calculations and processing their results.[1] Given this circumstance, the industrial implementation of the results of dynamic 3D modeling of high-speed diamond grinding processes can be effectively carried out using expert systems that allow solving problem situations based on optimizing the manufacturing conditions and operating modes of diamond wheels.

The efficiency of using super hard materials (SHM) grinding wheels is determined, first of all, by the productivity of the process and the quality of parts processing. In our opinion, the decisive factor in improving the efficiency of diamond-abrasive tools is the use of scientifically based grinding regimes, which also make it possible to significantly increase processing productivity and reduce the specific consumption of diamonds in a wheel (this becomes especially important when sharpening tools from SHM).[1]

Since the processing of synthetic diamonds is accompanied by low productivity and high values of the specific consumption of diamond wheels, the utilization rate of diamond grains in these processes does not exceed 5-10%. The use of synthetic diamonds as a blade tool requires their sharpening and finishing. The use of high-speed grinding is especially important for the processing of superhard materials, since at significant processing speeds, not only productivity increases, but also the accuracy of tool sharpening. This is due to the high impact speeds of the grains on a comparable, or the same hardness, workpiece material and the formation of micro-edges on the cutting grains.[1]

2. Research methodology

To apply the methods of optimization theory, it is necessary to propose a correct formulation of the problem and choose the most appropriate method for its numerical solution. The correct formulation of the optimization problem [2] necessarily includes: determination of the boundaries of the system; the formulation of the characteristic criterion, the choice of independent variables, the construction of a model of the system. The choice of a method for solving an optimization problem is determined taking into account the peculiarities of its mathematical formulation.[3]

As applied to high-speed diamond grinding, the above scheme for solving optimization problems takes the following form. The boundary of the object under study separates this object from other objects interacting with it and allows you to separate the characteristics (object parameters into external and internal). When considering the problems of optimization of high-speed grinding, the boundary of the object under study is chosen by the content of the problem under consideration. The boundary of the object under study can be determined by the spatial area of the cutting tool when solving problems of choosing the optimal characteristics of diamond wheels at the stage of their design and manufacture. When solving problems of optimizing high-speed grinding processes using available cutting tools, taking into account the characteristics of the material being processed, the boundary of the object under study can be determined by the area covering the processing diamond grain with the material being processed around their contact interaction.

Based on the features of synthetic diamond as a tool material[1, 4], as well as the processes of shaping tools based on them, a methodological approach to research was determined based on the widespread use of 3D modeling of the processing process in conjunction with the experimental data obtained to assess the adequacy of the models. All elements of the studied systems (diamond grain, metal phase, binder, coating of diamond grains) and processing conditions are considered in interaction. Dynamic 3D modeling is used to determine the performance parameters and consider the processes of destruction of diamond grains and a binder during high-speed grinding.

To implement the tasks of studying deformation in the grinding zone, the calculation systems ANSYS, LS-DYNA, CosmosWorks, Abaqus were used. The construction of 3D models was used by the system of automation of design work (CAD) in three dimensions SolidWorks (Fig. 2)

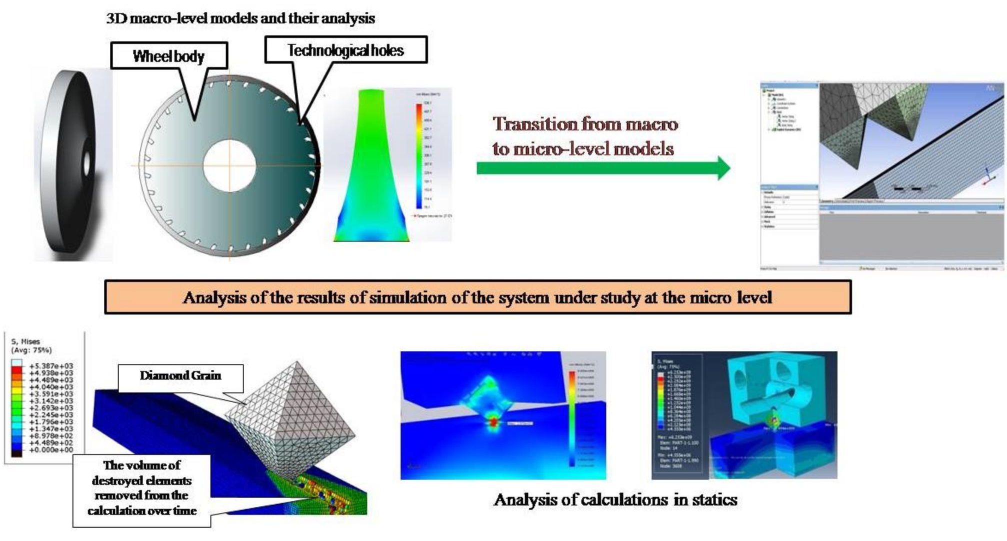
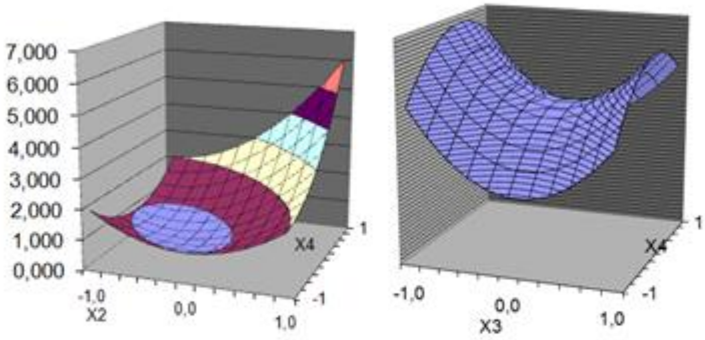


Fig. 2 Simulation of high-speed grinding process at macro and micro levels

Using the experiment planning technique [5, 6], preliminary results were obtained for the values of the characteristic function, which can be optimized (Fig. 3)



(a) (b)

Fig. 3 - Examples of constructing the characteristic function of the process of high-speed diamond grinding using the FEM: a - minimum at a stationary point, b - saddle stationary point

The tasks of optimizing the processes of SHM high-speed grinding shows that such a function (Fig. 3) can be quite accurately represented by approximation in the form of a quadratic function (1), which in the general case of n independent variables (factors) is written as follows[7, 8]:

$$Y = b_0 + \sum_{i=1}^{n} b_i X_i + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} b_{ij} X_i X_j + \sum_{i=1}^{n} b_{ii} X_i^2 \tag{1}$$

To solve the problem of finding the minimum and maximum of a function using a computer, it is advisable to represent the quadratic function (1) of independent variables n in matrix-vector form as follows[9]:

$$Y = \frac{1}{2}\{X\}^T [A]\{X\} + \{B\}^T \{X\} + B_0 \tag{2}$$

where $[A]$ - given matrix of numerical coefficients, $\{B\}$ - given vector of numerical coefficients, B_0 - specified numerical factor.

Matrix $[A]$, vector $\{B\}$ and numerical factors B_0 of matrix-vector form (2) corresponding to the quadratic function (1) are written as follows[10]:

$$[A]=\begin{bmatrix}2b_{11} & b_{12} & b_{13} & \cdots & b_{1n}\\ b_{12} & 2b_{22} & b_{23} & \cdots & b_{2n}\\ b_{13} & b_{23} & 2b_{33} & \cdots & b_{3n}\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ b_{1n} & b_{2n} & b_{3n} & \cdots & 2b_{nn}\end{bmatrix},\{B\}=\begin{Bmatrix}b_1\\ b_2\\ b_3\\ \vdots\\ b_n\end{Bmatrix}, B_0=b_0 \quad (3)$$

For the characteristic function (2), the stationary condition is written as follows:

$$[A]\{X\}=\{B\} \quad (4)$$

The minimum and maximum conditions for function (2) take the following form, respectively:

$$\{X\}[A][X]\geq 0, \quad (5)$$

$$\{X\}[A][X]\leq 0, \quad (6)$$

3. Maximizing the performance of high-speed SHM diamond grinding

The production process is primarily characterized by its productivity. The main goal when using high-speed grinding processes is to increase the productivity of the process, this is especially true when processing superhard materials, since the hardness of the workpiece is comparable or the same as the hardness of the machining tool.

In accordance with the initial data obtained from the results of 3D modeling (simulation analysis in Fig.2 and Fig. 3), the matrix [A] and the vector {B} from expression (3) are presented as:

$$[A]=\begin{bmatrix}-7,66 & -0,001 & 0,01 & -0,001\\ -0,001 & -6,5 & -0,02 & 0,06\\ 0,01 & -0,02 & -24,1 & 0,32\\ -0,001 & 0,06 & 0,32 & -19,04\end{bmatrix},\{B\}=\begin{Bmatrix}0,02\\ -0,07\\ -6,79\\ 0,51\end{Bmatrix}, B_0=55,91 \quad (7)$$

As a result of solving the system (7), a stationary point X_{stat} was obtained:

$$X_{stat}=\begin{Bmatrix}\text{-2.24194024298796E-0003}\\ \text{9.70029544277527E-0003}\\ \text{2.81441311184647E-0001}\\ \text{-2.20249223085138E-0002}\end{Bmatrix}, Y_{stat}=\text{5.3025585390789573E+001} \quad (8)$$

where X_{stat} - vector of coordinates of the stationary point of the function, Y_{stat} - the value of the characteristic function (1) at a stationary point.

The maximum value of the characteristic function at the boundary points is calculated and presented as:

$$X_{bound} = \begin{Bmatrix} 1.00000000000000E+0000 \\ 1.00000000000000E+0000 \\ -1.00000000000000E+0000 \\ 1.00000000000000E+0000 \end{Bmatrix}, Y_{\text{boumd}} = 3.42579999999999996E+001 \quad (9)$$

where X_{boumd} - vector of coordinates of the function point on the boundaries of the considered interval, Y_{boumd} - the value of the characteristic function (1) at the boundary point of the considered interval.

As can be seen from (8) and (9), the maximum value of the productivity of the SHM high-speed diamond grinding process is achieved at a stationary point almost in the middle of the considered interval of independent variables (factors). The maximum value of productivity is more than 1.5 times greater than its maximum value at the boundaries of the interval.

4. Accounting and risk assessment in the process of quantitative analysis and decision-making in expert systems

The results of modeling the processes of high-speed diamond grinding and sintering of diamond wheels contain some error, acquired as a result of the hypotheses and assumptions adopted in the simulation; due to the error of the initial data and their inconsistency with real parameters and characteristics; due to errors acquired when using numerical methods and other factors that are difficult to take into account. Given the above, the results shown in Figures 2 and 3 are approximations and include these uncertainties. Naturally, the use of these results is possible if the accumulated error is small. Accordingly, it is necessary to propose an approach to substantiate the error of the results obtained [11]. The main criterion may be verification with the results of specially designed laboratory experiments and field tests. Such experiments are associated with significant material costs, require high-tech equipment that ensures control measurements with a certain accuracy [12, 13], since various physical parameters are used in the optimality criterion, and those that cannot be measured directly (for example, strain). At the same time, it should be remembered that the results of field experiments and tests also have some error and their use is possible after evaluating such an error. It seems more practical, in our case, to apply the methods of mathematical statistics and consider the results obtained as one of the many possible implementations, i.e. estimate the probability that the result obtained has a predetermined error. In what follows, we will use this approach.

To do this, we use the basic scheme of statistical tests, which we use according to the scheme of the well-known sampling method.[14] In accordance with this method, we consider the object under study (in this case, the results of optimizing the processes of high-speed grinding and sintering of diamond wheels) in the form of a general population, which is a set of possible results corresponding to various initial data. Thus, the errors in the result of optimizing the processes of high-speed grinding and sintering of diamond wheels will be presented as a result of the error in the initial data [15].

The idea of the sampling method is that, having data on a finite (often very small) number N of elements of the general population, to evaluate the properties of the general population itself. These N elements about which information is known is called the sample, and the number N itself is called the sample size.[16]

Let us designate the possible optimal values of the characteristic function that form the sample as follows:

$$Y_1, Y_2, \dots Y_N \, , \tag{10}$$

where k=1,2…N – possible optimal values of the characteristic function (1)
With the values (9) we can approximately determine the mathematical expectation and variance of the sample:

$$\overline{Y} = \frac{1}{N}\sum_{k=1}^{N} Y_k \tag{11}$$

$$s^2 = \frac{1}{N^{-1}}\sum_{k=1}^{N}\left(Y_k - \overline{Y}\right)^2 \tag{12}$$

As is known from mathematical statistics [17], the true mean value of the general population with probability 1-p is determined by the inequality:

$$\overline{Y} - \frac{s}{\sqrt{N}} t^N_{1-p/2} \le Y \le \overline{Y} + \frac{s}{\sqrt{N}} t^N_{1-p/2} \tag{13}$$

where p – significance level, $t^N_{1-p/2}$ - quantile of the Student's distribution corresponding to N-1 degrees of freedom and probability 1-p/2 [17], which for the corresponding initial data can be found from special mathematical (statistical) tables.
As the double inequality (13) shows, in order to estimate the average value of the general population, it is necessary to know information about two of its elements. Thus, the estimation of the error in determining the optimal parameters of the characteristic function (indicators of high-speed grinding and sintering of diamond wheels) is reduced to determining a sample of its possible values.
To construct a sample of values (12), we use the Monte Carlo method, to which we take into account the uncertainty of information about the sources of errors. Taking as sources of errors the assumptions adopted in mathematical models, numerical methods (including the FEM), which were used to approximately determine the characteristic function (1) in the form of a quadratic function (2).
Let us consider the influence of possible errors in the calculation of the diagonal elements of the matrix [A] from (2) on the optimal value of the characteristic function.
In accordance with the Monte Carlo method, it is necessary to use a generator of random variables and determine the set of possible random values of the diagonal elements of the matrix [A] from (2) in their various combinations. As is known, a random variable generator can be constructed by functional transformation of a random variable distributed according to a uniform law. Taking into account this circumstance, we estimate the error in calculating the diagonal elements of the matrix by the minimum possible and maximum possible values, which is sufficient to determine the uniformly distributed probability density function.
Considering that the number of diagonal elements of the matrix A is equal to n, we obtain as a result a selection of N=2n possible combinations of possible values that characterize the permissible values of the diagonal elements of the matrix A.
The Monte Carlo method for assessing the impact of the risk caused by the error in the simulation in the FEM of the processes of high-speed grinding and sintering of diamond wheels is implemented by repeatedly solving the problem of optimizing the characteristic function (1) describing the listed processes.

As an example, let us consider the estimation of the error of solution (8) of the problem of maximizing the productivity of SHM high-speed diamond grinding, considered in [1]. In accordance with the above approach, we assume that the diagonal coefficients of matrix (3), which determines the characteristic function, have a relative error of ±20%.
Considering that n=4 we get N=24=16. We accept the significance level p=0.05, which corresponds to a probability of 95%. As is known [16, 17], the quantile of Student's distribution corresponding to this case with 15 degrees of freedom and significance level p=0,05, $t^N_{1-p/2}$ =2,13
Let us consider as an example of maximizing the characteristic function of high-speed diamond grinding performance (8) and, in accordance with the above approach, write the matrix [A] from (8) in the form:

$$[A]_{i,j,k,l} = \begin{bmatrix} a_{11}(i) & -0,001 & 0,01 & -0,001 \\ -0,001 & a_{22}(j) & -0,02 & 0,06 \\ 0,01 & -0,02 & a_{33}(k) & 0,32 \\ -0,001 & 0,06 & 0,32 & a_{44}(l) \end{bmatrix}, \{B\} = \begin{Bmatrix} 0,02 \\ -0,07 \\ -6,79 \\ 0,51 \end{Bmatrix}, B_0 = 55,91 \qquad (14)$$

where

$$a_{11}(i) = -7,66(0,8+0,4(i-1)), i = 1,2$$
$$a_{22}(j) = -6,5(0,8+0,4(j-1)), j = 1,2$$
$$a_{33}(k) = -24,1(0,8+0,4(k-1)), k = 1,2$$
$$a_{44}(l) = -6,5(0,8+0,4(l-1)), l = 1,2 \qquad (15)$$

Let’s consider a general cycle that implements the enumeration of matrix values for the implementation of the Monte Carlo algorithm for solving risk assessment problems when studying the processes of high-speed grinding and sintering of diamond wheels to develop an expert system for choosing the optimal characteristics of diamond wheels and processing modes (Fig. 4).

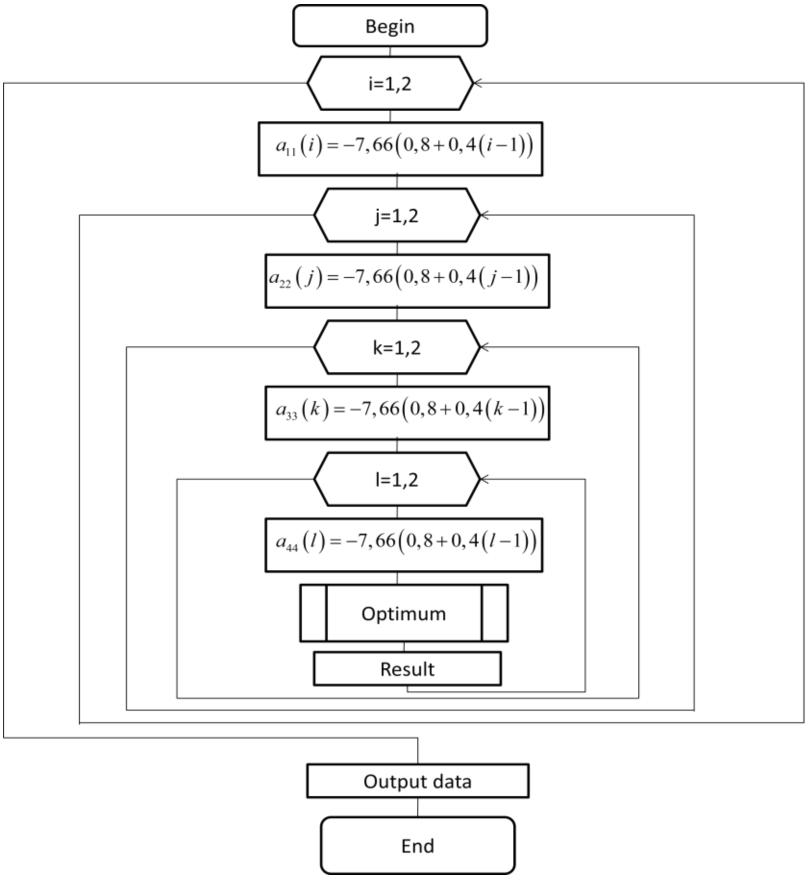


Fig. 4 - The general cycle that implements the enumeration of matrix values for the implementation of the Monte Carlo algorithm for solving problems of risk assessment when studying the processes of high-speed grinding and sintering of diamond wheels

The calculation results are shown in table 1.

Table 1 Calculation results according to the algorithm shown in Figure 4

s	i	j	k	l	Calculated Values Ys
1	1	1	1	1	5.23063588881460E+0001
2	1	1	1	2	5.23115249045368E+0001
3	1	1	2	1	5.34988382129410E+0001
4	1	1	2	2	5.35050272651754E+0001
5	1	2	1	1	5.23067214406208E+0001
6	1	2	1	2	5.23118936036018E+0001
7	1	2	2	1	5.34992271840527E+0001
8	1	2	2	2	5.35054232186763E+0001
9	2	1	1	1	5.23063810168473E+0001
10	2	1	1	2	5.23115470529779E+0001
11	2	1	2	1	5.34988635995559E+0001
12	2	1	2	2	5.35050526764440E+0001
13	2	2	1	1	5.23067435586006E+0001

14	2	2	1	2	5.23119157412378E+0001
15	2	2	2	1	5.34992525588109E+0001
16	2	2	2	2	5.35054486179902E+0001

Based on the results of the calculation, we obtain the mathematical expectation and variance:

$$\bar{Y} = 5.2905638721263479\text{E} + 001 \quad (16)$$

$$s^2 = 3.7954552900854943\text{E} - 001 \quad (17)$$

Using the values (16) and (17) according to formula (13), with a probability of 0.95, we obtain that the optimal (maximum) value will satisfy the inequality:

$$5.2577580026957214\text{E+}001 \le Y \le 5.3233697415569743\text{E+}001 \quad (18)$$

It is interesting that the value (8) obtained earlier does not coincide with the mathematical expectation (16), however, it is within the interval (18), i.e. it is obtained with a probability of 95%. If we assume that the distribution density of the optimal (maximum) value Y has a normal law, then, having the mathematical expectation (16) and variance (17), we can construct this distribution density and it will have the form, as shown in Fig. 5.

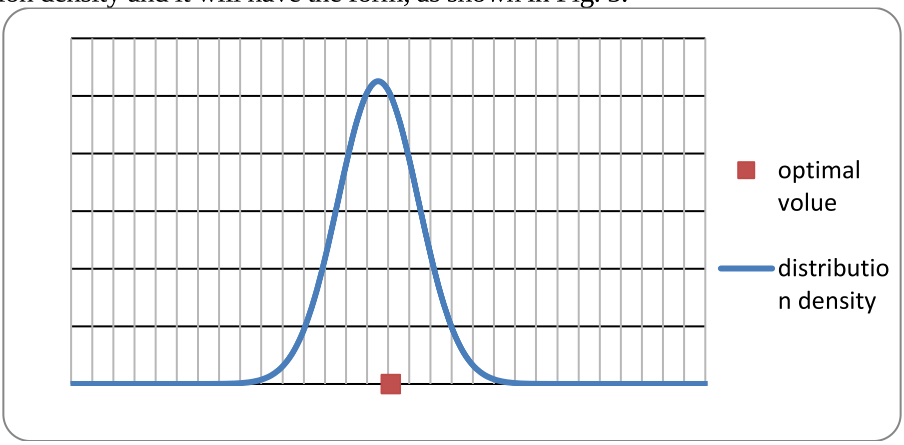


Fig. 5 - Density of distribution of the obtained optimization results and the optimal value for an example of optimizing the performance of high-speed diamond grinding

When developing an expert system, this algorithm is supposed to be used to estimate the probability for all characteristic functions that approximately describe the processes of STM ultra-high-speed grinding and diamond wheel sintering. Although the probability value itself may have some error, it shows at least the stability of the selected mode of operation of the diamond wheel, i.e. the ability to provide parameters close to optimal values in case of deviations in the system.

5. Conclusion

Fundamental mathematical approaches to optimization issues applicable to the study of the process of high-speed diamond processing of SHM are considered.

It is shown that in order to estimate the error of the results, it seems to apply the methods of mathematical statistics and consider the results obtained as one of the many possible implementations, i.e. estimate the probability that the result obtained has a predetermined error.

Special software has been developed that combines all the simulation results into an expert system that allows, with a certain probability and error, to predict the main indicators of the grinding process, depending on the various properties of the tool and processing modes.

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