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AN INTRODUCTION TO FERMATEAN FUZZY GRAPH STRUCTURE AND ITS APPLICATIONS IN DECISION MAKING

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Abstract: Fuzzy graphs find numerous applications in real life. Fermatean Fuzzy Sets (FFS) is on extension of the Ortho pair fuzzy sets which can be able to carried out uncertain evaluation more actively in decision – making environment one of the extensions of fuzzy graph is fermatean fuzzy graph. The concept of fermatean fuzzy structures is introduced and investigate in this article. Some interesting operations, cartesian product, union, composition, lexicographic product of fermatean fuzzy structures are defined with suitable examples. Finally, decision making applications has to be explained using fermatean fuzzy graph structures.

Keywords: Fuzzy Graph, Fuzzy Set, Fermatean Fuzzy Sets, Pythagoras Fuzzy Sets,
Cartesian Product, Union, Composition, Lexicographic Product, Decision Making.

AMS Subject Classification. 03E72, 03E75, 54A05, 03F55

1. Introduction: The father of graph theory was the great Swiss mathematician Leonhard Euler who's famous 1736 paper. "The seven bridges of Konigberg "was the first treatise on the subject. The fuzzy graph theory as a generalization of Euler's graph theory was first introduced by Rosenfeld in 1975. the fuzzy relations between fuzzy sets was first considered by Rosenfeld and he developed the body of fuzzy graphs obtaining analogs of several graph theoretical concepts fermatean fuzzy environment is the modern tool. For handling uncertainty in many decision-making problems. Atanassov [1] established the fuzzy sets to intuitionistic fuzzy sets which incorporated the degree of hesitation called hesitation margin $\pi = 1 - \alpha - \beta$ with membership values α and a non- membership values β. Shannon and Atanassov [10] proposed the concept of intuitionistic fuzzy graph by considering intuitionistic fuzzy relations on IFSS. To overcome this situations Yager developed a new type of fuzzy sets called the Pythagoras fuzzy sets which satisfies the constraint that the sum of the squares of membership grades and the non-membership grades is Yuval to less than one but they still have obvious shortcomings in which Pythagorean fuzzy set and intuitionistic fuzzy set who failed to over a situation in which the sum of the membership grade and non – membership grade is greater than 1 and sum of the squares is still greater than 1, in order to solve this situation Senapati and yager [11] developed a modern idea called fermatean fuzzy set and has the constraints as the sum of the third power of the membership grades and the non- membership grades is less than 1. Senapati and Yager[11] define basic operations over fermatean fuzzy sets in 2019. Yager proposed a q – rung Ortho pair fuzzy sets, as a new conception of Ortho pair fuzzy sets in 2017 . In this article, Fuzzy graphs find numerous applications in real life. fermatean fuzzy sets (FFS) is on extension of the Ortho pair fuzzy sets which can be able to carried out uncertain evaluation more actively in decision – making environment one of the extensions of fuzzy graph is fermatean fuzzy graph. the concept of fermatean fuzzy structures is introduced and investigate in this article. Some interesting operations, cartesian product, union, composition, lexicographic product of fermatean fuzzy structures are defined with suitable examples. Finally, decision making applications has to be explained using fermatean fuzzy graph structures.

2.Preliminaries

Definition 2.1 [L.A. Zadah]:A Fuzzy set (class) A in X is characterized by a membership (characteristics) function $\alpha_A(x)$ which associates with each point in X a real number in the interval [0,1], with the value of $\alpha_A(x)$ at x representing the grade of membership of x in A.

Example 2.2: let X = {a, b, c, d} be a given set. then the fuzzy set is

x	A	b	c	d
$\alpha_A(x)$	0.2	0.4	0.7	0.8

Definition -2.3 [K.T Atanassov]: Let X be a non-empty set. An intuitionistic fuzzy set A in X is an object having the form A = { (x, $\alpha_A(x)$,$\beta_A(x)$ / x $\in X$ } where the functions $\alpha_A(x)$,$\beta_A(x)$*:* $X \to [0,1]$ define respectively the degree of membership and degree of non-membership of the element x$\in X$ to the set A and for every element xϵX , 0$\leq$ $\alpha_A(x)$ +$\beta_A(x) \leq 1$.

Example 2.3: in the case of election for a public office a candidate can have vote for (membership), vote against (non- membership) and undecided (hesitancy) by the electoral.

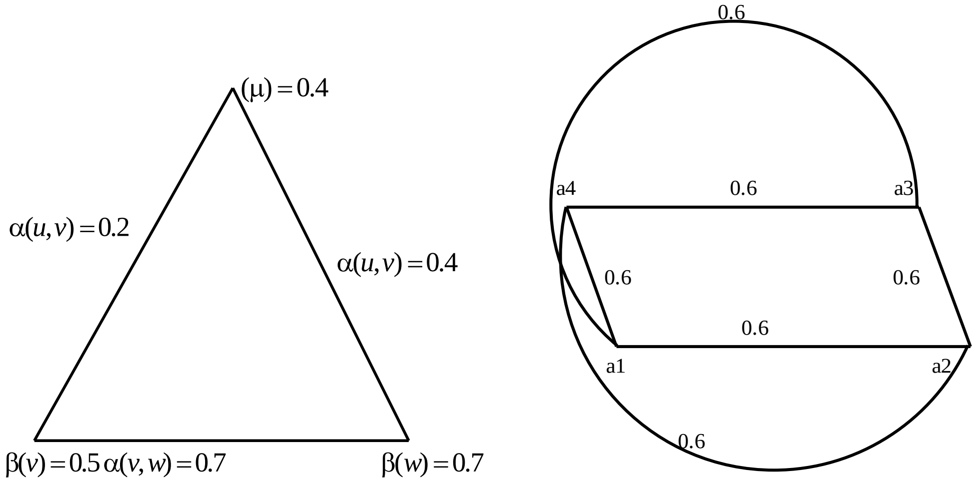
Definition 2.4: [R.R. Yager] :A Pythagoras fuzzy sets (PFS) A on a universe of discourse X is a structure having the form as A = {x, $\alpha_A(x)$,$\beta_A(x)$/ x $\in X$ } where $\alpha_A(x)$*:* $X \to [0,1]$indicates the degree of membership and $\beta_A(x)$ *:* $X \to [0,1]$indicates the degree of non-membership of every element x $\in X$ to the set A respectively, with the constraints:0$\leq$ $(\alpha_A(x))^2$ +$(\beta_A(x))^2 \leq 1$.

Definition 2.5 :[Senapati and Yager]:A fermatean fuzzy graph (FFG) on a universal set X is a pair G: (P, Q) where P is fermatean fuzzy set on X and Q is a fermatean fuzzy relation on X Such that; A fuzzy graph $G(\alpha,\beta)$ on $G(V,E)$ is a pair of functions $\alpha: V \to [0,1]$ and $\beta: V \times V \to [0,1]$ where for all $u,v,\in V$. We have

$\alpha(uv) \leq \min\{\beta(u),\beta(v)\}, \beta(u)=0.4, \beta(v)=0.5, \beta(w)=0.7,$

$\alpha(u,v)=0.2, \alpha(u,v)=0.4, \alpha(v,w)=0.3$

Example 2.6:



Fuzzy Graph

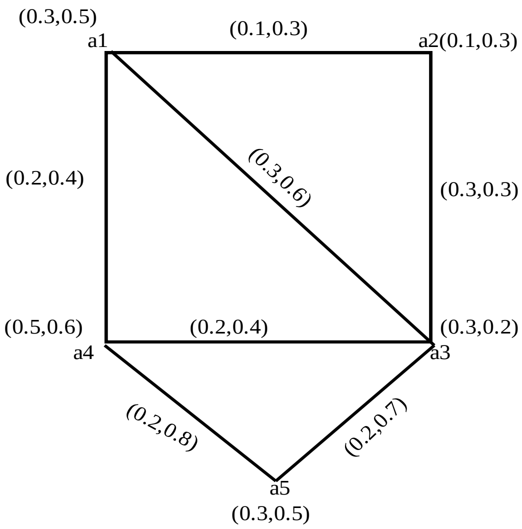
Definition 2.7: An intuitionistic fuzzy graph is defined as $G=(V,E,\alpha,B)$, where
(i) $V=\{v_1,v_2...v_n\}$ be a non-empty set such that $\alpha_1:V\to[0,1]$ and $\beta_1:V\to[0,1]$ denote the degree of membership and non-membership of the element $\beta,\in V$ respectively and $O\le\alpha_1(v_1)+\beta,(v_1)\le 1$ for every $V_i\in V, i=1,2...n$
(ii)E $\subset$ V x V where $\alpha_2:$ V x V $\to[0,1]$ and $\beta_2:V\times V\to[0,\ l]$ are such that
$\alpha_2(v_iv_j)\le\min\{\alpha_1(v_i),(\alpha_1(v_j)\}$
$\beta_2(v_i,v_j)\le\max\{\beta_2(V_i),\beta_2(v_j)\}$ and $O\le\alpha_2(v_i,v_j)+\beta_2(v_iv_j)\le 1,$
$O\le\alpha_2(v_iv_j),\beta_2(v_iv_j),\pi(v_iv_j)\le 1$ where $\pi(v_iv_j)=1-\alpha_2(v_i,v_j)-\beta_2(v_iv_j)$ for every $(v_i,v_j)\in E, E=1,2,...n.$

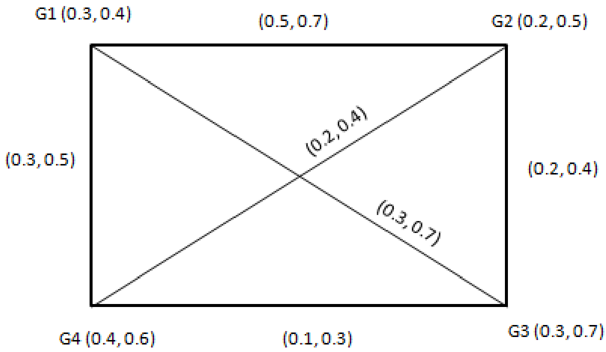
Definition 2.8: A Pythagorean fuzzy graph on a Universal set X is a pair G=(P,Q) where 'p' is Pythagorean fuzzy set on X and Q is a Pythagorean fuzzy relation on X such that $\alpha_Q^2(u,v)\le\min\{\alpha_P^2(u),\alpha_P^2(v)\}$ and $\beta_Q^2(u,v)\ge\max\{\beta_P^2(u),\beta_P^2(v)\}$ and $O\le\alpha_Q^2(u,v)+\beta_Q^2(u,v)\le 1$ for all $u,V\in X$ where $\alpha_Q:XXX\to[0,1]$ indicates degree of membership and degree of non-membership of Q, correspondingly.

Definition 2.9: A fermatean fuzzy graph (FFG) on a Universal set X is pair G: (P,Q) where P is fermatean fuzzy set on X and Q is a fermatean fuzzy relation on X such that,

$$\begin{cases}\alpha_Q^3(u,v)\le\min\{\alpha_p^3(u),\alpha_p^3(v)\}\\ \beta_Q^3(u,v)\ge\max\{\beta_P^3(u),\beta_P^3(v)\}\end{cases}$$ and $O\le\alpha_Q^3(u,v)+\beta_Q^3(u,v)\le 1$ for all $u,V\in X$,

where $\alpha_Q:$ X x X $\to[0,1]$, $\beta_Q:$ X x X $\to[0,1]$ indicates degree of membership and degree of non-membership of Q, corresponding. Here P is the fermatean fuzzy vertex set of G and Q is the fermatean fuzzy edge at of G.





Fermatean fuzzy graph G

Section-3 Fermatean fuzzy graph structures

Definition 3.1:Let $G^*=(U,E_1,E_{2,\ldots\ldots}E_n)$ be a graph structure (GS). Then $\tilde{G}_s=(A,B_1,B_{2,\ldots\ldots}B_n)$ is called an fermatean fuzzy graph structure (FFGS) of G with underlying vertex set U if the following conditions are satisfied;

(i) A is a fermatean fuzzy set on U with $\alpha_A : U \to (0,\ 1)$ and $B_A : U \to (0,\ l)$, the degree of membership and the degree of non-membership of $x \in U$, respectively, such that $O \le \alpha_A^3(x) + \beta_A^3(x) \le 1, Vx \in U$.

(ii) B_i is fermatean fuzzy set on E, for all i and the membership functions

$\alpha_{B_1} : E_1 \to (0,\ l)$ and $B_{B_1} : E_1 \to (0,1)$ are restricted by

$\alpha_{B_i}^3(xy) \le \min\left\{\alpha_A^3(x), \alpha_A^3(y)\right\}$

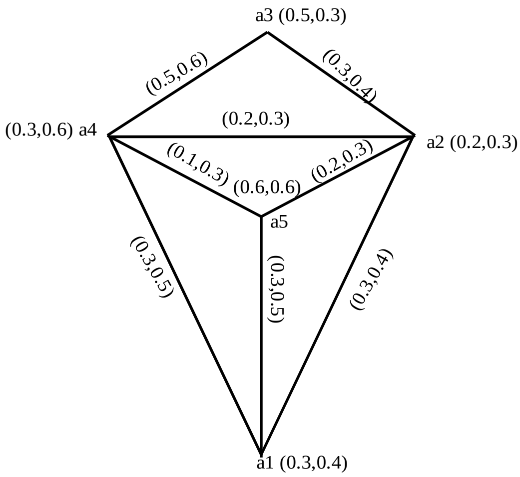
$\beta_{B_i}^3(xy) \ge \max\left\{\beta_A^3(x), \beta_A^3(y)\right\}$ such that $O \le \alpha_{B_i}^3(xy) + B_{B_i}^3(xy) \le 1$ for all $xy \in E_i CU \times U,\ i = 1, 2, \ldots..n$. Now we will illustrate the example of fermatean fuzzy structure.

Example 3.2: Let $G^*=(U, E_1,E_2)$ be a GS such that $U=\{a_1,a_2,a_3,a_4,a_5\}$, $E_1=\{a_1a_4, a_3a_5\}$and $E_2=\{a_1a_3,\ a_1a_2,\ a_2a_3,\ a_3a_4,\ a_2a_4,\ a_5a_4\}$. Let A, B, and B_2 be fermatean fuzzy subsets of U_1, E_1, and E_2 respectively, such that

$A=\{(a_1, 0.3, 0.4), (a_2, 0.2, 0.3), (a_3, 0.5, 0.3), (a_4, 0.3, 0.6), (a_5, 0.6, 0.6)\}$

$B_1=\{(a_1a_4, 0.3, 0.5), (a_2a_5, 0.2, 0.3)\}$and

$B_2=\{(a_1a_5, 0.3, 0.5), (a_1a_2, 0.3, 0.4), (a_2a_3, 0.3, 0.4), (a_3a_4, 0.5, 0.6), (a_2a_4, 0.2, 0.3), (a_5a_4, 0.1, 0.3)\}$. It is easy to see from simple calculations that $G_5=(A,B_1,B_2)$ is a fermatean fuzzy graph structure as shown in fig-1



Definition: 3.3: Let $\tilde{G}S_1$ and $\tilde{G}S_2$ be respective fermatean fuzzy structure of graph structure G_1^* and G_2^*. Then the calculation product of $\tilde{G}S_1$ and $\tilde{G}S_2$ is defined as $\tilde{G}S_1$ x $\tilde{G}S_2$ is

(i) $\alpha^3_{(A_1\times A_2)}(xy) = (\alpha^3_{A_1}\times\alpha^3_{A_2})(xy) = \min\left\{\alpha^3_{A_1}(x), \alpha^3_{A_2}(x)\right\}$

$\beta^3_{(A_1\times A_2)}(xy) = (\beta^3_{A_1}\times\beta^3_{A_2})(xy) = \max\left\{\beta^3_{A_1}(x), \beta^3_{A_2}(x)\right\}$

(ii) $\alpha^3_{(B_1\times B_2)}\left((xy_1)(xy_2)\right) = \left(\alpha^3_{B_{1i}}\times\alpha_{2i}\right)\left((xy_1)(xy_2)\right) = \min\{\alpha^3_{A_1}(x), \alpha^3_{B_2}(y_1y_2)\}$

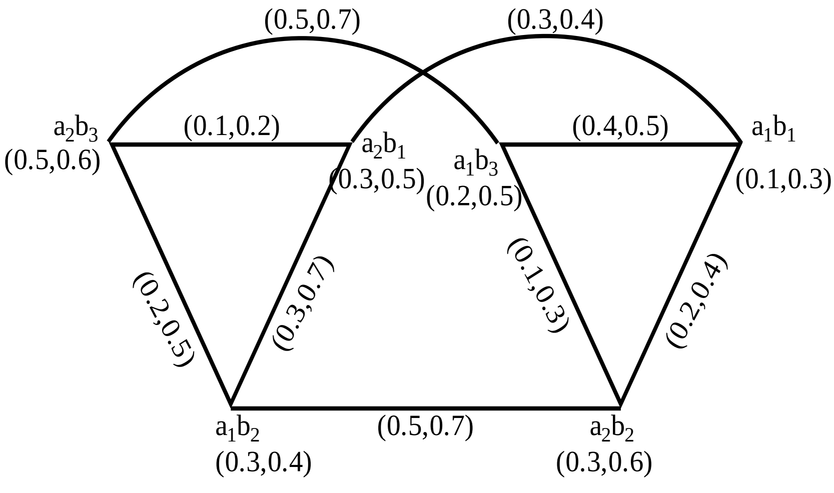
$\beta^3_{(B_{1i}\times B_{2i})}\left((xy_1)(xy_2)\right) = \left(\beta^3_{B1i}\times\beta^3_{B2i}\right)\left((xy_1)(xy_2)\right) = \max\left\{B^3_A(x), B^3_{B2}(y_1y_2)\right\}$

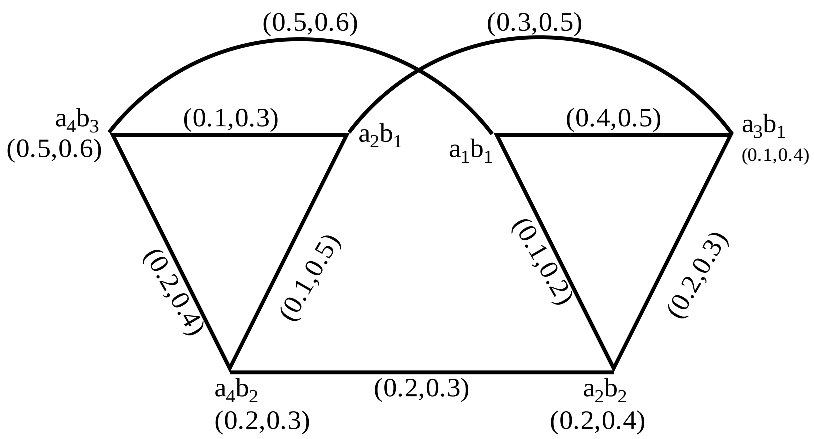
(iii) $\alpha^3_{B1i}\times B_{2i}\left((x_1y)(x_2y)\right) = (\alpha^3_{B1i}\times\alpha^3_{B2i})\left((x_1y)(x_2y)\right) = \min\left\{\alpha^3_{A_2}(y), \alpha^3_{B1i}(x_1y_2)\right\}$

$\beta^3_{B_{1i}}\times\beta_{2i}\left((x_1y)(x_2y)\right) = (\beta^3_{B_{1i}}\times\beta^3_{B_{2i}})\left((x_1y)(x_2y)\right) = \max\left\{B_{A_2}(y), B_{Bi}(x_1y_2)\right\}$

for all $x\in U_1, y\in U_2, x_1x_2\in E_{1i}$

Example 3.4: Let $\tilde{G}S_1$ and $\tilde{G}_2$ be respective fermatean fuzzy structure of graph structures $\tilde{G}S_1$ and $\tilde{G}_2$, where $U_1=\{a_1,a_2,a_3,a_4\}$.$U_2=\{b_1,b_2,b_3\}$,$E_{11}=\{a_1,a_2\}$, $E_{12}=\{a3,a4\}$, $E_{21}=\{b_1,b_2,b_1,b_3\}$,$E_{22}=\{b_2,b_3\}$, then $\tilde{G}S_1 \text{x} \tilde{G}_s$ is diagrammatically represented as





Cartesian product of FFGS

Proportion 3.5: Let G*, be the certain product of graph structure G_1^* and G_2^*. Let $\widetilde{GS}_1$ and $\widetilde{GS}_2$ be respective fermatean fuzzy graph structure of G_1^* and G_2^*. Then $\widetilde{GS}_1$ and $\widetilde{GS}_2$ is a fermatean fuzzy graph structure of G*.

Proof:_ It is clear that A_1xA_2 is fermatean fuzzy set of U_1xU_2 and B_{1i} x B_{2i} is a fermatean fuzzy set of E_{1i} x E_{2i}. First we have to show that

$B_{1i}xB_{2i}$ is formation fuzzy relation A_1xA_2. For this, we discuss some following cases.

Case(i): When $u \in U_1, b_1b_2 \in E_{2i}$

$$= \min\left\{\alpha^3_{A_1}(u), \alpha^3_{B_2}(b_1b_2)\right\}$$

$$\leq \min\left\{\alpha^3_{A_1}(u), \min\left\{\alpha^3_{B_2}(b_1), \alpha^3_{B_2}(b_2)\right\}\right\}$$

$$= \min\left\{\min\left\{\alpha^3_{A_1}(u), \alpha^3_{A_2}(b_1)\right\}, \min\left\{\alpha_{A_1}(u), \alpha^3_{A_2}(b_2)\right\}\right\}$$

$$= \min\left\{\alpha^3_{(A_1\times A_2)}(ub_1), \alpha^3_{(A_1\times A_2)}(ub_2)\right\}$$

$$\beta^3_A(\beta_{1i}\times\beta_{2i})\left((ub_1)(ub_2)\right)$$

$$= \max\left\{\beta^3_{A_1}(u), \beta^3_{B_2}(b_1b_2)\right\}$$

$$\geq \max\left\{\beta^3_{A_1}(u), \max\left\{\beta^3_{B_2}(b_1), \beta^3_{B_2}(b_2)\right\}\right\}$$

$$= \max\left\{\max\left\{\beta^3_{A_1}(u), \beta^3_{A_2}(u)\right\}, \min\left\{\beta^3_{A_1}(u), \beta^3_{A_2}(u)\right\}\right\}$$

$$= = \max\left\{\beta^3_{(A_1\times A_2)}(ub_1), \beta^3_{(A_1\times A_2)}(ub_2)\right\}$$

Case(ii): When $u \in U_2, b_1b_2 \in E_{1i}$

$$\alpha^3_{(B_{1i}\times B_{2i})} = \left(\left(b_1u\right)\left(b_2u\right)\right)$$

$$= \min\left\{\alpha^3_{A_2}(u), \alpha^3_{B_{1i}}\left(b_1b_2\right)\right\}$$

$$\leq \min\left\{\alpha^3_{A_2}(u), \min\left\{\alpha^3_{B_{1i}}(b1), \alpha^3_{B_{1i}}(b2)\right\}\right\}$$

$$= \min\left\{\min\left\{\alpha^3_{A_2}(u), \alpha^3_{A_1}(b_1)\right\}, \min\left\{\alpha^3_{A_2}(u), \alpha^3_{A_1}(b_2)\right\}\right\}$$

$$= \min\left\{\alpha^3_{(A_1\times A_2)}(b_1u), \alpha^3_{(A_1\times A_2)}(b_2u)\right\}$$

$$\alpha^3_{(B_{1i}\times B_{2i})} = \left(\left(b_1u\right)\left(b_2u\right)\right)$$

$$= \max\left\{\beta^3_{A_2}(u), \beta^3_{B_{1i}}\left(b_1b_2\right)\right\}$$

$$\geq \max\left\{\beta^3_{A_2}(u), \max\left\{\beta^3_{B_{1i}}(b_1), \beta^3_{B_{1i}}(b_1)\right\}, \max\left\{\beta^3_{A_2}(u), \beta^3_{A_1}(b_2)\right\}\right\}$$

$$= \max\left\{\beta^3_{(A_1\times A_2)}(b_1u), \beta^3_{(A_1\times A_2)}(b_2u)\right\}$$

For $b_1u, b_2u \in U_1 \times U_2$. both cases hold for i=1,2,......n. Hence, $B_{1i} \times B_{2i}$ is a fermatean fuzzy relation on A_1xA_2 and so, $\tilde{G}S_1 \mathrm{x} \tilde{G}S_2$ is fermatean fuzzy set of G.
Hence the proof.

Definition 3.6: Let $\tilde{G}S_1$ and $\tilde{G}S_2$ be two fermatean fuzzy graph structure of graph structures G_1* and G_2^* and let $U_1 \cap U_2 = \phi$. The union $\tilde{G}S_1 \cup \tilde{G}S_2$ =($A_1 \cup A_2$,B_{11} UB_{21},B_{12}UB_{1n}…..B_{2n}), Where $A_1 \cup A_2$ is defined by

$$\alpha^3_A(A_1 \cup A_2)(x) = (\alpha^3_{A1} \cup \alpha^3_{A2})(x) = \max\left\{\alpha^3_{A1}(x), \alpha^3_{A2}(x)\right\}$$

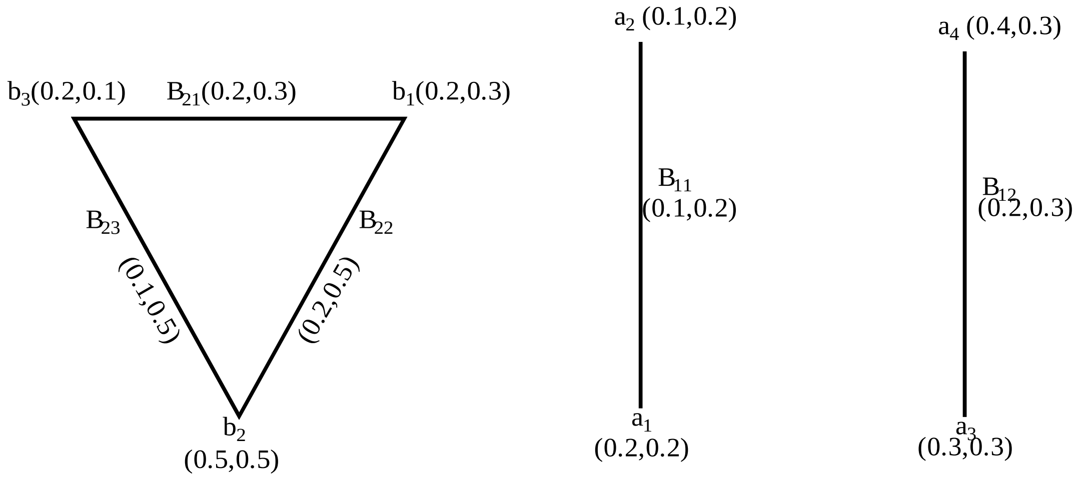
$$\beta^3_A(A_1 \cup A_2)(x) = (\beta^3_{A1} \cup \beta^3_{A2})(x) = \max\left\{\beta^3_{A1}(x), \beta^3_{A2}(x)\right\}$$

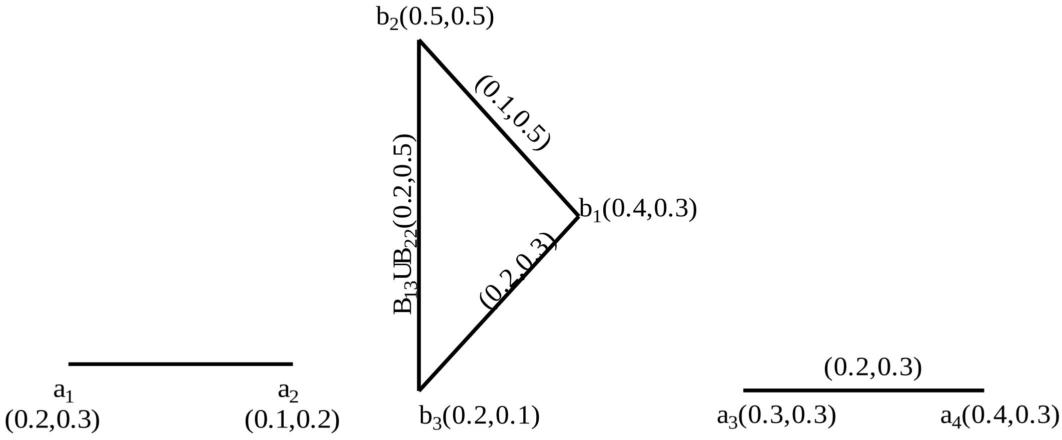
for all $x \in U_1 \cup U_2$ assuming $\alpha^3_{Aj}(x) = o, \beta^3_{Aj}(x) = 1{,}16 x \notin U_j \; j = 1,2,..$ And B_{1i} U B_{2i} for i=1,2......n defined by $\alpha^3(B_1 \cup B_2)(xy) = (\alpha^3_{B_{1i}} \cup \alpha^3_{B_2})(xy) = \max\left\{\alpha^3_{B_{1i}}(xy), \alpha^3_{B_{2i}}\right\}$

$$B_3(B_1 \cup B_2)(xy) = (B^3_{B_{1i}} \cup \alpha^3_{B_2})(xy) = \max\left\{B^3_{B_1}(xy), B^3_{B_2}(xy)\right\}$$

$xy \in E_{1i} \cup E_{2i}$ assuming $\alpha_{B_{ji}}(xy) = 0, B^3_{B_{ji}}(xy) = 1$ for $xy \notin E_{ji}$ j=1,2.

Example 3.7 : Let $\tilde{G}S_1$ and $\tilde{G}S_2$ be two fermatean fuzzy graph structures as shown below.





Proportion 3.8: Let G* be the Union of graph structure G_1^* and G_2^*. Let G_{S1} and G_{S2} be two fermatean fuzzy graph structures of G_1*and G_2^*. Then $\tilde{G}S_2 \cup \tilde{G}S_2$ is a fermatean fuzzy graph structure of G*.

Proof: We have to prove that $\tilde{G}S_1 \cup \tilde{G}S_2$ is fermatean fuzzy graph structures of G*. Clearly, $A_1 \cup A_2$ is fermatean fuzzy set of $U_1 \cup U_2$ and $B_1 \cup B_2$ is fermatean fuzzy set of $E_{1i} \cup E_{2i}$. We discuss the following cases.

Case(i): Suppose $u_1, u_2 \in U_1$, then by def (2)

$$\alpha^3_{A_2}(u_1) = \alpha^3_{A_2}(u_2) = \alpha^3_{D_2}(u_1, u_2) = 0$$

$$\beta^3_{A_2}(u_1) = 1, \beta^3_{A_2}(u_2) = \beta^3_{B_2}(u_1, u_2) = 1.$$

So we have

$$\begin{aligned}
\alpha^3_{(B_1 \cup B_2)}(u_1u_2) &= \max\left\{\alpha^3_{B_{1i}}(u_1u_2), \alpha^3_{B_2}(u_1u_2)\right\} \\
&= \max\left\{\alpha^3_{B_{1i}}(u_1u_2), 0\right\} \\
&\leq \left[\min\left\{\alpha^3_{A_1}(u), \alpha^3_{A_1}(u_2)\right\}, 0\right] \\
&= \min\left\{\max\left\{\alpha^3_{A_1}(u_1), 0\right\}, \max\left\{\alpha^3_{A_1}(u_2), 0\right\}\right\} \\
&= \min\left\{\alpha^3_{A_1 \cup A_2}(u_1), \alpha^3_{A_1 \cup A_2}(u_2)\right\}
\end{aligned}$$

Also,

$$\begin{aligned}
\beta^3_{(B_{1i} \cup B_{2i})}(u_1u_2) &= \min\left\{\beta^3_{B_{1i}}(u_1u_2), \beta^3_{B1i}(u_1u_2)\right\} \\
&= \min\left\{\beta^3_{B_{1i}}(u_1u_2), 1\right\} \\
&\geq \min\left\{\max \beta^3_{A_1}(u_1), \beta^3_{A_2}(y_2), 1\right\} \\
&= \max\left\{\min\left\{\beta^3_{A_1}(u_1), 1\right\}, \min\left\{\beta^3_{A_1}(u_2), 1\right\}\right\} \\
&= \max\left\{\beta^3_{(A_1 \cup A_2)}(u_1), \beta^3_{(A_1 \cup A_2)}(u_2)\right\}
\end{aligned}$$

for $u_1u_2 \in E_{1i} \cup E_{2i}$

Case(ii): When $u_1u_2 \in \cup_2$, then by def (2),

$$\alpha^3_{A_1}(u_1) = \alpha^3_{A_2}(u_2) = \alpha^3_{B_{1i}}(u_1u_2) = 0,$$

$$B^3_{A_1}(u_1) = B^3_{A_2}(u_2) = B^3_{B_{1i}}(u_1u_2) = 1,$$

We have,

$$\begin{aligned}
\alpha^3_{(B_{1i} \cup B_{2i})}(u_1u_2) &= \max\left\{\alpha^3_{B_{1i}}(u_1u_2), \alpha^3_{B_{2i}}(u_1u_2)\right\} \\
&= \max\left\{0, \alpha^3_{B_{2i}}(u_1u_2)\right\} \\
&\leq \max\left\{0, \min\left\{\alpha^3_{A_2}(u_1), \alpha^3_{A_2}(u_2)\right\}\right\} \\
&= \min\left\{\max\left\{\alpha^3_{A_1}(u_1), \alpha^3_{A_2}(u_1)\right\}, \max\left\{\alpha^3_{A_1}(u_2), \alpha^3_{A_2}(u_2)\right\}\right\} \\
&= \min\left\{\alpha^3_{(A_1 \cup A_2)}(u_1), \alpha^3_{(A_1 \cup A_2)}(u_2)\right\}
\end{aligned}$$

Also

$$\begin{aligned}
\beta^3_{(B_{1i} \cup B_{2i})}(u_1u_2) &= \min\left\{\beta^3_{B_{1i}}(u_1u_2), \beta^3_{B_{2i}}(u_1u_2)\right\} \\
&= \min\left\{1, \beta^3_{B_{2i}}(u_1u_2)\right\}
\end{aligned}$$

$$\leq \min\left\{1, \max\left\{\beta^3_{A_2}(u_1), \beta^3_{A_2}(u_2)\right\}\right\}$$
$$= \max\left\{\min\left\{1, \beta^3_{A_2}(u_1)\right\}, \min\left\{1, \beta^3_{A_2}(u_2)\right\}\right\}$$
$$= \max\left\{\min\left\{\beta^3_{A_1}(u_1), \beta^3_{A_2}(u_2)\right\}, \min\left\{\beta^3_{A_1}(u_2), \beta^3_{A_2}(u_2)\right\}\right\}$$
$$= \max\left\{\beta^3_{A_1 \cup A_2}(u_1), \beta^3_{(A_1 \cup A_2)}(u_2)\right\}$$

for $u_1u_2 \in E_{1i} \cup E_{2i}$

Hence, $\widetilde{G}S_1 \cup \widetilde{G}S_2$ is a fermatean fuzzy graph structure of G_1^* and G_2^*.

Definition 3.9: (Composition): Let $\widetilde{G}S_1$ and $\widetilde{G}S_2$ be two fermatean fuzzy graph structures of G_1^* and G_2^* the composition of $\widetilde{G}S_1$ and $\widetilde{G}S_2$ is defined by

$$\alpha^3_{(A_1 o A_2)}(xy) = \left(\alpha^3_{A_1} o \alpha^3_{A_2}\right)(xy) = \min\left\{\alpha^3_{A_1}(x), \alpha^3_{A_2}(y)\right\}$$
$$\beta^3_{(A_1 o A_2)}(xy) = \left(\beta^3_{A_1} o \beta^3_{A_2}\right)(xy) = \max\left\{\beta^3_{A_1}(x), \beta^3_{A_2}(y)\right\}, xy \in U_1 x U_2$$
$$\alpha^3_{(B_1 o B_2)}(xy_1)(xy_2) = \left(\alpha^3_{B_1} o \alpha^3_{B_2}\right)(xy_1)(xy_2)$$
$$= \min\left\{\alpha^3_{A_1}(x), \alpha^3_{B_2}(y_1y_2)\right\}$$
$$\beta^3_{(B_1 o B_2)}(xy_1)(xy_2) = \left(\beta^3_{B_1} o B^3_{B_2}\right)(xy_1)(xy_2)$$
$$= \min\left\{\beta^3_{A_1}(x), \beta^3_{B_2}(y_1y_2)\right\}$$

$x \in U_1, y_1y_2 \in E_{2i}$

$$\alpha^3_{(B_1 o B_2)}(x_1y)(x_2y) = \left(\alpha^3_{B_1} o \alpha^3_{B_2}\right)(x_1y)(x_2y)$$
$$= \min\left\{\alpha^3_{A_1}(x), \alpha^3_{B_1}(x_1x_2)\right\}$$
$$\beta^3_{(B_1 o B_2)}(x_1y)(x_2y) = \left(\beta^3_{B_1} o \beta^3_{B_2}\right)(x_1y)(x_2y)$$
$$= \max\left\{\beta^3_{A_2}(y), \beta^3_{B_{1i}}(x_1x_2)\right\}$$

$y \in U_1, x_1x_2 \in E_{1i}$

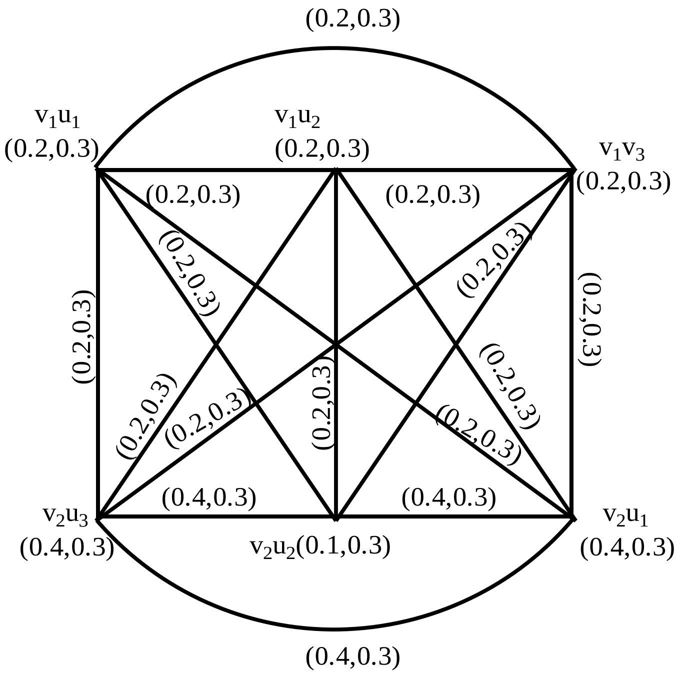
$$\alpha^3_{(B_1 o B_2)}(x_1y_1)(x_2y_2) = \left(\alpha^3_{B_1} o \alpha^3_{B_2}\right)((x_1y)(x_2y))$$
$$= \min\left\{\alpha^3_{A_2}(y_1), \alpha^3_{A_{2i}}(y_2), \alpha^3_{B_{1i}}(x_1x_2)\right\}$$
$$\beta^3_{(B_1 o B_2)}(x_1y_1)(x_2y_2) = \left(\beta^3_{B_{1i}} o \beta^3_{B_{2i}}\right)(x_1y_1)(x_2y_2)$$
$$= \max\left\{\beta^3_{A_2}(y_1), \beta^3_{B_{2i}}(y_2), \beta^3_{B_{1i}}(x_1x_2)\right\}$$

$x_1x_2 \in E_{1i}, y_1y_2 \in U$ such that $y_1 \neq y_2$.

Example 3.10: Let $\widetilde{G}S_1$ and $\widetilde{G}S_2$ be the two fermatean fuzzy graphs defined as follows:

G_1^*={{(V_1,0.2,0.3),(V_2,0.4,0.3)},{V_1V_2,0.2,0.3)},
{(V_1,0.4,0.1),(V_2,0.7,0.2),(V_3,0.3,0.4)}
{V_1V2,0.4,0.1),(V_2V_3,0.2,0.3)(V_3V_1,0.2,0.2)}}

G_2*={(u_1,0.7,0.2),(u_2,0.5,0.3),(u_3,0.8,0.1),
(u_1u_2,0.5,0.2),(u_2u_3,0.4,0.2),(u_3u_1,0.6,0.1)}



Composition of fermatean ***fuzzy graph***

Proportion 3.11: Let G* be the composition of graph structures G_1^* and G_2^*. Let $\tilde{G}S_1$ and $\tilde{G}S_2$ be two fermatean fuzzy graph structures of G_1^* and G_2^*. Then $\tilde{G}S_1$ and $\tilde{G}S_2$ is also a fermatean fuzzy graph structures of G*.

Proof: It is enough to show $A_1 o A_2$ is clearly a fermatean fuzzy set of $U_1 o U_2$ and $B_{1i} o B_{2i}$ is fermatean fuzzy set of $E_{1i} o E_{2i}$.Therefore, We have to prove $B_1 o B_2$ is fermatean fuzzy relation on $A_1 o A_2$

Case(i): Suppose $u \in U_1,\ b_1b_2 \in E_{2i}$.

$$\begin{aligned}
\alpha^3_{(B_1oB_2)}\left((ub_1)(ub_2)\right) &= \min\left\{\alpha^3_{A_1}(u), \alpha^3_{B_2}(b_1b_2)\right\} \\
&\le \min\left\{\alpha^3_{A_1}(u), \left\{\min\{\alpha^3_{A_2}(b_1), \alpha^3_{A_2}(b_2)\}\right\}\right\} \\
&= \min\left\{\min\{\alpha^3_{A_1}(u), \alpha^3_{A_2}(b_1)\}, \min\left\{\alpha^3_{A_1}(u), \alpha^3_{A_2}(b_2)\right\}\right\} \\
&= \min\left\{\alpha^3_{(A_1oA_2)}(ub_1), \alpha^3_{(A_1oA_2)}(ub_2)\right\}
\end{aligned}$$

$$\begin{aligned}
\beta^3_{(B_1oB_2)}((ub_1)(ub_2)) &= \max\left\{\beta^3_{A_1}(u), \beta^3_{B_2}(b_1b_2)\right\} \\
&\ge \max\left\{\beta^3_{A_1}(u), \left\{\max\{\beta^3_{A_2}(b_1), \beta^3_{A_2}(b_2)\}\right\}\right\} \\
&= \max\left\{\beta^3_{(A_1oA_2)}(ub_1), \beta^3_{(A_1oA_2)}(ub_2)\right\}, \text{for } ub_1, ub_2 \in U_1 o U_2
\end{aligned}$$

Case(ii):

When $u \in U_2, b_1b_2 \in E_{1i}$

$$\begin{aligned}
\alpha^3_{(B_1oB_2)}((b_1u)(b_2u)) &= \min\left\{\alpha^3_{A_2}(u), \alpha^3_{B_{1i}}(b_1b_2)\right\} \\
&\le \min\left\{\alpha^3_{A_2}(u), \min\left\{\alpha^3_{A_1}(b_1), \alpha^3_{A_1}(b_2)\right\}\right\} \\
&= \min\left\{\min\left\{\alpha^3_{A_2}(u), \alpha^3_{A_1}(b_1), \min\left\{\alpha^3_{A_2}(u), \alpha^3_{A_1}(b_2)\right\}\right\}\right\} \\
&= \min\left\{\alpha^3_{(A_1oA_2)}(b_1u), \alpha^3_{(A_1oA_2)}(b_2u)\right\}
\end{aligned}$$

$$\beta^3_{(B_1 o B_2)}((b_1u)(b_2u)) = \max\left\{\beta^3_{A_2}(u), \beta^3_{B_{1i}}(b_1b_2)\right\}$$
$$\geq \max\{\beta^3_{A_2}(u), \max\{\beta^3_{A_1}(b_1), \beta^3_{A_1}(b_2)\}\}$$
$$= \max\left\{\max\{\beta^3_{A_2}(u), \beta^3_{A_1}(b_1)\}, \max\{\beta^3_{A_2}(u), \beta^3_{A_1}(b_2)\}\right\}$$
$$= \max\left\{\beta^3_{(A_1 o A_2)}(b_1u), \beta^3_{(A_1 o A_2)}(b_2u)\right\}$$

for all $b_1u, b_2u \in U_1 o U_2$

Case(iii): When $b_1b_2 \in E_{1i}, u_1u_2 \in U_2$ such that $u_1 \neq u_2$

$$\alpha^3_{(B_1 o B_2)}\left((b_1u_1)(b_2u_2)\right) = \min\left\{\alpha^3_{A_2}(u_1), \alpha^3_{A_2}(u_2), \alpha^3_{B_1}(b_1b_2)\right\}$$
$$\leq \min\left\{\min\{(\alpha^3_{A_2}(u_1), \alpha^3_{A_2}(u_2)\}, \min\{\alpha^3_{A_1}(b_1), \alpha^3_{A_1}(b_2)\right\}$$
$$= \min\left\{\min\{(\alpha^3_{A_2}(u_1), \alpha^3_{A_1}(b_2)\}, \min\{\alpha^3_{A_2}(u_2), \alpha^3_{A_1}(b_2)\right\}$$
$$= \min\left\{(\alpha^3_{(A_1 o A_2)}(b_1u), \alpha^3_{(A_1 o A_2)}(b_2u_2)\right\}$$
$$\beta^3_{(B_1 o B_2)}\left((b_1u_1)(b_2u_2)\right) = \max\left\{\beta^3_{A_2}(u_1), \beta^3_{A_2}(u_2), \beta^3_{B_1}(b_1b_2)\right\}$$
$$\geq \max\left\{\max\{\beta^3_{A_2}(u_1), \beta^3_{A_2}(u_2)\}, \max\{\beta^3_{A_1}(b_1), \beta^3_{A_1}(b_2)\}\right\}$$
$$= \max\left\{\max\{\beta^3_{A_2}(u_1), \beta^3_{A_1}(b_1)\}, \max\{\beta^3_{A_2}(u_2), \beta^3_{A_1}(b_2)\}\right\}$$
$$= \max\left\{\beta^3_{(B_1 o B_2)}(b_1u), \beta^3_{(B_1 o B_2)}(b_1u)\right\}$$

for $b_1u, b_2u \in U_1 o U_2$

Hence, $\tilde{G}S_1$ and $\tilde{G}S_2$ is fermatean fuzzy graph of G.

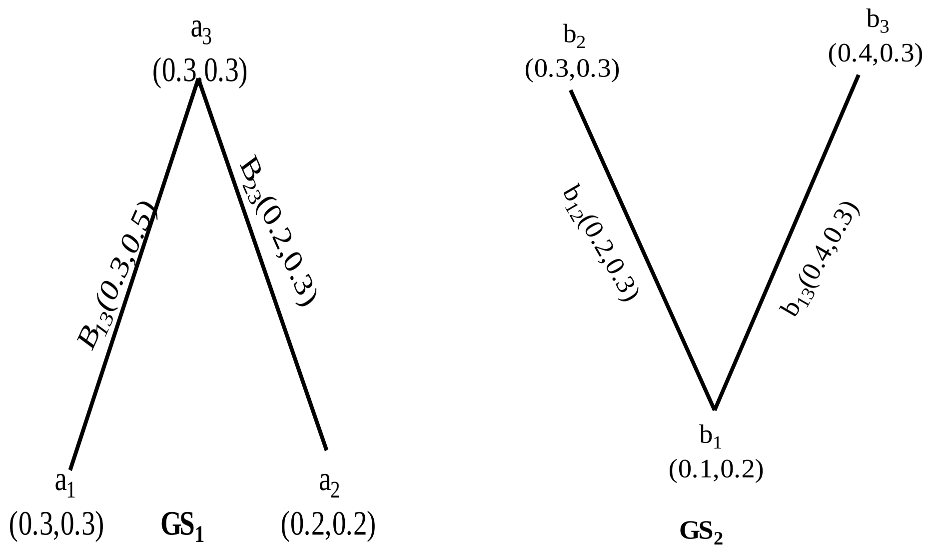
Definition 3.12: Lexicographic product of fermatean fuzzy graph.

Let $\tilde{G}S_1$ and $\tilde{G}S_2$ be two fermatean fuzzy graph structures of graph structures G_1^* and G_2^*.The lexicographic product of fermatean fuzzy graph is defined as

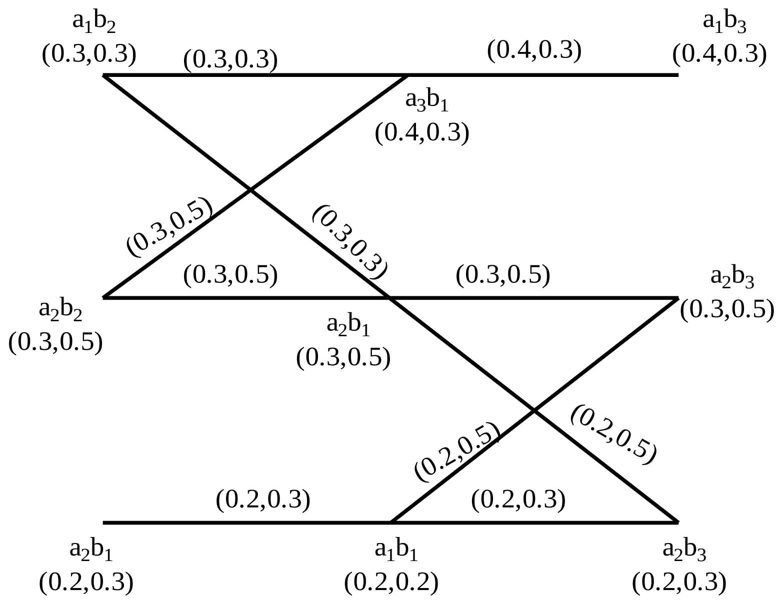
$$\alpha^3_{A_1.A_2}(xy) = \left(\alpha^3_{A_1}.\alpha^3_{A_2}\right)(xy) = \min\left\{\alpha^3_{A_1}(x), \alpha^3_{A_2}(y)\right\}$$
$$\beta^3_{B_{1i}.B_{2i}}((xy_1)(xy_2)) = \left(\alpha^3_{B_{1i}}.\alpha^3_{B_{2i}}\right)\left((xy_1)(xy_2)\right)$$
$$= \min\left\{\alpha_{A_1}(x), \alpha_{B_{2i}}(y_1y_2)\right\}$$
$$\beta^3_{B_{1i}.B_{2i}}((xy_1)(xy_2)) = \left(\beta^3_{B_{1i}}.\beta^3_{B_{2i}}\right)\left((xy_1)(xy_2)\right)$$
$$= \max\left\{\beta^3_{B_1}(x), \beta^3_{B_2}(y_1y_2)\right\}$$
$$\alpha^3_{(B_{1i}.B_{2i})}\left((x_1y_1)(x_2y_2)\right) = \left(\alpha^3_{B_{1i}}.\alpha_{B_{2i}}\right)\left((x_1y_1)(x_2y_2)\right)$$
$$= \min\left\{\alpha^3_{B_{1i}}(x_2x_2), \alpha^3_{B_{2i}}(y_2y_2)\right\}$$
$$\beta^3_{B_{1i}.B_{2i}}\left((x_1y_1)(x_2y_2)\right) = \left(\beta^3_{B_{1i}}.\beta^3_{B_{2i}}\right)\left((x_1y_1)(x_2y_2)\right)$$
$$= \max\left\{\beta^3_{B_{1i}}(x_1x_2), \beta^3_{B_{2i}}(y_1y_2)\right\}$$

for all $x_1x_2 \in E_1, y_1y_2 \in E_2$

Example 3.13: Let $\tilde{G}S_1$ and $\tilde{G}S_2$ be two fermatean fuzzy graph structure of G_1* and G_2* respectively.



Then the lexicographic product of $\tilde{G}S_1$ and $\tilde{G}S_2$



Proportion 3.14 : Let G* be the lexicographic product of graph structures G_1 *and G_2*. Let $\tilde{G}S_1$ and $\tilde{G}S_2$ be two fermatean fuzzy graph structures of G_1* and G_2*. Then $\tilde{G}S_1$ and $\tilde{G}S_2$ is fermatean fuzzy graph structures of G*.

Proof: It is enough to show that $A_1 o A_2$ is clearly fermatean fuzzy set of $U_1 o U_2$ and $B_{1i} o B_{2i}$ is fermatean fuzzy set of $E_{1i} o E_{2i}$. So the proof requires only to show that $B_{1i} o B_{2i}$ is fermatean fuzzy relation on $A_1 o A_2$. The following cases are arise.

Case(i): When $u \in U_1$, $b_1 b_2 \in E_{2i}$.

$$\begin{aligned}\alpha^3_{(B_1 o B_2)}\left((ub_1)(ub_2)\right) &= \min\left\{\alpha^3_{A_1}(u), \alpha^3_{B_{2i}}(b_1 b_2)\right\} \\ &\le \min\left\{\alpha^3_{A_1}(u), \min\left\{\alpha^3_{A_2}(b_1), \alpha^3_{A_2}(b_2)\right\}\right\} \\ &= \min\left\{\min\left\{\alpha^3_{A_1}(u), \alpha^3_{A_2}(b_1)\right\}, \min\left\{\alpha^3_{A_1}(u), \alpha^3_{A_2}(b_2)\right\}\right\} \\ &= \min\left\{\alpha^3_{A_1 o A_2}(ub_1), \alpha^3_{A_1 o A_2}(ub_2)\right\}\end{aligned}$$

$$\beta^3_{(B_{1i}\text{o}B_{2i})}\left((ub_1)(ub_2)\right)=\max\left\{\beta^3_{A_1}(u),\beta^3_{B_{2i}}(b_1b_2)\right\}$$
$$\geq \max\left\{\beta^3_{A_1}(u),\max\left\{\beta^3_{A_2}(b_1),\beta^3_{A_2}(b_2)\right\}\right\}$$
$$=\max\left\{\max\left\{\beta^3_{A_1}(u),\beta^3_{A_2}(u)\right\},\max\left\{\beta^3_{A_1}(u),\beta^3_{A_2}(u)\right\}\right\}$$
$$=\min\left\{\beta^3_{(A_1\text{o}A_2)}(ub_1),\beta^3_{(A_1\text{o}A_2)}(ub_2)\right\}$$

for all $ub_1,ub_2\in U_1\text{o}U_2$.

Case(ii): Suppose $a_1a_2\in E_{1i},b_1b_2\in E_{2i}$

$$\alpha^3_{B_{1i}\text{o}B_{2i}}\left((a_1b_1)(a_2b_2)\right)=\min\left\{\alpha^3_{B_{1i}}(a_1a_2),\alpha^3_{B_{2i}}(b_1b_2)\right\}$$
$$=\min\left\{\min\{\alpha^3_{A_1}(a_1),\alpha^3_{A_1}(a_2)\},\min\{\alpha^3_{A_1}(a_2),\alpha^3_{A_2}(a_2)\}\right\}$$
$$=\min\left\{\alpha^3_{A_1\text{o}A_2}(a_1b_1),\alpha^3_{A_1\text{o}A_2}(a_2b_2)\right\}$$

$$\beta^3_{(B_{1i}\text{o}B_{2i})}\left((a_1b_1)(a_2b_2)\right)=\max\left\{\beta^3_{B_{1i}}(a_2a_2),\beta^3_{B_{2i}}(b_2b_2)\right\}$$
$$\geq \max\left\{\max\{\beta^3_{B_{1i}}(a_1),\beta^3_{B_{1i}}(a_2)\},\max\{\beta^3_{B_{2i}}(a_1),\beta^3_{B_{2i}}(a_2)\}\right\}$$
$$=\max\left\{\max\{\beta^3_{A_1}(a_1),\beta^3_{A_2}(b_1)\},\max\{\beta^3_{A_1}(a_1),\beta^3_{A_2}(b_2)\}\right\}$$
$$=\min\left\{\beta^3_{(A_1\text{o}A_2)}(a_1b_1),\beta^3_{(A_1\text{o}A_2)}(a_2b_2)\right\}$$

for $a_1b_1,a_2b_2\in U_1\text{o}U_2$. Also holds for i=1,2,......n.

Hence, $B_{1i}\text{o}B_{2i}$ is fermatean fuzzy relation in $\mathrm{A}_1\text{o}\mathrm{A}_2$. $\tilde{G}S_1$ and $\tilde{G}S_2$ is fermatean fuzzy relation of G*.The proof is completed.

4. FERMATEAN FUZZY GRAPH DECISION MAKING APPROACH

Algorithm

Step-1:The choice parameters $\lambda_1\lambda_2......\lambda_p$ for the selection of objects.

Step-2:The fermatean fuzzy set (ϕ,M) over V and fermatean fuzzy relation (N,M) on V.

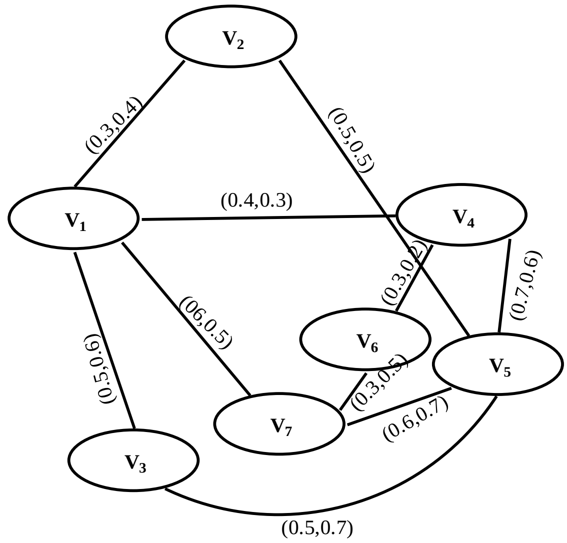
Step-3:Form the adjacency matrix w.r.t the parameters.

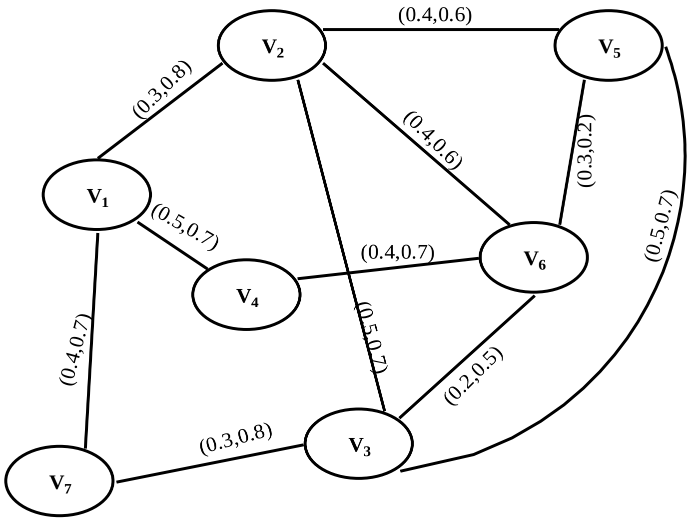
Step-4:Compute the resultant adjacency matrix.

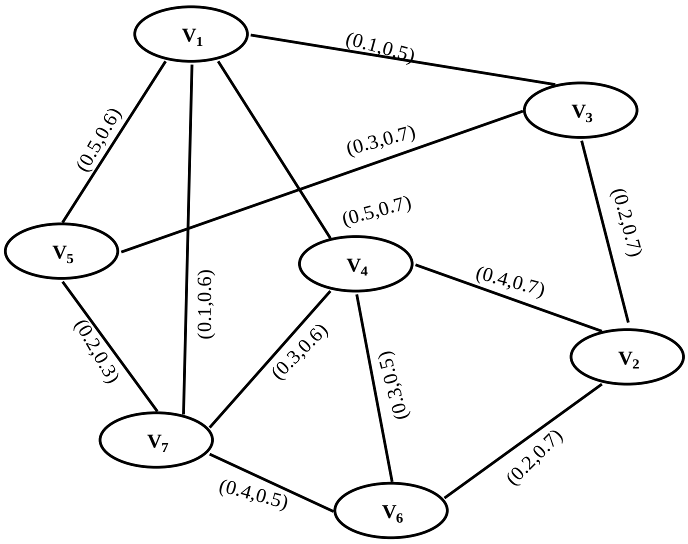
Step-5:Calculate the score value S_{ij}.

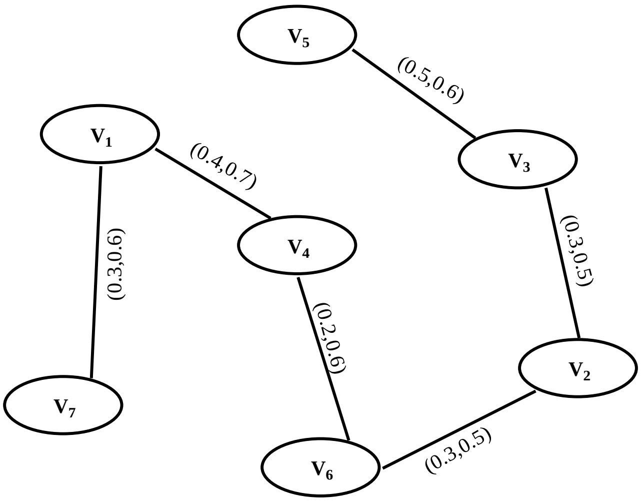
Step-6:Calculate the choice value $\sum S_{ij}$ of each objects.

Step-7:Calculate the decision is maximum value.


V2
(0.3,0.4)
(0.5,0.5)
V1
(0.4,0.3)
V4
(0.3,0.2)
(0.7,0.6)
(06,0.5)
V6
(0.5,0.6)
V5
(0.3,0.5)
V7
(0.6,0.7)
V3
(0.5,0.7)


(0.4,0.6)
V2
V5
(0.3,0.8)
(0.4,0.6)
(0.3,0.2)
V1
(0.5,0.7)
(0.5,0.7)
(0.4,0.7)
V6
V4
(0.5,0.7)
(0.4,0.7)
(0.2,0.5)
(0.3,0.8)
V3
V7


V1
(0.1,0.5)
V3
(0.5,0.6)
(0.3,0.7)
(0.2,0.7)
(0.5,0.7)
V5
V4
(0.4,0.7)
(0.1,0.6)
V2
(0.2,0.3)
(0.3,0.6)
(0.3,0.5)
V7
(0.2,0.7)
(0.4,0.5)
V6



A software house wants to select the best software development product, a project with the greatest business impact, the least effort required and the highest probability of success. V={V_1,V_2,V_3,V_4,V_5,V_6,V_7} where,

Let V_1={E-Commerce development protect}

V_2 = {Custom development protect}

V_3 = {Applications development protect}

V_4 = {Game development Protect}

V_5 = {Browser development protect}

V_6 = {Web development protect}

V_7 = {System integration protect} be the set of software developments profits under considerations.

M={ l_1, l_2, l_3}Where

l_1={ensure protect success}

l_2={technologically feasibility}

l_3=(Commercial viability be the set of decision parameters correspond to the venture set V=={V_1,V_2,V_3,.........V_7}.

We consider on fermatean fuzzy graph $G=\{\phi,\psi,M\}$ where (ϕ,M) is fermatean fuzzy set over V which describes the membership and non-membership values of the protects based the given parameters. (ψ,M) is fermatean fuzzy set over $E\subseteq VxV$ describes the membership and non-membership values of the relationship between two protects corresponding to the given parameter $l_1 l_2$ *and* l_3.

A fermatean fuzzy graph G={$H_{(C1)}$,} is given in Table -1.

ϕ	V_1	V_2	V_3	V_4	V_5	V_6	V_7
l_1	(0.3,0.4)	(0.2,0.3)	(0.1,0.4)	(0.2,0.4)	(0.2,0.5)	(0.3,0.5)	(0.2,0.6)
l_2	(0.1,0.3)	(0.1,0.4)	(0.2,0.4)	(0.3,0.4)	(0.3,0.5)	(0.2,0.6)	(0.1,0.5)
l_3	(0.2,0.3)	(0.3,0.4)	(0.3,0.5)	(0.4,0.5)	(0.4,0.6)	(0.3,0.5)	(0.2,0.4)

Table-2

ϕ	V_1V_2	V_1V_3	V_1V_7	V_1V_5	V_2V_6
l_1	(0.3,0.4)	(0.5,0.6)	(0.6,0.5)	(0,0)	(0,0)
l_2	(0.3,0.8)	(0,0)	(0.4,0.7)	(0,0)	(0.4,0.6)
l_3	(0,0)	(0.1,0.5)	(0.1,0.6)	(0.5,0.6)	(0.2,0.7)

Table-3

ϕ	V_5V_2	V_4V_6	V_4V_5	V_4V_2
I_1	(0.5,0.5)	(0.3,0.2)	(0.7,0.6)	(0,0)
I_2	(0.4,0.6)	(0.4,0.7)	(0,0)	(0,0)
I_3	(0,0)	(0.3,0.5)	(0,0)	(0.4,0.7)

Table-4

ϕ	V_1V_4	V_5V_3	V_3V_4	V_6V_7
I_1	(0.4,0.3)	(0,0)	(0,0)	(0.3,0.5)
I_2	(0.5,0.7)	(0,0)	(0,0)	(0,0)
I_3	(0.5,0.7)	(0.3,0.7)	(0,0)	(0.4,0.5)

Table-5

ϕ	V_3V_7	V_6V_3	V_5V_7	V_6V_5	$V4V_7$
I_1	(0,0)	(0,0)	(0.6,0.7)	(0,0)	(0,0)
I_2	(0.3,0.8)	(0.2,0.5)	(0,0)	(0.3,0.2)	(0,0)
I_3	(0,0)	(0,0)	(0,0)	(0,0)	(0.3,0.6)

The adjacency matrix of the resultant fermatean fuzzy graph.

		V_1	V_2	V_3	V_4	V_5	V_6	V_7
H	V_1	(0,0)	(0,0)	(0,0)	(0.4,0.7)	(0,0)	(0,0)	(0.3,0.6)
	V_2	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0.3,0.5)	(0,0)
	V_3	(0,0)	(0.3,0.5)	(0,0)	(0,0)	(0.5,0.6)	(0,0)	(0,0)
	V_4	(0.4,0.7)	(0,0)	(0,0)	(0,0)	(0,0)	(0.2,0.6)	(0,0)
	V_5	(0,0)	(0,0)	(0.5,0.6)	(0,0)	(0.5,0.6)	(0,0)	(0,0)
	V_6	(0,0)	(0.3,0.5)	(0,0)	(0.4,0.6)	(0,0)	(0,0)	(0,0)
	V_7	(0.3,0.6)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)

The score values of resultant fermatean fuzzy graph title is computed with score function.

$$S_{ij} = \frac{\sqrt{\alpha_j^2 + (\beta_j - 1)^2 + \pi_j^2}}{\sqrt{\alpha_j^2 + (\beta_j - 1)^2 + \pi_j^2 + \sqrt{(\alpha_j - 1)^2 + (\beta_j) + \pi_j^2}}}$$

and $S_{ij} \in [011]$.

Table-6

	V_1	V_2	V_3	V_4	V_5	V_6	V_7	Total
V_1	0.5	0.5	0.5	0.73	0.5	0.5	0.43	**1.41**
V_2	0.5	0.5	0.5	0.5	0.5	0.53	0.5	0.83
V_3	0.5	0.53	0.5	0.5	0.47	0.5	0.5	1.25
V_4	0.73	0.5	0.5	0.5	0.5	0.41	0.5	1.39
V_5	0.5	0.5	0.47	0.5	0.47	0.5	0.5	1.19
V_6	0.5	0.53	0.5	0.41	0.5	0.5	0.5	1.19
V_7	0.43	0.5	0.5	0.5	0.5	0.5	0.5	0.73

From the Table-6, it follows that the maximum choice Value is 1.41 and so the optimal decision is to select the browser development protect after specifying weights for different parameters.

Conclusion: Graph theory is natural to deal with vagueness and uncertainly using the methods of fuzzy sets. Since fermatean fuzzy set has explained more advantages is handling vagueness and uncertainly then fuzzy set. In this article, we have applied the concept of fermatean fuzzy sets to graph structures. We also discussed with operations on fermatean fuzzy structures.

Future work: We will extend our work to Pythagorean fuzzy soft graph, Roughness various graph structures.

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