

Heat and Mass Transfer Performance of Dufour and Heat Source on Casson Fluid Flow Past an Inclined Oscillating Rotating Porous Plate with Chemical Reactions and Thermal Radiation

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Abstract: The current research presents an analytical interpretation of unsteady-free convective hydromagnetic boundary layers, focusing on how the Dufour effect, thermal radiation, and chemical reactions influence Casson fluid flow through a rotating porous medium, around an inclined oscillating plate within a uniform field. The governing equations are solved using the Laplace transform method, with the results presented visually through numerical values of Casson fluid velocity, temperature, and concentration at the plate, based on various key parameter values. The study examines how the Casson fluid behaves compared to Newtonian fluid, with the Casson fluid demonstrating a higher velocity. The findings also show that, as temperature decreases, parameters such as the Grashof number, Dufour effect, thermal radiation parameter (R), and chemical reaction parameter (K) increase. Additionally, increasing the chemical reaction parameter (K) causes a reduction in concentration. In analyzing the fluid flow, the study also measured the influence of Prandtl, Schmidt, thermal, and Grashof numbers. The results indicate that the velocity of the fluid decreases with higher radiation levels, but increases when the heat source and Grashof number are elevated. Similarly, rising radiation levels reduce temperature, while a stronger heat source increases it.

Keywords: Dufour, Heat source, Thermal Radiation, Chemical Reaction, Rotation

1. Introduction

The study of radiative heat and mass transport in convective flows is relevant for many industrial and technical applications. Mass transfer is a prevalent phenomenon in the chemical industry, as well as the physical and biological sciences. Heat transfer in porous media from different geometries has many scientific and practical applications, such as geothermal reservoirs, drying of porous solids, thermal insulation, enhanced oil recovery, packed-bed catalytic reactors, nuclear reactor cooling, and underground energy

transfer. Current research in radiative heat transfer heavily focuses on numerical simulations of coupled radiation and convection processes. Convective heat transfer in porous media has recently gained significant attention due to its technological relevance in several domains. For example, Muthucumaraswamy et al. [1] studied Radiative Flow Past a Parabolic Started Isothermal Vertical Plate with Uniform Mass Flux. A.K. Agrawal et al. [2] examined free convection due to thermal and mass diffusion in laminar flow of an accelerated infinite vertical plate in the presence of magnetic. Kataria et al. [3] explored the impact of radiation and chemical reactions on the flow of MHD Casson fluid over a vertical oscillating plate in a porous medium.

Vijayaragavan et al. [4] provided a theoretical solution to the problem of heat and mass transfer in MHD Casson fluid flow past a sloped porous plate with Dufour and chemical reactions. Kavitha et al. [5] investigated parabolic flow over a rotating isothermal plate with uniform temperature and mass diffusion in the presence of a chemical reaction. Selvaraj et al. [6] explored MHD parabolic flow over a rotating, accelerating isothermal plate, while Nanadakumar et al. [7] studied the combined Soret and MHD effects on parabolic flow past a quickly moving vertical plate. Jothi and Selvaraj [8] examined the impact of a heat source on MHD flow and a radiation-absorbing fluid moving over an exponentially accelerating plate surrounded by a porous medium. According to Lakshmikanth et al. [9], as radiation (R) increases, the temperature decreases, whereas an increase in the heat source (Q) raises the temperature. Maran [10] and his team studied mass and heat transfer over a vertical plate during chemical reactions and thermal diffusion, where MHD effects did not govern the convective flow. Aruna et al. [11] explored the Hall and magnetic effects on a fluid flowing past a parabolically accelerated vertical plate, considering both heat variations and uniform mass diffusion due to thermal radiation. Radha et al. [12] discussed the MHD effects on Casson fluid flow over a parabolically accelerated vertical plate with thermal diffusion in unsteady conditions. Muthucumaraswamy Rajamanickam et al. [13–14] analyzed free convection in MHD flow past a moving vertical plate with constant temperature during a chemical reaction, also investigating the Hall and rotational effects on heat and mass transfer in magnetohydrodynamic flow over a vertically oriented plate under exponential acceleration.

Hetnarski [15, 16] presented an algorithm for finding inverse Laplace transforms. Sarma and Ahmed [17] studied the Dufour effect in non-steady MHD flow through a porous medium over a vertical plate with rising temperature. Prakash and Selvaraj [18] investigated how radiation and heat impact MHD and parabolic motion in Casson fluids moving through a rotating porous medium on a vertical plate. Prabhakar et al. [19] used computational techniques to explore the effects of diffusion, thermodynamics, and rotation on heat-generating MHD Casson fluid flow past an oscillating porous plate. Ananda Reddy and Janardhan [20] corrected for the Soret and Dufour effects on MHD Casson fluid over a vertical plate in the presence of chemical reactions and radiation. Palani and Arutchelvi [21] analyzed the flow of an MHD nanofluid past an inclined plate, accounting for the Soret and Dufour effects. Kumar and Vempati [22, 23] studied the influence of MHD on the rapid starting of an infinite vertical plate under temperature variations in a horizontal magnetic field. Muthucumaraswamy Rajamanickam et al. [24, 25] examined chemical reaction effects on flow past a vertical plate undergoing rapid motion with varying temperatures and mass diffusion in the presence of a magnetic field. Patel and Harshad R. [26] investigated cross-

diffusion and heat generation effects on mixed convective MHD flow of Casson fluid through a porous medium with non-linear thermal radiation. Dhanalakshmi et al. [27] discussed molecular stability, kinetic energy, and energy-related factors in coumarin compounds through Gc-MS analysis. Karthikeyan et al. [28, 29] explored the rotational effects of parabolic flow past an isothermal vertical plate using MHD, while Gayathri et al. [30] studied the Soret and Dufour effects on unsteady MHD convective flow over an infinite vertical porous plate. In this study, we investigated the Dufour effects on MHD Casson fluid flow past an exponentially accelerating inclined oscillating plate, in comparison to a rotating fluid with constant temperature and mass diffusion

2. Mathematical Formulation

Consider an unsteady free hydromagnetic natural convective flow involving heat and mass transfer in a viscous, incompressible, electrically conductive, and optically thick radiative fluid. The fluid flows past an infinite inclined oscillating plate within a rotating system. The model incorporates the effects of the Hall current, rotation, and the presence of a porous medium, while also accounting for the Dufour effect. The coordinate system is chosen such that y' – axis is perpendicular to the plate and x' – axis is upward to the plate. The plate and fluid revolve along x' – axis in according with standardized angular velocity Ω . The fluid and plate initially rest at period along $t' \leq 0$ and are held at the uniform temperature T' . Also, on the surface of the plate, and at any point inside the fluid, species concentration at C'_∞ shall be preserved uniformly. At the time $t' \geq 0$, plate statistics moving to their own planes at $u' = u_0 \cos \omega t$ velocity. The flow temperature and species concentration on the plate surface is upturned to the uniform temperature about T'_w and uniform species concentration C'_w and consequently retained.

Under these conditions, the flow characteristics solely depend on y' and t . Plate temp is decreased or increased to $T'_\infty + (T'_w - T'_\infty) \frac{u_0^2 \bar{t}}{v}$ at $\bar{t} \geq 0$ and plate concentration is raised or lowered to $C'_\infty + (C'_w - C'_\infty) \frac{u_0^2 \bar{t}}{v}$ at $\bar{t} \geq 0$. The rheological state eq. for Cauchy stress tensor of Casson fluid is presented by

$$\tau_{ij} = \begin{cases} 2e_{ij} \left(\mu_B + \frac{py}{\sqrt{2\pi}} \right) & \pi > \pi_c \\ 2e_{ij} \left(\mu_B + \frac{py}{\sqrt{2\pi_c}} \right) & \pi < \pi_c \end{cases}$$

$$\frac{\partial u'}{\partial t'} - 2\Omega'v' = \vartheta \left(1 + \frac{1}{\gamma} \right) \frac{\partial^2 u'}{\partial y'^2} + g\beta_{T'}(T' - T'_\infty)\cos\alpha + g\beta_{C'}(C' - C'_\infty)\cos\alpha - \frac{\vartheta u'}{k'_1} \quad (1)$$

$$\frac{\partial v'}{\partial t'} + 2\Omega'u' = \vartheta \frac{\partial^2 v'}{\partial y'^2} - \frac{\vartheta u'}{k'_1} \quad (2)$$

$$\frac{\partial \theta'}{\partial t'} = \frac{k}{\rho C_p} \frac{\partial^2 \theta'}{\partial y'^2} - \frac{1}{\rho C_p} \frac{\partial q'_r}{\partial y'} + \frac{Q_0}{\rho C_p} (T' - T'_\infty)' + \frac{D_m K_T}{C_s C_p} \frac{\partial^2 C'}{\partial y'^2} \quad (3)$$

$$\frac{\partial C'}{\partial t'} = \frac{1}{S_c} \frac{\partial^2 C'}{\partial y'^2} - k'(C' - C'_\infty) \quad (4)$$

The initial and boundary conditions listed below

$$\left. \begin{aligned} u' = 0, v' = 0, T' = T'_\infty, C' = C'_\infty, \text{ for all } z' \geq 0, t' \leq 0 \\ u' = u_0 \cos \omega t, \quad T' = T'_\infty + (T'_w - T'_\infty) \frac{t'}{t_0}, C' = (C'_w - C'_\infty) \frac{t'}{t_0} \\ u' \rightarrow 0, \quad T' \rightarrow T'_\infty, C' \rightarrow C'_\infty \text{ as } z' \rightarrow \infty \text{ and } t' \geq 0 \end{aligned} \right\} (5)$$

As we have optically thick Casson fluid, we can use Rosseland approximation [26]

$$\frac{\partial q_r}{\partial z'} = -4a^* \sigma (T_\infty'^4 - T'^4) \quad (6)$$

It is assumed that the temperature variations within the flow are small enough T'^4 that can be approximated as a linear function of temperature. This is achieved by expanding T'^4 in a Taylor series around T_∞' and neglecting higher-order terms. Thus, the expression becomes

$$T'^4 \cong 4T_\infty'^3 T' - 3T_\infty'^4 \quad (7)$$

The consequent dimensionless aggregate is

$$\left. \begin{aligned} U &= \frac{u'}{U_0}, \quad V = \frac{v'}{U_0}, \quad t = \frac{t'}{t_0}, \quad y = \frac{y'}{U_0 t_0}, \quad \gamma = \frac{\mu_B \sqrt{2\pi c}}{P_y} \\ \theta &= \frac{T' - T_\infty'}{T_\infty' - T_\infty'}, \quad G_r = \frac{g\beta(T_\infty' - T_\infty')}{u_0}, \quad C = \frac{c' - c_\infty'}{c_\infty' - c_\infty'}, \quad G_c = \frac{g\beta(c_\infty' - c_\infty')}{c_\infty' - c_\infty'}, \quad Q = \frac{Q_0 v^2}{k U_0^2} \\ P_r &= \frac{\mu C_p}{k}, \quad K = \frac{\nu k_1}{U_0^2}, \quad S_c = \frac{\nu}{D}, \quad R = \frac{16a^* \sigma T_\infty'^3}{k U_0^2} \end{aligned} \right\} \quad (8)$$

By substituting the values from Eq. (6) and Eq. (7) into Eq. (3), as well as into Eqs. (1)–(4), and simplifying by dropping the notation (for clarity), we obtain the following non-dimensional equations:

$$\frac{\partial U}{\partial t} = \left(1 + \frac{1}{\gamma}\right) \frac{\partial^2 U}{\partial y^2} + 2\Omega v + Gr\theta(\cos\alpha) + GcC(\cos\alpha) - \frac{U}{k_1} \quad (9)$$

$$\frac{\partial V}{\partial t} = \left(1 + \frac{1}{\gamma}\right) \frac{\partial^2 V}{\partial y^2} - 2\Omega v - \frac{V}{k_1} \quad (10)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{P_r} \frac{\partial^2 \theta}{\partial y^2} - R\theta + Q\theta + Df \frac{\partial^2 C}{\partial y^2} \quad (11)$$

$$\frac{\partial C}{\partial t} = \frac{1}{S_c} \frac{\partial^2 C}{\partial y^2} - KC \quad (12)$$

To solve equations (1) and (2), use $q' = U + iV$ we get

$$\frac{\partial q'}{\partial t} = G_r \theta \cos(\alpha) + G_c C \cos(\alpha) + \left(1 + \frac{1}{\gamma}\right) \frac{\partial^2 q'}{\partial y^2} - m^* q' \quad (13)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{P_r} \frac{\partial^2 \theta}{\partial y^2} - R\theta + Q\theta + Df \frac{\partial^2 C}{\partial y^2} \quad (14)$$

$$\frac{\partial C}{\partial t} = \frac{1}{S_c} \frac{\partial^2 C}{\partial y^2} - KC \quad (15)$$

$$\text{Here } m^* = 2 \left[i\Omega + \frac{1}{k_1} \right]$$

The corresponding initial and boundary conditions are

$$\left. \begin{aligned} q' &= 0, \theta = 0, C = 0 \text{ for all } y \text{ and } t \leq 0 \\ q' &= \cos \omega t, \theta = 1, C = 1 \text{ for all } y, t \geq 0 \\ q' &\rightarrow 0, \theta \rightarrow 0, C \rightarrow 0 \text{ as } y \rightarrow \infty. \end{aligned} \right\} \quad (16)$$

3. Solution of the Problem

The dimensionless governing equations (14), (15), and (16), along with their initial and boundary conditions, can be solved using the Laplace transform technique we get the solutions

$$C = \frac{1}{2} \left[e^{-2\eta\sqrt{ScKt}} \operatorname{erfc}(\eta\sqrt{Sc} - \sqrt{Kt}) + e^{2\eta\sqrt{ScKt}} \operatorname{erfc}(\eta\sqrt{Sc} + \sqrt{Kt}) \right] \quad (17)$$

$$\theta = \theta_1 + \frac{PrDfSc}{(Sc-Pr)} \left(\frac{a+K}{a} \right) [\theta_2 + \theta_3] + \frac{PrDfSc}{(Sc-Pr)} \frac{K}{a} [\theta_5 + \theta_6] \quad (18)$$

$$\text{Where } \theta_1 = \frac{1}{2} \left[e^{-2\eta\sqrt{Pr(R-Q)t}} \operatorname{erfc}(\eta\sqrt{Pr} - \sqrt{(R-Q)t}) + e^{2\eta\sqrt{Pr(R-Q)t}} \operatorname{erfc}(\eta\sqrt{Pr} + \sqrt{(R-Q)t}) \right]$$

$$\theta_2 = \frac{e^{at}}{2} \left[e^{-2\eta\sqrt{Pr(a+R-Q)t}} \operatorname{erfc}(\eta\sqrt{Pr} - \sqrt{(a+R-Q)t}) + e^{2\eta\sqrt{Pr(a+R-Q)t}} \operatorname{erfc}(\eta\sqrt{Pr} + \sqrt{(a+R-Q)t}) \right]$$

$$\theta_3 = \frac{e^{at}}{2} \left[e^{-2\eta\sqrt{Sc(a+K)t}} \operatorname{erfc}(\eta\sqrt{Sc} - \sqrt{(a+K)t}) + e^{2\eta\sqrt{Sc(a+K)t}} \operatorname{erfc}(\eta\sqrt{Sc} + \sqrt{(a+K)t}) \right]$$

$$\theta_4 = \frac{1}{2} \left[e^{-2\eta\sqrt{ScKt}} \operatorname{erfc}(\eta\sqrt{Sc} - \sqrt{Kt}) + e^{2\eta\sqrt{ScKt}} \operatorname{erfc}(\eta\sqrt{Sc} + \sqrt{Kt}) \right]$$

$$\theta_5 = \frac{1}{2} \left[e^{-2\eta\sqrt{Pr(R-Q)t}} \operatorname{erfc}(\eta\sqrt{Pr} - \sqrt{(R-Q)t}) + e^{2\eta\sqrt{Pr(R-Q)t}} \operatorname{erfc}(\eta\sqrt{Pr} + \sqrt{(R-Q)t}) \right]$$

q'

$$= f_1 + f_2$$

$$+ \frac{Gr(\cos\alpha)}{a_1} \left[f_3 - f_4 + \frac{PrDfSc}{Sc-Pr} \left(\frac{k+a}{a} \right) \left\{ \frac{1}{a-c_1} (f_5 - f_6 - f_7 + f_8) - \frac{1}{a-d} (f_9 - f_{10} - f_{11} + f_{12}) \right\} \right. \\ \left. + \frac{PrDfSc}{Sc-Pr} \left(\frac{k}{a} \right) \left\{ \frac{1}{d} (-f_{13} + f_{14} + f_{15} - f_{16}) - \frac{1}{c_1} (-f_{17} + f_{18} + f_{19} - f_{20}) \right\} \right] \\ + \frac{Gc(\cos\alpha)}{a_1 b} [f_{21} - f_{22} - f_{23} + f_{24}] \quad (19)$$

where

$$f_1 = \left[e^{-2\eta\sqrt{\frac{(m^*-iw)t}{a_1}}} \operatorname{erfc}\left(\frac{\eta}{\sqrt{a_1}} - \sqrt{(m^*-iw)t}\right) + e^{2\eta\sqrt{\frac{(m^*-iw)t}{a_1}}} a_1 \operatorname{erfc}\left(\frac{\eta}{\sqrt{a_1}} + \sqrt{(m^*-iw)t}\right) \right]$$

$$f_2 = \frac{e^{iwt}}{4} \left[e^{-2\eta\sqrt{\frac{(m^*+iw)t}{a_1}}} \operatorname{erfc}\left(\frac{\eta}{\sqrt{a_1}} - \sqrt{(m^*+iw)t}\right) + e^{2\eta\sqrt{\frac{(m^*+iw)t}{a_1}}} \operatorname{erfc}\left(\frac{\eta}{\sqrt{a_1}} + \sqrt{(m^*+iw)t}\right) \right]$$

$$\begin{aligned}
f_3 &= \frac{e^{c_1 t}}{2} \left[e^{-2\eta \sqrt{\frac{(m^* + c_1)t}{a_1}}} \operatorname{erfc} \left(\eta - \sqrt{\frac{(m^* + c_1)t}{a_1}} \right) \right. \\
&\quad \left. + e^{2\eta \sqrt{\frac{(m^* + c_1)t}{a_1}}} \operatorname{erfc} \left(\eta + \sqrt{\frac{(m^* + c_1)t}{a_1}} \right) \right] \\
f_4 &= \frac{e^{c_1 t}}{2} \left[e^{-2\eta \sqrt{Pr(R+c_1-Q)t}} \operatorname{erfc} \left(\eta \sqrt{Pr} - \sqrt{(R+c_1-Q)t} \right) \right. \\
&\quad \left. + e^{2\eta \sqrt{Pr(R+c_1-Q)t}} \operatorname{erfc} \left(\eta \sqrt{Pr} + \sqrt{(R+c_1-Q)t} \right) \right] \\
f_5 &= \frac{e^{at}}{2} \left[e^{-2\eta \sqrt{\frac{(m^* + a)t}{a_1}}} \operatorname{erfc} \left(\eta - \sqrt{\frac{(m^* + a)t}{a_1}} \right) \right. \\
&\quad \left. + e^{2\eta \sqrt{\frac{(m^* + a)t}{a_1}}} \operatorname{erfc} \left(\eta + \sqrt{\frac{(m^* + a)t}{a_1}} \right) \right] \\
f_6 &= \frac{e^{c_1 t}}{2} \left[e^{-2\eta \sqrt{\frac{(m^* + c_1)t}{a_1}}} \operatorname{erfc} \left(\eta - \sqrt{\frac{(m^* + c_1)t}{a_1}} \right) \right. \\
&\quad \left. + e^{2\eta \sqrt{\frac{(m^* + c_1)t}{a_1}}} \operatorname{erfc} \left(\eta + \sqrt{\frac{(m^* + c_1)t}{a_1}} \right) \right] \\
f_7 &= \frac{e^{at}}{2} \left[e^{-2\eta \sqrt{Pr(R+a-Q)t}} \operatorname{erfc} \left(\eta \sqrt{Pr} - \sqrt{(R+a-Q)t} \right) \right. \\
&\quad \left. + e^{2\eta \sqrt{Pr(R+a-Q)t}} \operatorname{erfc} \left(\eta \sqrt{Pr} + \sqrt{(R+a-Q)t} \right) \right] \\
f_8 &= \frac{e^{c_1 t}}{2} \left[e^{-2\eta \sqrt{Pr(R+c_1-Q)t}} \operatorname{erfc} \left(\eta \sqrt{Pr} - \sqrt{(R+c_1-Q)t} \right) \right. \\
&\quad \left. + e^{2\eta \sqrt{Pr(R+c_1-Q)t}} \operatorname{erfc} \left(\eta \sqrt{Pr} + \sqrt{(R+c_1-Q)t} \right) \right] \\
f_9 &= \frac{e^{at}}{2} \left[e^{-2\eta \sqrt{\frac{(m^* + a)t}{a_1}}} \operatorname{erfc} \left(\eta - \sqrt{\frac{(m^* + a)t}{a_1}} \right) \right. \\
&\quad \left. + e^{2\eta \sqrt{\frac{(m^* + a)t}{a_1}}} \operatorname{erfc} \left(\eta + \sqrt{\frac{(m^* + a)t}{a_1}} \right) \right]
\end{aligned}$$

$$\begin{aligned}
f_{10} &= \frac{e^{dt}}{2} \left[e^{-2\eta\sqrt{\frac{(m^*+d)t}{a_1}}} \operatorname{erfc} \left(\eta - \sqrt{\frac{(m^*+d)t}{a_1}} \right) \right. \\
&\quad \left. + e^{2\eta\sqrt{\frac{(m^*+d)t}{a_1}}} \operatorname{erfc} \left(\eta + \sqrt{\frac{(m^*+d)t}{a_1}} \right) \right] \\
f_{11} &= \frac{e^{at}}{2} \left[e^{-2\eta\sqrt{Sc(a+k)t}} \operatorname{erfc} \left(\eta\sqrt{Sc} - \sqrt{(a+k)t} \right) \right. \\
&\quad \left. + e^{2\eta\sqrt{Sc(a+k)t}} \operatorname{erfc} \left(\eta\sqrt{Sc} + \sqrt{(a+k)t} \right) \right] \\
f_{12} &= \frac{e^{dt}}{2} \left[e^{-2\eta\sqrt{Sc(d+k)t}} \operatorname{erfc} \left(\eta\sqrt{Sc} - \sqrt{(d+k)t} \right) \right. \\
&\quad \left. + e^{2\eta\sqrt{Sc(d+k)t}} \operatorname{erfc} \left(\eta\sqrt{Sc} + \sqrt{(d+k)t} \right) \right] \\
f_{13} &= \frac{1}{2} \left[e^{-2\eta\sqrt{\frac{m^*t}{a_1}}} \operatorname{erfc} \left(\eta - \sqrt{\frac{m^*t}{a_1}} \right) + e^{2\eta\sqrt{\frac{m^*t}{a_1}}} \operatorname{erfc} \left(\eta + \sqrt{\frac{m^*t}{a_1}} \right) \right] \\
f_{14} &= \frac{e^{dt}}{2} \left[e^{-2\eta\sqrt{\frac{(m^*+d)t}{a_1}}} \operatorname{erfc} \left(\eta - \sqrt{\frac{(m^*+d)t}{a_1}} \right) \right. \\
&\quad \left. + e^{2\eta\sqrt{\frac{(m^*+d)t}{a_1}}} \operatorname{erfc} \left(\eta + \sqrt{\frac{(m^*+d)t}{a_1}} \right) \right] \\
f_{15} &= \frac{1}{2} \left[e^{-2\eta\sqrt{Sc}kt} \operatorname{erfc}(\eta\sqrt{Sc} - \sqrt{kt}) + e^{2\eta\sqrt{Sc}kt} \operatorname{erfc}(\eta\sqrt{Sc} + \sqrt{kt}) \right] \\
f_{16} &= \frac{e^{dt}}{2} \left[e^{-2\eta\sqrt{Sc(d+k)t}} \operatorname{erfc}(\eta\sqrt{Sc} - \sqrt{(d+k)t}) \right. \\
&\quad \left. + e^{2\eta\sqrt{Sc(d+k)t}} \operatorname{erfc}(\eta\sqrt{Sc} + \sqrt{(d+k)t}) \right] \\
f_{17} &= \frac{1}{2} \left[e^{-2\eta\sqrt{\frac{m^*t}{a_1}}} \operatorname{erfc} \left(\eta - \sqrt{\frac{m^*t}{a_1}} \right) + e^{2\eta\sqrt{\frac{m^*t}{a_1}}} \operatorname{erfc} \left(\eta + \sqrt{\frac{m^*t}{a_1}} \right) \right] \\
f_{18} &= \frac{e^{c_1t}}{2} \left[e^{-2\eta\sqrt{\frac{(m^*+c_1)t}{a_1}}} \operatorname{erfc} \left(\eta - \sqrt{\frac{(m^*+c_1)t}{a_1}} \right) \right. \\
&\quad \left. + e^{2\eta\sqrt{\frac{(m^*+c_1)t}{a_1}}} \operatorname{erfc} \left(\eta + \sqrt{\frac{(m^*+c_1)t}{a_1}} \right) \right]
\end{aligned}$$

$$\begin{aligned}
f_{19} &= \frac{1}{2} \left[e^{-2\eta\sqrt{Pr(R-Q)t}} \operatorname{erfc} \left(\eta\sqrt{Pr} - \sqrt{(R-Q)t} \right) \right. \\
&\quad \left. + e^{2\eta\sqrt{Pr(R-Q)t}} \operatorname{erfc} \left(\eta\sqrt{Pr} + \sqrt{(R-Q)t} \right) \right] \\
f_{20} &= \frac{e^{c_1 t}}{2} \left[e^{-2\eta\sqrt{Pr(R+c_1-Q)t}} \operatorname{erfc} \left(\eta\sqrt{Pr} - \sqrt{(R+c_1-Q)t} \right) \right. \\
&\quad \left. + e^{2\eta\sqrt{Pr(R+c_1-Q)t}} \operatorname{erfc} \left(\eta\sqrt{Pr} + \sqrt{(R+c_1-Q)t} \right) \right] \\
f_{21} &= \frac{e^{bt}}{2} \left[e^{-2\eta\sqrt{\frac{(m^*+b)t}{a_1}}} \operatorname{erfc} \left(\eta - \sqrt{\frac{(m^*+b)t}{a_1}} \right) \right. \\
&\quad \left. + e^{2\eta\sqrt{\frac{(m^*+b)t}{a_1}}} \operatorname{erfc} \left(\eta + \sqrt{\frac{(m^*+b)t}{a_1}} \right) \right] \\
f_{22} &= \frac{1}{2} \left[e^{-2\eta\sqrt{\frac{m^*t}{a_1}}} \operatorname{erfc} \left(\eta - \sqrt{\frac{m^*t}{a_1}} \right) + e^{2\eta\sqrt{\frac{m^*t}{a_1}}} \operatorname{erfc} \left(\eta + \sqrt{\frac{m^*t}{a_1}} \right) \right] \\
f_{23} &= \frac{e^{bt}}{2} \left[e^{-2\eta\sqrt{Sc(b+k)t}} \operatorname{erfc} \left(\eta\sqrt{Sc} - \sqrt{(b+k)t} \right) \right. \\
&\quad \left. + e^{2\eta\sqrt{Sc(b+k)t}} \operatorname{erfc} \left(\eta\sqrt{Sc} + \sqrt{(b+k)t} \right) \right] \\
f_{24} &= \frac{1}{2} \left[e^{-2\eta\sqrt{Sc}kt} \operatorname{erfc}(\eta\sqrt{Sc} - \sqrt{kt}) + e^{2\eta\sqrt{Sc}kt} \operatorname{erfc}(\eta\sqrt{Sc} + \sqrt{kt}) \right] \\
\operatorname{erfc}(a + ib) &= \operatorname{erf}(a) + \frac{\exp(-a^2)}{2a\pi} [1 - \cos(2ab) + i\sin(2ab)] \\
&\quad + \frac{2\exp(-a^2)}{\pi} \sum_{n=1}^{\infty} \frac{\exp(-\eta^2/4)}{\eta^2 + 4a^2} [f_n(a, b) + i g_n(a, b)] + \epsilon(a, b)
\end{aligned}$$

Appendix

$$\begin{aligned}
f_n &= 2a - 2 \operatorname{acosh}(nb) \cos(2ab) + n \sinh(nb) \sin(2ab) \quad \text{and} \\
g_n &= 2 \operatorname{acosh}(nb) \sin(2ab) + n \sinh(nb) \cos(2ab) \\
|\epsilon(a, b)| &\approx 10^{-16} |\operatorname{erf}(a + ib)|
\end{aligned}$$

$$A = \left(\frac{Pr(R-Q) - ScK}{Pr - Sc} \right), a_1 = 1 + \frac{1}{\gamma}, \eta = \frac{y}{2\sqrt{t}}, b = \frac{m^* + KSc a_1}{a_1 Sc - 1}, c = \frac{a_1 Pr(R-Q)}{1 - a_1 Pr}, d = \frac{ScK a_1 - m^*}{1 - Sc a_1},$$

erfc – Complementary error function

4. Results and Discussions

We are going to look at some numbers that show how different non-dimensional flow parameters affect the speed (q'), temperature (θ), and concentration (C) as they are plotted against the boundary layer coordinate (η) in Fig.1-Fig.16. Chemical reaction parameter (K), rotation parameter (R), Schmidt number (Sc), Grashof number (Gr), Permeability parameter

(k_1), Heat absorption coefficient (Q), Dufour number (D_f), Casson fluid parameter (γ), and time t .

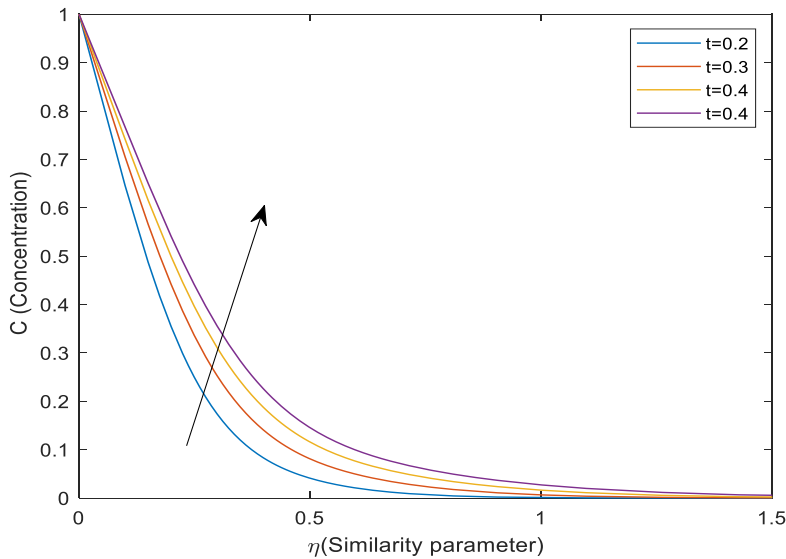


Fig.1 Concentration profile for distinct values of t and $Sc=2.01, K=0.5$

Fig. 1 ,it is observed that the concentration raises with an increase in the value of time t

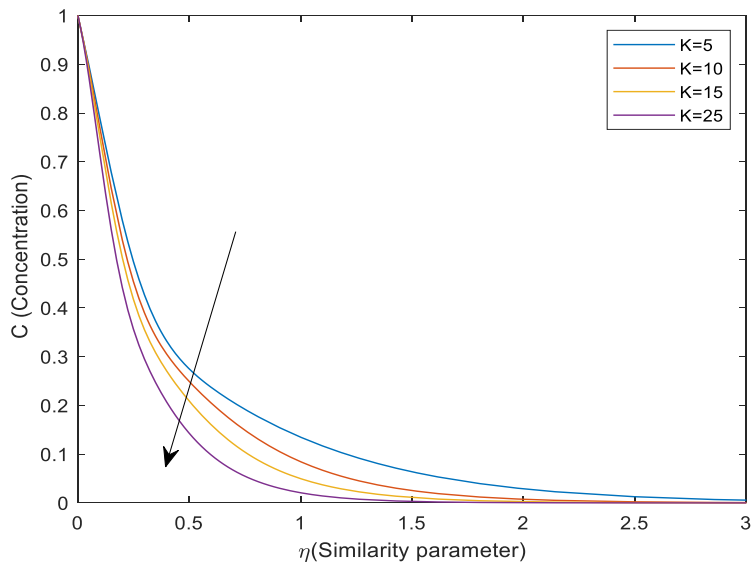


Fig.2 Concentration profile for distinct values of K and time $t = 0.2, Sc=2.01$

Fig.2 illustrates the concentration goes to down as increase the chemical reaction values

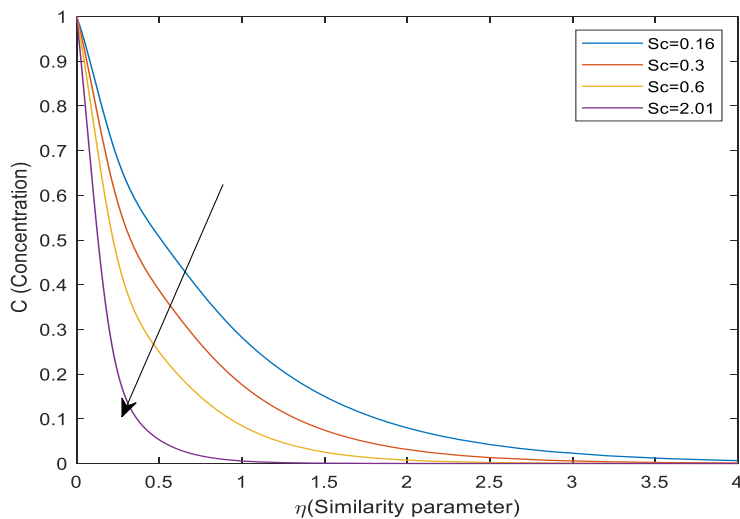


Fig.3 Concentration profile for different value of Sc and values $t=0.2, K=10$

Using the different values of Schmidt numbers, the result is seen in fig.3 when the Schmidt value is assumed to be high it may be noticed that the concentration down.

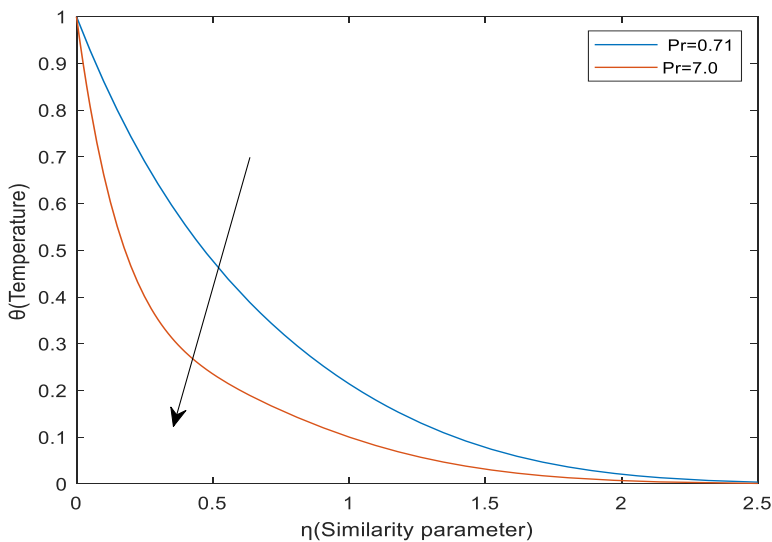


Fig .4 Temperature Profiles for several values of Pr and $Sc=0.6, R=5, Q=1, K=1, Df=1, t=0.2$

In Fig.4 shows that temperature decreases as in the values of pr values increases

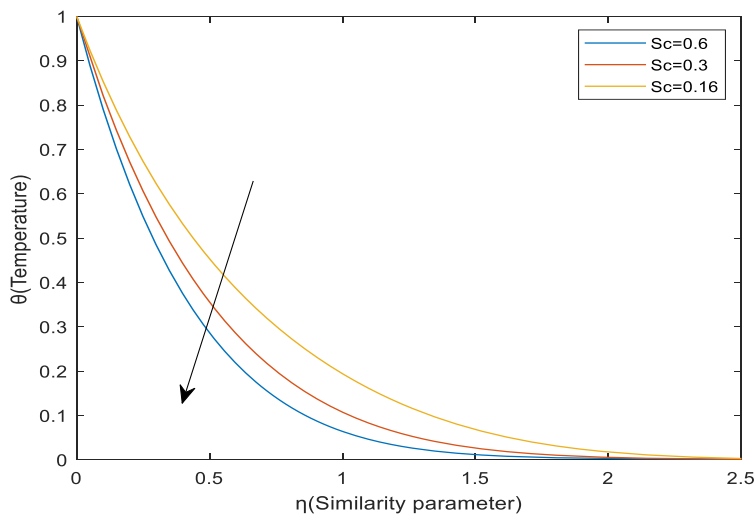


Fig.5 Temperature Profiles for distinguished values of Sc and $R=5, Q=1, K=1, Df=1, t=0.2, Pr=0.71$

Fig.5, it is observed that Schmidt number increases, indicating a tendency for the temperature to rise initially. However, the temperature subsequently fluctuates, revealing a change in trend, decrease in warm at this stage.

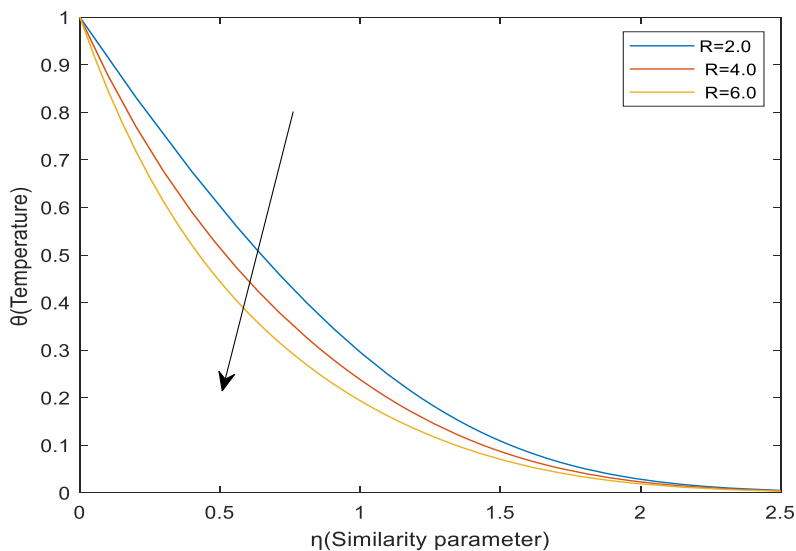
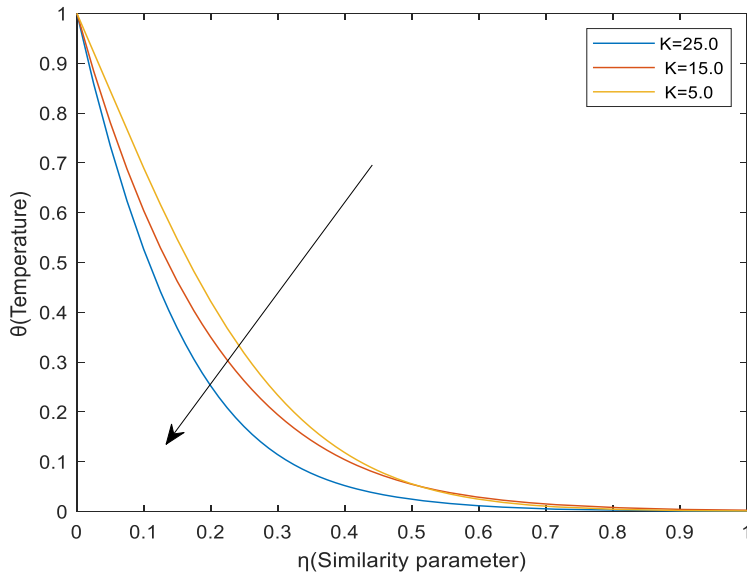


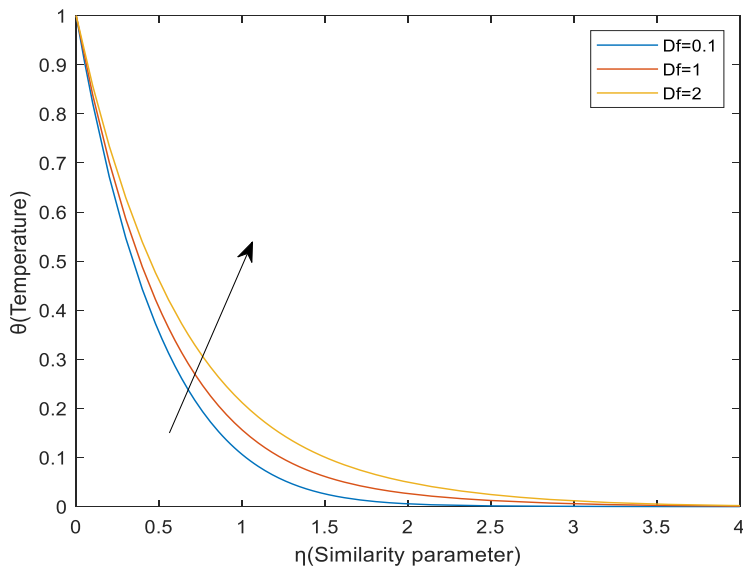
Fig.6 Temperature Profiles for several values of Chemical Reaction K and $Pr=0.71, Sc=0.6, Q=1, K=1, Df=1, t=0.2$

Fig. 6 illustrates the temperature profile shows that thermal radiation parameter R goes up, the temperature goes down



**Fig.7 Temperature profiles of distinct Chemical Reaction 'K'
and $Pr=0.71, Sc=0.6, R=2, Q=1, Df=1, t=0.2$**

In Fig. 7, it is observed that the chemical reaction values K goes up, then the temperature goes down



**Fig.8 Temperature Profiles for various values of Df and
 $Pr=0.71, Sc=0.6, R=6, Q=0.1, K=5, t=0.2$**

In Fig.8, we can explore the heat behaviour in more detailed by considering the thermal diffusion (Dufour) numbers ($Df = 0.1, 1, 2$). In general, as Dufour parameter increases, it means that the Dufour effect becomes more pronounced.

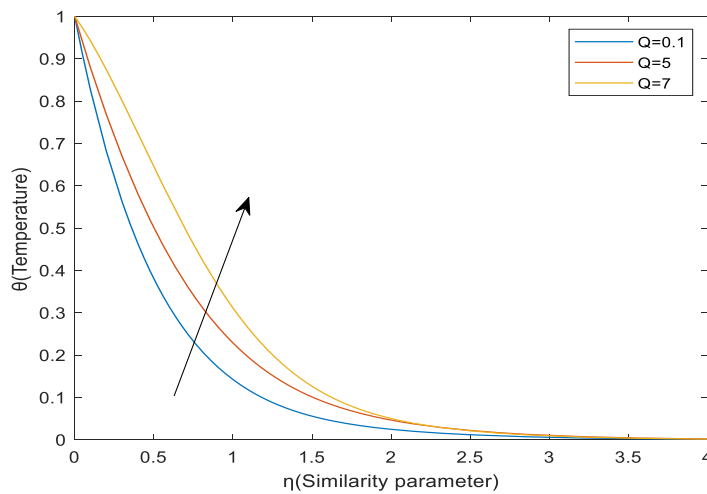


Fig.9. Temperature profile for different values of Q and $Pr=0.71, Sc=0.16, R=6, K=5, Df=1, t=0.2$

In Fig.9, it is observed that temperature increases with increases in the values of heat generation parameter Q

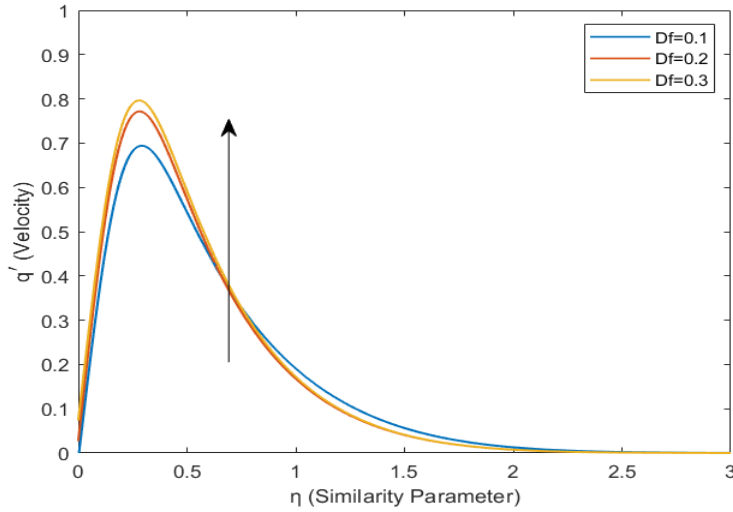


Fig.10 Velocity Profile for different values of Df and $r=5, Gc=10, Pr=7, Sc=0.1, R=5, Q=1, \gamma = 0.5, K=0.5, Df=0.1, h=1.5, t=0.1, w=30, \Omega =0.5, k1=0.4$

Fig.10 illustrates speed for thermal diffusion values ($Df = 0.1, 0.2, 0.3$), it is evident that the highest speed is achieved if Dufour number gets high

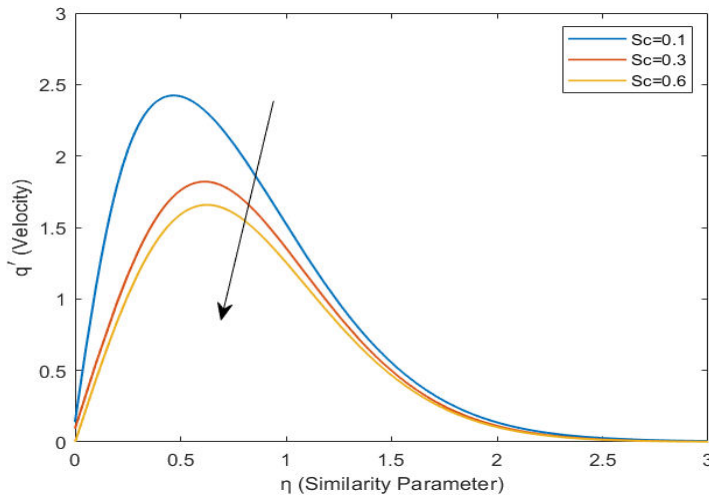


Fig. 11 Velocity for distinct values of Sc and $Gr=5, Gc=10, Pr=7, R=5, Q=1,$

$$\gamma = 0.5, K=0.5, Df=0.1, h=1.5, t=0.1, w=30, \Omega = 0.5, k1=0.4$$

In fig.11 displays plate's velocity contours at various Schmidt values. ($Sc= 0.1, 0.3, 0.6$). A plate's Schmidt number falls as speed rises.

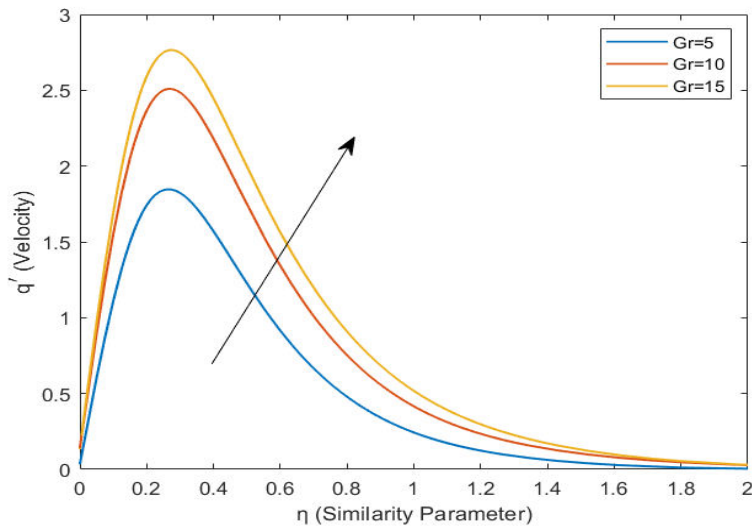


Fig.12 Velocity profile for different values of Gr and $Gc=10, Pr=7, Sc=0.1, R=5, Q=1,$

$$\gamma = 0.5, K=0.5, Df=0.1, h=1.5, t=0.1, w=30, \Omega = 0.5, k1=0.4$$

In fig.12 velocity contours of the plate are shown in the image above. Increasing velocity causes the Gr values to increase

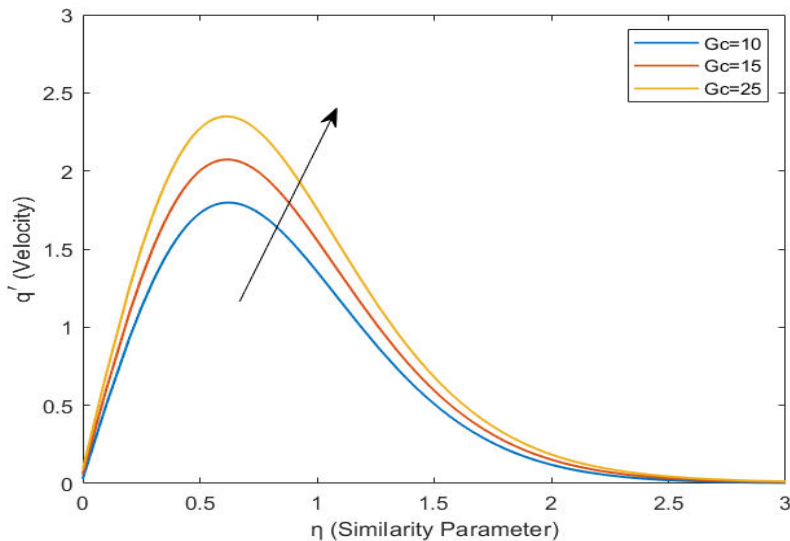


Fig.13 Velocity profile for different values of G_c and $Gr=10, Pr=7, Sc=0.1, M=10, R=5, Q=1, \gamma = 0.5, K=0.5, Df=0.1, h=1.5, t=0.1, w=30, \Omega =0.5, k_1=0.4$
In fig.13 velocity contours of the plate are shown in the image above. Increasing velocity causes the Gr values to increase.

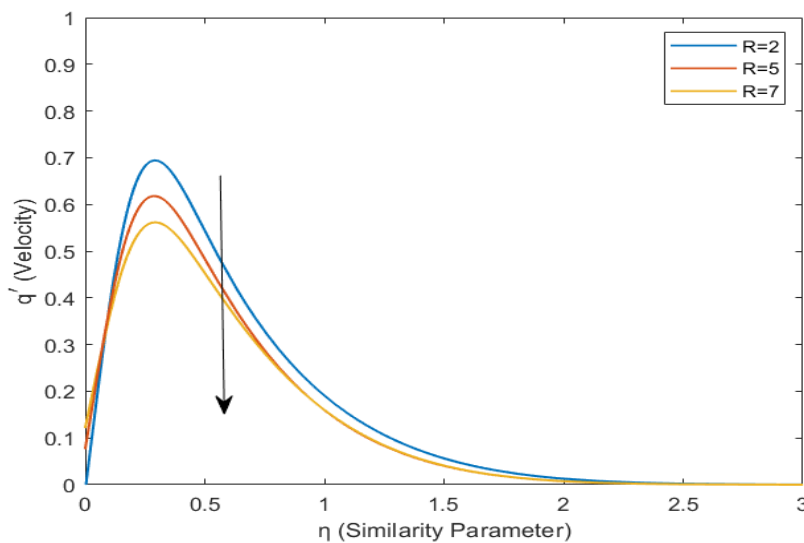


Fig.14 Velocity profile for different values of R and $G_c=10, Gr=10, Pr=7, Sc=0.1, R=5, Q=1, \gamma = 0.5, K=0.5, Df=0.1, h=1.5, t=0.1, w=30, \Omega =0.5, k_1=0.4$
In fig.14, it is observed that velocity of the plate goes to down with an increasing the values of thermal radiation parameter R

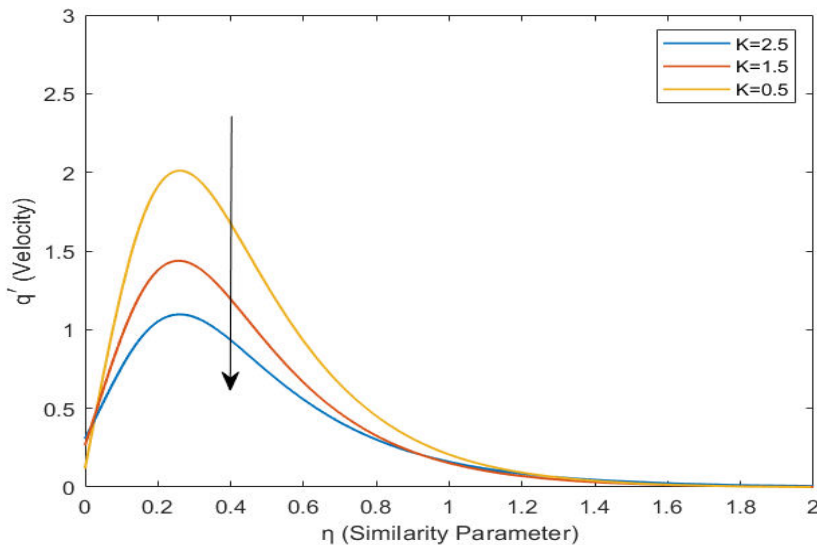


Fig.15 Velocity profile for different values of K and $Gc=10, Gr=10, Pr=7, Sc=0.1, R=5, Q=1, \gamma = 0.5, K=0.5, Df=0.1, h=1.5, t=0.1 w=30, \Omega =0.5, k1=0.4$

In fig.15, it is seen that velocity of the plate goes to down with an increasing the values of chemical reaction parameter K

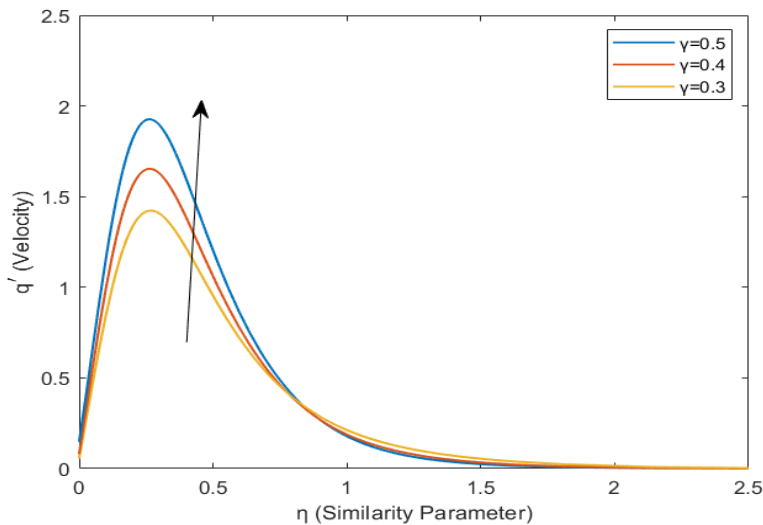


Fig.16 Velocity profile for different values of γ and $Gc=10, Gr=10, Pr=7, Sc=0.1, R=5, Q=1, K=0.5, Df=0.1, h=1.5, t=0.1 w=30, \Omega =0.5, k1=0.4$

In fig.16, it is seen that velocity of the plate goes to high performance with an increasing the values of Casson fluid parameter

5. Conclusion

This research explores the relationship between rotational and Dufour effects on Casson fluid flow, using Laplace transforms to understand equations and mathematical statements.

Graphs display findings, highlighting temperature, concentration, and speed schemes.

- The temperature profile tends to increase in the Dufour number (Df), heat generation (Q), and time (t), and experiences a reversal in the Prandtl number (Pr), Schmidt number (Sc), chemical reaction parameter (K), and thermal radiation parameter (R).
- The concentration boundary layer decreased and reversed on time (t) as the Schmidt number (Sc) and chemical reaction parameter (K) rose.
- The speed slows down and the Dufour number goes up as the values of the thermal gradient number (Gr), the Casson fluid parameter (Y), the chemical reaction parameter (K), the Schmidt number (Sc), the thermal radiation parameter (R), and the Prandtl number (Pr) go up.

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