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Integrating IVIFS and Butcher's RK Method for Multiple Attribute Group Decision Making: Mobile Brand Selection

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Abstract:
Problems in MAGDM (Multiple Attribute Group Decision Making) are examined using IVIFS (Interval-Valued Intuitionistic Fuzzy Sets). To aggregate the Interval-Valued Intuitionistic Fuzzy Decision Matrices (IVIFDM), subjective geometric & hybrid geometric operators are employed. Decision-maker weights are determined using Runge-Kutta methods, which are applied to the decision-making process. A new Extended Hamming Distance formula is used to rank the alternatives. A numerical instancehad been given to demonstrate effectiveness of proposed method.
Keywords: Multiple Attribute Group Decision Making (MAGDM), Interval Valued Intuitionistic Fuzzy Set (IVIFS), Butcher 's Seventh Order Runge-Kutta Method, Weighted Geometric and Hybrid Geometric Operator

Introduction

In recent years, the realm of decision-making has increasingly embraced the complexities inherent in uncertainty and imprecision. Intuitionistic fuzzy sets, introduced by Atanassov (1986), provide a robust framework for modeling such uncertainties, capturing both membership and non-membership degrees (Atanassov, 1989). This dual representation enhances traditional fuzzy set theory, making it particularly suitable for MAGDM scenarios where subjective judgments and preferences are critical.A variety of methods have emerged to address MAGDM problems using intuitionistic fuzzy sets, each leveraging unique mathematical approaches. For instance, Akila and Robinson (2019) explore numerical methods for MAGDM involving intuitionistic triangular fuzzy sets, while Khan et al. (2023) introduced q-rung ortho-pair fuzzy aggregation operators which further refine decisionmaking processes. Additionally, researchers like Jiao et al. (2023) and Liu and Jiang (2020) have integrated advanced fuzzy techniques and distance measures into decision frameworks, demonstrating the versatility of intuitionistic fuzzy models.The significance of aggregation operators and distance measures in handling intuitionistic fuzzy information (Alcantud et al., 2020; Liu & Li, 2023). For example, Seikh and Mandal (2021) provide insights into Dombi aggregation operators, while Tiwari (2021) presents similarity measures tailored for interval-valued intuitionistic fuzzy sets. These advancements not only facilitate more nuanced decision-making but also enhance the applicability of fuzzy logic in real-world contexts, such as telecommunications (Rani et al., 2021) and financial investments (Zhao et al., 2021).As the integration of intuitionistic fuzzy sets continues to evolve, the implications for decision-making processes become increasingly profound, offering promising avenues for future research and practical applications. This paper aims to synthesize these developments, exploring the multifaceted applications and methodologies surrounding intuitionistic fuzzy sets in realm of MAGDM.

Preliminaries

This sectionwill present fundamental definitions of IVIFS& aggregation operators.

Definition 1: (IVIFS)

Consider a fixed set Z. An IVIFS $\grave{B}$ in K a object with form $\grave{B} = \{(k, [\mu_{\grave{B}L}(k), \mu_{\grave{B}U}(k)], [\gamma_{\grave{B}L}(k), \gamma_{\grave{B}U}(k)]), k \in \mathrm{K}\}$ where $[\mu_{\grave{B}L}(k), \mu_{\grave{B}U}(k)] : K \longrightarrow [0,1]$and $[\gamma_{\grave{B}L}(k), \gamma_{\grave{B}U}(k)]: K \longrightarrow [0,1]$ states"degree of membership & degree of non-membership respectively, for the element"$k \in \mathrm{K}$ to the set$\grave{B}$, a subset of K, each element $k \in \mathrm{K}, 0 \leq [\mu_{\grave{B}L}(k), \mu_{\grave{B}U}(k)] + [\gamma_{\grave{B}L}(k), \gamma_{\grave{B}U}(k)] \leq 1$.

Definition 2: (IVIFWG Operator)

Interval Valued Intuitionistic Fuzzy Weighted Geometric (IVIFWG) Operator is described as follows: $W_j = IVIFWG(W_{1j}, W_{2j}, \ldots \ldots, W_{mj}) = W_{1_j}^{w_1} \oplus W_{2_j}^{w_2} \oplus \ldots \ldots . W_{m_j}^{w_m}$.

$$= \langle \left[\prod_{i=1}^{m} a_{ij}^{w_i}, \prod_{i=1}^{m} b_{ij}^{w_i} \right], \left[1 - \prod_{i=1}^{m} (1 - c_{ij})^{w_i}, 1 - \prod_{i=1}^{m} (1 - d_{ij})^{w_i} \right] \rangle, j = 1,2,.n$$

Definition 3: (IVIFHG Operator).

The definition of Interval Valued Intuitionistic Fuzzy Hybrid Geometric (IVIFHG) Operator is as follows:

$$W_j = IVIFHG_{\alpha,\lambda}(W_{ij}{}^{(1)}, W_{ij}{}^{(2)}, \ldots \ldots, W_{ij}{}^{m}) = \left(W_{ij}^{\sigma(1)} \right)^{\alpha_1} \otimes \left(W_{ij}^{\sigma(2)} \right)^{\alpha_2} \otimes \ldots \otimes \left(W_{ij}^{\sigma(k)} \right)^{\alpha_k}.$$

$$= \langle \left[\prod_{k=1}^{n} \left(a_{ij}^{\sigma(k)} \right)^{\alpha_k}, \prod_{k=1}^{n} \left(b_{ij}^{\sigma(k)} \right)^{\alpha_k} \right], \left[1 - \prod_{k=1}^{n} \left(1 - c_{ij}^{\sigma(k)} \right)^{\alpha_k}, 1 - \prod_{k=1}^{n} \left(1 - d_{ij}^{\sigma(k)} \right)^{\alpha_k}, \right] \rangle,$$

where $\alpha = (\alpha_1, \alpha_2, \ldots \alpha_1)^T$; a weight vector of IVIFHG operator with $\alpha_k > 0 (k = 1,2, \ldots 1)$ and$\sum_{k=1}^{l} \alpha_k = 1 \& \dot{r_{ij}} = \langle [\dot{a_{ij}}, \dot{b_{ij}}], [\dot{c_{ij}}, \dot{d_{ij}}] \rangle \dot{s_{ij}}^{\sigma(k)} = \langle \left[\dot{a_{ij}}^{\sigma(k)}, \dot{b_{ij}}^{\sigma(k)} \right], \left[\dot{c_{ij}}^{\sigma(k)}, \dot{d_{ij}}^{\sigma(k)} \right] \rangle$ isk^{th}highest of weighted IVIFHG $\dot{s_{ij}}^{(k)} = (s_{ij}^{(k)})^{l\lambda_k}, i = 1,2, \ldots m, j = 1,2, \ldots . n$.

Definition4:

Letusconsidertwo IVIFS: $A = \langle [\mu_{AL}(x), \mu_{AU}(x)], [\gamma_{AL}(x), \gamma_{AU}(x)] \rangle$ $and\ B = \langle [\mu_{BL}(x), \mu_{BU}(x)], [\gamma_{BL}(x), \gamma_{BU}(x)] \rangle$. Consider the perfect IVIFSs (termed as PositiveIdealSolution: PIS&NegativeIdealSolutio: NIS):$r^{\sim +} = ([1,1], [0,0])\ and\ r^{\sim -} = ([0,0], [1,1])$.

Definition 5: Normalized Hamming Distance (NHD) Formula:

Let $a_1 = ([a_1, b_1, c_1]; \overline{v_{a1}}, \overline{v_{a1}})$, $a_2 = ([a_2, b_2, c_2]; \overline{v_{a2}}, \overline{v_{a2}})$ be 2 ITrFNs. Next, we defined the NHD between $a_1$and $a_2$given below:

$d(a_1, a_2) = \frac{1}{8}[(1 + \overline{v_{a1}} - \overline{v_{a1}} - \overline{\pi_{a1}})a_1 - (1 + \overline{v_{a2}} - \overline{v_{a2}} - \overline{\pi_{a2}})a_2 + (1 + \overline{v_{a1}} - \overline{v_{a1}} - \overline{\pi_{a1}})b_1 - (1 + \overline{v_{a2}} - \overline{v_{a2}} - \overline{\pi_{a2}})b_2 + (1 + \overline{v_{a1}} - \overline{v_{a1}} - \overline{\pi_{a1}})c_1 - (1 + \overline{v_{a2}} - \overline{v_{a2}} - \overline{\pi_{a2}})c_2]$.

Assigning Weights to Decision Makers Using Different Numerical Methods

In numerical methods, a value u_j represents an approximation of the solution u(t) at the point t_j. These values form the numerical solutions to initial value problem. Examine the initial value problem over interval using various Runge-Kutta methods.

Butcher`s Seventh Order Runge-Kutta "Method:

Initial Value Problem given as $\frac{dy}{dx} = f(x, y), y(x_i) = y_i$

$$K_1 = f(x_m, y_m)$$

$$K_2 = f\left(x_m + \frac{1}{6}h, y_m + \frac{1}{6}K_1 h\right)$$

$$K_3 = f\left(x_m + \frac{1}{3}h, y_m + \frac{1}{3}K_2 h\right)$$

$$K_4 = f\left(x_m + \frac{1}{2}h, y_m + \frac{1}{8}K_1 h + \frac{1}{8}K_3 h\right)$$

$$K_5 = f\left(x_m + \frac{2}{11}h,\ y_m + 0.11119K_1 h + 0.11269K_3 h - 0.04207K_4 h\right)$$

$$K_6 = f\left(x_m + \frac{2}{3},\ y_m + 1.66255K_1 h - 6.2962K_3 h + 2.36566K_4 h + 6.2598K_5 h\right)$$

$K_7 = f\left(x_m + \frac{6}{7}, \, y_m + 10.275K_1h + 3.62099K_3h - 1.14095K_4h + 3.08853K_5h + 0.43856K_6h\right)$
$K_8 = f(x_m, \, y_m + 0.03265K_1h - 0.1781K_4h - 0.08904K_5h - 0.16436K_6h + 0.04283K_7h)$
$K_9 = f(x_m + h, \, y_m - 3.53\,K_1h - 8.87\,K_3h + 4.57K_4h + 8.20\,K_5h - 1.42\,K_6h + 0.73K_7h$
$+ 1.31K_8h\,)$
$y_{(n+1)} = y_n + 0.31hK_1 + 0.29hK_5 + 0.9hK_6 + 0.22hK_7 + 0.05hK_8 + 0.04hK_9$

Problem proposed by Decision maker 1:
For the initial Value problem $\frac{dy}{dx} = (1 + xy), y(0) = 2$ with h=0.1 on the interval use the Seventh Order Runge-Kutta Method.
Solution:
Given $\frac{dy}{dx} = (1 + xy), y(0) = 2$ with h=0.1
For $j = 0, x_0 = 0, y_0 = 2$
$K_1 = 1 + 0 = 1$
$K_2 = 1 + (0.01666666667 * 1.0166667) = 1.016944$
$K_3 = 1 + (0.03333333 * 1.033898) = 1.034463$
$K_4 = 1.051272$
$K_5 = 1.18516$
$K_6 = 1.934132$
$K_7 = 2.759188$
$K_8 = 1$
$K_9 = 0.910677$
The approximate value of $Y_1 = y(0.1) = \ 1.13713358$
Similarly, we have
$Y_2 = y(0.2) = \ 1.309215902897$
$Y_3 = y(0.3) = \ 1.535662107305$
$Y_4 = y(0.4) = \ 1.814725503459$
The Weighting Vectors are$W = (0.195416, 0.226065, 0.265166, 0.313353)$

Problem proposed by Decision maker 2:
Solve the initial Value problem $u^{`} = -2tu^2, u(0) = 1$ with h=0.2 using Fifth and Sixth order Runge-Kutta method.
Runge-Kutta Fifth Order method:
$K_1 = 0.1\big(-2(0)(1^2)\big) = 0$
$K_2 = 0.1(-2 * (0.025 * 1^2) = -0.05$
$K_3 = -0.04994$
$K_4 = -0.0995$
$K_5 = -0.14833$
$K_6 = -0.32373$
$Y_1 = y(0.1) = 0.9887764492$
Similarly, we have
$Y_2 = y(0.2) = \ 0.960955045834$
$Y_3 = y(0.3) = \ 0.907280916143$
$Y_4 = y(0.4) = \ 0.846382357577$
The Weighting Vectors are$\gamma = (0.266992, 0.259480, 0.244986, 0.228542)$

Runge-Kutta Sixth Order method:
$K_1 = 0$
$K_2 = -0.2$
$K_3 = -0.0995$
$K_4 = -0.13215$
$K_5 = -0.03451$

$K_6 = -1.05108$
$K_7 = -2.17796$
$Y_1 = y(0.1) = 0.956020122684$
Similarly, we have
$Y_2 = y(0.2) = 0.8973426900$
$Y_3 = y(0.3) = 0.5449213821$
$Y_4 = y(0.4) = 0.5357776305$
The Weighting Vectors are $W^{`} = (0.325836, 0.305837, 0.185723, 0.182603)$

MAGDM Problem Solving Algorithm:
Step 1: The IVIFWG "operator & Interval valued Intuitionistic fuzzy decision" matrix"is provided. $r_{ij}^{\sim(k)} = \left(\left[a_{ij}^{(k)}, b_{ij}^{(k)}, c_{ij}^{(k)}\right]; \mu_{ij}^{(k)}, \gamma_{ij}^{(k)}\right) = IVIFWG_w\left(r_{i1}^{\sim(k)}, r_{i2}^{\sim(k)}, \dots\dots, r_{in}^{\sim(k)}\right) i = 1,2 \dots n;\ k = 1,2, \dots\dots$.to determine the individual value$r_i^{\sim(k)}$.
Step 2: To compute totalIVIF values $r_j^{\sim}, (j = 1,2, \dots n)$ is alternativeuse IVIFHG operator.
Step 3: Compute difference among aggregate overall values $r_i^{\sim} = ([a_i, b_i, c_i];\ \mu_i, \gamma_i)$ and Interval Valued Intuitionistic fuzzy (IVIF) positive ideal value $r_i^{\sim+} = ([a^{\sim+}, b^{\sim+}, c^{\sim+}];\ \mu^{\sim+}, \gamma^{\sim+}) = ([1,1,1]; 1,0)$using the formula.
Step 4: Sort the alternatives in ascending order.
Numerical Illustration
An example is considered to implement the proposed method.
In this case, we use Butcher`s Fifth, Sixth and Seventh Order Runge-Kutta Method to find the weights.

Let us consider that a buyer wants to buy a mobilephone in a shop.Initially the buyer considers 5 possible alternatives: A_1 is Xiaomi,A_2 is OPPO, A_3 is Samsung, A_4 is Apple, and A_5isVivo. The buyer consults with a group of experts to assess these options. This panel of specialistsdetermines that the key factor is economic environments. After analysis, they identify four key attributes:
E_1: Display
E_2: Size and ergonomics
E_3: Storage
E_4: Compatibility
The possible alternatives (A_1, A_2, A_3, A_4, A_5) are to be examinedby employing intuitionistic fuzzy numbers by weight vector $W = (0.19542, 0.22606, 0.26516, 0.31335)^T$
under four attributes and the weighting vector $\gamma = (0.26699, 0.25948, 0.24498, 0.22854) W^{`} = (0.32584, 0.30584, 0.18573, 0.18260)$respectively. The decision matrices $R = (r_{2ij})_{4X4}^{k}$are

$$R_1 = \left(\begin{bmatrix} ([0.6,0.7]; 0.2,0.3) ([0.4,0.5]; 0.4,0.5) ([0.4,0.5]; 0.3,0.4) ([0.3,0.4]; 0.4,0.5) \\ ([0.3,0.4]; 0.3,0.4) ([0.1,0.2]; 0.2,0.3) ([0.6,0.7]; 0.1,0.3) ([0.1,0.2]; 0.6,0.7) \\ ([0.7,0.8]; 0.1,0.2) ([0.3,0.4]; 0.5,0.6) ([0.5,0.8]; 0.1,0.2) ([0.1,0.2]; 0.5,0.8) \\ ([0.5,0.6]; 0.1,0.3) ([0.2,0.3]; 0.4,0.6) ([0.4,0.5]; 0.2,0.3) ([0.2,0.3]; 0.4,0.5) \\ ([0.1,0.2]; 0.5,0.7) ([0.6,0.7]; 0.1,0.2) ([0.5,0.6]; 0.1,0.2) ([0.1,0.2]; 0.5,0.6) \end{bmatrix}\right)$$

$$R_2 = \left(\begin{bmatrix} ([0.4,0.5]; 0.2,0.4) ([0.3,0.5]; 0.4,0.5) ([0.4,0.6]; 0.3,0.4) ([0.3,0.4]; 0.4,0.6) \\ ([0.3,0.4]; 0.4,0.6) ([0.1,0.3]; 0.3,0.7) ([0.6,0.8]; 0.1,0.2) ([0.1,0.2]; 0.6,0.8) \\ ([0.6,0.7]; 0.1,0.2) ([0.3,0.4]; 0.4,0.5) ([0.7,0.8]; 0.1,0.2) ([0.1,0.2]; 0.7,0.8) \\ ([0.5,0.6]; 0.1,0.3) ([0.2,0.3]; 0.6,0.7) ([0.4,0.6]; 0.3,0.4) ([0.3,0.4]; 0.4,0.6) \\ ([0.1,0.3]; 0.3,0.5) ([0.6,0.8]; 0.1,0.2) ([0.5,0.6]; 0.2,0.4) ([0.2,0.4]; 0.5,0.6) \end{bmatrix}\right)$$

$$R_3 = \left(\begin{bmatrix} ([0.4,0.7]; 0.1,0.2) ([0.4,0.5]; 0.2,0.4) ([0.2,0.4]; 0.3,0.4) ([0.3,0.4]; 0.2,0.4) \\ ([0.3,0.5]; 0.3,0.4) ([0.2,0.4]; 0.4,0.5) ([0.6,0.8]; 0.1,0.2) ([0.1,0.2]; 0.6,0.8) \\ ([0.6,0.7]; 0.1,0.2) ([0.4,0.5]; 0.3,0.4) ([0.5,0.7]; 0.1,0.3) ([0.1,0.3]; 0.5,0.7) \\ ([0.5,0.6]; 0.1,0.3) ([0.1,0.2]; 0.7,0.8) ([0.5,0.7]; 0.2,0.3) ([0.2,0.3]; 0.5,0.7) \\ ([0.3,0.5]; 0.4,0.5) ([0.6,0.7]; 0.2,0.3) ([0.6,0.8]; 0.1,0.2) ([0.1,0.2]; 0.6,0.8) \end{bmatrix}\right)$$

$$R_4 = \left(\begin{bmatrix} ([0.6,0.7];0.2,0.3)([0.4,0.5];0.4,0.5)([0.4,0.5];0.3,0.4)([0.3,0.4];0.4,0.5) \\ ([0.3,0.4];0.3,0.4)([0.1,0.2];0.2,0.3)([0.6,0.7];0.1,0.3)([0.1,0.3];0.6,0.7) \\ ([0.7,0.8];0.1,0.2)([0.3,0.4];0.5,0.6)([0.5,0.8];0.1,0.2)([0.1,0.2];0.5,0.8) \\ ([0.5,0.6];0.1,0.3)([0.2,0.3];0.4,0.6)([0.4,0.5];0.2,0.3)([0.2,0.3];0.4,0.5) \\ ([0.1,0.2];0.5,0.7)([0.6,0.7];0.1,0.2)([0.5,0.6];0.1,0.2)([0.1,0.2];0.5,0.6) \end{bmatrix}\right)$$

Step 1: By implementing the algorithm, we derive the collective decision matrix with IVIF values

$$\tilde{r_{11}} = \Big[[(0.6)^{0.1954155} * (0.4)^{0.226065} * (0.4)^{0.265166} * (0.3)^{0.313352}, (0.7)^{0.1954155} * (0.5)^{0.22065} * (0.5)^{0.265166} * (0.4)^{0.313352}]; 1 - [(1-0.2)^{0.1954155} * (1-0.4)^{0.226065} * (1-0.3)^{0.265166} * (1-0.4)^{0.313352}], 1 - [(1-0.3)^{0.1954155} * (1-0.5)^{0.226065} * (1-0.4)^{0.265166} * (1-0.5)^{0.313352}]\Big]$$

$\tilde{r_{11}} = ([0.39565901, 0.49791896]; 0.33882221, 0.43956933)$

Similarly,

$\tilde{r_{12}} = ([0.19933172, 0.36249594]; 0.35286814, 0.47915436)$
$\tilde{r_{13}} = ([0.28730895, 0.44297615]; 0.34454348, 0.55702385)$
$\tilde{r_{14}} = ([0.28748598, 0.39334534]; 0.29904238, 0.44490677)$
$\tilde{r_{15}} = ([0.22975042, 0.35525598]; 0.33262808, 0.46847894)$
$\tilde{r_{21}} = ([0.34250410, 0.48923700]; 0.33882221, 0.49292474)$
$\tilde{r_{22}} = ([0.19933172, 0.36250173]; 0.39074322, 0.63749827)$
$\tilde{r_{23}} = ([0.30479999, 0.43156659]; 0.41799450, 0.53410469)$
$\tilde{r_{24}} = ([0.32643398, 0.45177284]; 0.38268600, 0.53441116)$
$\tilde{r_{25}} = ([0.285448606, 0.49248265]; 0.30918828, 0.45581868)$
$\tilde{r_{31}} = ([0.30414948, 0.46931172]; 0.20985195, 0.36530333)$
$\tilde{r_{32}} = ([0.23314647, 0.40410305]; 0.39361497, 0.55957358)$
$\tilde{r_{33}} = ([0.29751648, 0.49745866]; 0.29274118, 0.46790442)$
$\tilde{r_{34}} = ([0.26077233, 0.39238655]; 0.43398537, 0.59561358)$
$\tilde{r_{35}} = ([0.29887488, 0.45860058]; 0.37207129, 0.54139942)$
$\tilde{r_{41}} = ([0.39565901, 0.49791896]; 0.33882221, 0.43956933)$
$\tilde{r_{42}} = ([0.19933172, 0.36249594]; 0.35286814, 0.47915436)$
$\tilde{r_{43}} = ([0.28730895, 0.44297615]; 0.34454348, 0.55702385)$
$\tilde{r_{44}} = ([0.28748598, 0.39334534]; 0.29904238, 0.44490677)$
$\tilde{r_{45}} = ([0.22975042, 0.35525598]; 0.33262808, 0.46847894)$

Step 2: Use the Hybrid Geometric operator with the current intuitionistic triangular fuzzy matrix.

$z_1 = ([0.5962557243, 0.6964162389]; 0.3386714268, 0.6599878869)$
$z_2 = ([0.4476689681, 0.6036508928]; 0.6052125968, 0.7307335252)$
$z_3 = ([0.5379510925, 0.665946491]; 0.5918126548, 0.7266542044)$
$z_4 = ([0.5374383598, 0.6366931811]; 0.583209435, 0.7010931998)$
$z_5 = ([0.5028208923, 0.6377522474]; 0.5714026661, 0.6867708629)$

Step 3: Compute the distance formula between $\tilde{z_i} = ([a_i, b_i, c_i]; \mu_i, \gamma_i)$ and $z^{\sim +} = ([1,1,1]; 1,0)$

$d(\tilde{z_1}, z^{\sim +}) = 0.3905522355$
$d(\tilde{z_2}, z^{\sim +}) = 0.3409319942$
$d(\tilde{z_3}, z^{\sim +}) = 0.3218795438$
$d(\tilde{z_4}, z^{\sim +}) = 0.3288088518$
$d(\tilde{z_5}, z^{\sim +}) = 0.3370683668$

Step 4: Sort theoption, $A_k (k = 1, 2, 3, 4)$

$$A_3 < A_4 < A_5 < A_2 < A_1$$

As a result, the best option is A_3
A_3: is the finest option.

Conclusion

Various Runge-Kutta methods are used to compute numerical solutions for ordinary differential equations and to establish decision maker weights in MAGDM problems with IVIF numbers. Aggregation of values is performed using operators such as the IVIFWG and IVIFHG. The MAGDM problem is tackled to select the best option, with alternatives ranked according to a distance formula. The proposed method's effectiveness is demonstrated.

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