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Measuring Relative Efficiency of Life Insurance Companies with Data Envelopment and Fuzzy Data Envelopment Analysis

S. JASMINE RATHI *[1], V. PRAKASH[2]

[1]*Department of Statistics, Madras Christian College, University of Madras, Chennai, India.*

[2]*Department of Statistics, Dr.Ambedkar Govt. Arts College, University of Madras, Chennai, India.*

Abstract

Insurance sector in India is one of the flourishing sectors of the economy. The insurance sector plays a vital role as it minimizes the risk and mobilizes the resource. Studying the efficiency of the insurance companies is highly important because it helps in determining how insurance companies respond to the various challenges and how many of them are likely to survive those challenges in the event of their occurrence. So, it becomes necessary to analyse the performance of life insurance companies in India with the application of Data Envelopment Analysis (DEA) and its extension Fuzzy Data Envelopment Analysis (FDEA). The empirical results of the Conventional DEA models and Fuzzy DEA models are computed to trace the performance of decision making unit at a possibility level. The computed fuzzy efficiency scores need to be ranked in order to determine how each DMU performs relative to the other DMUs in an uncertain environment. Chen and Klein (1997) ranking method were applied to rank the fuzzy numbers. To compare the two ranking methods the concept of Spearman rank correlation is applied. From this study the author infer that fuzzy DEA is more efficient than conventional DEA (CRS and VRS) Approach.

Keywords: Data Envelopment Analysis (DEA), Fuzzy Data Envelopment Analysis (FDEA), Life insurance sector, Fuzzy Numbers, Chen and Klein Ranking Index, Spearman rank Correlation

I. Introduction

The insurance sector has played a significant part in India's financial system.it has also been promoting an objective to build an efficient, effective and a stable economic environment in India. The insurance sector of India has observed and witnessed tremendous change. Earlier there was only public insurance companies but with the entry of private insurance companies the picture of insurance sector changed.There was only one life insurance firm functioning in India till 2000, and there was no fierce competition in the industry. After formation of IRDA in the year 1999 many private companies entered in to life insurance market in India. Vital changes were brought in by the private players. Several insurance companies in the country are expanding their operations across both public and private sectors.The insurance industry provides financial security as well as financial intermediation to both individuals and businesses in the economy, hence improving the nation's financial and economic development. It is necessary that the companies must be efficient to survive in the competitive sector There are many tools related for the performance measurement. Of these, Data Envelopment Analysis (DEA) is a powerful and popularly used tool for measuring the performance of unique set of organizations. So, the above facts motivated the author to evaluate the technical efficiency of life insurance companies usingConventional DEA(CRS and VRS) and Fuzzy DEA approach.

Data Envelopment Analysis (DEA) is a mathematical modelling technique based on linear programming approach to assess the relative efficiency of a group of similar decision making units (DMUs). Decision making units are described by multiple inputs and multiple outputs. Basically, there are two formulations of DEA namely Charnes Cooper and Rhodes (1978) CCR model and Banker, Charnes Cooper (1984) BCC model. Each model has two orientations namely input-oriented and output-oriented model. In the first case, DEA methods defines frontier by seeking the maximum possible proportional reduction in input usage, with output level remain constant for each DMU. However, in the output-oriented situation, the highest proportionate growth in output production is achieved with the input level unchanged. CCR model is based on Constant Return to Scale (CRS) assumption whereas BCC model is based on Variable Return to Scale (VRS) assumption. Both CCR and BCC models are developed in terms of multiplier and envelopment version. When inputs and outputs are emphasized multiplier versions can be used in an application. Envelopment versions are used when the relations among the DMUs are to be emphasized. Fuzzy Data Envelopment Analysis (FDEA) was developed with the combination of fuzzy set theory - Zadeh (1965) and DEA (CCR and BCC) models. It is more capable of dealing with imprecise and ambiguous data.

FDEA model can be used to deal with all kinds of input and output under fuzzy numbers.Crisp input and output are fundamentally indispensable in Conventional DEA. However the observed value of the input and output data in real world situations are sometimes imprecise or vague. Due to the uncertainty, the data have complex input and output so we use the concept of fuzzy parametric programming to deal with DEA framework under fuzzy environment. (Despotis and Smirlis, 2002) and (Wang et al, 2005) generated the following models to create lower and upper bounds of interval efficiency of each DMU in order to mention the uncertainty state in the condition that was known to locate between lower and upper bounds The α-level set (α-cut) of a fuzzy set $\tilde{A}$ is acrisp subset of X and is applied and is denoted by

$$\tilde{A}_\alpha = \{x \mid \mu_A(x) \geq \alpha \text{and } x \in X\}$$

Each α-cut level of a fuzzy number is a closed interval which can be represented as [L(α), U(α)], where L(α) is lower bound and U(α) is upper bound at a defined α-cut level α. Fuzzy numbers are determined by a family of α-cut levels. Therefore, the interval of confidence at a given α-cut level, where L is the lower bound and U is upper bound, can be characterized as:

$$\forall\ \alpha \in [0:1], A_\alpha = [L = \alpha(a^m - a^l) + a^l, U = a^u - \alpha(a^u - a^m)]$$

If $\tilde{A} = (a^m, a^l, a^u)$then, is called a triangular fuzzy number. A triangular fuzzy number has the following membership function:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a^l}{a^m-a^l}, & a^l \leq x \leq a^m, \\ 1, & x = a^m, \\ \frac{a^u-x}{a^u-a^m}, & a^m \leq x \leq a^u, \\ 0, & \text{Otherwise} \end{cases} \quad (1)$$

Based on the concepts of fuzzy set theory, fuzzy DEA was generated to measure the efficiency of uncertain data using the models based on α-level based approach.

The objective of this study is to analyse the technical efficiency of life insurance companies for the financial year 2018- 2019 using DEA and Fuzzy DEA approach. The author examined 24 life insurance businesses from the Insurance Regulatory and Development Authority of India's (IRDA) annual report in order to achieve these goals. The rest of this paper is organized as follows: Section II deals with the Review of literature. III describe the Data Structure, Methodology is presented in Section IV, Empirical Investigations are carried out in Section V and Findings and Conclusions are outlined in Section VI.

II Review of Literature

Farrell (1957)and others suggested a measure of technical efficiency in terms of deviation of observed points from the points on the frontier constructed from the observed points. Zadeh L.A (1965) developed fuzzy set algebra which is the formal body of theory that allows the treatment of imprecise and vague estimates in uncertain environments. R.E Bellman and Zadeh L.A (1970) developed a general theory of decision making in a fuzzy environment. The result presented in this article was considered as an initial attempt at constructing a conceptual framework for such a theory. Charnes et al. (1978 and 1979) have made pioneering contributions for the study of DEA. H. J. Zimmerman (1978)presented the application of fuzzy linear programming approaches to the linear vector maximum problem. It reveals that the solutions obtained by fuzzy linear programming are always efficient solutions. Banker (1984) for the first time has developed procedures for estimating most productive scale size using DEA. Banker et al. (1984) introduced a new separate variable which makes it possible to determine whether operations are conducted in regions of increasing, constant, or decreasing returns to scale (in multiple input and multiple output situations). Zhu (1996) based on Russel measure developed some weighted non radial CCR models by specifying a proper set of "preference weights" that reflect the degree of disability of the potential adjustments of current inputs or output levels. This approach gives a scalar efficiency score for DMU to secure comparability. Chen C-B and Klein CM (1997) proposed a ranking method based on an area measurement method. This study considers the application of α-cuts and fuzzy arithmetic operations to the fuzzy weighted average method which can be used to rank aggregated fuzzy utilities. Kahraman and Tolga (1998)proposed a fuzzy mathematical programming by assuming that the values of inputs and outputs in DEA are not known with certainty. Despotis and Smirlis (2002) developed and provided an alternative approach for dealing with imprecise data in DEA. In this approach the authors transformed a non-linear DEA model to a linear programming equivalent as in IDEA. Kao and Liu (2000a)[8] developed a procedure to measure the efficiency of DMUs with fuzzy observations. This study suggested the uses of α-cut level sets to transform fuzzy data into interval data so that the fuzzy model becomes a family of conventional crisp DEA models. Leon et al. (2003) have made use of fuzzy linear programming to provide a new approach to the problem of assessing efficiency with DEA models. This approach has been based on two ranking methods that compare α-cuts. Ying-Ming Wang et al. (2005)have developed a new pair of interval DEA models for dealing with imprecise data such as interval data, ordinal preference information, fuzzy data, and their mixture. They claimed that their interval DEA models are much easier to understand and more convenient to use comparatively with IDEA models developed by Despotis and Smirlis. Kao and Liu (2005) applied fuzzy data envelopment analysis model (FDEA) to calculate the efficiency scores of a sample of missionary firms in Taiwan. S.H. Razavi et al. (2013) extended the original DEA models namely CCR and BCC to a fuzzy version and its solution approach is developed with a basic idea to transform the original DEA model to an equivalent linear parametric programming model applying the notion of α-cuts. Pejman Peykani, Emran Mohammadi, Ali Emrouznejad, Mir Saman Pishvaee, Mohsen Rostamy – Malkhalifeh (2019) proposed a novel fuzzy DEA model which determines the attitude of the DMUs through optimistic and pessimistic parameters. Natthan Singh et al. (2020) analysed the performance of 8 selected paper mills in India with the application of FDEA.

III. Data structure

The data used in this study is a secondary source which is obtained from the Annual report of life insurance companies published by The Insurance Regulatory and Development Authority of India (IRDAI). 24 life insurance companies from all around India make up the sample. The study relates to a period of 2018 – 2019. Each life insurance company is considered as a Decision-MakingUnit and it includes three inputs and three outputs which are described below: Inputs (in Crores) - Operating expenses, Policy Liabilities and Fixed Asset. Outputs (in Crores)- Net Premium, Income from Investment and Investments (shareholders& policy holders).

IV. Methodology

Data Envelopment Analysis (DEA) is a linear programming technique used for measuring the relative efficiency of a homogeneous set of decision-making units by analyzing their multiple inputs with multiple outputs. Basically, there are two formulations of DEA namely Charnes Cooper and Rhodes (1978) CCR model and Banker, Charnes Cooper (1984) BCC model. The BCC model is an extension of the CCR model in which a convex curve that passes through every efficient DMU represents the efficient frontier set. DEA can be output-oriented or input-oriented.In the first case, DEA methods define frontier by seeking the maximum possible proportional reduction in input usage, with output level remains constant, for each DMU. However, in output-oriented case, maximum proportional increase in output production with input level being fixed. Input oriented (CCR and BCC) model is used to compute the overall technical efficiency scores and ranking of decision-making units. And the same are presented below:

$Min\ \theta_k$

Subject to,

$$\sum_{j=1}^{n} x_{ij}\lambda_j \leq \theta_k\, x_{ik} \qquad i = 1,2, \dots\dots, m$$

$$\sum_{j=1}^{n} y_{rj}\lambda_j \geq y_{rk} \qquad r = 1,2, \dots\dots, s$$

$$\lambda_j \geq 0 \qquad j = 1,2 \dots\dots, n$$

where

$\theta_k$is the input- oriented technical efficiency of the k^{th} DMU

'λ' is the weight assigned to the j^{th}DMU where $(j = 1,2, \dots, n)$

$x_{ik}$is the amount of i^{th} input required by the k^{th} DMU, where $(i = 1,2, \dots, m)$

y_{rk} is the amount of r^{th}output produced by the k^{th} DMU, where $(r = 1,2, \dots, s)$

The aforementioned model is solved to determine the efficiency score of DMUs θ_k and λ_j.if $\theta_k = 1, \lambda_j = 1$ for $\lambda_j = \lambda_k$ and $\lambda_j = 0$for all other DMUsthen the observed DMU is efficient Otherwise, if $(\lambda_k < 1)$the observed DMU is inefficient.

Min θ_k

Subject to,

$$\sum_{j=1}^{n} x_{ij}\, \lambda_j \leq \theta_k\, x_{ik} \qquad i = 1,2, \dots, m$$

$$\sum_{j=1}^{n} y_{rj}\, \lambda_j \geq y_{rk} \qquad r = 1,2, \dots, s$$

$$\sum_{j=1}^{n} \lambda_j = 1 \qquad j = 1,2, \dots, n$$

$$\lambda_j \geq 0$$

where $\theta_k$is the input- oriented technical efficiency of the k^{th} DMU

'λ' is the weight assigned to the j^{th}DMU where $(j = 1,2, \dots, n)$and other notations are similar to the CCR model. Solving the above model if the optimal value of the objective functions satisfies $\theta_k^* = 1$ and has no slacks then the DMU_k is BCC efficient otherwise it is BCC inefficient. Saati et al(2002) present a fuzzy variant of the CCR model with triangular fuzzy numbers. and the same is elaborated below:

$(E_j)^L_{\alpha i}$ = Min θ

Subject to

$[\ \theta(\alpha x^m_{i0} + (1-\alpha)x^l_{i0}] \geq \left[\sum_{j=1}^{n} \lambda_j \left(\alpha x^m_{ij} + (1-\alpha)x^l_{ij}\right)\right] \quad \forall\ i,$

$[\ \theta(\alpha y^m_{r0} + (1-\alpha)y^l_{r0} \leq \left[\sum_{j=1}^{n} \lambda_j \left(\alpha y^m_{rj} + (1-\alpha)y^l_{rj}\right)\right] \forall\ i,$

$\lambda_j \geq 0$

$(E_j)^U_{\alpha i}$ = Min θ

Subject to

$[\ \theta(\alpha x^m_{i0} + (1-\alpha)x^u_{i0})] \geq \left[\sum_{j=1}^{n} \lambda_j\ (\alpha x^m_{ij} + (1-\alpha)x^u_{ij}\right] \forall\ i,$

$[\ \theta(\alpha y^m_{r0} + (1-\alpha)y^u_{r0})] \leq \left[\sum_{j=1}^{n} \lambda_j\ (\alpha y^m_{rj} + (1-\alpha)y^u_{rj}\right] \forall\ i,$

$\lambda_j \geq 0$

By solving the above models, the fuzzy efficiency scores can be obtained and these computed fuzzy efficiency scores are viewed as a fuzzy variable in the range [0,1]. The computed fuzzy efficiency scores need to be ranked in order to determine how each DMU performs relative to the other DMUs in an uncertain environment. many methods for ranking fuzzy numbers have been proposed, of which the ranking method proposed by Chen and Klein (1997) is considered in this study and described below.

Chen -Klein ranking method

Let $(E_j)^L_{\alpha i}$ be the lower bound technical efficiency scores and $(E_j)^U_{\alpha i}$ be the upper bound technical efficiency scores. Chen and Klein (1997) developed the following index to rank fuzzy numbers:

$$I_j = \frac{\sum_{i=0}^{N} \quad ((E_j)^U_{\alpha i} - c)}{\sum_{i=0}^{N} \quad ((E_j)^U_{\alpha i} - c) - \sum_{i=0}^{N} \quad ((E_j)^L_{\alpha i} - d)},\ N \to \infty \quad (2)$$

where $c = min_{i,j}\{(E_{ji})^L_{\propto i}\}$ and $d = max_{i,j}\{(E_{ji})^U_{\propto i}\}$. The lower bound and upper bound efficiency indices are represented by $(E_{ji})^L_{\propto i}$ and $(E_{ji})^U_{\propto i}$.

The larger value of the ranking index indicates the fuzzy number is more preferred. Ranking indices for each DMU is computed using Chen - Klein method,the value of N should be used is that N=3 0r 4 is sufficient to discriminate the difference. To compare the two ranking methods the concept of Spearman rank correlation is applied.

$$\rho = 1 - 6\left[\frac{\sum D_i^2 + \frac{1}{12}(m_1^3 - m_1) + \frac{1}{12}(m_2^3 - m_2) + \cdots}{n(n^2-1)}\right] \quad (3)$$

where m_i is the number of repetitions of i^{th} rank.

V. Empirical investigation

Empirical investigation for the data structure presented in Section III is carried out and the results are presented below. Efficiency scores of Conventional DEA model and Fuzzy CCR DEA model are presented in the following tables.

Table 1: Efficiency Scores of CRS and VRS Model

S.No.	DMUs	CRS-TE	VRS-TE
1.	Aviva	**1.000**	**1.000**
2.	HDFC	0.917	**1.000**
3.	Birla Sun	0.942	0.955
4.	ICICI Prudential	0.356	0.852
5.	Kotak	0.947	**1.000**

6.	IndiaFirst	**1.000**	**1.000**
7.	LIC	**1.000**	**1.000**
8.	Aegon	**1.000**	**1.000**
9.	Shriram	0.838	0.848
10.	IDBI Federal	0.971	**1.000**
11.	Sahara	**1.000**	**1.000**
12.	Edelweiss	**1.000**	**1.000**
13.	SBI	**1.000**	**1.000**
14.	Max Life	**1.000**	**1.000**
15.	Exide	**1.000**	**1.000**
16.	TATA AIA	0.945	0.964
17.	DHFL Pramerica	**1.000**	**1.000**
18.	Canara HSBC	**1.000**	**1.000**
19.	Bajaj Allianz	**1.000**	**1.000**
20.	Bharti AXA	**1.000**	**1.000**
21.	Future Generali	0.757	0.784
22.	Reliance Nippon	**1.000**	**1.000**
23.	Star Union	0.878	0.880
24.	PNB Met	0.780	0.781
25.	**Average**	**0.9304**	**0.9598**

Table 2: Fuzzy efficiency scores of Life Insurance Companies under α-values

DMUs	$\alpha = 0$	$\alpha = 0.25$	$\alpha = 0.50$	$\alpha = 0.75$	$\alpha = 1$
1	(0.356, 1.000)	(0.460, 1.000)	(0.601, 1.000)	(0.797, 1.000)	(1.000, 1.000)
2	(0.549, 1.000)	(0.626, 1.000)	(0.727, 1.000)	(0.834, 0.994)	(0.916, 0.916)
3	(0.270, 1.000)	(0.395, 1.000)	(0.553, 1.000)	(0.760, 1.000)	(0.941, 0.941)
4	(0.213, 0.475)	(0.257, 0.442)	(0.302, 0.412)	(0.329, 0.383)	(0.356, 0.356)
5	(0.641, 1.000)	(0.717, 1.000)	(0.808, 1.000)	(0.886, 1.000)	(0.947, 0.947)
6	(0.807, 1.000)	(0.983, 1.000)	(1.000, 1.000)	(1.000, 1.000)	(1.000, 1.000)
7	(0.377, 1.000)	(0.689, 1.000)	(1.000, 1.000)	(1.000, 1.000)	(1.000, 1.000)

8	(0.753, 1.000)	(0.814, 1.000)	(0.880, 1.000)	(0.952, 1.000)	(1.000, 1.000)
9	(0.551, 1.000)	(0.625, 1.000)	(0.715, 0.955)	(0.785, 0.894)	(0.838, 0.838)
10	(0.677, 1.000)	(0.750, 1.000)	(0.821, 1.000)	(0.895, 1.000)	(0.971, 0.971)
11	(0.248, 1.000)	(0.401, 1.000)	(0.607, 1.000)	(0.855, 1.000)	(1.000, 1.000)
12	(0.493, 1.000)	(0.637, 1.000)	(0.825, 1.000)	(0.960, 1.000)	(1.000, 1.000)
13	(0.643, 1.000)	(0.887, 1.000)	(1.000, 1.000)	(1.000, 1.000)	(1.000, 1.000)
14	(0.788, 1.000)	(0.858, 1.000)	(0.927, 1.000)	(1.000, 1.000)	(1.000, 1.000)
15	**(1.000, 1.000)**	**(1.000, 1.000)**	**(1.000, 1.000)**	**(1.000, 1.000)**	**(1.000, 1.000)**
16	(0.238, 1.000)	(0.372, 1.000)	(0.551, 1.000)	(0.775, 1.000)	(0.945, 0.945)
17	(0.871, 1.000)	(0.931, 1.000)	(0.993, 1.000)	(1.000, 1.000)	(1.000, 1.000)
18	**(1.000, 1.000)**	**(1.000, 1.000)**	**(1.000, 1.000)**	**(1.000, 1.000)**	**(1.000, 1.000)**
19	(0.467, 1.000)	(0.611, 1.000)	(0.799, 1.000)	(1.000, 1.000)	(1.000, 1.000)
20	(0.821, 1.000)	(0.886, 1.000)	(0.955, 1.000)	(1.000, 1.000)	(1.000, 1.000)
21	(0.443, 0.984)	(0.526, 0.921)	(0.631, 0.863)	(0.708, 0.808)	(0.756, 0.756)
22	(0.808, 1.000)	(0.868, 1.000)	(0.932, 1.000)	(0.997, 1.000)	(1.000, 1.000)
23	(0.444, 1.000)	(0.516, 1.000)	(0.613, 1.000)	(0.741, 0.938)	(0.878, 0.878)
24	(0.418, 1.000)	(0.512, 0.954)	(0.633, 0.892)	(0.727, 0.834)	(0.779, 0.779)

The above results indicate that there are two DMUs namely Exide Life (DMU15) and Canara HSBC Life (DMU18) insurance companies with upper and lower bounds of efficiency score of 1 at all levels of α. Hence, they are efficient insurance companies surely. These insurance companies of course are also efficient under the conventional crisp DEA model. However, there are 20 inefficient insurance companies with at least one upper bound to be 1 still has the possibility to be efficient. The other two insurance companies namely ICICI Prudential Life (DMU4) and Future Generali (DMU21) are unlikely to be efficient since all of their upper and lower bounds are less than 1.

Further in order to determine how each DMU performs relative to the other DMUs in an uncertain environment, the computed fuzzy efficiency scores need to be ranked. The ranking of the computed fuzzy efficiency score is then compared with the ranking of conventional DEA scores in order to discriminate which decision-making units are sensitive to the variation of the inputs or outputs variable measurement inaccuracy(recall bias). We use Chen - Klein's method (1997) to compute an index I, when N=4 (the number of α-cuts) for ranking fuzzy efficiency scores of all the DMUs. The Chen Klein index is computed, and it is presented in the following table.

Table 4: Chen - Klein Index

DMU	Lower bound efficiency scores							Upper bound efficiency scores							N=4
	Alpha 0	Alpha 0.25	Alpha 0.5	Alpha 0.75	Alpha 1	d	L	Alpha 0	Alpha 0.25	Alpha 0.5	Alpha 0.75	Alpha 1	C	U-c	Index
1	0.3564	0.4600	0.6018	0.7976	1.0000	1.00	0.000	1.0000	1.0000	1.0000	1.0000	1.0000	0.213	0.786	0.6381
2	0.5491	0.6268	0.7277	0.8349	0.9168	1.00	0.083	1.0000	1.0000	1.0000	0.9942	0.9168	0.213	0.703	0.7134
3	0.2708	0.3959	0.5533	0.7608	0.9418	1.00	0.058	1.0000	1.0000	1.0000	1.0000	0.9418	0.213	0.728	0.6091
4	0.2134	0.2571	0.3024	0.3297	0.3562	1.00	0.643	0.4758	0.4428	0.4120	0.3832	0.3562	0.213	0.142	0.2289
5	0.6410	0.7173	0.8089	0.8868	0.9471	1.00	0.052	1.0000	1.0000	1.0000	1.0000	0.9471	0.213	0.733	0.7688
6	0.8075	0.9833	1.0000	1.0000	1.0000	1.00	0.000	1.0000	1.0000	1.0000	1.0000	1.0000	0.213	0.786	0.9376
7	0.3776	0.6892	1.0000	1.0000	1.0000	1.00	0.000	1.0000	1.0000	1.0000	1.0000	1.0000	0.213	0.786	0.7712
8	0.7531	0.8141	0.8808	0.9520	1.0000	1.00	0.000	1.0000	1.0000	1.0000	1.0000	1.0000	0.213	0.786	0.8398
9	0.5513	0.6251	0.7155	0.7851	0.8382	1.00	0.161	1.0000	1.0000	0.9551	0.8948	0.8382	0.213	0.624	0.6937
10	0.6776	0.7502	0.8214	0.8957	0.9711	1.00	0.028	1.0000	1.0000	1.0000	1.0000	0.9711	0.213	0.757	0.7863
11	0.2486	0.4015	0.6070	0.8553	1.0000	1.00	0.000	1.0000	1.0000	1.0000	1.0000	1.0000	0.213	0.786	0.6250
12	0.4937	0.6376	0.8256	0.9604	1.0000	1.00	0.000	1.0000	1.0000	1.0000	1.0000	1.0000	0.213	0.786	0.7439
13	0.6437	0.8875	1.0000	1.0000	1.0000	1.00	0.000	1.0000	1.0000	1.0000	1.0000	1.0000	0.213	0.786	0.8703
14	0.7886	0.8582	0.9271	1.0000	1.0000	1.00	0.000	1.0000	1.0000	1.0000	1.0000	1.0000	0.213	0.786	0.8807
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.00	0.000	1.0000	1.0000	1.0000	1.0000	1.0000	0.213	0.786	1.0000
16	0.2388	0.3722	0.5511	0.7750	0.9454	1.00	0.054	1.0000	1.0000	1.0000	1.0000	0.9454	0.213	0.732	0.6039
17	0.8710	0.9314	0.9936	1.0000	1.0000	1.00	0.000	1.0000	1.0000	1.0000	1.0000	1.0000	0.213	0.786	0.9391
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.00	0.000	1.0000	1.0000	1.0000	1.0000	1.0000	0.213	0.786	1.0000
19	0.4675	0.6112	0.7998	1.0000	1.0000	1.00	0.000	1.0000	1.0000	1.0000	1.0000	1.0000	0.213	0.786	0.7372
20	0.8219	0.8860	0.9559	1.0000	1.0000	1.00	0.000	1.0000	1.0000	1.0000	1.0000	1.0000	0.213	0.786	0.9034
21	0.4430	0.5261	0.6316	0.7083	0.7566	1.00	0.243	0.9841	0.9218	0.8632	0.8083	0.7566	0.213	0.543	0.6169
22	0.8089	0.8684	0.9326	0.9977	1.0000	1.00	0.000	1.0000	1.0000	1.0000	1.0000	1.0000	0.213	0.786	0.8890
23	0.4444	0.5165	0.6136	0.7416	0.8784	1.00	0.121	1.0000	1.0000	1.0000	0.9384	0.8784	0.213	0.665	0.6468
24	0.4189	0.5129	0.6337	0.7271	0.7796	1.00	0.220	1.0000	0.9543	0.8924	0.8342	0.7796	0.213	0.566	0.6234
Min	**0.2134**	**0.2571**	**0.3024**	**0.3297**	**0.3562**		**Min**	**0.4758**	**0.4428**	**0.4120**	**0.3832**	**0.3562**			
Max	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**		**Max**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**			

Based on the ranking indices the efficiencies of the evaluated DMUs are ranked accordingly. The Chen-Klein ranking is then compared to the ranking of the Crisp technical efficiency scores from the Conventional DEA (CCR) model and all these are presented in the following table.

Table 5: Ranking of Crisp and Fuzzy efficiency scores

Rank	DMU	Insurance Companies	Chen-Klein Index	CCR-Technical Efficiency
1	**15**	**Exide**	**1.000**	**1.0000**
2	**18**	**Canara HSBC**	**1.000**	**1.0000**
3	17	DHFL Pramerica	0.939	1.0000
4	6	IndiaFirst	0.937	1.0000
5	20	Bharti AXA	0.903	1.0000
6	22	Reliance Nippon	0.889	1.0000
7	14	Max Life	0.880	1.0000
8	13	SBI	0.870	1.0000
9	8	Aegon	0.839	1.0000
10	10	IDBI Federal	0.786	0.971
11	7	LIC	0.771	1.000
12	5	Kotak	0.768	0.947
13	12	Edelweiss	0.744	1.000
14	19	Bajaj Allianz	0.737	1.000
15	2	HDFC	0.713	0.916
16	9	Shriram	0.693	0.838
17	23	Star Union	0.646	0.878
18	1	Aviva	0.638	1.000
19	11	Sahara	0.625	1.000
20	24	PNB Met	0.623	0.779
21	21	Future Generali	0.617	0.756
22	3	Birla Sun	0.609	0.941
23	16	Tata AIA	0.604	0.945
24	4	ICICI Prudential	0.228	0.356

From the result of the above table, it may be observed that Chen-Klein index identifies 2 decision making units 15 (Exide Life) and 18 (Canara HSBC) are efficient whose efficiency score is 1 and hence defining the frontier. But the Conventional DEA model identifies 14 decision making units (6,7,13,14,15,17,18,19,20,22,12,8,11 and 1) are efficient and so defining the frontier. It is interesting to note that the two decision making units 15 and 18 are efficient under fuzzy and conventional DEA model. However, there are 12 decision making units (17,6,20,22,14,13,8,7,12,19,1,11) are efficient with efficiency score of 1 under non fuzzy conditions while the ranking indices are less than one with rankings between 3 to 19 under fuzzy conditions. That is these decision making units gets at least one lower bound value is less than one under fuzzy conditions. This may indicate that the fuzzy DEA approach might have higher ability to discriminate between efficient and inefficient insurance companies than the conventional DEA approach.Additionally, when compared to the ranking of Crisp technical efficiency ratings, the Chen-Klein ranking index produces comparable results. So, the author attempted Spearman's rank correlation of the two ranking methods and is found to be $\boldsymbol{\rho = 0.718}$ which is significant at < 1%. This indicates that there is a strong correlation between Chen-Klein ranking index and the Crisp technical efficiency scores.

VI. CONCLUSIONS

Accurate measurements of the inputs and outputs are necessary for traditional DEA techniques. Real-world situations, however, can have ambiguous or imprecise observed values for the input and output data. Inaccurate assessment could be caused by information that is unavailable, insufficient, or not measurable. The results of the CCR-DEA model indicate that On an average the set of life insurance companies included in this study are 93% efficient. BCC-DEA results reveal that On an average the set of life insurance companies are found to be 95% efficient. The results of Conventional DEA model indicates that VRS model perform efficiently than CRS model. α-cut level approach under Fuzzy CCR model provides Only two life insurance companies namely Exide Life (DMU15) and Canara HSBC Life (DMU18) remains efficient at all α-cut level and hence forms the frontier. The remaining life insurance companies are affected differently due to the imprecision in data. Fuzzy DEA scores are exactly same as conventional DEA scores for all DMUs at $\alpha = 1$. So, it may be inferred that the fuzzy DEA at α-cut level = 1 reduces to the CRS DEA. Chen-Klein ranking index reveals that the fuzzy DEA approach have higher ability to discriminate between efficient and inefficient life insurance companies.

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