Contra θ Open Maps and Homeomorphisms in Fuzzy Hypersoft Topological Spaces

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Abstract
In this paper, the concepts of fuzzy hypersoft contra θ open and fuzzy hypersoft contra θ closed mappings in fuzzy hypersoft topological spaces are introduced and their related properties are studied. Also, the relation between fuzzy hypersoft contra θ open, fuzzy hypersoft contra θ semi open, fuzzy hypersoft contra θ pre open, fuzzy hypersoft contra θ closed, fuzzy hypersoft contra θ semi closed and fuzzy hypersoft contra θ pre closed maps are analyzed with the examples. Moreover, the work is devoloped to fuzzy hypersoft contra θ homeomorphism, fuzzy hypersoft contra θ C homeomorphism and some of their characteristics are discussed.

Keywords: Fuzzy hypersoft contra θ open maps, fuzzy hypersoft contra θ closed maps, fuzzy hypersoft contra θ homeomorphism and fuzzy hypersoft conta θ C homeomorphism.
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1 Introduction

The real-world decision-making problems in medical diagnosis, engineering, economics, management computer science, artificial intelligence, social sciences, environmental science and sociology contain more uncertain and inadequate data. Traditional mathematical methods cannot deal with these kinds of problems due to imprecise data. To deal with the problems with uncertainty, Zadeh [28] introduced the fuzzy set in 1965 which contains the membership value in [0,1]. A fuzzy set is a set where each element of the universe belongs to it but with some value or degree of belongingness which lies between 0 and 1 and such values are called the membership value of an element in that set. The topological structure on fuzzy set was undertaken by Chang [9] as fuzzy topological space. Molodstov [11] introduced a new mathematical tool, soft set theory in 1999 to deal with uncertainties in which a soft set is a collection of approximate descriptions of an object. A soft set is a parameterized family of subsets where parameters are the properties, attributes or characteristics of the objects. The soft set theory has several applications in different fields such as decision-making, optimization, forecasting, data analysis etc. Shabir and Naz [21] presented soft topological spaces.

Smarandache [22] extended the notion of a soft set to a hypersoft set and then to plithogenic set by replacing a function with a multi-argument function described in the cartesian product with a different set of attributes. This new concept of hypersoft set is more flexible than the soft set and more suitable in decision-making issues involving a different kinds of attributes. Saeed et al. [18, 19] studied the fundamentals of hypersoft set theory by introducing aggregate operators, relations, functions, matrices and operations on hypersoft matrices. Abbas et al. [1] defined the basic operations on hypersoft sets and hypersoft point in the fuzzy, intuitionistic and neutrosophic environments. Ajay and Charisma [3] introduced fuzzy hypersoft topology, intuitionistic hypersoft topology and neutrosophic hypersoft topology. Neutrosophic hypersoft topology is the generalized framework which generalizes intuitionistic hypersoft topology and fuzzy hypersoft topology. Ajay et al. [4] defined fuzzy hypersoft semi-open sets and developed an application in multiattribute group decision-making. The concept of contra continuous function in general topology was introduced by Dontchev [10] in 1996. Vadivel et al. [25] introduced generalized fuzzy contra e-continuous functions in fuzzy topological spaces. Ahsan et al. [2] studied a theoretical and analytical approach for fundamental framework of composite mappings on fuzzy hypersoft classes.

Saha [20] defined δ-open sets and continuous maps in fuzzy topological spaces. Aranganayagi et al., Revathi et al., Surendra et al. and Vadivel et al. [5, 6, 12, 13, 14, 16, 23, 24, 26] introduced δ-open sets, e-open sets in neutrosophic, neutrosophic soft, fuzzy hypersoft, neutrosophic hypersoft topological spaces and studied its maps, separation axioms and compact spaces. In 2023, Revathi et al. [15] developed contra e-continuous maps in neutrosophic soft topological spaces. The class of sets namely, θ open sets are playing more important role in topological spaces, because of their applications in various fields of Mathematics and other real fields. In 1968 Velicko [27] defined θ open set in H-closed Topological Spaces. In [7, 8], Caldas et al. studied various kinds of θ open sets and their properties in topological spaces. Revathi et al. introduced θ open sets and studied its continuous, open and closed maps in fuzzy hypersoft topological spaces [17].

In this paper, we develop the concepts of fuzzy hypersoft contra θ open and fuzzy hypersoft contra θ closed mappings in fuzzy hypersoft topological spaces and some of their related properties are analyzed with examples. Added to that, fuzzy hypersoft contra θ semi open maps, fuzzy hypersoft contra θ pre open maps, fuzzy hypersoft contra θ semi closed maps, fuzzy hypersoft contra θ pre closed maps, fuzzy hypersoft contra θ homeomorphism and fuzzy hypersoft contra θ C homeomorphism are developed and the relation between them are discussed.

2 Preliminaries

Definition 2.1 *[28] Let $\mathfrak{M}$ be an initial universe. A function λ from $\mathfrak{M}$ into the unit interval I is called a fuzzy set in $\mathfrak{M}$. For every $\mathfrak{m} \in \mathfrak{M}, \lambda(\mathfrak{m}) \in I$ is called the grade of membership of $\mathfrak{m}$ in λ. Some authors say that λ is a fuzzy subset of $\mathfrak{M}$ instead of saying that λ is a fuzzy set in $\mathfrak{M}$. The class of all fuzzy sets from $\mathfrak{M}$ into the closed unit interval I will be denoted by $I^{\mathfrak{M}}$.*

Definition 2.2 *[11] Let $\mathfrak{M}$ be an initial universe, Q be a set of parameters and $\mathcal{P}(\mathfrak{M})$ be the power set of $\mathfrak{M}$. A pair $(\widetilde{H}, Q)$ is called the a soft set over $\mathfrak{M}$ where $\widetilde{H}$ is a mapping $\widetilde{H}: Q \to \mathcal{P}(\mathfrak{M})$. In other words, the soft set is a parametrized family of subsets of the set $\mathfrak{M}$.*

Definition 2.3 *[22] Let $\mathfrak{M}$ be an initial universe and $\mathcal{P}(\mathfrak{M})$ be the power set of $\mathfrak{M}$. Consider $q_1, q_2, q_3, \ldots, q_n$ for $n \geq 1$, be n distinct attributes, whose corresponding attribute values are respectively the sets $Q_1, Q_2, \ldots, Q_n$ with $Q_i \cap Q_j = \emptyset$, for $i \neq j$ and $i, j \in \{1, 2, \ldots, n\}$. Then the pair $(\widetilde{H}, Q_1 \times Q_2 \times \ldots \times Q_n)$ where $\widetilde{H}: Q_1 \times Q_2 \times \ldots \times Q_n \to \mathcal{P}(\mathfrak{M})$ is called a hypersoft set over $\mathfrak{M}$.*

Definition 2.4 *[1] Let $\mathfrak{M}$ be an initial universal set and $Q_1, Q_2, \ldots, Q_n$ be pairwise disjoint sets of parameters. Let $\mathcal{P}(\mathfrak{M})$ be the set of all fuzzy sets of $\mathfrak{M}$. Let E_i be the nonempty subset of the pair Q_i for each $i = 1, 2, \ldots, n$. A fuzzy hypersoft set (briefly, FHSs) over $\mathfrak{M}$ is defined as the pair $(\widetilde{H}, E_1 \times E_2 \times \ldots \times E_n)$ where $\widetilde{H}: E_1 \times E_2 \times \ldots \times E_n \to \mathcal{P}(\mathfrak{M})$ and $\widetilde{H}(E_1 \times E_2 \times \ldots \times E_n) = \{(q, \langle \mathfrak{m}, \mu_{\widetilde{H}(q)}(\mathfrak{m}) \rangle : \mathfrak{m} \in \mathfrak{M}) : q \in E_1 \times E_2 \times \ldots \times E_n \subseteq Q_1 \times Q_2 \times \ldots \times Q_n\}$ where $\mu_{\widetilde{H}(q)}(\mathfrak{m})$ is the membership value such that $\mu_{\widetilde{H}(q)}(\mathfrak{m}) \in [0,1]$.*

Definition 2.5 *[1] Let $\mathfrak{M}$ be an universal set and $(\widetilde{H}, \wedge_1)$ and $(\widetilde{G}, \wedge_2)$ be two FHSs's over $\mathfrak{M}$. Then $(\widetilde{H}, \wedge_1)$ is the fuzzy hypersoft subset of $(\widetilde{G}, \wedge_2)$ if $\mu_{\widetilde{H}(q)}(\mathfrak{m}) \leq \mu_{\widetilde{G}(q)}(\mathfrak{m})$.* It is denoted by $(\widetilde{H}, \wedge_1) \subseteq (\widetilde{G}, \wedge_2)$.

Definition 2.6 *[1] Let $\mathfrak{M}$ be an universal set and $(\widetilde{H}, \wedge_1)$ and $(\widetilde{G}, \wedge_2)$ be FHSs's over $\mathfrak{M}$. $(\widetilde{H}, \wedge_1)$ is equal to $(\widetilde{G}, \wedge_1)$ if $\mu_{\widetilde{H}(q)}(\mathfrak{m}) = \mu_{\widetilde{G}(q)}(\mathfrak{m})$.*

Definition 2.7 *[1] A FHSs $(\widetilde{H}, \wedge)$ over the universe set $\mathfrak{M}$ is said to be null fuzzy hypersoft set if $\mu_{\widetilde{H}(q)}(\mathfrak{m}) = 0$, $\forall q \in \wedge$ and $\mathfrak{m} \in \mathfrak{M}$. It is denoted by $\tilde{0}_{(\mathfrak{M},Q)}$.*

A $FHSs$ $(\widetilde{G}, \wedge)$ over the universal set $\mathfrak{M}$ is said to be absolute fuzzy hypersoft set if $\mu_{\widetilde{H}(q)}(\mathfrak{m}) = 1$ $\forall q \in \wedge$ and $\mathfrak{m} \in \mathfrak{M}$. It is denoted by $\tilde{1}_{(\mathfrak{M},Q)}$.

Clearly, $\tilde{0}^c_{(\mathfrak{M},Q)} = \tilde{1}_{(\mathfrak{M},Q)}$ and $\tilde{1}^c_{(\mathfrak{M},Q)} = \tilde{0}_{(\mathfrak{M},Q)}$.

Definition 2.8 *[1] Let $\mathfrak{M}$ be an universal set and $(\widetilde{H}, \wedge)$ be FHSs over $\mathfrak{M}$. $(\widetilde{H}, \wedge)^C$ is the complement of $(\widetilde{H}, \wedge)$ if $\mu^C_{\widetilde{H}(q)}(\mathfrak{m}) = \tilde{1}_{(\mathfrak{M},Q)} - \mu_{\widetilde{H}(q)}(\mathfrak{m})$ where $\forall q \in \wedge$ and $\forall \mathfrak{m} \in \mathfrak{M}$. It is clear that $((\widetilde{H}, \wedge)^C)^C = (\widetilde{H}, \wedge)$.*

Definition 2.9 *[1] Let $\mathfrak{M}$ be the universal set and $(\widetilde{H}, \wedge_1)$ and $(\widetilde{G}, \wedge_2)$ be FHSs's over $\mathfrak{M}$. Extended union $(\widetilde{H}, \wedge_1) \cup (\widetilde{G}, \wedge_2)$ is defined as*

$\mu((\widetilde{H}, \wedge_1) \cup (\widetilde{G}, \wedge_2)) =$ $\mu_{\widetilde{H}(q)}(\mathfrak{m})$ if $q \in \wedge_1 - \wedge_2$ $\mu_{\widetilde{G}(q)}(\mathfrak{m})$ if $q \in \wedge_2 - \wedge_1$ $max\{\mu_{\widetilde{H}(q)}(\mathfrak{m}), \mu_{\widetilde{G}(q)}(\mathfrak{m})\}$ if $q \in \wedge_1 \cap \wedge_2$

Definition 2.10 *[1, 3] Let $\mathfrak{M}$ be the universal set and $(\widetilde{H}, \wedge_1)$ and $(\widetilde{G}, \wedge_2)$ be FHSs's over $\mathfrak{M}$. Extended intersection $(\widetilde{H}, \wedge_1) \cap (\widetilde{G}, \wedge_2)$ is defined as*

$\mu\left(\left(\widetilde{H}, \wedge_1\right) \cap \left(\widetilde{G}, \wedge_2\right)\right) = \mu_{\widetilde{H}(q)}(\mathfrak{m})$ $if q \in \wedge_1 - \wedge_2$ $\mu_{\widetilde{G}(q)}(\mathfrak{m})$
$if q \in \wedge_2 - \wedge_1$ $min\{\mu_{\widetilde{H}(q)}(\mathfrak{m}), \mu_{\widetilde{G}(q)}(\mathfrak{m})\}$ $if q \in \wedge_1 \cap \wedge_2$

Definition 2.11 *[3] Let $(\mathfrak{M}, Q)$ be the family of all $FHSs$'s over the universe set $\mathfrak{M}$ and $\tau \subseteq FHSs(\mathfrak{M}, Q)$. Then τ is said to be a fuzzy hypersoft topology (briefly, $FHSt$) on $\mathfrak{M}$ if*

1. $\tilde{0}_{(\mathfrak{M},Q)}$ and $\tilde{1}_{(\mathfrak{M},Q)}$ belongs to τ

2. the union of any number of $FHSs$'s in τ belongs to τ
3. the intersection of finite number of $FHSs$'s in τ belongs to τ.

Then $(\mathfrak{M}, Q, \tau)$ is called a fuzzy hypersoft toplogical space (briefly, $FHSts$) over $\mathfrak{M}$. Each member of τ is said to be fuzzy hypersoft open set (briefly, $FHSos$). A $FHSs$ $(\widetilde{H},\wedge)$ is called a fuzzy hypersoft closed set (briefly, $FHScs$) if its complement $(\widetilde{H},\wedge)^C$ is $FHSos$.

Definition 2.12 *[3] Let $(\mathfrak{M}, Q, \tau)$ be a $FHSts$ over $\mathfrak{M}$ and $(\widetilde{H},\wedge)$ be a $FHSs$ in $\mathfrak{M}$. Then,*

1. the fuzzy hypersoft interior (briefly, $FHSint$) of $(\widetilde{H},\wedge)$ is defined as $FHSint(\widetilde{H},\wedge) = \cup \{(\tilde{G},\wedge) : (\tilde{G},\wedge) \subseteq (\widetilde{H},\wedge)$ where $(\tilde{G},\wedge)$ is $FHSos\}$.
2. the fuzzy hypersoft closure (briefly, $FHScl$) of $(\widetilde{H},\wedge)$ is defined as $FHScl(\widetilde{H},\wedge) = \cap \{(\tilde{G},\wedge) : (\tilde{G},\wedge) \supseteq (\widetilde{H},\wedge)$ where $(\tilde{G},\wedge)$ is $FHScs\}$.

Definition 2.13 *[4] Let $(\mathfrak{M}, Q, \tau)$ be a $FHSts$ over $\mathfrak{M}$ and $(\widetilde{H},\wedge)$ be a $FHSs$ in $\mathfrak{M}$. Then, $(\widetilde{H},\wedge)$ is called the fuzzy hypersoft semiopen set (briefly, $FHSSos$) if $(\widetilde{H},\wedge) \subseteq FHScl(int(\widetilde{H},\wedge))$.*

A $FHSs$ $(\widetilde{H},\wedge)$ is called a fuzzy hypersoft semiclosed set (briefly, $FHSScs$) if its complement $(\widetilde{H},\wedge)^c$ is a $FHSSos$.

Definition 2.14 *[2] Let $(\mathfrak{M}, L)$ and $(\mathfrak{N}, M)$ be classes of $FHSs$'s over $\mathfrak{M}$ and $\mathfrak{N}$ with attributes L and M respectively. Let $\omega: \mathfrak{M} \to \mathfrak{N}$ and $\nu: L \to M$ be mappings. Then a FHS mappings $\mathfrak{h} = (\omega, \nu): (\mathfrak{M}, L) \to (\mathfrak{N}, M)$ is defined as follows, for a $FHSs$ $(\widetilde{H},\wedge)_A$ in $(\mathfrak{M}, L)$, $f(\widetilde{H},\wedge)_A$ is a $FHSs$ in $(\mathfrak{N}, M)$ obtained as follows, for $\beta \in \nu(L) \subseteq M$ and $\mathfrak{n} \in \mathfrak{N}$, $\mathfrak{h}(\widetilde{H},\wedge)_A(\beta)(\mathfrak{n}) = \bigcup_{\alpha \in \nu^{-1}(\beta) \cap \ A, s \in \omega^{-1}(\mathfrak{n})} (\alpha)\mu_s \mathfrak{h}(\widetilde{H},\wedge)_A$ is called a fuzzy hypersoft image of a $FHSs$ $(\widetilde{H},\wedge)$. Hence $((\widetilde{H},\wedge)_A, \mathfrak{h}(\widetilde{H},\wedge)_A) \in \mathfrak{h}$, where $(\widetilde{H},\wedge)_A \subseteq (\mathfrak{M}, L), \mathfrak{h}(\widetilde{H},\wedge)_A \subseteq (\mathfrak{N}, M)$.*

Definition 2.15 *[2] If $\mathfrak{h}: (\mathfrak{M}, L) \to (\mathfrak{N}, M)$ be a FHS mapping, then FHS class $(\mathfrak{M}, L)$ is called the domain of $\mathfrak{h}$ and the FHS class $(\tilde{G},\wedge) \in (\mathfrak{N}, M) : (\tilde{G},\wedge) = \mathfrak{h}(\widetilde{H},\wedge))$ for some $(\widetilde{H},\wedge) \in (\mathfrak{M}, L)$ is called the range of $\mathfrak{h}$. The FHS class $(\mathfrak{N}, M)$ is called co-domain of $\mathfrak{h}$.*

Definition 2.16 *[2] If $\mathfrak{h}: (\mathfrak{M}, L) \to (\mathfrak{N}, M)$ be a FHS mapping and $(\tilde{G},\wedge)_B$, a $FHSs$ in $(\mathfrak{N}, M)$ where $\omega: \mathfrak{M} \to \mathfrak{N}$, $\nu: L \to M$ and $B \subseteq M$. Then $\mathfrak{h}^{-1}(\tilde{G},\wedge)_B$ is a $FHSs$ in $(\mathfrak{M}, L)$ defined as follows, for $\alpha \in \nu^{-1}(B) \subseteq L$ and $\mathfrak{m} \in \mathfrak{M}$, $\mathfrak{h}^{-1}(\tilde{G},\wedge)_B(\alpha)(\mathfrak{m}) = (\nu(\alpha))\mu_p(\mathfrak{m})\mathfrak{h}^{-1}(\tilde{G},\wedge)_B$ is called a FHS inverse image of $(\tilde{G},\wedge)_B$.*

Definition 2.17 *[2] Let $\mathfrak{h} = (\omega, \nu)$ be a FHS mapping of a FHS class $(\mathfrak{M}, L)$ into a FHS class $(\mathfrak{N}, M)$. Then [(i)]*

1. $\mathfrak{h}$ is said to be a one-one (or injection) FHS mapping if for both $\omega: \mathfrak{M} \to \mathfrak{N}$ and $\nu: L \to M$ are one-one.
2. $\mathfrak{h}$ is said to be a onto (or surjection) FHS mapping if for both $\omega: \mathfrak{M} \to \mathfrak{N}$ and

$\nu: L \to M$ are onto.

If $\mathfrak{h}$ is both one-one and onto, then $\mathfrak{h}$ is called a one-one onto (or bijective) correspondance of FHS mapping.

Definition 2.18 *[2] If* $\mathfrak{h} = (\omega, \nu): (\mathfrak{M}, L) \to (\mathfrak{N}, M)$ *and* $g = (m, n): (\mathfrak{N}, M) \to (P, N)$ *are two FHS mappings, then their composite* $\mathfrak{g} \circ \mathfrak{h}$ *is a FHS mapping of* $(\mathfrak{M}, L)$ *into* (P, N) *such that for every* $(\widetilde{H}, \wedge)_A \in (\mathfrak{M}, L)$, $(\mathfrak{g} \circ \mathfrak{h})(\widetilde{H}, \wedge)_A = \mathfrak{g}(\mathfrak{h}(\widetilde{H}, \wedge)_A)$. *For* $\beta \in n(M) \subseteq N$ *and* $p \in P$, *it is defined as* $\mathfrak{g}(\mathfrak{h}(\widetilde{H}, \wedge)_A(\beta)(p) = \cup_{\alpha \in n^{-1}(\beta) \cap \mathfrak{h}(A), s \in m^{-1}(p)} (\alpha)\mu_s$.

Definition 2.19 *[2] Let* $\mathfrak{h} = (\omega, \nu)$ *be a FHS mapping where* $\omega: \mathfrak{M} \to \mathfrak{M}$ *and* $\nu: L \to L$. *Then* $\mathfrak{h}$ *is said to be a* FHS *identity mapping if for both* ω *and* ν *are identity mappings.*

Definition 2.20 *[2] A one-one onto FHS mapping* $\mathfrak{h} = (\omega, \nu): (\mathfrak{M}, L) \to (\mathfrak{N}, M)$ *is called FHS invertable mapping. Its FHS inverse mapping is denoted by* $\mathfrak{h}^{-1} = (\omega^{-1}, \nu^{-1}): (\mathfrak{M}, L) \to (\mathfrak{N}, M)$.

3 Fuzzy Hypersoft Contra θ Open Mapping

In this section, fuzzy hypersoft contra θ open maps are introduced and their characteristics are studied.

Definition 3.1 *A mapping* $\mathfrak{h}: (\mathfrak{M}, L, \tau) \to (\mathfrak{N}, M, \sigma)$ *is fuzzy hypersoft contra* θ *open (resp.* θ *semi open &* θ *pre open) map (in short,* $FHScontra\theta O$ *(resp.* $FHScontra\theta\mathcal{S}O$ *&* $FHScontra\theta\mathcal{P}O$*)) if the image of each* $FHSos$ *of* $(\mathfrak{M}, L, \tau)$ *is* $FHS\theta cs$ *(resp.* $FHS\theta\mathcal{S}cs$ *&* $FHS\theta\mathcal{P}cs$*) in* $(\mathfrak{N}, M, \sigma)$.

Proposition 3.1 *The statements hold but the converse is not.*

1. Each $FHScontra\theta O$ is a $FHScontra\theta\mathcal{S}O$.
2. Each $FHScontra\theta O$ is a $FHScontraO$.
3. Each $FHScontra\theta\mathcal{S}O$ is a $FHScontra\mathcal{S}O$.
4. Each $FHScontraO$ is a $FHScontra\mathcal{P}O$.
5. Each $FHScontra\mathcal{S}O$ is a $FHScontra\theta\mathcal{P}O$.
6. Each $FHScontra\mathcal{P}O$ is a $FHScontra\theta\mathcal{P}O$.

Proof. Consider the map $\mathfrak{h}: (\mathfrak{M}, L, \tau) \to (\mathfrak{N}, M, \sigma)$

1. Let $(\widetilde{H}, \wedge)$ be a $FHSos$ in $\mathfrak{M}$. As $\mathfrak{h}$ is $FHScontra\theta O$, $\mathfrak{h}(\widetilde{H}, \wedge)$ is $FHS\theta cs$ in $\mathfrak{N}$. Since all $FHS\theta cs$ are $FHS\theta\mathcal{S}cs$, $\mathfrak{h}(\widetilde{H}, \wedge)$ is $FHS\theta\mathcal{S}cs$ in $\mathfrak{N}$. Hence $\mathfrak{h}$ is a $FHScontra\theta\mathcal{S}O$.
2. Let $(\widetilde{H}, \wedge)$ be a $FHSos$ in $\mathfrak{M}$. As $\mathfrak{h}$ is $FHScontra\theta O$, $\mathfrak{h}(\widetilde{H}, \wedge)$ is $FHS\theta cs$ in $\mathfrak{N}$. Since all $FHS\theta cs$ are $FHScs$, $\mathfrak{h}(\widetilde{H}, \wedge)$ is $FHScs$ in $\mathfrak{N}$. Hence $\mathfrak{h}$ is a $FHScontraO$.
3. Let $(\widetilde{H}, \wedge)$ be a $FHSos$ in $\mathfrak{M}$. As $\mathfrak{h}$ is $FHScontra\theta\mathcal{S}O$, $\mathfrak{h}(\widetilde{H}, \wedge)$ is $FHS\theta\mathcal{S}cs$ in $\mathfrak{N}$. Since all $FHS\theta\mathcal{S}cs$ are $FHS\mathcal{S}cs$, $\mathfrak{h}(\widetilde{H}, \wedge)$ is $FHS\mathcal{S}cs$ in $\mathfrak{N}$. Hence $\mathfrak{h}$ is a $FHScontra\mathcal{S}O$.
4. Let $(\widetilde{H}, \wedge)$ be a $FHSos$ in $\mathfrak{M}$. As $\mathfrak{h}$ is $FHScontraO$, $\mathfrak{h}(\widetilde{H}, \wedge)$ is $FHScs$ in $\mathfrak{N}$.

Since all $FHScs$ are $FHS\mathcal{P}cs$, $\mathfrak{h}(\widetilde{H},\wedge)$ is $FHS\mathcal{P}cs$ in $\mathfrak{N}$. Hence $\mathfrak{h}$ is a $FHScontra\mathcal{P}O$.

5. Let $(\widetilde{H},\wedge)$ be a $FHSos$ in $\mathfrak{M}$. As $\mathfrak{h}$ is $FHScontra\mathcal{S}O$, $\mathfrak{h}(\widetilde{H},\wedge)$ is $FHS\mathcal{S}cs$ in $\mathfrak{N}$. Since all $FHS\mathcal{S}cs$ are $FHS\theta\mathcal{P}cs$, $\mathfrak{h}(\widetilde{H},\wedge)$ is $FHS\theta\mathcal{P}cs$ in $\mathfrak{N}$. Hence $\mathfrak{h}$ is a $FHScontra\theta\mathcal{P}O$.

6. Let $(\widetilde{H},\wedge)$ be a $FHSos$ in $\mathfrak{M}$. As $\mathfrak{h}$ is $FHScontra\mathcal{P}O$, $\mathfrak{h}(\widetilde{H},\wedge)$ is $FHS\mathcal{P}cs$ in $\mathfrak{N}$. Since all $FHS\mathcal{P}cs$ are $FHS\theta\mathcal{P}cs$, $\mathfrak{h}(\widetilde{H},\wedge)$ is $FHS\theta\mathcal{P}cs$ in $\mathfrak{N}$. Hence $\mathfrak{h}$ is a $FHScontra\theta\mathcal{P}O$.

Example 3.1 *Let* $\mathfrak{M} = \{\mathfrak{m}_1, \mathfrak{m}_2\}$ *and* $\mathfrak{N} = \{\mathfrak{n}_1, \mathfrak{n}_2\}$ *be the* FHS *initial universes and the attributes be* $L = Q_1 \times Q_2$ *and* $M = Q_{1'} \times Q_{2'}$ *respectively. The attributes are given as:*
$Q_1 = \{a_1, a_2, a_3\}, Q_2 = \{b_1, b_2\}$
$Q_{1'} = \{c_1, c_2, c_3\}, Q_{2'} = \{d_1, d_2\}$.

Let $(\mathfrak{M}, L), (\mathfrak{N}, M)$ be the classes of FHS sets. Let the $FHSs$ $(\widetilde{H}_1,\wedge_3)$ over the universe $\mathfrak{M}$ be defined as

$$(\widetilde{H}_1,\wedge_3) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{n}_1}{0.3}, \frac{\mathfrak{n}_2}{0.2}\}\rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{n}_1}{0.5}, \frac{\mathfrak{n}_2}{0.3}\}\rangle, \\ \langle (a_1, b_1), \{\frac{\mathfrak{n}_1}{0.4}, \frac{\mathfrak{n}_2}{0.5}\}\rangle \end{Bmatrix}$$

$\tau = \{\tilde{0}_{(\mathfrak{M},Q)}, \tilde{1}_{(\mathfrak{M},Q)}, (\widetilde{H}_1,\wedge_3)\}$ is $FHSts$.

Let the $FHSs$'s $(\tilde{G}_1,\wedge_1), (\tilde{G}_2,\wedge_2), (\tilde{G}_3,\wedge_2), (\tilde{G}_4,\wedge_3), (\tilde{G}_5,\wedge_3), (\tilde{G}_6,\wedge_3), (\tilde{G}_7,\wedge_3)$ over the universe $\mathfrak{N}$ be

$$(\tilde{G}_1,\wedge_1) = \begin{Bmatrix} \langle (c_2, d_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.6}\}\rangle, \\ \langle (c_1, d_2), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_2,\wedge_2) = \begin{Bmatrix} \langle (c_2, d_1), \{\frac{\mathfrak{m}_1}{0.2}, \frac{\mathfrak{m}_2}{0.3}\}\rangle, \\ \langle (c_2, d_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.4}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_3,\wedge_2) = \begin{Bmatrix} \langle (c_2, d_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.7}\}\rangle, \\ \langle (c_2, d_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.6}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_4,\wedge_3) = \begin{Bmatrix} \langle (c_2, d_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.6}\}\rangle, \\ \langle (c_1, d_2), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle (c_2, d_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.4}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_5,\wedge_3) = \begin{Bmatrix} \langle (c_2, d_1), \{\frac{\mathfrak{m}_1}{0.2}, \frac{\mathfrak{m}_2}{0.3}\}\rangle, \\ \langle (c_1, d_2), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle (c_2, d_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.4}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_6,\wedge_3) = \begin{Bmatrix} \langle (c_2,d_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.7}\}\rangle, \\ \langle (c_1,d_2), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle (c_2,d_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.6}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_7,\wedge_3) = \begin{Bmatrix} \langle (c_2,d_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.6}\}\rangle, \\ \langle (c_1,d_2), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle (c_2,d_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.6}\}\rangle \end{Bmatrix}$$

$\sigma = \{\tilde{0}_{(\mathfrak{N},Q)}, \tilde{1}_{(\mathfrak{N},Q)}, (\tilde{G}_1,\wedge_1), (\tilde{G}_2,\wedge_2), (\tilde{G}_3,\wedge_2), (\tilde{G}_4,\wedge_3), (\tilde{G}_5,\wedge_3), (\tilde{G}_6,\wedge_3), (\tilde{G}_7,\wedge_3)\}$ is $FHSts$.

Let $\mathfrak{h} = (\omega, \nu): (\mathfrak{M}, L) \to (\mathfrak{N}, M)$ be a FHS mapping as follows:
$\omega(\mathfrak{m}_1) = \mathfrak{n}_2, \omega(\mathfrak{m}_2) = \mathfrak{n}_1$,
$\nu(a_1,b_1) = (c_2,d_1), \nu(a_2,b_1) = (c_1,d_2),\ \nu(a_1,b_2) = (c_2,d_2)$
$\mathfrak{h}(\widetilde{H}_1,\wedge_3) = (\tilde{G}_6,\wedge_3)^c$

$(\widetilde{H}_1,\wedge_3)$ is $FHSos$ in $\mathfrak{M}$ and $\mathfrak{h}(\widetilde{H}_1,\wedge_3) = (\tilde{G}_6,\wedge_3)^c$ is $FHS\theta\mathcal{S}cs$ in $\mathfrak{N}$.

$\therefore$ $\mathfrak{h}$ is $FHScontra\theta\mathcal{S}O$ mapping but $\mathfrak{h}$ is not $FHScontra\theta O$ mapping because $(\widetilde{H}_1,\wedge_3)$ is $FHSos$ in $\mathfrak{M}$ but $\mathfrak{h}(\widetilde{H}_1,\wedge_3) = (\tilde{G}_6,\wedge_3)^c$ is not $FHS\theta cs$ in $\mathfrak{N}$.

Example 3.2 *Let* $\mathfrak{M} = \{\mathfrak{m}_1, \mathfrak{m}_2\}$ *and* $\mathfrak{N} = \{\mathfrak{n}_1, \mathfrak{n}_2\}$ *be the* FHS *initial universes and the attributes be* $L = Q_1 \times Q_2$ *and* $M = Q_{1'} \times Q_{2'}$ *respectively. The attributes are given as:*
$Q_1 = \{a_1, a_2, a_3\}, Q_2 = \{b_1, b_2\}$
$Q_{1'} = \{c_1, c_2, c_3\}, Q_{2'} = \{d_1, d_2\}$.

Let $(\mathfrak{M}, L), (\mathfrak{N}, M)$ be the classes of FHS sets. Let the $FHSs(\widetilde{H}_1,\wedge_1)$ over the universe $\mathfrak{M}$ be defined as

$$(\widetilde{H}_1,\wedge_1) = \begin{Bmatrix} \langle (a_1,b_1), \{\frac{\mathfrak{n}_1}{0.4}, \frac{\mathfrak{n}_2}{0.2}\}\rangle, \\ \langle (a_2,b_1), \{\frac{\mathfrak{n}_1}{0.5}, \frac{\mathfrak{n}_2}{0.3}\}\rangle, \\ \end{Bmatrix}$$

$\tau = \{\tilde{0}_{(\mathfrak{M},Q)}, \tilde{1}_{(\mathfrak{M},Q)}, (\widetilde{H}_1,\wedge_1)\}$ is $FHSts$.

Let the $FHSs$'s $(\tilde{G}_1,\wedge_1), (\tilde{G}_2,\wedge_2), (\tilde{G}_3,\wedge_2), (\tilde{G}_4,\wedge_3), (\tilde{G}_5,\wedge_3), (\tilde{G}_6,\wedge_3), (\tilde{G}_7,\wedge_3)$ over the universe $\mathfrak{N}$ be

$$(\tilde{G}_1,\wedge_1) = \begin{Bmatrix} \langle (c_2,d_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.6}\}\rangle, \\ \langle (c_1,d_2), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_2,\wedge_2) = \begin{Bmatrix} \langle (c_2,d_1), \{\frac{\mathfrak{m}_1}{0.2}, \frac{\mathfrak{m}_2}{0.3}\}\rangle, \\ \langle (c_2,d_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.4}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_3,\wedge_2) = \begin{Bmatrix} \langle (c_2,d_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.7}\}\rangle, \\ \langle (c_2,d_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.6}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_4,\wedge_3) = \begin{Bmatrix} \langle (c_2,d_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.6}\}\rangle, \\ \langle (c_1,d_2), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle (c_2,d_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.4}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_5,\wedge_3) = \begin{Bmatrix} \langle (c_2,d_1), \{\frac{\mathfrak{m}_1}{0.2}, \frac{\mathfrak{m}_2}{0.3}\} \rangle, \\ \langle (c_1,d_2), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\} \rangle, \\ \langle (c_2,d_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.4}\} \rangle \end{Bmatrix}$$

$$(\tilde{G}_6,\wedge_3) = \begin{Bmatrix} \langle (c_2,d_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.7}\} \rangle, \\ \langle (c_1,d_2), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\} \rangle, \\ \langle (c_2,d_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.6}\} \rangle \end{Bmatrix}$$

$$(\tilde{G}_7,\wedge_3) = \begin{Bmatrix} \langle (c_2,d_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.6}\} \rangle, \\ \langle (c_1,d_2), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\} \rangle, \\ \langle (c_2,d_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.6}\} \rangle \end{Bmatrix}$$

$\sigma = \{\tilde{0}_{(\mathfrak{N},Q)}, \tilde{1}_{(\mathfrak{N},Q)}, (\tilde{G}_1,\wedge_1), (\tilde{G}_2,\wedge_2), (\tilde{G}_3,\wedge_2), (\tilde{G}_4,\wedge_3), (\tilde{G}_5,\wedge_3), (\tilde{G}_6,\wedge_3), (\tilde{G}_7,\wedge_3)\}$ is $FHSts$.

Let $\mathfrak{h} = (\omega, \nu): (\mathfrak{M}, L) \to (\mathfrak{N}, M)$ be a FHS mapping as follows:
$\omega(\mathfrak{m}_1) = \mathfrak{n}_2, \omega(\mathfrak{m}_2) = \mathfrak{n}_1,$
$\nu(a_1,b_1) = (c_2,d_1), \nu(a_2,b_1) = (c_1,d_2),\ \nu(a_1,b_2) = (c_2,d_2)$
$\mathfrak{h}(\widetilde{H}_1,\wedge_1) = (\tilde{G}_1,\wedge_1)^c$

$(\widetilde{H}_1,\wedge_1)$ is $FHSos$ in $\mathfrak{M}$ and $\mathfrak{h}(\widetilde{H}_1,\wedge_1) = (\tilde{G}_1,\wedge_1)^c$ is $FHScs$ in $\mathfrak{N}$.

$\therefore$ $\mathfrak{h}$ is $FHScontraO$ mapping but $\mathfrak{h}$ is not $FHScontra\theta O$ mapping because $(\widetilde{H}_1,\wedge_1)$ is $FHSos$ in $\mathfrak{M}$ but $\mathfrak{h}(\widetilde{H}_1,\wedge_1) = (\tilde{G}_1,\wedge_1)^c$ is not $FHS\theta cs$ in $\mathfrak{N}$.

Example 3.3 *Let $\mathfrak{M} = \{\mathfrak{m}_1, \mathfrak{m}_2\}$ and $\mathfrak{N} = \{\mathfrak{n}_1, \mathfrak{n}_2\}$ be the FHS initial universes and the attributes be $L = Q_1 \times Q_2$ and $M = Q_{1'} \times Q_{2'}$, respectively. The attributes are given as:*
$Q_1 = \{a_1, a_2, a_3\}, Q_2 = \{b_1, b_2\}$
$Q_{1'} = \{c_1, c_2, c_3\}, Q_{2'} = \{d_1, d_2\}$.

Let $(\mathfrak{M}, L), (\mathfrak{N}, M)$ be the classes of FHS sets. Let the $FHSs$ $(\widetilde{H}_1,\wedge_1)$ over the universe$\mathfrak{N}$ be defined as

$$(\widetilde{H}_1,\wedge_3) = \begin{Bmatrix} \langle (c_2,d_1), \{\frac{\mathfrak{n}_1}{0.4}, \frac{\mathfrak{n}_2}{0.2}\} \rangle, \\ \langle (c_1,d_2), \{\frac{\mathfrak{n}_1}{0.5}, \frac{\mathfrak{n}_2}{0.3}\} \rangle, \\ \langle (c_2,d_2), \{\frac{\mathfrak{n}_1}{0.5}, \frac{\mathfrak{n}_2}{0.6}\} \rangle \end{Bmatrix}$$

$\tau = \{\tilde{0}_{(\mathfrak{N},Q)}, \tilde{1}_{(\mathfrak{N},Q)}, (\widetilde{H}_1,\wedge_3)\}$ is $FHSts$.

Let the $FHSs$'s $(\tilde{G}_1,\wedge_1), (\tilde{G}_2,\wedge_2), (\tilde{G}_3,\wedge_2), (\tilde{G}_4,\wedge_3), (\tilde{G}_5,\wedge_3), (\tilde{G}_6,\wedge_3), (\tilde{G}_7,\wedge_3)$ over the universe $\mathfrak{N}$ be

$$(\tilde{G}_1,\wedge_1) = \begin{Bmatrix} \langle (c_2,d_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.6}\} \rangle, \\ \langle (c_1,d_2), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\} \rangle \end{Bmatrix}$$

$$(\tilde{G}_2,\wedge_2) = \begin{Bmatrix} \langle (c_2,d_1), \{\frac{\mathfrak{m}_1}{0.2}, \frac{\mathfrak{m}_2}{0.3}\} \rangle, \\ \langle (c_2,d_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.4}\} \rangle \end{Bmatrix}$$

$$(\tilde{G}_3,\wedge_2) = \begin{Bmatrix} \langle (c_2,d_1),\{\frac{\mathfrak{m}_1}{0.8},\frac{\mathfrak{m}_2}{0.7}\}\rangle, \\ \langle (c_2,d_2),\{\frac{\mathfrak{m}_1}{0.5},\frac{\mathfrak{m}_2}{0.6}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_4,\wedge_3) = \begin{Bmatrix} \langle (c_2,d_1),\{\frac{\mathfrak{m}_1}{0.8},\frac{\mathfrak{m}_2}{0.6}\}\rangle, \\ \langle (c_1,d_2),\{\frac{\mathfrak{m}_1}{0.7},\frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle (c_2,d_2),\{\frac{\mathfrak{m}_1}{0.5},\frac{\mathfrak{m}_2}{0.4}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_5,\wedge_3) = \begin{Bmatrix} \langle (c_2,d_1),\{\frac{\mathfrak{m}_1}{0.2},\frac{\mathfrak{m}_2}{0.3}\}\rangle, \\ \langle (c_1,d_2),\{\frac{\mathfrak{m}_1}{0.7},\frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle (c_2,d_2),\{\frac{\mathfrak{m}_1}{0.5},\frac{\mathfrak{m}_2}{0.4}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_6,\wedge_3) = \begin{Bmatrix} \langle (c_2,d_1),\{\frac{\mathfrak{m}_1}{0.8},\frac{\mathfrak{m}_2}{0.7}\}\rangle, \\ \langle (c_1,d_2),\{\frac{\mathfrak{m}_1}{0.7},\frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle (c_2,d_2),\{\frac{\mathfrak{m}_1}{0.5},\frac{\mathfrak{m}_2}{0.6}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_7,\wedge_3) = \begin{Bmatrix} \langle (c_2,d_1),\{\frac{\mathfrak{m}_1}{0.8},\frac{\mathfrak{m}_2}{0.6}\}\rangle, \\ \langle (c_1,d_2),\{\frac{\mathfrak{m}_1}{0.7},\frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle (c_2,d_2),\{\frac{\mathfrak{m}_1}{0.5},\frac{\mathfrak{m}_2}{0.6}\}\rangle \end{Bmatrix}$$

$\sigma = \{\tilde{0}_{(\mathfrak{N},Q)}, \tilde{1}_{(\mathfrak{N},Q)}, (\tilde{G}_1,\wedge_1), (\tilde{G}_2,\wedge_2), (\tilde{G}_3,\wedge_2), (\tilde{G}_4,\wedge_3), (\tilde{G}_5,\wedge_3), (\tilde{G}_6,\wedge_3), (\tilde{G}_7,\wedge_3)\}$ is $FHSts$.

Let $\mathfrak{h} = (\omega, \nu): (\mathfrak{M}, L) \to (\mathfrak{N}, M)$ be a FHS mapping as follows:
$\omega(\mathfrak{m}_1) = \mathfrak{n}_2, \omega(\mathfrak{m}_2) = \mathfrak{n}_1$,
$\nu(a_1,b_1) = (c_2,d_1), \nu(a_2,b_1) = (c_1,d_2),\ \nu(a_1,b_2) = (c_2,d_2)$
$\mathfrak{h}(\widetilde{H}_1,\wedge_3) = (\tilde{G}_4,\wedge_3)^c$

$(\widetilde{H}_1,\wedge_3)$ is $FHSos$ in $\mathfrak{M}$ and $\mathfrak{h}(\widetilde{H}_1,\wedge_3) = (\tilde{G}_4,\wedge_3)^c$ is $FHSScs$ in $\mathfrak{N}$.

$\therefore$ $\mathfrak{h}$ is $FHScontraSO$ mapping but $\mathfrak{h}$ is not $FHScontra\theta SO$ mapping because $(\widetilde{H}_1,\wedge_3)$ is $FHSos$ in $\mathfrak{M}$ but $\mathfrak{h}(\widetilde{H}_1,\wedge_3) = (\tilde{G}_4,\wedge_3)^c$ is not $FHS\theta Scs$ in $\mathfrak{N}$.

Example 3.4 *Let $\mathfrak{M} = \{\mathfrak{m}_1, \mathfrak{m}_2\}$ and $\mathfrak{N} = \{\mathfrak{n}_1, \mathfrak{n}_2\}$ be the FHS initial universes and the attributes be $L = Q_1 \times Q_2$ and $M = Q_{1'} \times Q_{2'}$ respectively. The attributes are given as:*
$Q_1 = \{a_1, a_2, a_3\}, Q_2 = \{b_1, b_2\}$
$Q_{1'} = \{c_1, c_2, c_3\}, Q_{2'} = \{d_1, d_2\}$.

Let $(\mathfrak{M}, L), (\mathfrak{N}, M)$ be the classes of FHS sets. Let the $FHSs$ $(\widetilde{H}_1,\wedge_1)$ over the universe $\mathfrak{M}$ be defined as

$$(\widetilde{H}_1,\wedge_3) = \begin{Bmatrix} \langle (a_1,b_1),\{\frac{\mathfrak{n}_1}{0.3},\frac{\mathfrak{n}_2}{0.2}\}\rangle, \\ \langle (a_2,b_1),\{\frac{\mathfrak{n}_1}{0.5},\frac{\mathfrak{n}_2}{0.7}\}\rangle, \\ \langle (a_1,b_2),\{\frac{\mathfrak{n}_1}{0.4},\frac{\mathfrak{n}_2}{0.5}\}\rangle \end{Bmatrix}$$

$\tau = \{\tilde{0}_{(\mathfrak{M},Q)}, \tilde{1}_{(\mathfrak{M},Q)}, (\widetilde{H}_1,\wedge_3)\}$ is $FHSts$.

Let the $FHSs$'s $(\tilde{G}_1,\wedge_1), (\tilde{G}_2,\wedge_2), (\tilde{G}_3,\wedge_2), (\tilde{G}_4,\wedge_3), (\tilde{G}_5,\wedge_3), (\tilde{G}_6,\wedge_3), (\tilde{G}_7,\wedge_3), (\tilde{G}_8,\wedge_3)$

over the universe $\mathfrak{N}$ be

$$(\tilde{G}_1,\wedge_1) = \begin{Bmatrix} \langle (c_2,d_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.6}\}\rangle, \\ \langle (c_1,d_2), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_2,\wedge_2) = \begin{Bmatrix} \langle (c_2,d_1), \{\frac{\mathfrak{m}_1}{0.2}, \frac{\mathfrak{m}_2}{0.3}\}\rangle, \\ \langle (c_2,d_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.4}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_3,\wedge_2) = \begin{Bmatrix} \langle (c_2,d_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.7}\}\rangle, \\ \langle (c_2,d_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.6}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_4,\wedge_3) = \begin{Bmatrix} \langle (c_2,d_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.6}\}\rangle, \\ \langle (c_1,d_2), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle (c_2,d_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.4}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_5,\wedge_3) = \begin{Bmatrix} \langle (c_2,d_1), \{\frac{\mathfrak{m}_1}{0.2}, \frac{\mathfrak{m}_2}{0.3}\}\rangle, \\ \langle (c_1,d_2), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle (c_2,d_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.4}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_6,\wedge_3) = \begin{Bmatrix} \langle (c_2,d_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.7}\}\rangle, \\ \langle (c_1,d_2), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle (c_2,d_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.6}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_7,\wedge_3) = \begin{Bmatrix} \langle (c_2,d_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.6}\}\rangle, \\ \langle (c_1,d_2), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle (c_2,d_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.6}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_8,\wedge_3) = \begin{Bmatrix} \langle (c_2,d_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.7}\}\rangle, \\ \langle (c_1,d_2), \{\frac{\mathfrak{m}_1}{0.3}, \frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle (c_2,d_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.6}\}\rangle \end{Bmatrix}$$

$\sigma =$
$\{\tilde{0}_{(\mathfrak{N},Q)}, \tilde{1}_{(\mathfrak{N},Q)}, (\tilde{G}_1,\wedge_1), (\tilde{G}_2,\wedge_2), (\tilde{G}_3,\wedge_2), (\tilde{G}_4,\wedge_3), (\tilde{G}_5,\wedge_3), (\tilde{G}_6,\wedge_3), (\tilde{G}_7,\wedge_3), (\tilde{G}_7,\wedge_3)\}$ is $FHSts$.

Let $\mathfrak{h} = (\omega,\nu): (\mathfrak{M},L) \to (\mathfrak{N},M)$ be a FHS mapping as follows:

$\omega(\mathfrak{m}_1) = \mathfrak{n}_2, \omega(\mathfrak{m}_2) = \mathfrak{n}_1,$

$\nu(a_1,b_1) = (c_2,d_1), \nu(a_2,b_1) = (c_1,d_2),\ \nu(a_1,b_2) = (c_2,d_2)$

$\mathfrak{h}(\widetilde{H}_1,\wedge_3) = (\tilde{G}_8,\wedge_3)^c$

$(\widetilde{H}_1,\wedge_3)$ is $FHSos$ in $\mathfrak{M}$ and $\mathfrak{h}(\widetilde{H}_1,\wedge_3) = (\tilde{G}_8,\wedge_3)^c$ is $FHS\mathcal{P}cs$ in $\mathfrak{N}$.

$\therefore$ $\mathfrak{h}$ is $FHScontra\mathcal{P}O$ mapping but $\mathfrak{h}$ is not $FHScontraO$ mapping because $(\widetilde{H}_1,\wedge_3)$ is $FHSos$ in $\mathfrak{M}$ but $\mathfrak{h}(\widetilde{H}_1,\wedge_3) = (\tilde{G}_8,\wedge_3)^c$ is not $FHScs$ in $\mathfrak{N}$.

Example 3.5 *Let* $\mathfrak{M} = \{\mathfrak{m}_1, \mathfrak{m}_2\}$ *and* $\mathfrak{N} = \{\mathfrak{n}_1, \mathfrak{n}_2\}$ *be the FHS initial universes and the attributes be* $L = Q_1 \times Q_2$ *and* $M = Q_{1'} \times Q_{2'}$ *respectively. The attributes are given as:*

$Q_1 = \{a_1, a_2, a_3\}, Q_2 = \{b_1, b_2\}$

$Q_{1'} = \{c_1, c_2, c_3\}, Q_{2'} = \{d_1, d_2\}.$

Let $(\mathfrak{M}, L), (\mathfrak{N}, M)$ be the classes of FHS sets. Let the $FHSs$ $(\widetilde{H}_1,\wedge_3)$ over the universe $\mathfrak{M}$ be defined as

$$(\widetilde{H}_1,\wedge_3) = \begin{Bmatrix} \langle(a_1,b_1),\{\frac{\mathfrak{n}_1}{0.7},\frac{\mathfrak{n}_2}{0.8}\}\rangle, \\ \langle(a_2,b_1),\{\frac{\mathfrak{n}_1}{0.5},\frac{\mathfrak{n}_2}{0.7}\}\rangle, \\ \langle(a_1,b_2),\{\frac{\mathfrak{n}_1}{0.6},\frac{\mathfrak{n}_2}{0.5}\}\rangle \end{Bmatrix}$$

$\tau = \{\tilde{0}_{(\mathfrak{M},Q)}, \tilde{1}_{(\mathfrak{M},Q)}, (\widetilde{H}_1,\wedge_3)\}$ is $FHSts$.

Let the $FHSs$'s $(\tilde{G}_1,\wedge_1), (\tilde{G}_2,\wedge_2), (\tilde{G}_3,\wedge_2), (\tilde{G}_4,\wedge_3), (\tilde{G}_5,\wedge_3), (\tilde{G}_6,\wedge_3), (\tilde{G}_7,\wedge_3), (\tilde{G}_8,\wedge_3)$ over the universe $\mathfrak{N}$ be

$$(\tilde{G}_1,\wedge_1) = \begin{Bmatrix} \langle(c_2,d_1),\{\frac{\mathfrak{m}_1}{0.8},\frac{\mathfrak{m}_2}{0.6}\}\rangle, \\ \langle(c_1,d_2),\{\frac{\mathfrak{m}_1}{0.7},\frac{\mathfrak{m}_2}{0.5}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_2,\wedge_2) = \begin{Bmatrix} \langle(c_2,d_1),\{\frac{\mathfrak{m}_1}{0.2},\frac{\mathfrak{m}_2}{0.3}\}\rangle, \\ \langle(c_2,d_2),\{\frac{\mathfrak{m}_1}{0.5},\frac{\mathfrak{m}_2}{0.4}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_3,\wedge_2) = \begin{Bmatrix} \langle(c_2,d_1),\{\frac{\mathfrak{m}_1}{0.8},\frac{\mathfrak{m}_2}{0.7}\}\rangle, \\ \langle(c_2,d_2),\{\frac{\mathfrak{m}_1}{0.5},\frac{\mathfrak{m}_2}{0.6}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_4,\wedge_3) = \begin{Bmatrix} \langle(c_2,d_1),\{\frac{\mathfrak{m}_1}{0.8},\frac{\mathfrak{m}_2}{0.6}\}\rangle, \\ \langle(c_1,d_2),\{\frac{\mathfrak{m}_1}{0.7},\frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle(c_2,d_2),\{\frac{\mathfrak{m}_1}{0.5},\frac{\mathfrak{m}_2}{0.4}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_5,\wedge_3) = \begin{Bmatrix} \langle(c_2,d_1),\{\frac{\mathfrak{m}_1}{0.2},\frac{\mathfrak{m}_2}{0.3}\}\rangle, \\ \langle(c_1,d_2),\{\frac{\mathfrak{m}_1}{0.7},\frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle(c_2,d_2),\{\frac{\mathfrak{m}_1}{0.5},\frac{\mathfrak{m}_2}{0.4}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_6,\wedge_3) = \begin{Bmatrix} \langle(c_2,d_1),\{\frac{\mathfrak{m}_1}{0.8},\frac{\mathfrak{m}_2}{0.7}\}\rangle, \\ \langle(c_1,d_2),\{\frac{\mathfrak{m}_1}{0.7},\frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle(c_2,d_2),\{\frac{\mathfrak{m}_1}{0.5},\frac{\mathfrak{m}_2}{0.6}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_7,\wedge_3) = \begin{Bmatrix} \langle(c_2,d_1),\{\frac{\mathfrak{m}_1}{0.8},\frac{\mathfrak{m}_2}{0.6}\}\rangle, \\ \langle(c_1,d_2),\{\frac{\mathfrak{m}_1}{0.7},\frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle(c_2,d_2),\{\frac{\mathfrak{m}_1}{0.5},\frac{\mathfrak{m}_2}{0.6}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_8,\wedge_3) = \begin{Bmatrix} \langle(c_2,d_1),\{\frac{\mathfrak{m}_1}{0.2},\frac{\mathfrak{m}_2}{0.3}\}\rangle, \\ \langle(c_1,d_2),\{\frac{\mathfrak{m}_1}{0.3},\frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle(c_2,d_2),\{\frac{\mathfrak{m}_1}{0.5},\frac{\mathfrak{m}_2}{0.4}\}\rangle \end{Bmatrix}$$

$\sigma = \{\tilde{0}_{(\mathfrak{N},Q)}, \tilde{1}_{(\mathfrak{N},Q)}, (\tilde{G}_1,\wedge_1), (\tilde{G}_2,\wedge_2), (\tilde{G}_3,\wedge_2), (\tilde{G}_4,\wedge_3), (\tilde{G}_5,\wedge_3), (\tilde{G}_6,\wedge_3), (\tilde{G}_7,\wedge_3)\}$ is $FHSts$.

Let $\mathfrak{h} = (\omega, \nu): (\mathfrak{M}, L) \to (\mathfrak{N}, M)$ be a FHS mapping as follows:

$\omega(\mathfrak{m}_1) = \mathfrak{n}_2, \omega(\mathfrak{m}_2) = \mathfrak{n}_1,$

$\nu(a_1,b_1) = (c_2,d_1), \nu(a_2,b_1) = (c_1,d_2),\ \nu(a_1,b_2) = (c_2,d_2)$

$\mathfrak{h}(\widetilde{H}_1,\wedge_3) = (\tilde{G}_8,\wedge_3)^c$

$(\widetilde{H}_1,\wedge_3)$ is $FHSos$ in $\mathfrak{M}$ and $\mathfrak{h}(\widetilde{H}_1,\wedge_3) = (\tilde{G}_8,\wedge_3)^c$ is $FHS\theta\mathcal{P}cs$ in $\mathfrak{N}$.

$\therefore$ $\mathfrak{h}$ is $FHScontra\theta\mathcal{P}O$ mapping but $\mathfrak{h}$ is not $FHScontra\mathcal{S}O$ mapping because $(\widetilde{H}_1,\wedge_3)$ is $FHSos$ in $\mathfrak{M}$ but $\mathfrak{h}(\widetilde{H}_1,\wedge_3) = (\tilde{G}_8,\wedge_3)^c$ is not $FHS\mathcal{S}cs$ in $\mathfrak{N}$.

Example 3.6 *Let* $\mathfrak{M} = \{\mathfrak{m}_1, \mathfrak{m}_2\}$ *and* $\mathfrak{N} = \{\mathfrak{n}_1, \mathfrak{n}_2\}$ *be the FHS initial universes and the attributes be* $L = Q_1 \times Q_2$ *and* $M = Q_{1'} \times Q_{2'}$ *respectively. The attributes are given as:*

$Q_1 = \{a_1, a_2, a_3\}, Q_2 = \{b_1, b_2\}$

$Q_{1'} = \{c_1, c_2, c_3\}, Q_{2'} = \{d_1, d_2\}$.

Let $(\mathfrak{M}, L), (\mathfrak{N}, M)$ be the classes of FHS sets. Let the $FHSs$ $(\widetilde{H}_1,\wedge_3)$ over the universe $\mathfrak{M}$ be defined as

$$(\widetilde{H}_1,\wedge_3) = \begin{Bmatrix} \langle (a_1,b_1), \{\frac{\mathfrak{n}_1}{0.6}, \frac{\mathfrak{n}_2}{0.8}\}\rangle, \\ \langle (a_2,b_1), \{\frac{\mathfrak{n}_1}{0.5}, \frac{\mathfrak{n}_2}{0.7}\}\rangle, \\ \langle (a_1,b_2), \{\frac{\mathfrak{n}_1}{0.6}, \frac{\mathfrak{n}_2}{0.5}\}\rangle \end{Bmatrix}$$

$\tau = \{\tilde{0}_{(\mathfrak{M},Q)}, \tilde{1}_{(\mathfrak{M},Q)}, (\widetilde{H}_1,\wedge_3)\}$ is $FHSts$.

Let the $FHSs$'s $(\tilde{G}_1,\wedge_1), (\tilde{G}_2,\wedge_2), (\tilde{G}_3,\wedge_2), (\tilde{G}_4,\wedge_3), (\tilde{G}_5,\wedge_3), (\tilde{G}_6,\wedge_3), (\tilde{G}_7,\wedge_3), (\tilde{G}_8,\wedge_3)$ over the universe $\mathfrak{N}$ be

$$(\tilde{G}_1,\wedge_1) = \begin{Bmatrix} \langle (c_2,d_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.6}\}\rangle, \\ \langle (c_1,d_2), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_2,\wedge_2) = \begin{Bmatrix} \langle (c_2,d_1), \{\frac{\mathfrak{m}_1}{0.2}, \frac{\mathfrak{m}_2}{0.3}\}\rangle, \\ \langle (c_2,d_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.4}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_3,\wedge_2) = \begin{Bmatrix} \langle (c_2,d_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.7}\}\rangle, \\ \langle (c_2,d_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.6}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_4,\wedge_3) = \begin{Bmatrix} \langle (c_2,d_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.6}\}\rangle, \\ \langle (c_1,d_2), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle (c_2,d_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.4}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_5,\wedge_3) = \begin{Bmatrix} \langle (c_2,d_1), \{\frac{\mathfrak{m}_1}{0.2}, \frac{\mathfrak{m}_2}{0.3}\}\rangle, \\ \langle (c_1,d_2), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle (c_2,d_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.4}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_6,\wedge_3) = \begin{Bmatrix} \langle (c_2,d_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.7}\}\rangle, \\ \langle (c_1,d_2), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle (c_2,d_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.6}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_7,\wedge_3) = \begin{Bmatrix} \langle (c_2,d_1),\{\frac{\mathfrak{m}_1}{0.8},\frac{\mathfrak{m}_2}{0.6}\}\rangle, \\ \langle (c_1,d_2),\{\frac{\mathfrak{m}_1}{0.7},\frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle (c_2,d_2),\{\frac{\mathfrak{m}_1}{0.5},\frac{\mathfrak{m}_2}{0.6}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_8,\wedge_3) = \begin{Bmatrix} \langle (c_2,d_1),\{\frac{\mathfrak{m}_1}{0.2},\frac{\mathfrak{m}_2}{0.4}\}\rangle, \\ \langle (c_1,d_2),\{\frac{\mathfrak{m}_1}{0.3},\frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle (c_2,d_2),\{\frac{\mathfrak{m}_1}{0.5},\frac{\mathfrak{m}_2}{0.4}\}\rangle \end{Bmatrix}$$

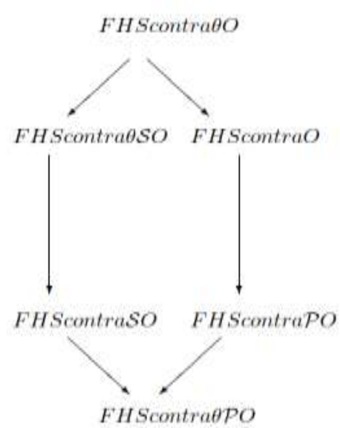
$\sigma = \{\tilde{0}_{(\mathfrak{N},Q)}, \tilde{1}_{(\mathfrak{N},Q)}, (\tilde{G}_1,\wedge_1), (\tilde{G}_2,\wedge_2), (\tilde{G}_3,\wedge_2), (\tilde{G}_4,\wedge_3), (\tilde{G}_5,\wedge_3), (\tilde{G}_6,\wedge_3), (\tilde{G}_7,\wedge_3)\}$ is $FHSts$.

Let $\mathfrak{h} = (\omega,\nu): (\mathfrak{M},L) \to (\mathfrak{N},M)$ be a FHS mapping as follows:
$\omega(\mathfrak{m}_1) = \mathfrak{n}_2, \omega(\mathfrak{m}_2) = \mathfrak{n}_1,$
$\nu(a_1,b_1) = (c_2,d_1), \nu(a_2,b_1) = (c_1,d_2),\ \nu(a_1,b_2) = (c_2,d_2)$
$\mathfrak{h}(\tilde{H}_1,\wedge_3) = (\tilde{G}_8,\wedge_3)^c$

$(\tilde{H}_1,\wedge_3)$ is $FHSos$ in $\mathfrak{M}$ and $\mathfrak{h}(\tilde{H}_1,\wedge_3) = (\tilde{G}_8,\wedge_3)^c$ is $FHS\theta\mathcal{P}cs$ in $\mathfrak{N}$.

$\therefore$ $\mathfrak{h}$ is $FHScontra\theta\mathcal{P}O$ mapping but $\mathfrak{h}$ is not $FHScontra\mathcal{P}O$ mapping because $(\tilde{H}_1,\wedge_3)$ is $FHSos$ in $\mathfrak{M}$ but $\mathfrak{h}(\tilde{H}_1,\wedge_3) = (\tilde{G}_8,\wedge_3)^c$ is not $FHS\mathcal{P}cs$ in $\mathfrak{N}$.

Remark 3.1 *From the results discussed above, the following diagram is obtained.*



Theorem 3.1 *A mapping* $\mathfrak{h}: (\mathfrak{M},L,\tau) \to (\mathfrak{N},M,\sigma)$ *is* $FHScontra\theta O$ *iff for every* $FHSs$ $(\tilde{H},\wedge)$ *of* $(\mathfrak{M},L,\tau)$, $\mathfrak{h}(FHSint(\tilde{H},\wedge)) \supseteq FHS\theta cl(\mathfrak{h}(\tilde{H},\wedge))$.

Proof. Necessity: Assume $\mathfrak{h}$ is a $FHScontra\theta O$ mapping and $(\tilde{H},\wedge)$ is a $FHSos$ in $(\mathfrak{M},L,\tau)$. Now, $FHSint(\tilde{H},\wedge) \subseteq (\tilde{H},\wedge)$ implies $\mathfrak{h}(FHSint(\tilde{H},\wedge)) \subseteq \mathfrak{h}(\tilde{H},\wedge)$. Since $\mathfrak{h}$ is a $FHScontra\theta O$ mapping, $\mathfrak{h}(FHSint(\tilde{H},\wedge))$ is $FHS\theta cs$ in $(\mathfrak{N},M,\sigma)$ such that $\mathfrak{h}(FHSint(\tilde{H},\wedge)) \supseteq \mathfrak{h}(\tilde{H},\wedge)$. Therefore $\mathfrak{h}(FHSint(\tilde{H},\wedge)) \supseteq FHS\theta cl(\mathfrak{h}(\tilde{H},\wedge))$.

Sufficiency: Assume $(\tilde{H},\wedge)$ is a $FHSos$ of $(\mathfrak{M},L,\tau)$. Then we have $\mathfrak{h}(\tilde{H},\wedge) = \mathfrak{h}(FHSint(\tilde{H},\wedge)) \supseteq FHS\theta cl(\mathfrak{h}(\tilde{H},\wedge))$. But $FHS\theta cl\ (\mathfrak{h}(\tilde{H},\wedge)) \supseteq \mathfrak{h}(\tilde{H},\wedge)$. So, $\mathfrak{h}(\tilde{H},\wedge) = FHS\theta cl(\tilde{H},\wedge)$ which implies $\mathfrak{h}(\tilde{H},\wedge)$ is a $FHS\theta cs$ of $(\mathfrak{N},M,\sigma)$ and hence $\mathfrak{h}$ is a $FHScontra\theta O$.

Theorem 3.2 *If $\mathfrak{h}: (\mathfrak{M}, L, \tau) \to (\mathfrak{N}, M, \sigma)$ is a $FHScontra\theta O$ mapping, then $FHSint(\mathfrak{h}^{-1}(\tilde{G},\wedge)) \subseteq \mathfrak{h}^{-1}(FHS\theta cl(\tilde{G},\wedge))$ for every $FHSs$ $(\tilde{G},\wedge)$ of $(\mathfrak{N}, M, \sigma)$.*

Proof. Consider a $FHSs$ $(\tilde{G},\wedge)$ in $(\mathfrak{N}, M, \sigma)$. We know that $FHSint(\mathfrak{h}^{-1}(\tilde{G},\wedge))$ is a $FHSos$ in $(\mathfrak{M}, L, \tau)$. Since $\mathfrak{h}$ is $FHSScontra\theta O$, $\mathfrak{h}(FHSint(\mathfrak{h}^{-1}(\tilde{G},\wedge)))$ is $FHS\theta cs$ in $(\mathfrak{N}, M, \sigma)$ and hence $\mathfrak{h}(FHSint(\mathfrak{h}^{-1}(\tilde{G},\wedge))) \subseteq FHS\theta cl(\mathfrak{h}(\mathfrak{h}^{-1}(\tilde{G},\wedge))) \subseteq FHS\theta cl(\tilde{G},\wedge)$. Thus $FHSint(\mathfrak{h}^{-1}(\tilde{G},\wedge)) \subseteq \mathfrak{h}^{-1}(FHS\theta cl(\tilde{G},\wedge))$.

Theorem 3.3 *A mapping $\mathfrak{h}: (\mathfrak{M}, L, \tau) \to (\mathfrak{N}, M, \sigma)$ is $FHScontra\theta O$ iff for each $FHSs$ $(\tilde{G},\wedge)$ of $(\mathfrak{N}, M, \sigma)$ and for each $FHSos$ $(\widetilde{H},\wedge)$ of $(\mathfrak{M}, L, \tau)$ containing $\mathfrak{h}^{-1}(\tilde{G},\wedge)$, there is a $FHS\theta os$ $(\tilde{A},\wedge)$ of $(\mathfrak{N}, M, \sigma)$ such that $(\tilde{G},\wedge) \subseteq (\tilde{A},\wedge)$ and $\mathfrak{h}^{-1}(\tilde{A},\wedge) \subseteq (\widetilde{H},\wedge)$.*

Proof. Necessity: Let $\mathfrak{h}$ be a $FHScontra\theta O$ mapping. Consider a $FHScs$ $(\tilde{G},\wedge)$ in $(\mathfrak{N}, M, \sigma)$ and a $FHSos$ $(\widetilde{H},\wedge)$ in $(\mathfrak{M}, L, \tau)$ such that $\mathfrak{h}^{-1}(\tilde{G},\wedge) \subseteq (\widetilde{H},\wedge)$. Then $(\tilde{A},\wedge) = (\mathfrak{h}(\widetilde{H},\wedge)^{c})^{c}$ is $FHS\theta os$ of $(\mathfrak{N}, M, \sigma) \ni \mathfrak{h}^{-1}(\tilde{A},\wedge) \subseteq (\widetilde{H},\wedge)$.

Sufficiency: Assume $(\widetilde{H},\wedge)$ is a $FHSos$ of $(\mathfrak{M}, L, \tau)$. So $\mathfrak{h}^{-1}((\mathfrak{h}(\widetilde{H},\wedge))^{c}) \subseteq (\widetilde{H},\wedge)^{c}$ and $(\widetilde{H},\wedge)^{c}$ is $FHScs$ in $(\mathfrak{M}, L, \tau)$. By presumption, there is a $FHS\theta os$ $(\tilde{A},\wedge)$ of $(\mathfrak{N}, M, \sigma)$ such that $(\mathfrak{h}(\widetilde{H},\wedge))^{c} \subseteq (\tilde{A},\wedge)$ and $\mathfrak{h}^{-1}(\tilde{A},\wedge) \subseteq (\widetilde{H},\wedge)^{c}$. Therefore $(\widetilde{H},\wedge) \subseteq (\mathfrak{h}^{-1}(\tilde{A},\wedge))^{c}$. Hence $(\tilde{A},\wedge)^{c} \subseteq \widetilde{H},\wedge) \subseteq \mathfrak{h}((\mathfrak{h}^{-1}(\tilde{A},\wedge))^{c}) \subseteq (\tilde{A},\wedge)^{c}$ which implies $\mathfrak{h}(\widetilde{H},\wedge) = (\tilde{A},\wedge)^{c}$. As $(\tilde{A},\wedge)^{c}$ is $FHS\theta cs$ of $(\mathfrak{N}, M, \sigma)$, $\mathfrak{h}(\widetilde{H},\wedge)$ is $FHS\theta cs$ in $(\mathfrak{N}, M, \sigma)$ and hence $\mathfrak{h}$ is $FHScontra\theta O$ mapping.

Theorem 3.4 *A mapping $\mathfrak{h}: (\mathfrak{M}, L, \tau) \to (\mathfrak{N}, M, \sigma)$ is $FHScontra\theta O$ iff $\mathfrak{h}^{-1}(FHS\theta cl(\tilde{G},\wedge)) \supseteq FHSint(\mathfrak{h}^{-1}(\tilde{G},\wedge))$ for every $FHSs$ $(\tilde{G},\wedge)$ of $(\mathfrak{N}, M, \sigma)$.*

Proof. Necessity: Let $\mathfrak{h}$ be a $FHScontra\theta O$ mapping. For any $FHSs$ $(\tilde{G},\wedge)$ of $(\mathfrak{N}, M, \sigma)$, $\mathfrak{h}^{-1}(\tilde{G},\wedge) \subseteq FHScl(\mathfrak{h}^{-1}(\tilde{G},\wedge))$. Therefore by Theorem 3.3, there exists a $FHS\theta os$ $(\tilde{A},\wedge)$ in $(\mathfrak{N}, M, \sigma)$ $\ni$ $(\tilde{G},\wedge) \supseteq (\tilde{A},\wedge)$ & $\mathfrak{h}^{-1}(\tilde{A},\wedge) \supseteq FHSint(\mathfrak{h}^{-1}(\tilde{G},\wedge))$. Hence $\mathfrak{h}^{-1}(FHS\theta cl(\tilde{G},\wedge)) \supseteq \mathfrak{h}^{-1}(\tilde{A},\wedge) \supseteq FHSint(\mathfrak{h}^{-1}(\tilde{G},\wedge))$.

Sufficiency: Let $(\tilde{G},\wedge)$ be a $FHSs$ in $(\mathfrak{N}, M, \sigma)$ and $(\widetilde{H},\wedge)$ be a $FHScs$ of $(\mathfrak{M}, L, \tau)$ containing $\mathfrak{h}^{-1}(\tilde{G},\wedge)$.

Put $(\tilde{A},\wedge) = FHScl(\tilde{G},\wedge)$, then $(\tilde{G},\wedge) \subseteq (\tilde{A},\wedge)$ and $(\tilde{A},\wedge)$ is $FHS\theta cs$ and $\mathfrak{h}^{-1}(\tilde{A},\wedge) \subseteq FHSint(\mathfrak{h}^{-1}(\tilde{G},\wedge)) \subseteq (\widetilde{H},\wedge)$. Thus by Theorem 3.3, $\mathfrak{h}$ is $FHS\theta O$ mapping.

Theorem 3.5 *If $\mathfrak{h}: (\mathfrak{M}, L, \tau) \to (\mathfrak{N}, M, \sigma)$ and $\mathfrak{g}: (\mathfrak{N}, M, \sigma) \to (P, N, \rho)$ be two fuzzy hypersoft mappings and $\mathfrak{g} \circ \mathfrak{h}: (\mathfrak{M}, L, \tau) \to (P, N, \rho)$ is $FHScontra\theta O$ mapping. If $\mathfrak{g}: (\mathfrak{N}, M, \sigma) \to (P, N, \rho)$ is $FHScontra\theta Irr$, then $\mathfrak{h}: (\mathfrak{M}, L, \tau) \to (\mathfrak{N}, M, \sigma)$ is $FHS\theta O$ mapping.*

Proof. Let $(\widetilde{H},\wedge)$ be a $FHSos$ in $(\mathfrak{M}, L, \tau)$. Then $\mathfrak{g} \circ \mathfrak{h}(\widetilde{H},\wedge)$ is $FHS\theta cs$ of (P, N, ρ) because $\mathfrak{g} \circ \mathfrak{h}$ is $FHScontra\theta O$ mapping. As $\mathfrak{g}$ is $FHScontra\theta Irr$ and $\mathfrak{g} \circ \mathfrak{h}(\widetilde{H},\wedge)$ is $FHS\theta cs$ of (P, N, ρ), $\mathfrak{g}^{-1}(\mathfrak{g} \circ \mathfrak{h}(\widetilde{H},\wedge)) = \mathfrak{h}(\widetilde{H},\wedge)$ is $FHS\theta os$ in $(\mathfrak{N}, M, \sigma)$. Hence $\mathfrak{h}$ is $FHS\theta O$ mapping.

Theorem 3.6 *If $\mathfrak{h}: (\mathfrak{M}, L, \tau) \to (\mathfrak{N}, M, \sigma)$ is $FHSO$ & $\mathfrak{g}: (\mathfrak{N}, M, \sigma) \to (P, N, \rho)$ is $FHScontra\theta O$ mappings, then $\mathfrak{g} \circ \mathfrak{h}: (\mathfrak{M}, L, \tau) \to (P, N, \rho)$ is $FHScontra\theta O$.*

Proof. Let $(\widetilde{H},\wedge)$ be a $FHSos$ in $(\mathfrak{M}, L, \tau)$. Then $\mathfrak{h}(\widetilde{H},\wedge)$ is a $FHSos$ of $(\mathfrak{N}, M, \sigma)$ because $\mathfrak{h}$ is a $FHSO$ mapping. As $\mathfrak{g}$ is $FHScontra\theta O$, $\mathfrak{g}(\mathfrak{h}(\widetilde{H},\wedge)) = (\mathfrak{g} \circ \mathfrak{h})(\widetilde{H},\wedge)$ is a $FHS\theta cs$ of (P, N, ρ). Thus $\mathfrak{g} \circ \mathfrak{h}$ is $FHScontra\theta O$ mapping.

4 Fuzzy Hypersoft Contra θ Closed Mapping

In this section, fuzzy hypersoft contra θ closed maps are introduced and some of its properties are discussed.

Definition 4.1 *A mapping* $\mathfrak{h}: (\mathfrak{M}, L, \tau) \to (\mathfrak{N}, M, \sigma)$ *is fuzzy hypersoft contra* θ *closed (resp.* θ *semi closed &* θ *pre closed) (in short,* $FHScontra\theta C$ *(resp.* $FHScontra\theta\mathcal{S}C$ *&* $FHScontra\theta\mathcal{P}C$ *)) if the image of each* $FHScs$ *of* $(\mathfrak{M}, L, \tau)$ *is* $FHS\theta os$ *(resp.* $FHS\theta\mathcal{S}os, FHS\theta\mathcal{P}os$*) in* $(\mathfrak{N}, M, \sigma)$

Proposition 4.1 *The statements hold but the converse is not.*

1. Each $FHScontra\theta C$ is a $FHScontra\theta\mathcal{S}C$.
2. Each $FHScontra\theta C$ is a $FHScontraC$.
3. Each $FHScontra\theta\mathcal{S}C$ is a $FHScontra\mathcal{S}C$.
4. Each $FHScontraC$ is a $FHScontra\mathcal{P}C$.
5. Each $FHScontra\mathcal{S}C$ is a $FHScontra\theta\mathcal{P}C$.
6. Each $FHScontra\mathcal{P}C$ is a $FHScontra\theta\mathcal{P}C$.

Proof. Consider the map $\mathfrak{h}: (\mathfrak{M}, L, \tau) \to (\mathfrak{N}, M, \sigma)$

1. Let $(\widetilde{H},\wedge)$ be a $FHScs$ in $\mathfrak{M}$. As $\mathfrak{h}$ is $FHScontra\theta C$, $\mathfrak{h}(\widetilde{H},\wedge)$ is $FHS\theta os$ in $\mathfrak{N}$. Since all $FHS\theta os$ are $FHS\theta\mathcal{S}os$, $\mathfrak{h}(\widetilde{H},\wedge)$ is $FHS\theta\mathcal{S}os$ in $\mathfrak{N}$. Hence $\mathfrak{h}$ is a $FHScontra\theta\mathcal{S}C$.
2. Let $(\widetilde{H},\wedge)$ be a $FHScs$ in $\mathfrak{M}$. As $\mathfrak{h}$ is $FHScontra\theta C$, $\mathfrak{h}(\widetilde{H},\wedge)$ is $FHS\theta os$ in $\mathfrak{N}$. Since all $FHS\theta os$ are $FHSos$, $\mathfrak{h}(\widetilde{H},\wedge)$ is $FHSos$ in $\mathfrak{N}$. Hence $\mathfrak{h}$ is a $FHScontraC$.
3. Let $(\widetilde{H},\wedge)$ be a $FHScs$ in $\mathfrak{M}$. As $\mathfrak{h}$ is $FHScontra\theta\mathcal{S}C$, $\mathfrak{h}(\widetilde{H},\wedge)$ is $FHS\theta\mathcal{S}os$ in $\mathfrak{N}$. Since all $FHS\theta\mathcal{S}os$ are $FHS\mathcal{S}os$, $\mathfrak{h}(\widetilde{H},\wedge)$ is $FHS\mathcal{S}os$ in $\mathfrak{N}$. Hence $\mathfrak{h}$ is a $FHScontra\mathcal{S}C$.
4. Let $(\widetilde{H},\wedge)$ be a $FHScs$ in $\mathfrak{M}$. As $\mathfrak{h}$ is $FHScontraC$, $\mathfrak{h}(\widetilde{H},\wedge)$ is $FHSos$ in $\mathfrak{N}$. Since all $FHSos$ are $FHS\mathcal{P}os$, $\mathfrak{h}(\widetilde{H},\wedge)$ is $FHS\mathcal{P}os$ in $\mathfrak{N}$. Hence $\mathfrak{h}$ is a $FHScontra\mathcal{P}C$.
5. Let $(\widetilde{H},\wedge)$ be a $FHScs$ in $\mathfrak{M}$. As $\mathfrak{h}$ is $FHScontra\mathcal{S}C$, $\mathfrak{h}(\widetilde{H},\wedge)$ is $FHS\mathcal{S}os$ in $\mathfrak{N}$. Since all $FHS\mathcal{S}os$ are $FHS\theta\mathcal{P}os$, $\mathfrak{h}(\widetilde{H},\wedge)$ is $FHS\theta\mathcal{P}os$ in $\mathfrak{N}$. Hence $\mathfrak{h}$ is a $FHScontra\theta\mathcal{P}C$.
6. Let $(\widetilde{H},\wedge)$ be a $FHScs$ in $\mathfrak{M}$. As $\mathfrak{h}$ is $FHScontra\mathcal{P}C$, $\mathfrak{h}(\widetilde{H},\wedge)$ is $FHS\mathcal{P}os$ in $\mathfrak{N}$. Since all $FHS\mathcal{P}os$ are $FHS\theta\mathcal{P}os$, $\mathfrak{h}(\widetilde{H},\wedge)$ is $FHS\theta\mathcal{P}os$ in $\mathfrak{N}$. Hence $\mathfrak{h}$ is a $FHScontra\theta\mathcal{P}C$.

Example 4.1 *Let* $\mathfrak{M} = \{\mathfrak{m}_1, \mathfrak{m}_2\}$ *and* $\mathfrak{N} = \{\mathfrak{n}_1, \mathfrak{n}_2\}$ *be the* FHS *initial universes and the attributes be* $L = Q_1 \times Q_2$ *and* $M = Q_{1'} \times Q_{2'}$ *respectively. The attributes are given as:*

$Q_1 = \{a_1, a_2, a_3\}, Q_2 = \{b_1, b_2\}$

$Q_{1'} = \{c_1, c_2, c_3\}, Q_{2'} = \{d_1, d_2\}$.

Let $(\mathfrak{M}, L), (\mathfrak{N}, M)$ be the classes of FHS sets. Let the $FHSs$ $(\widetilde{H}_1,\wedge_3)$ over the universe $\mathfrak{M}$ be defined as

$$(\widetilde{H}_1,\wedge_3) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{n}_1}{0.3}, \frac{\mathfrak{n}_2}{0.2}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{n}_1}{0.5}, \frac{\mathfrak{n}_2}{0.3}\} \rangle, \\ \langle (a_1, b_1), \{\frac{\mathfrak{n}_1}{0.4}, \frac{\mathfrak{n}_2}{0.5}\} \rangle \end{Bmatrix}$$

$\tau = \{\tilde{0}_{(\mathfrak{M},Q)}, \tilde{1}_{(\mathfrak{M},Q)}, (\widetilde{H}_1,\wedge_3)\}$ is $FHSts$.

Let the $FHSs$'s $(\tilde{G}_1,\wedge_1), (\tilde{G}_2,\wedge_2), (\tilde{G}_3,\wedge_2), (\tilde{G}_4,\wedge_3), (\tilde{G}_5,\wedge_3), (\tilde{G}_6,\wedge_3), (\tilde{G}_7,\wedge_3)$ over the universe $\mathfrak{N}$ be

$$(\tilde{G}_1,\wedge_1) = \begin{Bmatrix} \langle(c_2,d_1),\{\frac{\mathfrak{m}_1}{0.8},\frac{\mathfrak{m}_2}{0.6}\}\rangle, \\ \langle(c_1,d_2),\{\frac{\mathfrak{m}_1}{0.7},\frac{\mathfrak{m}_2}{0.5}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_2,\wedge_2) = \begin{Bmatrix} \langle(c_2,d_1),\{\frac{\mathfrak{m}_1}{0.2},\frac{\mathfrak{m}_2}{0.3}\}\rangle, \\ \langle(c_2,d_2),\{\frac{\mathfrak{m}_1}{0.5},\frac{\mathfrak{m}_2}{0.4}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_3,\wedge_2) = \begin{Bmatrix} \langle(c_2,d_1),\{\frac{\mathfrak{m}_1}{0.8},\frac{\mathfrak{m}_2}{0.7}\}\rangle, \\ \langle(c_2,d_2),\{\frac{\mathfrak{m}_1}{0.5},\frac{\mathfrak{m}_2}{0.6}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_4,\wedge_3) = \begin{Bmatrix} \langle(c_2,d_1),\{\frac{\mathfrak{m}_1}{0.8},\frac{\mathfrak{m}_2}{0.6}\}\rangle, \\ \langle(c_1,d_2),\{\frac{\mathfrak{m}_1}{0.7},\frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle(c_2,d_2),\{\frac{\mathfrak{m}_1}{0.5},\frac{\mathfrak{m}_2}{0.4}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_5,\wedge_3) = \begin{Bmatrix} \langle(c_2,d_1),\{\frac{\mathfrak{m}_1}{0.2},\frac{\mathfrak{m}_2}{0.3}\}\rangle, \\ \langle(c_1,d_2),\{\frac{\mathfrak{m}_1}{0.7},\frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle(c_2,d_2),\{\frac{\mathfrak{m}_1}{0.5},\frac{\mathfrak{m}_2}{0.4}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_6,\wedge_3) = \begin{Bmatrix} \langle(c_2,d_1),\{\frac{\mathfrak{m}_1}{0.8},\frac{\mathfrak{m}_2}{0.7}\}\rangle, \\ \langle(c_1,d_2),\{\frac{\mathfrak{m}_1}{0.7},\frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle(c_2,d_2),\{\frac{\mathfrak{m}_1}{0.5},\frac{\mathfrak{m}_2}{0.6}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_7,\wedge_3) = \begin{Bmatrix} \langle(c_2,d_1),\{\frac{\mathfrak{m}_1}{0.8},\frac{\mathfrak{m}_2}{0.6}\}\rangle, \\ \langle(c_1,d_2),\{\frac{\mathfrak{m}_1}{0.7},\frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle(c_2,d_2),\{\frac{\mathfrak{m}_1}{0.5},\frac{\mathfrak{m}_2}{0.6}\}\rangle \end{Bmatrix}$$

$\sigma = \{\tilde{0}_{(\mathfrak{N},Q)}, \tilde{1}_{(\mathfrak{N},Q)}, (\tilde{G}_1,\wedge_1), (\tilde{G}_2,\wedge_2), (\tilde{G}_3,\wedge_2), (\tilde{G}_4,\wedge_3), (\tilde{G}_5,\wedge_3), (\tilde{G}_6,\wedge_3), (\tilde{G}_7,\wedge_3)\}$ is $FHSts$.

Let $\mathfrak{h} = (\omega, \nu): (\mathfrak{M}, L) \to (\mathfrak{N}, M)$ be a FHS mapping as follows:
$\omega(\mathfrak{m}_1) = \mathfrak{n}_2, \omega(\mathfrak{m}_2) = \mathfrak{n}_1$,
$\nu(a_1, b_1) = (c_2, d_1), \nu(a_2, b_1) = (c_1, d_2),\ \nu(a_1, b_2) = (c_2, d_2)$
$\mathfrak{h}(\widetilde{H}_1,\wedge_3)^c = (\tilde{G}_6,\wedge_3)$

$(\widetilde{H}_1,\wedge_3)^c$ is $FHScs$ in $\mathfrak{M}$ and $\mathfrak{h}(\widetilde{H}_1,\wedge_3)^c = (\tilde{G}_6,\wedge_3)$ is $FHS\theta Sos$ in $\mathfrak{N}$.

∴ $\mathfrak{h}$ is $FHScontra\theta SC$ mapping but $\mathfrak{h}$ is not $FHScontra\theta C$ mapping because $(\widetilde{H}_1,\wedge_3)^c$ is $FHScs$ in $\mathfrak{M}$ but $\mathfrak{h}(\widetilde{H}_1,\wedge_3)^c = (\tilde{G}_6,\wedge_3)$ is not $FHS\theta os$ in $\mathfrak{N}$.

Example 4.2 *Let* $\mathfrak{M} = \{\mathfrak{m}_1, \mathfrak{m}_2\}$ *and* $\mathfrak{N} = \{\mathfrak{n}_1, \mathfrak{n}_2\}$ *be the* FHS *initial universes and the attributes be* $L = Q_1 \times Q_2$ *and* $M = Q_{1'} \times Q_{2'}$ *respectively. The attributes are given as:*
$Q_1 = \{a_1, a_2, a_3\}, Q_2 = \{b_1, b_2\}$
$Q_{1'} = \{c_1, c_2, c_3\}, Q_{2'} = \{d_1, d_2\}$.

Let $(\mathfrak{M}, L), (\mathfrak{N}, M)$ be the classes of FHS sets. Let the $FHSs$ $(\widetilde{H}_1,\wedge_1)$ over the universe $\mathfrak{M}$ be defined as

$$(\widetilde{H}_1,\wedge_1) = \begin{Bmatrix} \langle(a_1,b_1),\{\frac{\mathfrak{n}_1}{0.4},\frac{\mathfrak{n}_2}{0.2}\}\rangle, \\ \langle(a_2,b_1),\{\frac{\mathfrak{n}_1}{0.5},\frac{\mathfrak{n}_2}{0.3}\}\rangle, \\ \end{Bmatrix}$$

$\tau = \{\tilde{0}_{(\mathfrak{M},Q)}, \tilde{1}_{(\mathfrak{M},Q)}, (\widetilde{H}_1,\wedge_1)\}$ is $FHSts$.

Let the $FHSs$'s $(\tilde{G}_1,\wedge_1), (\tilde{G}_2,\wedge_2), (\tilde{G}_3,\wedge_2), (\tilde{G}_4,\wedge_3), (\tilde{G}_5,\wedge_3), (\tilde{G}_6,\wedge_3), (\tilde{G}_7,\wedge_3)$ over the universe $\mathfrak{N}$ be

$$(\tilde{G}_1,\wedge_1) = \begin{Bmatrix} \langle(c_2,d_1),\{\frac{\mathfrak{m}_1}{0.8},\frac{\mathfrak{m}_2}{0.6}\}\rangle, \\ \langle(c_1,d_2),\{\frac{\mathfrak{m}_1}{0.7},\frac{\mathfrak{m}_2}{0.5}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_2,\wedge_2) = \begin{Bmatrix} \langle(c_2,d_1),\{\frac{\mathfrak{m}_1}{0.2},\frac{\mathfrak{m}_2}{0.3}\}\rangle, \\ \langle(c_2,d_2),\{\frac{\mathfrak{m}_1}{0.5},\frac{\mathfrak{m}_2}{0.4}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_3,\wedge_2) = \begin{Bmatrix} \langle(c_2,d_1),\{\frac{\mathfrak{m}_1}{0.8},\frac{\mathfrak{m}_2}{0.7}\}\rangle, \\ \langle(c_2,d_2),\{\frac{\mathfrak{m}_1}{0.5},\frac{\mathfrak{m}_2}{0.6}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_4,\wedge_3) = \begin{Bmatrix} \langle(c_2,d_1),\{\frac{\mathfrak{m}_1}{0.8},\frac{\mathfrak{m}_2}{0.6}\}\rangle, \\ \langle(c_1,d_2),\{\frac{\mathfrak{m}_1}{0.7},\frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle(c_2,d_2),\{\frac{\mathfrak{m}_1}{0.5},\frac{\mathfrak{m}_2}{0.4}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_5,\wedge_3) = \begin{Bmatrix} \langle(c_2,d_1),\{\frac{\mathfrak{m}_1}{0.2},\frac{\mathfrak{m}_2}{0.3}\}\rangle, \\ \langle(c_1,d_2),\{\frac{\mathfrak{m}_1}{0.7},\frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle(c_2,d_2),\{\frac{\mathfrak{m}_1}{0.5},\frac{\mathfrak{m}_2}{0.4}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_6,\wedge_3) = \begin{Bmatrix} \langle(c_2,d_1),\{\frac{\mathfrak{m}_1}{0.8},\frac{\mathfrak{m}_2}{0.7}\}\rangle, \\ \langle(c_1,d_2),\{\frac{\mathfrak{m}_1}{0.7},\frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle(c_2,d_2),\{\frac{\mathfrak{m}_1}{0.5},\frac{\mathfrak{m}_2}{0.6}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_7,\wedge_3) = \begin{Bmatrix} \langle(c_2,d_1),\{\frac{\mathfrak{m}_1}{0.8},\frac{\mathfrak{m}_2}{0.6}\}\rangle, \\ \langle(c_1,d_2),\{\frac{\mathfrak{m}_1}{0.7},\frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle(c_2,d_2),\{\frac{\mathfrak{m}_1}{0.5},\frac{\mathfrak{m}_2}{0.6}\}\rangle \end{Bmatrix}$$

$\sigma = \{\tilde{0}_{(\mathfrak{N},Q)}, \tilde{1}_{(\mathfrak{N},Q)}, (\tilde{G}_1,\wedge_1), (\tilde{G}_2,\wedge_2), (\tilde{G}_3,\wedge_2), (\tilde{G}_4,\wedge_3), (\tilde{G}_5,\wedge_3), (\tilde{G}_6,\wedge_3), (\tilde{G}_7,\wedge_3)\}$ is $FHSts$.

Let $\mathfrak{h} = (\omega,\nu): (\mathfrak{M}, L) \to (\mathfrak{N}, M)$ be a FHS mapping as follows:

$\omega(\mathfrak{m}_1) = \mathfrak{n}_2, \omega(\mathfrak{m}_2) = \mathfrak{n}_1,$

$\nu(a_1,b_1) = (c_2,d_1), \nu(a_2,b_1) = (c_1,d_2),\ \nu(a_1,b_2) = (c_2,d_2)$

$\mathfrak{h}(\widetilde{H}_1,\wedge_1)^c = (\tilde{G}_1,\wedge_1)$

$(\widetilde{H}_1,\wedge_1)^c$ is $FHScs$ in $\mathfrak{M}$ and $\mathfrak{h}(\widetilde{H}_1,\wedge_1)^c = (\tilde{G}_1,\wedge_1)$ is $FHSos$ in $\mathfrak{N}$.

$\therefore$ $\mathfrak{h}$ is $FHScontraC$ mapping but $\mathfrak{h}$ is not $FHScontra\theta C$ mapping because $(\widetilde{H}_1,\wedge_1)^c$ is $FHScs$ in $\mathfrak{M}$ but $\mathfrak{h}(\widetilde{H}_1,\wedge_1)^c = (\tilde{G}_1,\wedge_1)$ is not $FHS\theta os$ in $\mathfrak{N}$.

Example 4.3 *Let* $\mathfrak{M} = \{\mathfrak{m}_1,\mathfrak{m}_2\}$ *and* $\mathfrak{N} = \{\mathfrak{n}_1,\mathfrak{n}_2\}$ *be the* FHS *initial universes and the*

attributes be $L = Q_1 \times Q_2$ *and* $M = Q_{1'} \times Q_{2'}$ *respectively. The attributes are given as:*
$Q_1 = \{a_1, a_2, a_3\}, Q_2 = \{b_1, b_2\}$
$Q_{1'} = \{c_1, c_2, c_3\}, Q_{2'} = \{d_1, d_2\}$.

Let $(\mathfrak{M}, L), (\mathfrak{N}, M)$ be the classes of FHS sets. Let the $FHSs$ $(\widetilde{H}_1,\wedge_1)$ over the universe $\mathfrak{N}$ be defined as

$$(\widetilde{H}_1,\wedge_3) = \begin{Bmatrix} \langle (c_2, d_1), \{\frac{\mathfrak{n}_1}{0.4}, \frac{\mathfrak{n}_2}{0.2}\}\rangle, \\ \langle (c_1, d_2), \{\frac{\mathfrak{n}_1}{0.5}, \frac{\mathfrak{n}_2}{0.3}\}\rangle, \\ \langle (c_2, d_2), \{\frac{\mathfrak{n}_1}{0.5}, \frac{\mathfrak{n}_2}{0.6}\}\rangle \end{Bmatrix}$$

$\tau = \{\tilde{0}_{(\mathfrak{N},Q)}, \tilde{1}_{(\mathfrak{N},Q)}, (\widetilde{H}_1,\wedge_3)\}$ is $FHSts$.

Let the $FHSs$'s $(\tilde{G}_1,\wedge_1), (\tilde{G}_2,\wedge_2), (\tilde{G}_3,\wedge_2), (\tilde{G}_4,\wedge_3), (\tilde{G}_5,\wedge_3), (\tilde{G}_6,\wedge_3), (\tilde{G}_7,\wedge_3)$ over the universe $\mathfrak{N}$ be

$$(\tilde{G}_1,\wedge_1) = \begin{Bmatrix} \langle (c_2, d_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.6}\}\rangle, \\ \langle (c_1, d_2), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_2,\wedge_2) = \begin{Bmatrix} \langle (c_2, d_1), \{\frac{\mathfrak{m}_1}{0.2}, \frac{\mathfrak{m}_2}{0.3}\}\rangle, \\ \langle (c_2, d_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.4}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_3,\wedge_2) = \begin{Bmatrix} \langle (c_2, d_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.7}\}\rangle, \\ \langle (c_2, d_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.6}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_4,\wedge_3) = \begin{Bmatrix} \langle (c_2, d_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.6}\}\rangle, \\ \langle (c_1, d_2), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle (c_2, d_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.4}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_5,\wedge_3) = \begin{Bmatrix} \langle (c_2, d_1), \{\frac{\mathfrak{m}_1}{0.2}, \frac{\mathfrak{m}_2}{0.3}\}\rangle, \\ \langle (c_1, d_2), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle (c_2, d_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.4}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_6,\wedge_3) = \begin{Bmatrix} \langle (c_2, d_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.7}\}\rangle, \\ \langle (c_1, d_2), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle (c_2, d_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.6}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_7,\wedge_3) = \begin{Bmatrix} \langle (c_2, d_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.6}\}\rangle, \\ \langle (c_1, d_2), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle (c_2, d_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.6}\}\rangle \end{Bmatrix}$$

$\sigma = \{\tilde{0}_{(\mathfrak{N},Q)}, \tilde{1}_{(\mathfrak{N},Q)}, (\tilde{G}_1,\wedge_1), (\tilde{G}_2,\wedge_2), (\tilde{G}_3,\wedge_2), (\tilde{G}_4,\wedge_3), (\tilde{G}_5,\wedge_3), (\tilde{G}_6,\wedge_3), (\tilde{G}_7,\wedge_3)\}$ is $FHSts$.

Let $\mathfrak{h} = (\omega, \nu): (\mathfrak{M}, L) \to (\mathfrak{N}, M)$ be a FHS mapping as follows:
$\omega(\mathfrak{m}_1) = \mathfrak{n}_2, \omega(\mathfrak{m}_2) = \mathfrak{n}_1$,
$\nu(a_1, b_1) = (c_2, d_1), \nu(a_2, b_1) = (c_1, d_2),\ \nu(a_1, b_2) = (c_2, d_2)$
$\mathfrak{h}(\widetilde{H}_1,\wedge_3)^c = (\tilde{G}_4,\wedge_3)$

$(\widetilde{H}_1,\wedge_3)^c$ is $FHScs$ in $\mathfrak{M}$ and $\mathfrak{h}(\widetilde{H}_1,\wedge_3)^c = (\tilde{G}_4,\wedge_3)$ is $FHS\mathcal{S}os$ in $\mathfrak{N}$.

$\therefore$ $\mathfrak{h}$ is $FHScontra\mathcal{S}C$ mapping but $\mathfrak{h}$ is not $FHScontra\theta\mathcal{S}C$ mapping because $(\widetilde{H}_1,\wedge_3)^c$ is $FHScs$ in $\mathfrak{M}$ but $\mathfrak{h}(\widetilde{H}_1,\wedge_3)^c = (\tilde{G}_4,\wedge_3)$ is not $FHS\theta\mathcal{S}os$ in $\mathfrak{N}$.

Example 4.4 *Let* $\mathfrak{M} = \{\mathfrak{m}_1, \mathfrak{m}_2\}$ *and* $\mathfrak{N} = \{\mathfrak{n}_1, \mathfrak{n}_2\}$ *be the* FHS *initial universes and the attributes be* $L = Q_1 \times Q_2$ *and* $M = Q_{1'} \times Q_{2'}$ *respectively. The attributes are given as:*
$Q_1 = \{a_1, a_2, a_3\}, Q_2 = \{b_1, b_2\}$
$Q_{1'} = \{c_1, c_2, c_3\}, Q_{2'} = \{d_1, d_2\}$.

Let $(\mathfrak{M}, L), (\mathfrak{N}, M)$ be the classes of FHS sets. Let the $FHSs$ $(\widetilde{H}_1,\wedge_1)$ over the universe $\mathfrak{M}$ be defined as

$$(\widetilde{H}_1,\wedge_3) = \begin{Bmatrix} \langle (a_1,b_1), \{\frac{\mathfrak{n}_1}{0.3}, \frac{\mathfrak{n}_2}{0.2}\}\rangle, \\ \langle (a_2,b_1), \{\frac{\mathfrak{n}_1}{0.5}, \frac{\mathfrak{n}_2}{0.7}\}\rangle, \\ \langle (a_1,b_2), \{\frac{\mathfrak{n}_1}{0.4}, \frac{\mathfrak{n}_2}{0.5}\}\rangle \end{Bmatrix}$$

$\tau = \{\tilde{0}_{(\mathfrak{M},Q)}, \tilde{1}_{(\mathfrak{M},Q)}, (\widetilde{H}_1,\wedge_3)\}$ is $FHSts$.

Let the $FHSs$'s $(\tilde{G}_1,\wedge_1), (\tilde{G}_2,\wedge_2), (\tilde{G}_3,\wedge_2), (\tilde{G}_4,\wedge_3), (\tilde{G}_5,\wedge_3), (\tilde{G}_6,\wedge_3), (\tilde{G}_7,\wedge_3), (\tilde{G}_8,\wedge_3)$ over the universe $\mathfrak{N}$ be

$$(\tilde{G}_1,\wedge_1) = \begin{Bmatrix} \langle (c_2,d_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.6}\}\rangle, \\ \langle (c_1,d_2), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_2,\wedge_2) = \begin{Bmatrix} \langle (c_2,d_1), \{\frac{\mathfrak{m}_1}{0.2}, \frac{\mathfrak{m}_2}{0.3}\}\rangle, \\ \langle (c_2,d_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.4}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_3,\wedge_2) = \begin{Bmatrix} \langle (c_2,d_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.7}\}\rangle, \\ \langle (c_2,d_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.6}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_4,\wedge_3) = \begin{Bmatrix} \langle (c_2,d_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.6}\}\rangle, \\ \langle (c_1,d_2), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle (c_2,d_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.4}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_5,\wedge_3) = \begin{Bmatrix} \langle (c_2,d_1), \{\frac{\mathfrak{m}_1}{0.2}, \frac{\mathfrak{m}_2}{0.3}\}\rangle, \\ \langle (c_1,d_2), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle (c_2,d_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.4}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_6,\wedge_3) = \begin{Bmatrix} \langle (c_2,d_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.7}\}\rangle, \\ \langle (c_1,d_2), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle (c_2,d_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.6}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_7,\wedge_3) = \begin{Bmatrix} \langle (c_2,d_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.6}\}\rangle, \\ \langle (c_1,d_2), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle (c_2,d_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.6}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_8,\wedge_3) = \begin{Bmatrix} \langle (c_2,d_1),\{\frac{\mathfrak{m}_1}{0.8},\frac{\mathfrak{m}_2}{0.7}\}\rangle, \\ \langle (c_1,d_2),\{\frac{\mathfrak{m}_1}{0.3},\frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle (c_2,d_2),\{\frac{\mathfrak{m}_1}{0.5},\frac{\mathfrak{m}_2}{0.6}\}\rangle \end{Bmatrix}$$

$\sigma =$
$\{\tilde{0}_{(\mathfrak{N},Q)}, \tilde{1}_{(\mathfrak{N},Q)}, (\tilde{G}_1,\wedge_1), (\tilde{G}_2,\wedge_2), (\tilde{G}_3,\wedge_2), (\tilde{G}_4,\wedge_3), (\tilde{G}_5,\wedge_3), (\tilde{G}_6,\wedge_3), (\tilde{G}_7,\wedge_3), (\tilde{G}_7,\wedge_3)\}$ is $FHSts$.

Let $\mathfrak{h} = (\omega,\nu): (\mathfrak{M},L) \to (\mathfrak{N},M)$ be a FHS mapping as follows:
$\omega(\mathfrak{m}_1) = \mathfrak{n}_2, \omega(\mathfrak{m}_2) = \mathfrak{n}_1,$
$\nu(a_1,b_1) = (c_2,d_1), \nu(a_2,b_1) = (c_1,d_2),\ \nu(a_1,b_2) = (c_2,d_2)$
$\mathfrak{h}(\widetilde{H}_1,\wedge_3)^c = (\tilde{G}_8,\wedge_3)$

$(\widetilde{H}_1,\wedge_3)^c$ is $FHScs$ in $\mathfrak{M}$ and $\mathfrak{h}(\widetilde{H}_1,\wedge_3)^c = (\tilde{G}_8,\wedge_3)$ is $FHS\mathcal{P}os$ in $\mathfrak{N}$.

$\therefore$ $\mathfrak{h}$ is $FHScontra\mathcal{P}C$ mapping but $\mathfrak{h}$ is not $FHScontraC$ mapping because $(\widetilde{H}_1,\wedge_3)^c$ is $FHScs$ in $\mathfrak{M}$ but $\mathfrak{h}(\widetilde{H}_1,\wedge_3)^c = (\tilde{G}_8,\wedge_3)$ is not $FHSos$ in $\mathfrak{N}$.

Example 4.5 *Let* $\mathfrak{M} = \{\mathfrak{m}_1,\mathfrak{m}_2\}$ *and* $\mathfrak{N} = \{\mathfrak{n}_1,\mathfrak{n}_2\}$ *be the* FHS *initial universes and the attributes be* $L = Q_1 \times Q_2$ *and* $M = Q_{1'} \times Q_{2'}$ *respectively. The attributes are given as:*
$Q_1 = \{a_1,a_2,a_3\}, Q_2 = \{b_1,b_2\}$
$Q_{1'} = \{c_1,c_2,c_3\}, Q_{2'} = \{d_1,d_2\}$.

Let $(\mathfrak{M},L),(\mathfrak{N},M)$ be the classes of FHS sets. Let the $FHSs$ $(\widetilde{H}_1,\wedge_3)$ over the universe $\mathfrak{M}$ be defined as

$$(\widetilde{H}_1,\wedge_3) = \begin{Bmatrix} \langle (a_1,b_1),\{\frac{\mathfrak{n}_1}{0.7},\frac{\mathfrak{n}_2}{0.8}\}\rangle, \\ \langle (a_2,b_1),\{\frac{\mathfrak{n}_1}{0.5},\frac{\mathfrak{n}_2}{0.7}\}\rangle, \\ \langle (a_1,b_2),\{\frac{\mathfrak{n}_1}{0.6},\frac{\mathfrak{n}_2}{0.5}\}\rangle \end{Bmatrix}$$

$\tau = \{\tilde{0}_{(\mathfrak{M},Q)}, \tilde{1}_{(\mathfrak{M},Q)}, (\widetilde{H}_1,\wedge_3)\}$ is $FHSts$.

Let the $FHSs$'s $(\tilde{G}_1,\wedge_1), (\tilde{G}_2,\wedge_2), (\tilde{G}_3,\wedge_2), (\tilde{G}_4,\wedge_3), (\tilde{G}_5,\wedge_3), (\tilde{G}_6,\wedge_3), (\tilde{G}_7,\wedge_3), (\tilde{G}_8,\wedge_3)$ over the universe $\mathfrak{N}$ be

$$(\tilde{G}_1,\wedge_1) = \begin{Bmatrix} \langle (c_2,d_1),\{\frac{\mathfrak{m}_1}{0.8},\frac{\mathfrak{m}_2}{0.6}\}\rangle, \\ \langle (c_1,d_2),\{\frac{\mathfrak{m}_1}{0.7},\frac{\mathfrak{m}_2}{0.5}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_2,\wedge_2) = \begin{Bmatrix} \langle (c_2,d_1),\{\frac{\mathfrak{m}_1}{0.2},\frac{\mathfrak{m}_2}{0.3}\}\rangle, \\ \langle (c_2,d_2),\{\frac{\mathfrak{m}_1}{0.5},\frac{\mathfrak{m}_2}{0.4}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_3,\wedge_2) = \begin{Bmatrix} \langle (c_2,d_1),\{\frac{\mathfrak{m}_1}{0.8},\frac{\mathfrak{m}_2}{0.7}\}\rangle, \\ \langle (c_2,d_2),\{\frac{\mathfrak{m}_1}{0.5},\frac{\mathfrak{m}_2}{0.6}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_4,\wedge_3) = \begin{Bmatrix} \langle (c_2,d_1),\{\frac{\mathfrak{m}_1}{0.8},\frac{\mathfrak{m}_2}{0.6}\}\rangle, \\ \langle (c_1,d_2),\{\frac{\mathfrak{m}_1}{0.7},\frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle (c_2,d_2),\{\frac{\mathfrak{m}_1}{0.5},\frac{\mathfrak{m}_2}{0.4}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_5,\wedge_3) = \begin{Bmatrix} \langle (c_2,d_1), \{\frac{\mathfrak{m}_1}{0.2}, \frac{\mathfrak{m}_2}{0.3}\} \rangle, \\ \langle (c_1,d_2), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\} \rangle, \\ \langle (c_2,d_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.4}\} \rangle \end{Bmatrix}$$

$$(\tilde{G}_6,\wedge_3) = \begin{Bmatrix} \langle (c_2,d_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.7}\} \rangle, \\ \langle (c_1,d_2), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\} \rangle, \\ \langle (c_2,d_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.6}\} \rangle \end{Bmatrix}$$

$$(\tilde{G}_7,\wedge_3) = \begin{Bmatrix} \langle (c_2,d_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.6}\} \rangle, \\ \langle (c_1,d_2), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\} \rangle, \\ \langle (c_2,d_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.6}\} \rangle \end{Bmatrix}$$

$$(\tilde{G}_8,\wedge_3) = \begin{Bmatrix} \langle (c_2,d_1), \{\frac{\mathfrak{m}_1}{0.2}, \frac{\mathfrak{m}_2}{0.3}\} \rangle, \\ \langle (c_1,d_2), \{\frac{\mathfrak{m}_1}{0.3}, \frac{\mathfrak{m}_2}{0.5}\} \rangle, \\ \langle (c_2,d_2), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.4}\} \rangle \end{Bmatrix}$$

$\sigma = \{\tilde{0}_{(\mathfrak{N},Q)}, \tilde{1}_{(\mathfrak{N},Q)}, (\tilde{G}_1,\wedge_1), (\tilde{G}_2,\wedge_2), (\tilde{G}_3,\wedge_2), (\tilde{G}_4,\wedge_3), (\tilde{G}_5,\wedge_3), (\tilde{G}_6,\wedge_3), (\tilde{G}_7,\wedge_3)\}$ is $FHSts$.

Let $\mathfrak{h} = (\omega,\nu): (\mathfrak{M},L) \to (\mathfrak{N},M)$ be a FHS mapping as follows:
$\omega(\mathfrak{m}_1) = \mathfrak{n}_2, \omega(\mathfrak{m}_2) = \mathfrak{n}_1,$
$\nu(a_1,b_1) = (c_2,d_1), \nu(a_2,b_1) = (c_1,d_2),\ \nu(a_1,b_2) = (c_2,d_2)$
$\mathfrak{h}(\widetilde{H}_1,\wedge_3)^c = (\tilde{G}_8,\wedge_3)$

$(\widetilde{H}_1,\wedge_3)^c$ is $FHScs$ in $\mathfrak{M}$ and $\mathfrak{h}(\widetilde{H}_1,\wedge_3)^c = (\tilde{G}_8,\wedge_3)$ is $FHS\theta\mathcal{P}os$ in $\mathfrak{N}$.
∴ $\mathfrak{h}$ is $FHScontra\theta\mathcal{P}C$ mapping but $\mathfrak{h}$ is not $FHScontra\mathcal{S}C$ mapping because $(\widetilde{H}_1,\wedge_3)^c$ is $FHScs$ in $\mathfrak{M}$ but $\mathfrak{h}(\widetilde{H}_1,\wedge_3)^c = (\tilde{G}_8,\wedge_3)$ is not $FHSSos$ in $\mathfrak{N}$.

Example 4.6 *Let $\mathfrak{M} = \{\mathfrak{m}_1,\mathfrak{m}_2\}$ and $\mathfrak{N} = \{\mathfrak{n}_1,\mathfrak{n}_2\}$ be the FHS initial universes and the attributes be $L = Q_1 \times Q_2$ and $M = Q_{1'} \times Q_{2'}$ respectively. The attributes are given as:*
$Q_1 = \{a_1,a_2,a_3\}, Q_2 = \{b_1,b_2\}$
$Q_{1'} = \{c_1,c_2,c_3\}, Q_{2'} = \{d_1,d_2\}.$

Let $(\mathfrak{M},L),(\mathfrak{N},M)$ be the classes of FHS sets. Let the $FHSs$ $(\widetilde{H}_1,\wedge_3)$ over the universe $\mathfrak{M}$ be defined as

$$(\widetilde{H}_1,\wedge_3) = \begin{Bmatrix} \langle (a_1,b_1), \{\frac{\mathfrak{n}_1}{0.6}, \frac{\mathfrak{n}_2}{0.8}\} \rangle, \\ \langle (a_2,b_1), \{\frac{\mathfrak{n}_1}{0.5}, \frac{\mathfrak{n}_2}{0.7}\} \rangle, \\ \langle (a_1,b_2), \{\frac{\mathfrak{n}_1}{0.6}, \frac{\mathfrak{n}_2}{0.5}\} \rangle \end{Bmatrix}$$

$\tau = \{\tilde{0}_{(\mathfrak{M},Q)}, \tilde{1}_{(\mathfrak{M},Q)}, (\widetilde{H}_1,\wedge_3)\}$ is $FHSts$.

Let the $FHSs$'s $(\tilde{G}_1,\wedge_1), (\tilde{G}_2,\wedge_2), (\tilde{G}_3,\wedge_2), (\tilde{G}_4,\wedge_3), (\tilde{G}_5,\wedge_3), (\tilde{G}_6,\wedge_3), (\tilde{G}_7,\wedge_3), (\tilde{G}_8,\wedge_3)$ over the universe $\mathfrak{N}$ be

$$(\tilde{G}_1,\wedge_1) = \begin{Bmatrix} \langle (c_2,d_1),\{\frac{\mathfrak{m}_1}{0.8},\frac{\mathfrak{m}_2}{0.6}\}\rangle, \\ \langle (c_1,d_2),\{\frac{\mathfrak{m}_1}{0.7},\frac{\mathfrak{m}_2}{0.5}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_2,\wedge_2) = \begin{Bmatrix} \langle (c_2,d_1),\{\frac{\mathfrak{m}_1}{0.2},\frac{\mathfrak{m}_2}{0.3}\}\rangle, \\ \langle (c_2,d_2),\{\frac{\mathfrak{m}_1}{0.5},\frac{\mathfrak{m}_2}{0.4}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_3,\wedge_2) = \begin{Bmatrix} \langle (c_2,d_1),\{\frac{\mathfrak{m}_1}{0.8},\frac{\mathfrak{m}_2}{0.7}\}\rangle, \\ \langle (c_2,d_2),\{\frac{\mathfrak{m}_1}{0.5},\frac{\mathfrak{m}_2}{0.6}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_4,\wedge_3) = \begin{Bmatrix} \langle (c_2,d_1),\{\frac{\mathfrak{m}_1}{0.8},\frac{\mathfrak{m}_2}{0.6}\}\rangle, \\ \langle (c_1,d_2),\{\frac{\mathfrak{m}_1}{0.7},\frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle (c_2,d_2),\{\frac{\mathfrak{m}_1}{0.5},\frac{\mathfrak{m}_2}{0.4}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_5,\wedge_3) = \begin{Bmatrix} \langle (c_2,d_1),\{\frac{\mathfrak{m}_1}{0.2},\frac{\mathfrak{m}_2}{0.3}\}\rangle, \\ \langle (c_1,d_2),\{\frac{\mathfrak{m}_1}{0.7},\frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle (c_2,d_2),\{\frac{\mathfrak{m}_1}{0.5},\frac{\mathfrak{m}_2}{0.4}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_6,\wedge_3) = \begin{Bmatrix} \langle (c_2,d_1),\{\frac{\mathfrak{m}_1}{0.8},\frac{\mathfrak{m}_2}{0.7}\}\rangle, \\ \langle (c_1,d_2),\{\frac{\mathfrak{m}_1}{0.7},\frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle (c_2,d_2),\{\frac{\mathfrak{m}_1}{0.5},\frac{\mathfrak{m}_2}{0.6}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_7,\wedge_3) = \begin{Bmatrix} \langle (c_2,d_1),\{\frac{\mathfrak{m}_1}{0.8},\frac{\mathfrak{m}_2}{0.6}\}\rangle, \\ \langle (c_1,d_2),\{\frac{\mathfrak{m}_1}{0.7},\frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle (c_2,d_2),\{\frac{\mathfrak{m}_1}{0.5},\frac{\mathfrak{m}_2}{0.6}\}\rangle \end{Bmatrix}$$

$$(\tilde{G}_8,\wedge_3) = \begin{Bmatrix} \langle (c_2,d_1),\{\frac{\mathfrak{m}_1}{0.2},\frac{\mathfrak{m}_2}{0.4}\}\rangle, \\ \langle (c_1,d_2),\{\frac{\mathfrak{m}_1}{0.3},\frac{\mathfrak{m}_2}{0.5}\}\rangle, \\ \langle (c_2,d_2),\{\frac{\mathfrak{m}_1}{0.5},\frac{\mathfrak{m}_2}{0.4}\}\rangle \end{Bmatrix}$$

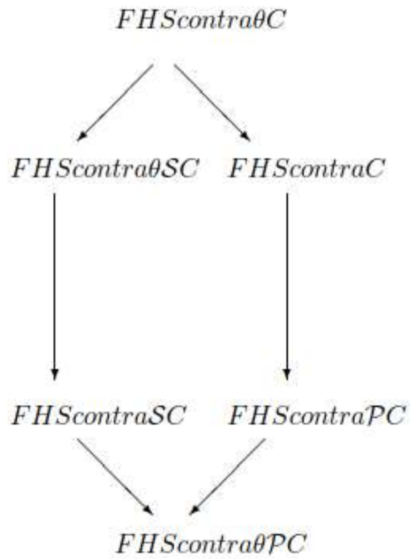
$\sigma = \{\tilde{0}_{(\mathfrak{N},Q)}, \tilde{1}_{(\mathfrak{N},Q)}, (\tilde{G}_1,\wedge_1), (\tilde{G}_2,\wedge_2), (\tilde{G}_3,\wedge_2), (\tilde{G}_4,\wedge_3), (\tilde{G}_5,\wedge_3), (\tilde{G}_6,\wedge_3), (\tilde{G}_7,\wedge_3)\}$ is $FHSts$.

Let $\mathfrak{h} = (\omega, \nu): (\mathfrak{M}, L) \to (\mathfrak{N}, M)$ be a FHS mapping as follows:
$\omega(\mathfrak{m}_1) = \mathfrak{n}_2, \omega(\mathfrak{m}_2) = \mathfrak{n}_1,$
$\nu(a_1,b_1) = (c_2,d_1), \nu(a_2,b_1) = (c_1,d_2),\ \nu(a_1,b_2) = (c_2,d_2)$
$\mathfrak{h}(\widetilde{H}_1,\wedge_3)^c = (\tilde{G}_8,\wedge_3)$

$(\widetilde{H}_1,\wedge_3)^c$ is $FHScs$ in $\mathfrak{M}$ and $\mathfrak{h}(\widetilde{H}_1,\wedge_3)^c = (\tilde{G}_8,\wedge_3)$ is $FHS\theta\mathcal{P}os$ in $\mathfrak{N}$.

$\therefore$ $\mathfrak{h}$ is $FHScontra\theta\mathcal{P}C$ mapping but $\mathfrak{h}$ is not $FHScontra\mathcal{P}C$ mapping because $(\widetilde{H}_1,\wedge_3)^c$ is $FHScs$ in $\mathfrak{M}$ but $\mathfrak{h}(\widetilde{H}_1,\wedge_3)^c = (\tilde{G}_8,\wedge_3)$ is not $FHS\mathcal{P}os$ in $\mathfrak{N}$.

Remark 4.1 *From the results discussed above, the following diagram is obtained.*



Theorem 4.1 *A mapping $\mathfrak{h}: (\mathfrak{M}, L, \tau) \to (\mathfrak{N}, M, \sigma)$ is $FHScontra\theta C$ iff for each $FHSs$ $(\tilde{G},\wedge)$ of $(\mathfrak{N}, M, \sigma)$ and for each $FHScs$ $(\widetilde{H},\wedge)$ of $(\mathfrak{M}, L, \tau)$ containing $\mathfrak{h}^{-1}(\tilde{G},\wedge)$, there is a $FHS\theta cs$ $(\tilde{A},\wedge)$ of $(\mathfrak{N}, M, \sigma)$ such that $(\tilde{G},\wedge) \subseteq (\tilde{A},\wedge)$ and $\mathfrak{h}^{-1}(\tilde{A},\wedge) \subseteq (\widetilde{H},\wedge)$.*

Proof. Necessity: Let $\mathfrak{h}$ be a $FHScontra\theta C$ mapping. Consider a $FHSos$ $(\tilde{G},\wedge)$ in $(\mathfrak{N}, M, \sigma)$ and a $FHScs$ $(\widetilde{H},\wedge)$ in $(\mathfrak{M}, L, \tau)$ such that $\mathfrak{h}^{-1}(\tilde{G},\wedge) \subseteq (\widetilde{H},\wedge)$. Then $(\tilde{A},\wedge) = 1_{(\mathfrak{N},Q)} - \mathfrak{h}^{-1}((\widetilde{H},\wedge)^c)$ is $FHS\theta cs$ of $(\mathfrak{N}, M, \sigma)$ such that $\mathfrak{h}^{-1}(\tilde{A},\wedge) \subseteq (\widetilde{H},\wedge)$.

Sufficiency: Assume $(\widetilde{H},\wedge)$ is a $FHScs$ of $(\mathfrak{M}, L, \tau)$. Then $(\mathfrak{h}(\widetilde{H},\wedge))^c$ is a $FHSs$ of $(\mathfrak{N}, M, \sigma)$ and $(\widetilde{H},\wedge)^c$ is $FHSos$ in $(\mathfrak{M}, L, \tau)$ such that $\mathfrak{h}^{-1}((\mathfrak{h}(\widetilde{H},\wedge))^c) \subseteq (\widetilde{H},\wedge)^c$. By hypothseis, there is a $FHS\theta cs$ $(\tilde{A},\wedge)$ of $(\mathfrak{N}, M, \sigma)$ such that $(\mathfrak{h}(\widetilde{H},\wedge))^c \subseteq (\tilde{A},\wedge)$ and $\mathfrak{h}^{-1}(\tilde{A},\wedge) \subseteq (\widetilde{H},\wedge)^c$. Therefore $(\widetilde{H},\wedge) \subseteq (\mathfrak{h}^{-1}(\tilde{A},\wedge))^c$. Hence $(\tilde{A},\wedge)^c \subseteq \mathfrak{h}(\widetilde{H},\wedge) \subseteq \mathfrak{h}((\mathfrak{h}^{-1}(\tilde{A},\wedge))^c) \subseteq (\tilde{A},\wedge)^c$ which implies $\mathfrak{h}(\widetilde{H},\wedge) = (\tilde{A},\wedge)^c$. Since $(\tilde{A},\wedge)^c$ is $FHS\theta os$ of $(\mathfrak{N}, M, \sigma)$, $\mathfrak{h}(\widetilde{H},\wedge)$ is $FHS\theta os$ in $(\mathfrak{N}, M, \sigma)$ and hence $\mathfrak{h}$ is $FHScontra\theta C$ mapping.

Theorem 4.2 *If $\mathfrak{h}: (\mathfrak{M}, L, \tau) \to (\mathfrak{N}, M, \sigma)$ is $FHSC$ and $\mathfrak{g}: (\mathfrak{N}, M, \sigma) \to (P, N, \rho)$ is $FHScontra\theta C$ mappings. Then $\mathfrak{g} \circ \mathfrak{h}: (\mathfrak{M}, L, \tau) \to (P, N, \rho)$ is $FHScontra\theta C$ map.*

Proof. Let $(\widetilde{H},\wedge)$ be a $FHScs$ in $(\mathfrak{M}, L, \tau)$. As $\mathfrak{h}$ is $FHSC$ mapping, $\mathfrak{h}(\widetilde{H},\wedge)$ is $FHScs$ in $(\mathfrak{N}, M, \sigma)$. As $\mathfrak{g}$ is $FHScontra\theta C$ mapping, $(\mathfrak{g} \circ \mathfrak{h})(\widetilde{H},\wedge) = \mathfrak{g}(\mathfrak{h}(\widetilde{H},\wedge))$ is $FHS\theta os$ in (P, N, ρ). Hence $\mathfrak{g} \circ \mathfrak{h}$ is $FHScontra\theta C$ mapping.

Theorem 4.3 *If $\mathfrak{h}: (\mathfrak{M}, L, \tau) \to (\mathfrak{N}, M, \sigma)$ is $FHScontra\theta C$ map, then $FHS\theta int(\mathfrak{h}(\widetilde{H},\wedge)) \supseteq \mathfrak{h}(FHSint(\widetilde{H},\wedge))$.*

Proof. The proof is obvious from Definition 4.1 .

Theorem 4.4 *Let $\mathfrak{h}: (\mathfrak{M}, L, \tau) \to (\mathfrak{N}, M, \sigma)$ and $\mathfrak{g}: (\mathfrak{N}, M, \sigma) \to (P, N, \rho)$ be $FHScontra\theta C$ mappings. If every $FHS\theta os$ of $(\mathfrak{N}, M, \sigma)$ is $FHSos$, then $\mathfrak{g} \circ \mathfrak{h}: (\mathfrak{M}, L, \tau) \to (P, N, \rho)$ is $FHS\theta C$.*

Proof. Let $(\widetilde{H},\wedge)$ be a $FHScs$ in $(\mathfrak{M}, L, \tau)$. As $\mathfrak{h}$ is $FHScontra\theta C$ mapping, $\mathfrak{h}(\widetilde{H},\wedge)$ is $FHS\theta os$ in $(\mathfrak{N}, M, \sigma)$. By presumption, $\mathfrak{h}(\widetilde{H},\wedge)$ is $FHSos$ of $(\mathfrak{N}, M, \sigma)$. As $\mathfrak{g}$ is

$FHScontra\theta C$ mapping, $\mathfrak{g}(\mathfrak{h}(\widetilde{H},\wedge)) = (\mathfrak{g} \circ \mathfrak{h})(\widetilde{H},\wedge)$ is $FHS\theta cs$ in (P, N, ρ). Hence $\mathfrak{g} \circ \mathfrak{h}$ is $FHS\theta C$ mapping.

Theorem 4.5 *Let* $\mathfrak{h}: (\mathfrak{M}, L, \tau) \to (\mathfrak{N}, M, \sigma)$ *be a bijective mapping. Then the following statements are equivalent: [(i)]*

1. $\mathfrak{h}$ is a $FHScontra\theta O$ mapping.
2. $\mathfrak{h}$ is a $FHScontra\theta C$ mapping.
3. $\mathfrak{h}^{-1}$ is $FHScontra\theta Cts$ mapping.

Proof. (i) ⇒ (ii):Let us assume that $\mathfrak{h}$ is a $FHScontra\theta O$ mapping. By definition, if $(\widetilde{H},\wedge)$ is a $FHSos$ in $(\mathfrak{M}, L, \tau)$, by presumption $\mathfrak{h}(\widetilde{H},\wedge)$ is a $FHS\theta cs$ in $(\mathfrak{N}, M, \sigma)$. But now, $(\widetilde{H},\wedge)$ is $FHScs$ in $(\mathfrak{M}, L, \tau)$. So, $1_{(\mathfrak{M},Q)} - (\widetilde{H},\wedge)$ is a $FHSos$ in $(\mathfrak{M}, L, \tau)$. By assumption, $\mathfrak{h}(1_{(\mathfrak{M},Q)} - (\widetilde{H},\wedge))$ is a $FHS\theta cs$ in $(\mathfrak{N}, M, \sigma)$. Hence, $1_{(\mathfrak{N},Q)} - \mathfrak{h}(1_{(\mathfrak{M},Q)} - (\widetilde{H},\wedge))$ is a $FHS\theta os$ in $(\mathfrak{N}, M, \sigma)$. Thus, $\mathfrak{h}$ is a $FHScontra\theta C$ mapping.

(ii) ⇒ (iii): Let $(\widetilde{H},\wedge)$ be a $FHScs$ in $(\mathfrak{M}, L, \tau)$. By assumption, $\mathfrak{h}(\widetilde{H},\wedge)$ is a $FHS\theta os$ in $(\mathfrak{N}, M, \sigma)$. Hence, $\mathfrak{h}(\widetilde{H},\wedge) = (\mathfrak{h}^{-1})^{-1}(\widetilde{H},\wedge)$. So $\mathfrak{h}^{-1}$ is a $FHS\theta os$ in $(\mathfrak{N}, M, \sigma)$. Thus, $\mathfrak{h}^{-1}$ is $FHScontra\theta Cts$.

(iii) ⇒ (i): Let $(\widetilde{H},\wedge)$ be a $FHSos$ in $(\mathfrak{M}, L, \tau)$. By assumption, $(\mathfrak{h}^{-1})^{-1}(\widetilde{H},\wedge) = \mathfrak{h}(\widetilde{H},\wedge)$ is a $FHScontra\theta O$ mapping.

5 Fuzzy hypersoft contra $\boldsymbol{\theta}$ homeomorphism

In this section, the concept of fuzzy hypersoft contra θ homeomorphism is introduced and its properties are discussed.

Definition 5.1 *A bijection* $\mathfrak{h}: (\mathfrak{M}, L, \tau) \to (\mathfrak{N}, M, \sigma)$ *is called a* $FHScontra\theta$ *homeomorphism (resp. contra homeomorphism) (in short,* $FHS\theta contraHom$ *(resp.* $FHScontraHom$*)) if* $\mathfrak{h}$ *and* $\mathfrak{h}^{-1}$ *are* $FHScontra\theta Cts$ *(resp.* $FHScontraCts$*) mappings.*

Theorem 5.1 *Each* $FHScontra\theta Hom$ *is a* $FHScontraHom$*. But the converse is not true.*

Proof. Assume $\mathfrak{h}$ is $FHScontra\theta Hom$. Then $\mathfrak{h}$ and $\mathfrak{h}^{-1}$ are $FHScontra\theta Cts$. We know that each $FHScontra\theta Cts$ function is $FHScontraCts$. So, $\mathfrak{h}$ and $\mathfrak{h}^{-1}$ are $FHScontraCts$. Thus, $\mathfrak{h}$ is a $FHScontraHom$.

Example 5.1 *Let* $\mathfrak{M} = \{\mathfrak{m}_1, \mathfrak{m}_2\}$ *and* $\mathfrak{N} = \{\mathfrak{n}_1, \mathfrak{n}_2\}$ *be the* FHS *initial universes and the attributes be* $L = Q_1 \times Q_2$ *and* $M = Q_{1'} \times Q_{2'}$*, respectively. The attributes are given as:*
$Q_1 = \{a_1, a_2, a_3\}, Q_2 = \{b_1, b_2\}$
$Q_{1'} = \{c_1, c_2, c_3\}, Q_{2'} = \{d_1, d_2\}$.

Let $(\mathfrak{M}, L), (\mathfrak{N}, M)$ be the classes of FHS sets. Let the $FHSs$'s $(\widetilde{H}_1,\wedge_1)$, $(\widetilde{H}_2,\wedge_2)$, $(\widetilde{H}_3,\wedge_2)$, $(\widetilde{H}_4,\wedge_3)$,$(\widetilde{H}_5,\wedge_3)$,$(\widetilde{H}_6,\wedge_3)$,$(\widetilde{H}_7,\wedge_3)$, over the universe $\mathfrak{M}$ be

$$(\widetilde{H}_1,\wedge_1) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.8}, \frac{\mathfrak{m}_2}{0.6}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.5}\} \rangle \end{Bmatrix}$$

$$(\widetilde{H}_2,\wedge_2)=\begin{Bmatrix}\langle(a_1,b_1),\{\frac{\mathfrak{m}_1}{0.2},\frac{\mathfrak{m}_2}{0.3}\}\rangle,\\ \langle(a_1,b_2),\{\frac{\mathfrak{m}_1}{0.5},\frac{\mathfrak{m}_2}{0.4}\}\rangle\end{Bmatrix}$$

$$(\widetilde{H}_3,\wedge_3)=\begin{Bmatrix}\langle(a_1,b_1),\{\frac{\mathfrak{m}_1}{0.8},\frac{\mathfrak{m}_2}{0.6}\}\rangle,\\ \langle(a_2,b_1),\{\frac{\mathfrak{m}_1}{0.7},\frac{\mathfrak{m}_2}{0.5}\}\rangle\\ \langle(a_1,b_2),\{\frac{\mathfrak{m}_1}{0.5},\frac{\mathfrak{m}_2}{0.4}\}\rangle\end{Bmatrix}$$

$$(\widetilde{H}_4,\wedge_3)=\begin{Bmatrix}\langle(a_1,b_1),\{\frac{\mathfrak{m}_1}{0.2},\frac{\mathfrak{m}_2}{0.3}\}\rangle,\\ \langle(a_2,b_1),\{\frac{\mathfrak{m}_1}{0.7},\frac{\mathfrak{m}_2}{0.5}\}\rangle\\ \langle(a_1,b_2),\{\frac{\mathfrak{m}_1}{0.5},\frac{\mathfrak{m}_2}{0.4}\}\rangle\end{Bmatrix}$$

$\tau=\{\tilde{0}_{(\mathfrak{M},Q)},\tilde{1}_{(\mathfrak{M},Q)},(\widetilde{H}_1,\wedge_1),(\widetilde{H}_2,\wedge_2),(\widetilde{H}_3,\wedge_3),(\widetilde{H}_4,\wedge_3)\}$ is $FHSts$.

Let the $FHSs's$ $(\tilde{G}_1,\wedge_1),(\tilde{G}_2,\wedge_1)$, $(\tilde{G}_3,\wedge_2),(\tilde{G}_4,\wedge_3),(\tilde{G}_5,\wedge_3)$, $(\tilde{G}_6,\wedge_3)$, $(\tilde{G}_7,\wedge_3)$, over the universe $\mathfrak{N}$ be

$$(\tilde{G}_1,\wedge_1)=\begin{Bmatrix}\langle(c_2,d_1),\{\frac{\mathfrak{n}_1}{0.4},\frac{\mathfrak{n}_2}{0.2}\}\rangle,\\ \langle(c_1,d_2),\{\frac{\mathfrak{n}_1}{0.5},\frac{\mathfrak{n}_2}{0.3}\}\rangle\end{Bmatrix}$$

$\sigma=\{\tilde{0}_{(\mathfrak{N},Q)},\tilde{1}_{(\mathfrak{N},Q)},(\tilde{G}_1,\wedge_1)\}$ is $FHSts$.

Let $\mathfrak{h}=(\omega,\nu):(\mathfrak{M},L)\to(\mathfrak{N},M)$ be a FHS mapping as follows:

$\omega(\mathfrak{m}_1)=\mathfrak{n}_2,\omega(\mathfrak{m}_2)=\mathfrak{n}_1,$

$\nu(a_1,b_1)=(c_2,d_1),\nu(a_2,b_1)=(c_1,d_2),\ \nu(a_1,b_2)=(c_2,d_2)$

$$\mathfrak{h}(\widetilde{H}_1,\wedge_1)=(\tilde{G}_1,\wedge_1)^c$$

and $\mathfrak{h}^{-1}(\tilde{G}_1,\wedge_1)=(\widetilde{H}_1,\wedge_1)^c$

Here $\mathfrak{h}^{-1}$ is $FHScontraCts$ because $(\widetilde{H}_1,\wedge_1)$ is $FHSos$ in $\mathfrak{M}$ and $\mathfrak{h}(\widetilde{H}_1,\wedge_1)=(\tilde{G}_1,\wedge_1)^c$ is $FHScs$ in $\mathfrak{N}$. Also, $\mathfrak{h}$ is $FHScontraCts$ because $(\tilde{G}_1,\wedge_1)$ is $FHSos$ in $\mathfrak{N}$ and $\mathfrak{h}^{-1}(\tilde{G}_1,\wedge_1)$ is $FHScs$ in $\mathfrak{M}$. Hence $\mathfrak{h}$ is $FHScontraHom$. But $\mathfrak{h}$ is not $FHScontra\theta Hom$ because $(\tilde{G}_1,\wedge_1)$ is $FHSos$ in $\mathfrak{N}$ but $\mathfrak{h}^{-1}(\tilde{G}_1,\wedge_1)$ is not $FHS\theta cs$ in $\mathfrak{M}$.

Theorem 5.2 *Consider a bijective mapping* $\mathfrak{h}:(\mathfrak{M},L,\tau)\to(\mathfrak{N},M,\sigma)$. *The followings statements are equivalent if* $\mathfrak{h}$ *is* $FHScontra\theta Cts$.

1. $\mathfrak{h}$ is a $FHScontra\theta C$ mapping.
2. $\mathfrak{h}$ is a $FHScontra\theta O$ mapping.
3. $\mathfrak{h}^{-1}$ is a $FHScontra\theta Hom$.

Proof. (i) ⇒ (ii) : Let $\mathfrak{h}$ be a bijective mapping and a $FHScontra\theta C$ mapping. Therefore, $\mathfrak{h}^{-1}$ is a $FHScontra\theta Cts$ mapping. As each $FHSos$ in $(\mathfrak{M},L,\tau)$ is a $FHS\theta cs$ in $(\mathfrak{N},M,\sigma)$. Hence $\mathfrak{h}$ is a $FHScontra\theta O$ mapping.

(ii) ⇒ (iii) : Assume $\mathfrak{h}$ is a bijective and $FHScontra\theta O$ mapping. Also, $\mathfrak{h}^{-1}$ is a $FHScontra\theta Cts$ mapping. Therefore, $\mathfrak{h}$ and $\mathfrak{h}^{-1}$ are $FHScontra\theta Cts$. Thus, $\mathfrak{h}$ is a $FHScontra\theta Hom$.

(iii) ⇒ (i): Assume $\mathfrak{h}$ is a $FHScontra\theta Hom$. So, $\mathfrak{h}$ and $\mathfrak{h}^{-1}$ are $FHScontra\theta Cts$. As every $FHScs$ in $(\mathfrak{M},L,\tau)$ is a $FHS\theta os$ in $(\mathfrak{N},M,\sigma)$, $\mathfrak{h}$ is a $FHScontra\theta C$ mapping.

Theorem 5.3 *Let* $\mathfrak{h}: (\mathfrak{M}, L, \tau) \to (\mathfrak{N}, M, \sigma)$ *and* $\mathfrak{g}: (\mathfrak{N}, M, \sigma) \to (P, N, \rho)$ *be two* FHS *mappings. Then the following hold: [(i)]*

1. If $\mathfrak{g} \circ \mathfrak{h}$ is $FHScontra\theta O$ and $\mathfrak{h}$ is $FHSCts$, then $\mathfrak{g}$ is $FHScontra\theta O$.
2. If $\mathfrak{g} \circ \mathfrak{h}$ is $FHSO$ and $\mathfrak{g}$ is $FHScontra\theta Cts$, then $\mathfrak{h}$ is $FHScontra\theta O$.

Proof. The proof is obvious

6 Fuzzy Hypersoft contra θ C homeomorphism

The fuzzy hypersoft contra θ C homeomorphism is introduced in this section and some of its properties are analyzed.

Definition 6.1 *A bijection* $\mathfrak{h}: (\mathfrak{M}, L, \tau) \to (\mathfrak{N}, M, \sigma)$ *is called a* $FHScontra\theta$ *C homeomorphism (in short,* $FHScontra\theta CHom$*) if* $\mathfrak{h}$ *and* $\mathfrak{h}^{-1}$ *are* $FHScontra\theta Irr$ *mappings.*

Theorem 6.1 *Each* $FHScontra\theta CHom$ *is a* $FHScontra\theta Hom$*. But not conversely.*

Proof. Consider a $FHSos$ $(\tilde{G}, \wedge)$ in $(\mathfrak{N}, M, \sigma)$. Then $(\tilde{G}, \wedge)$ is a $FHS\theta os$ in $(\mathfrak{N}, M, \sigma)$. By presumption, $\mathfrak{h}^{-1}(\tilde{G}, \wedge)$ is a $FHS\theta cs$ in $(\mathfrak{M}, L, \tau)$. Therefore, $\mathfrak{h}$ is a $FHScontra\theta Cts$ mapping. So, $\mathfrak{h}$ and $\mathfrak{h}^{-1}$ are $FHSScontra\theta Cts$ mappings. Thus, $\mathfrak{h}$ is a $FHScontra\theta Hom$.

Example 6.1 *Let* $\mathfrak{M} = \{\mathfrak{m}_1, \mathfrak{m}_2\}$ *and* $\mathfrak{N} = \{\mathfrak{n}_1, \mathfrak{n}_2\}$ *be the* FHS *initial universes and the attributes be* $L = Q_1 \times Q_2$ *and* $M = Q_{1'} \times Q_{2'}$*, respectively. The attributes are given as:*

$Q_1 = \{a_1, a_2\}, Q_2 = \{b_1\}$

$Q_{1'} = \{c_1, c_2\}, Q_{2'} = \{d_1\}$.

Let $(\mathfrak{M}, L), (\mathfrak{N}, M)$ be the classes of FHS sets. Let the $FHSs$'s $(\tilde{H}_1, \wedge)$, $(\tilde{H}_2, \wedge)$, $(\tilde{H}_3, \wedge)$ over the universe $\mathfrak{M}$ be

$$(\tilde{H}_1, \wedge) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.1}, \frac{\mathfrak{m}_2}{0.2}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{m}_1}{0.3}, \frac{\mathfrak{m}_2}{0.4}\} \rangle \end{Bmatrix}$$

$$(\tilde{H}_2, \wedge) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.9}, \frac{\mathfrak{m}_2}{0.8}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{m}_1}{0.7}, \frac{\mathfrak{m}_2}{0.6}\} \rangle \end{Bmatrix}$$

$$(\tilde{H}_3, \wedge) = \begin{Bmatrix} \langle (a_1, b_1), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.5}\} \rangle, \\ \langle (a_2, b_1), \{\frac{\mathfrak{m}_1}{0.5}, \frac{\mathfrak{m}_2}{0.5}\} \rangle \\ \end{Bmatrix}$$

$\tau = \{\tilde{0}_{(\mathfrak{M},Q)}, \tilde{1}_{(\mathfrak{M},Q)}, (\tilde{H}_1, \wedge), (\tilde{H}_2, \wedge)\}$ is $FHSts$.

Let the $FHSs's$ $(\tilde{G}_1, \wedge), (\tilde{G}_2, \wedge)$, $(\tilde{G}_3, \wedge)$, over the universe $\mathfrak{N}$ be

$$(\tilde{G}_1, \wedge) = \begin{Bmatrix} \langle (c_2, d_1), \{\frac{\mathfrak{n}_1}{0.8}, \frac{\mathfrak{n}_2}{0.9}\} \rangle, \\ \langle (c_1, d_2), \{\frac{\mathfrak{n}_1}{0.6}, \frac{\mathfrak{n}_2}{0.5}\} \rangle \end{Bmatrix}$$

$$(\tilde{G}_2, \wedge) = \begin{Bmatrix} \langle (c_2, d_1), \{\frac{\mathfrak{n}_1}{0.2}, \frac{\mathfrak{n}_2}{0.1}\} \rangle, \\ \langle (c_1, d_2), \{\frac{\mathfrak{n}_1}{0.4}, \frac{\mathfrak{n}_2}{0.3}\} \rangle \end{Bmatrix}$$

$$(\tilde{G}_3,\wedge) = \begin{Bmatrix} \langle(c_2,d_1),\{\frac{\mathfrak{n}_1}{0.5},\frac{\mathfrak{n}_2}{0.5}\}\rangle, \\ \langle(c_1,d_2),\{\frac{\mathfrak{n}_1}{0.5},\frac{\mathfrak{n}_2}{0.5}\}\rangle \end{Bmatrix}$$

$\sigma = \{\tilde{0}_{(\mathfrak{N},Q)}, \tilde{1}_{(\mathfrak{N},Q)}, (\tilde{G}_1,\wedge), (\tilde{G}_2,\wedge), (\tilde{G}_3,\wedge)\}$ is $FHSts$.

Let $\mathfrak{h} = (\omega,\nu): (\mathfrak{M}, L) \to (\mathfrak{N}, M)$ be a FHS mapping as follows:

$\omega(\mathfrak{m}_1) = \mathfrak{n}_2, \omega(\mathfrak{m}_2) = \mathfrak{n}_1,$

$$\nu(a_1,b_1) = (c_2,d_1), \nu(a_2,b_1) = (c_1,d_2)$$

$$\mathfrak{h}(\tilde{H}_1,\wedge) = (\tilde{G}_1,\wedge)^c$$

$$\mathfrak{h}(\tilde{H}_2,\wedge) = (\tilde{G}_2,\wedge)^c$$

$$\mathfrak{h}(\tilde{H}_3,\wedge) = (\tilde{G}_3,\wedge)^c$$

and

$$\mathfrak{h}^{-1}(\tilde{G}_1,\wedge) = (\tilde{H}_1,\wedge)^c$$

$$\mathfrak{h}^{-1}(\tilde{G}_2,\wedge) = (\tilde{H}_2,\wedge)^c$$

$$\mathfrak{h}^{-1}(\tilde{G}_3,\wedge) = (\tilde{H}_3,\wedge)^c$$

Here $\mathfrak{h}$ is $FHScontra\theta Hom$ but not $FHScontra\theta CHom$ because $(\tilde{G}_3,\wedge)$ is $FHS\theta os$ in $\mathfrak{N}$ but $\mathfrak{h}^{-1}(\tilde{G}_3,\wedge) = (\tilde{H}_3,\wedge)$ is not $FHS\theta cs$ in $\mathfrak{M}$.

Theorem 6.2 *If* $\mathfrak{h}: (\mathfrak{M}, L, \tau) \to (\mathfrak{N}, M, \sigma)$ *is a* $FHScontra\theta CHom$ *, then* $FHS\theta int(\mathfrak{h}^{-1}(\tilde{G},\wedge)) \subseteq \mathfrak{h}^{-1}(FHScl(\tilde{G},\wedge))$ *for each* $FHSs$ $(\tilde{G},\wedge)$ *in* $(\mathfrak{N}, M, \sigma)$.

Proof. Consider a $FHSs$ $(\tilde{G},\wedge)$ in $(\mathfrak{N}, M, \sigma)$. Since, $FHScl(\tilde{G},\wedge)$ is a $FHScs$ in $(\mathfrak{N}, M, \sigma)$ and every $FHScs$ is a $FHS\theta cs$ in $(\mathfrak{N}, M, \sigma)$. As $\mathfrak{h}$ is $FHScontra\theta Irr$, $\mathfrak{h}^{-1}(FHScl(\tilde{G},\wedge))$ is a $FHS\theta os$ in $(\mathfrak{M}, L, \tau)$. Then, $FHS\theta int(\mathfrak{h}^{-1}(FHScl(\tilde{G},\wedge))) = \mathfrak{h}^{-1}(FHScl(\tilde{G},\wedge))$. Here, $FHS\theta int(\mathfrak{h}^{-1}(\tilde{G},\wedge)) \subseteq FHS\theta int(\mathfrak{h}^{-1}(FHScl(\tilde{G},\wedge))) = \mathfrak{h}^{-1}(FHScl(\tilde{G},\wedge))$. Therefore,

$FHS\theta int(\mathfrak{h}^{-1}(\tilde{G},\wedge)) \subseteq \mathfrak{h}^{-1}(FHScl(\tilde{G},\wedge))$ for every $FHSs$ $(\tilde{G},\wedge)$ in $(\mathfrak{N}, M, \sigma)$.

Theorem 6.3 *Let* $\mathfrak{h}: (\mathfrak{M}, L, \tau) \to (\mathfrak{N}, M, \sigma)$ *be a* $FHScontra\theta CHom$. *Then* $FHS\theta int(\mathfrak{h}^{-1}(\tilde{G},\wedge)) \subseteq \mathfrak{h} - 1(FHS\theta cl(\tilde{G},\wedge))$ *for each* $FHSs$ $(\tilde{G},\wedge)$ *in* $(\mathfrak{N}, M, \sigma)$.

Proof. As $\mathfrak{h}$ is a $FHScontra\theta CHom$, $\mathfrak{h}$ is a $FHScontra\theta Irr$ mapping. Consider a $FHSs$ $(\tilde{G},\wedge)$ in $(\mathfrak{N}, M, \sigma)$. It is obvious that, $FHS\theta cl(\tilde{G},\wedge)$ is a $FHS\theta cs$ in $(\mathfrak{N}, M, \sigma)$. As $\mathfrak{h}^{-1}(\tilde{G},\wedge) \subseteq \mathfrak{h}^{-1}(FHS\theta cl(\tilde{G},\wedge))$, we have

$$FHS\theta int(\mathfrak{h}^{-1}(\tilde{G},\wedge)) \subseteq FHS\theta int(\mathfrak{h}^{-1}(FHS\theta cl(\tilde{G},\wedge)))$$

$$\subseteq \mathfrak{h}^{-1}(FHS\theta cl(\tilde{G},\wedge)).$$

Thus, $FHS\theta int(\mathfrak{h}^{-1}(\tilde{G},\wedge)) \subseteq \mathfrak{h}^{-1}(FHS\theta cl(\tilde{G},\wedge))$.

Theorem 6.4 *If* $\mathfrak{h}: (\mathfrak{M}, L, \tau) \to (\mathfrak{N}, M, \sigma)$ *and* $\mathfrak{g}: (\mathfrak{N}, M, \sigma) \to (P, N, \rho)$ *are* $FHScontra\theta CHom$*'s, then* $\mathfrak{g} \circ \mathfrak{h}$ *is a* $FHS\theta CHom$.

Proof. Assume that $\mathfrak{h}$ and $\mathfrak{g}$ are two $FHScontra\theta CHom$'s. Let $(\tilde{G},\wedge)$ be a $FHS\theta cs$ in (P, N, ρ). Then, $\mathfrak{h}^{-1}(\tilde{G},\wedge)$ is a $FHS\theta os$ in $(\mathfrak{N}, M, \sigma)$.Then by hypothrsis, $\mathfrak{h}^{-1}(\mathfrak{g}^{-1}(\tilde{G},\wedge))$ is a $FHS\theta cs$ in $(\mathfrak{M}, L, \tau)$. Hence, $(\mathfrak{g} \circ \mathfrak{h})^{-1}$ is a $FHS\theta Irr$ mapping. Assume $(\tilde{H},\wedge)$ is a $FHS\theta cs$ in $(\mathfrak{M}, L, \tau)$. Then, by hypothesis, $\mathfrak{g}(\mathfrak{h})$ is a $FHS\theta os$ in $(\mathfrak{N}, M, \sigma)$. Hence, $\mathfrak{g}(\mathfrak{h}(\tilde{G}, \Lambda))$ is a $FHS\theta cs$ in (P, N, ρ). This implies that $\mathfrak{g} \circ \mathfrak{h}$ is a $FHS\theta Irr$ mapping. Thus, $\mathfrak{g} \circ \mathfrak{h}$ is a $FHS\theta CHom$.

7 Conclusion

In this paper, $FHScontra\theta O$, $FHScontra\theta SO$, $FHScontra\theta\mathcal{P}O$, $FHScontra\theta C$, $FHScontra\theta SC$ and $FHScontra\theta\mathcal{P}C$ maps are introduced and their properties are discussed. Moreover, the relations between them are analyzed with the examples. Furthermore, the work was extended to include $FHScontra\theta Hom$, $FHScontra\theta CHom$ and some of its related characteristics are derived.

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