Nanotechnology Perceptions
ISSN 1660-6795
www.nano-ntp.com

The B (E2) transition Probabilities for $^{122\text{-}130}$Te (Tellurium) Even-Even Isotopes with the help of Cubic terms from Casimir Invariant Operators and IBM-1

Dr. Pinky Rani

Abstract- The interacting boson model-1 has been used to calculate the reduced electric transition probability B(E2) ↓ of even-even $^{122\text{-}130}$Te (Tellurium) isotopes with even neutrons from N = 70 to 78. The three-three boson interactions are also formed in the Hamiltonian from Casimir invariant operators. The parameters of best fit to measure the data are used from the experimental value of B (E2; $2_1^+ \rightarrow 0_1^+$) for even-even $^{122\text{-}130}$Te isotopes. The theoretical values are good in agreement especially with the experimental ones. The branching ratios B (E2; $4_1^+ \rightarrow 2_1^+$) / B (E2; $2_1^+ \rightarrow 0_1^+$) is less than 2 represents U (5) symmetry in $^{122\text{-}130}$Te isotopes.

Key words: Interacting Boson Model-1, even-even Tellurium, reduce electric transition probabilities, three-three interactions.

INTRODUCTION

The interacting boson model is suitable for describing the heavy atomic nuclei. This model was introduced within which seems to be relevant for the reason of deformed nuclei showing triaxial features. Also, describing the various nuclear properties like spins, energies of lower level, emission of gamma rays and multiple moments. The interacting boson model-1 is an interactive model developed by Iachello and Arima.

The interacting boson model (IBM) is an approach that provides an additional explanation of nuclear collective excitation. It is algebraic in character and employs group theoretical approaches to exploit symmetry. In terms of creation and annihilation boson operators, its Hamiltonian is expressed in second quantized form.

The s-d IBM exhibits three dynamical symmetries in the three vertices of casten triangle, U (5), SU (3) and O (6) appropriate for vibrational nuclei with spherical form, an axially symmetric deformed prolate rotor and γ-unstable axially asymmetric rotor respectively. The relation between the geometric collective model and the IBM established on the basis of an intrinsic coherent state formalism. The nuclear shape phase transitions between the three dynamical limits of the IBM have gained much theoretical interest in terms of geometric collective model and IBM.

The main objective of this research is to investigate $^{122\text{-}130}$Te transitional nuclei for the calculations of B (E2) reduced transition probabilities within framework of IBM-1 with the help of Casimir invariant operators.

THEORETICAL CONSIDERATION

One assumes that low-lying collective quadrupole states can be generated as states of a system of N bosons able to occupy two levels, one with the angular momentum J = 0, called s, and one with angular momentum J = 2, called d. The number N is the total number of active nucleon pairs. For calculate the properties of an even-even nucleus, one then needs to specify the appropriate operators.

Therefore, there are only two energies of this kind, namely ε_s and ε_d. If the bosons were independent of one another, a system of n_s s-bosons and n_d d-bosons would have the energy $n_s\varepsilon_s + n_d\varepsilon_d$.

The bosons are coupled symmetrically, allowed two bosons are s^2 (L=0), sd (L=2) and d^2 (L=0, 2, 4). Since for n states with a given angular momentum, one has n (n + 1)/2 interactions, seven independent two body interactions are found; three for L = 0, three for L = 2, and one for L = 4. This analysis can be extended to higher-order interactions.

The three-body interactions $(l_1l_2l_3; LM|H_3|l_1'l_2'l_3'; LM)$ allowed boson states are s^3 (L=0), sd^2 (L=2), s^2d (L=0,2,4) and d^3 (L=0,2,3,4,6), leading to 6+6+1+3+1=17 independent three-body interactions for L = 0,2,3,4,6, respectively.

A Hamiltonian that conserves the total number of bosons is of the generic for

$$\hat{H} = E_0 + \hat{H}^{(1)} + \hat{H}^{(2)} + \hat{H}^{(3)} + \ldots \qquad (1)$$

Where, the index refers to the order of the interaction in the generators of U (6). The first term E_0 is a constant which represents the binding energy of the core.

The most general Hamiltonian containing one-, two-, and three-body terms can be written as

$$H = \varepsilon_s N + (\varepsilon_d - \varepsilon_s) n_d + \tfrac{1}{2} \sum_{J=0,2,4} C_J \sqrt{(2J+1)}\, [[d^\dagger \times d^\dagger]^{(J)} \times [\tilde{d} \times \tilde{d}]^{(J)}]^{(0)} +$$
$$\sqrt{(\tfrac{1}{2})}\, v_2 ([[d^\dagger \times d^\dagger]^{(2)} \times \tilde{d}\, s]^{(0)} +$$
$$[s^\dagger d^\dagger \times [\tilde{d} \times \tilde{d}]^{(2)}]^{(0)}) + \tfrac{1}{2} v_0 ([[d^\dagger \times d^\dagger]^{(0)} \times ss]^{(0)} + [s^\dagger s^\dagger \times [\tilde{d} \times \tilde{d}]^{(0)}]^{(0)}) +$$
$$u_2 [d^\dagger s^\dagger \times \tilde{d}\, s]^{(0)} + \tfrac{1}{2} u_0 [s^\dagger s^\dagger \times ss]^{(0)} + \sqrt{5}/2\, A_2 (\{[d^\dagger \times d^\dagger]^{(2)} \times [\tilde{d} \times \tilde{d}]^{(2)}\}^{(0)} \times \{[d^\dagger \times \tilde{d}]^{(2)}\}^{(0)}) +$$
$$\sqrt{5}/2\, B_2 (\{[d^\dagger \times d^\dagger]^{(2)} \times [\tilde{d} \times \tilde{d}]^{(2)}\}^{(0)} \times \{[s^\dagger s + s s^\dagger]^{(2)}\}^{(0)}) +$$
$$p_2 (\{[d^\dagger \times d^\dagger]^{(2)} \times [\tilde{d} \times \tilde{d}]^{(2)}\}^{(0)} \times \{[d^\dagger s + s^\dagger \tilde{d}]^{(2)}\}^{(0)}) +$$
$$1/2\, p_0 [(\{[d^\dagger \times d^\dagger]^{(2)} \times [\tilde{s} \times \tilde{s}]^{(2)}\}^{(0)} \times \{[d^\dagger s]^{(2)}\}^{(0)}) + (\{[s^\dagger \times s^\dagger]^{(2)} \times [\tilde{d} \times \tilde{d}]^{(2)}\}^{(0)} \times \{[\tilde{d} s^\dagger]^{(2)}\}^{(0)})] +$$
$$q_2 (\{[d^\dagger \times d^\dagger]^{(2)} \times [\tilde{d} \times \tilde{d}]^{(2)}\}^{(0)} \times$$
$$\{[s^\dagger s]^{(2)}\}^{(0)}) +$$
$$\sqrt{5}\, D_2 (\{[s^\dagger \times s^\dagger]^{(2)} \times [\tilde{s} \times \tilde{s}]^{(2)}\}^{(0)} \times \{[d^\dagger \times \tilde{d}]^{(2)}\}^{(0)}) +$$
$$1/2\, q_0 \{[s^\dagger s^\dagger s^\dagger \times \tilde{s}\, \tilde{s}\, \tilde{s}]^{(2)}\}^{(0)} + r_0 (\{[s^\dagger s^\dagger \times \tilde{s}\tilde{s}]^{(2)}\}^{(0)} \times \{[s^\dagger \tilde{d} + d^\dagger\, \tilde{s}]^{(2)}\}^{(0)}) +$$
$$r_2 [(\{[d^\dagger \times d^\dagger]^{(2)} \times [\tilde{s} \times \tilde{s}]^{(2)}\}^{(0)} \times \{[s^\dagger \tilde{d}]^{(2)}\}^{(0)}) + (\{[s^\dagger \times s^\dagger]^{(2)} \times$$
$$[\tilde{d} \times \tilde{d}]^{(2)}\}^{(0)} \times \{[d^\dagger \tilde{s}]^{(2)}\}^{(0)})] \qquad (2)$$

Now, the Hamilton expressed in terms of Casimir operators

The number operator n_d of d-bosons, the number operator N of all bosons and the total angular momentum operator J^2 are written in the form of Casimir operator like as

$$n_d = \sqrt{5}\,[d^\dagger \times \tilde{d}]^{(0)} = \sum_\mu d^\dagger_\mu d_\mu, \qquad (3)$$

$$N = s^\dagger s + n_d,$$

$$J^2 = -\sqrt{3}.10\,[[d^\dagger \times \tilde{d}]^{(1)} \times [d^\dagger \times \tilde{d}]^{(1)}]^{(0)} \qquad (4)$$

The operator

$$T^2 = n_d\,(n_d + 3) - 5.\,[d^\dagger \times d^\dagger]^{(0)}\,[\tilde{d} \times \tilde{d}]^{(0)} \qquad (5)$$

It shows a certain relationship to the following operator

$$R^2 = N\,(N + 4) - (\sqrt{5}.\,[d^\dagger \times d^\dagger]^{(0)} - s^\dagger s^\dagger)\,(\sqrt{5}[\tilde{d} \times \tilde{d}]^{(0)} - ss). \qquad (6)$$

R^2 is identical with $C_{SO\,(6)}$

$$Q^2 = \sqrt{5}.\,[Q \times Q]^{(0)} = \sum_\mu (-1)^\mu Q_\mu Q_{-\mu}$$

With $Q_\mu = d^\dagger{}_\mu s + s^\dagger \tilde{d}_\mu -$

$$(\sqrt{7}/2)\,[d^\dagger \times \tilde{d}]^{(2)}{}_\mu \qquad (7)$$

is related to the quadruple moment of the nucleus.

Now we replace all components of the Hamilton operator (2) step by step by expressions constituted by the operators (3) up to (7).

$$H = \varepsilon_s N + (\varepsilon_d - \varepsilon_s)\,n_d + C_0(1/10)\,[-T^2 + n_d\,(n_d + 3)] + C_2(1/14)\,[2n_d^2 + 2T^2 - 4n_d - J^2]$$

$$+ C_4(3/14)\,[(6/5)\,n_d^2 - (12/5)\,n_d - (1/5)\,T^2 + (1/3)\,J^2]$$

$$- v_2(1/\sqrt{70})\,[Q^2 + (3/2)\,T^2 - R^2 - (1/8)\,J^2 - (1/2)\,n_d^2 - (5/2)\,n_d]$$

$$- v_0(1/\sqrt{20})\,[\,-R^2 + T^2 + 5N + 2Nn_d - 2n_d^2 - 4n_d + u_2(1/\sqrt{5})\,[Nn_d - n_d^2]$$

$$+ u_0\,(1/2)\,[N^2 - 2Nn_d - N + n_d^2 + n_d] + A_2(1/\sqrt{5})\,[n_d^3 + 5n_d^2 - n_dT^2 + (1/4)\,J^2n_d]$$

$$+ B_2(1/7)\,[2Nn_d^2 - 2n_d^3 - 9n_d^2 + 10Nn_d + 5n_d - 2T^2 N + 2T^2 n_d - T^2 + (1/2)\,J^2 N -$$

$$(1/2)\,J^2n_d + (1/4)\,J^2]$$

$$+ p_2(1/35)\,[Rn_d^2 + 5Rn_d - T^2R + (1/4)\,J^2R + Tn_d^2 + 5Tn_d - T^3 + (1/4)\,J^2T]$$

$$+ p_0\,(1/\sqrt{5})\,[-R^3 + RT^2 + 2Nn_dR + 4NR - 2n_d^2R - 4n_dR - TR^2 + T^3 + 2Nn_dT +$$

$$4NT - 2n_d^2T - 4n_dT]$$

$$+ q_2(2/7\sqrt{5})\,[n_d^2 + 5n_d - T^2 + (1/4)\,J^2] + D_2\,[N^2n_d + n_d^3 - 2Nn_d^2 - Nn_d + n_d^2]$$

$$+ r_0\,[N^2 R + n_d^2 - 2Nn_dR - NR + Tn_d + Rn_d - TN - 2Nn_dT + Tn_d^2 + N^2T]$$

$$+ q_0(1/2)\,[N^3 - n_d^3 - 3N^2 n_d + 3Nn_d^2 - N^2 - n_d^2 + 2Nn_d] \quad (8)$$

We summarize

$$H = \varepsilon_n N + \varepsilon_d' n_d + V_n N^2 + V_{nd} Nn_d + V_d n_d^2 + V_r R^2 + V_t T^2 + V_j J^2 + V_q Q^2 + V_{nd}^2 n_d^3 +$$
$$V_N N^3 + V_T T^3 + V_R R^3 + V_{nd1,T}\, nd\, T^2 + V_{nd,J}\, n_d J^2 + V_{N,nd1} Nn_d^2 + V_{T1,N} T^2 N +$$
$$V_{J,N} J^2 N + V_{N,nd2} N^2 n_d + V_{R,nd2} Rn_d^2 + V_{R,nd1} Rn_d + V_{T,R1} T^2 R + V_{J,R} J^2 R +$$
$$V_{T,nd2} Tn_d^2 + V_{T,nd} Tn_d + V_{J,T} J^2 T + V_{N,nd,R} Nn_d R + V_{N1,R} NR + V_{T,R2} TR^2 +$$
$$V_{N,nd,T} Nn_d T + V_{T,N} TN + V_{N2,R} N^2 R + V_{N2,T} N^2 T \quad (9)$$

The IBM-1 Hamiltonian (9) can be expressed as a linear combination of the U (6) and its subgroups linear and quadratic Casimir operators.

$$H = a_1 C_{1,U(5)} + a_1' C_{2,U(5)} + a_2 C_{1,U(6)} + a_2' C_{2,U(6)} + a_3 C_{1,U(6)} C_{1,U(5)} + a_4 C_{2,SO(5)} + a_5 C_{2,SO(3)} + a_6 C_{2,SO(6)} + a_7 C_{2,SU(3)} + b_1 [C_{1,U(5)}]^3 + b_2 C_{2,SO(5)} C_{1,U(5)} + b_2' C_{2,SO(3)} C_{1,U(5)} + b_3 C_{2,U(6)} C_{1,U(6)} + b_4 C_{1,U(6)} C_{2,U(5)} + b_5 C_{2,SO(5)} C_{1,U(6)} + b_6 C_{2,SO(3)} C_{1,U(6)} + b_7 [C_{1,U(6)}]^3 \quad (10)$$

The Casimir invariant operators of U (6) and its subgroups in the pattern are given below:

$C_{1,U(6)} = N$, $C_{1,U(5)} = n_d$, $C_{2,U(5)} = n_d (n_d + 4)$, $C_{2,U(6)} = N (N + 5)$,

$C_{2,SO(6)} = N (N + 4) - \{\sqrt{5}\, [d^\dagger \times d^\dagger]^{(0)} - s^\dagger s^\dagger\}\ \{\sqrt{5}\, [\tilde{d} \times \tilde{d}]^{(0)} - ss\}$

$C_{2,SO(5)} = n_d (n_d + 3) - 5\ \{[d^\dagger \times d^\dagger]^{(0)}\ [\tilde{d} \times \tilde{d}]^{(0)}\}$

$C_{2,SO(3)} = -10\sqrt{3}\ \{[d^\dagger \times \tilde{d}]^{(1)} \times [d^\dagger \times \tilde{d}]^{(1)}\}$

$C_{2,SU(3)} = \sum_\mu (-1)^\mu Q_\mu Q_{-\mu}$, Where $Q_\mu = \{d_\mu^\dagger \tilde{s} + s^\dagger \tilde{d}_\mu - \sqrt{7}/2\ [d^\dagger \times \tilde{d}]_\mu^{(2)}\}$

Transition operators are associated with the IBM-calculated collective states. The number of bosons must be conserved because the BE (2) transition operator must be a Hermitian tensor of rank two. Because there are only two operators that can be used in the lowest order with these constraints, the general E2 operator can be written as

$$T_m(E2) = \alpha_2\, [s^\dagger \tilde{d} + d^\dagger s]^{(2)}_m + \beta_2\, [d^\dagger \times \tilde{d}]^{(2)}_m \quad (11)$$

Where, α_2 plays the role of the effective boson charge and $\beta_2 = \sqrt{7}/2\ \alpha_2$. The BE (2) strength for the E2 transitions is given by

$$B\,(E2; L_i \rightarrow L_f) = 1/\,(2L_i + 1)^{1/2}\ |\langle L_f \,\|\, T_m(E2) \,\|\, L_i\rangle|^2 \quad (12)$$

The solution of the Hamiltonian, in either the eq. (2) or the eq. (11) form, may be attempted either analytically or numerically. Using the underlying group theoretic characteristics of this system, Arima and Iachello were able to solve the Hamiltonian analytically in the three-limiting cases stated previously. The five components of the L = 2 d-boson state, one component of the L = 0 s-boson state, and ten components for three-three boson interactions all span a linear vector space that provides a framework for totally symmetric representations of the group U (6), the special unitary group in six dimensions.

The reduced transition probabilities in IBM-1 are given for the limit U (5)-O (6).

$$B\,(E2;\,L + 2 \rightarrow L)\downarrow = 1/4\,\alpha_2^2\,(L + 2)\,(2N - L) \qquad (13)$$

Where, L is the state that nucleus translates to and N is the boson number, which is equal to half the number of valence nucleons. From the given experimental value of transition ($2^+ \rightarrow 0^+$), one can calculate the parameter α_2^2 for each isotope, where α_2^2 indicates the square of the effective charge. This value is used to calculate the transition 8^+ to 6^+, 6^+ to 4^+, 4^+ to 2^+ and 2^+ to 0^+. The value of B(E2) in units of $e^2\,b^2$, is related to B(E2) in units of Weisskopf single particle transition (w.u).

$$1\text{ w.u} = 5.94 \times 10^{-6} \times A^{4/3} \times B\,(E2)\,e^2\,b^2 \qquad (14)$$

Here e is the charge of electron and b (1 barn = 10^{-28} square meters) is the unit of area.

RESULTS AND DISCUSSION

Even-even nuclei with Z = 52 and N = 70-78 offer good chances to analyse the behaviour of total low-lying E2 strengths in the transitional region between deformed and spherical nuclei [15]. To determine the reduced transition probabilities strengths B (E2), the calculated absolute strengths B (E2) of the transitions within the ground state band can be fitted to the experimental ones. The vibrational limit U (5) has been used in IBM-1 to describe the Tellurium nuclei. Table 1 show the best fit values for the Hamiltonian parameters, as well as the estimated energy values for $^{122-130}Te$ isotopes, which are compared to experimental data. The value of effective charge (α_2) of IBM-1 was determined by normalizing the experimental data B (E2; $2_1^+ \rightarrow 0_1^+$) of each isotope using Eq. (13). From the given experimental value of transitions ($2_1^+ \rightarrow 0_1^+$), we have calculated the value of the parameter α_2^2 for each isotope and used this value to calculate the transitions from $4^+ \rightarrow 2^+$, $6^+ \rightarrow 4^+$, $8^+ \rightarrow 6^+$.

Isotopes	N	B (E2; $2_1^+ \rightarrow 0_1^+$) ($e^2\,b^2$)	E2SD ($e^2\,b^2$)	E2DD ($e^2\,b^2$)
^{122}Te	7	0.1331	0.1379	-0.095
^{124}Te	6	0.1416	0.1536	-0.107
^{126}Te	5	0.0994	0.1410	-0.0988
^{128}Te	4	0.0770	0.1387	-0.097
^{130}Te	3	2.212	1.226	-0.093

Table 1: The parameters which used in FBEM-code for $^{122-130}Te$ Isotopes

Various E2 reduced transition probabilities are examined experimentally. Theoretical and experimental data for proton charge are compared in Table-2 and Table-3. The theoretical B (E2) values agree with the experimental results within the specified errors.

Table-2: Theoretical and experimental values of B(E2) for $^{122-126}$Te even-even isotopes.

Spin Parity $J_i^\pi \to J_f^\pi$	$_{52}Te^{122}$		$_{52}Te^{124}$		$_{52}Te^{126}$	
	Experimental ($e^2 b^2$)	This work ($e^2 b^2$)	Experimental ($e^2 b^2$)	This work ($e^2 b^2$)	Experimental ($e^2 b^2$)	This work ($e^2 b^2$)
$2_1^+ \to 0_1^+$	0.144	0.1331	0.1153	0.1416	0.098	0.0994
$2_1^+ \to 0_2^+$	0.048	0.0443	0.038	0.047	0.0326	0.033
$2_2^+ \to 0_2^+$	0.0078	0.0072	0.0094	0.0092	0.0076	0.0078
$2_3^+ \to 0_3^+$	0.113	0.1116	0.1106	0.1036	0.0032	0.0033
$2_2^+ \to 2_1^+$	0.35	0.23	0.299	0.16	0.14	0.126
$4_1^+ \to 2_1^+$	0.196	0.2282	0.228	0.236	0.159	0.16
$4_2^+ \to 2_2^+$	0.149	0.146	0.1462	0.1483	0.0955	0.0937
$4_2^+ \to 2_3^+$	0.04	0.029	0.0152	0.0275	0.0086	0.0088
$6_1^+ \to 4_1^+$	0.308	0.285	0.2306	0.2832	0.1764	0.1789
$8_1^+ \to 6_1^+$	0.328	0.304	0.2306	0.2832	0.1568	0.1590

Table-3: B (E2) values for $^{128-130}$Te isotopes.

Spin Parity $J_i^\pi \to J_f^\pi$	$_{52}Te^{128}$		$_{52}Te^{130}$	
	Experimental ($e^2 b^2$)	This work ($e^2 b^2$)	Experimental ($e^2 b^2$)	This work ($e^2 b^2$)
$2_1^+ \to 0_1^+$	0.07	0.0770	2.569	2.212
$2_1^+ \to 0_2^+$	0.023	0.256	0.856	0.737
$2_2^+ \to 0_2^+$	0.00736	0.0075	0.642	0.553
$2_3^+ \to 0_3^+$	0.0034	0.0032	0.428	0.368
$2_2^+ \to 2_1^+$	0.049	0.04	0.584	0.553
$4_1^+ \to 2_1^+$	0.116	0.1154	2.849	2.949
$4_2^+ \to 2_2^+$	0.661	0.605	0.3248	0.2670
$4_2^+ \to 2_3^+$	0.0077	0.0084	0.198	0.2136
$6_1^+ \to 4_1$	0.105	0.1155	2.569	2.211
$8_1^+ \to 6_1^+$	0.07	0.077	0	0

The B (E2) values for the so-called cross-over $2_2^+ \rightarrow 0_2^+$ transition is well reproduced and they are very small. This indicates that in the theoretical B (E2) values, the particle and collective contributions are out of phase. Transitions from $2_2^+ \rightarrow 2_1^+$, B (E2) values are larger when compared to the ones in $2_2^+ \rightarrow 0_2^+$ transition for $^{122\text{-}130}$Te isotopes. This is due to the fact that both contributions are included in phase.

We have compared the ratio R = B (E2; $4_1^+ \rightarrow 2_1^+$) / B (E2; $2_1^+ \rightarrow 0_1^+$) of IBM-1 and the experimental values in the ground state bands as a function of angular momentum L. As a measure to quantify the evolution, it is shown that the results of R values increase with increasing the high spin states.

The branching ratios B (E2; $4_1^+ \rightarrow 2_1^+$) / B (E2; $2_1^+ \rightarrow 0_1^+$) is less than 2 represents U (5) symmetry, less than 1.42 for O (6) symmetry and zero for SU (3) symmetry. We investigate U (5) symmetry as B (E2; $4_1^+ \rightarrow 2_1^+$) / B (E2; $2_1^+ \rightarrow 0_1^+$) = 1.71(e^2 b^2), 1.67(e^2 b^2), 1.61(e^2 b^2), 1.49(e^2 b^2) and 1.33(e^2 b^2) of $_{52}Te^{122}$, $_{52}Te^{124}$, $_{52}Te^{126}$, $_{52}Te^{128}$ and $_{52}Te^{130}$ respectively.

CONCLUSION

In nuclear structural physics, the phonon concept is useful for explaining the excitation of collective lower states in vibrational nuclei near spherical nuclei. One can show the two phonon excitations by close the energy level 0_2^+ from the twice value of the energy level 2_1^+ and the closest of the energy levels (4_1^+, 2_2^+ and 0_2^+). A quantitative comparison with experimental data is not possible because these small components are not stable enough against small changes in the model parameters. Also, the reduced transition probabilities between $4^+ \rightarrow 2^+$, $6^+ \rightarrow 4^+$, $8^+ \rightarrow 6^+$ of even-

even Te (Z = 52, N = 70 to 78) have been studied within the framework of interacting boson model-1. It is found that electric quadrupole reduced transition probability are in good agreement with the experimental results for $^{122\text{-}130}$Te. These results are extremely valuable for generating nuclear data tables, makes it a good resource.

REFERENCES

[1] Arima , A., & Iachello, F. (1979). Interacting boson model collective nuclear states (the O(6) limit). *Ann. Phys.*, *123*, 468-492.

[2] Arima, A., & Iachello, F. (1975). Collecive nuclear states as representations of a SU (6) group. *Phys. Rev. Let., 35*, 1069-1072.

[3] Arima, A., & Iachello, F. (1976). Interacting boson model collective nuclear states (the vibrational limit). *Ann. Phys., 99*, 253-317.

[4] Arima, A., & Iachello, F. (1978). Interacting boson moel collective nuclear states (the rotational limit). *Ann. Phys., 111*, 201-238.

[5] Arima, A., & Iachello, F. (1978). New symmetry in sd boson model of nuclei: the group O(6). *Phys. Rev. Let., 40*, 385-387.

[6] Arima, A., & Iachello, F. (1981). The interacting boson model . *Ann. Rev. Nucl. Part. Sci., 31*, 75-105.

[7] Kucukbursa, A., & Manisa, K. (2005). IBM-1 calculations on the even-even $^{122\text{-}128}$Te isotopes. *Math. and computational applications* , 9.

[8] Iachello, F., & Arima, A. (1987). The interacting boson model. *Cambridge University Press, Cambridge*.

[9] Mordirosian, G., & Stewart, N. M. (1984). Low-energy nuclear states in ^{124}Te. *Z.. Phys., A315*, 213-222.

[10] Iachello, F. (1981). Nuclear structure/ Edited by K. Abrahams, K. Allaart and A. E. L. Dieperink. New York:. *Plenum Press* .

[11] Pfeifer, W. (2008). *An Introduction to the Interacting Boson Model of the atomic Nucleus.* Switzerland.
[12] Iachello, F. (1981). Group theory and nuclear spectroscopy. *Lecture notes in Physics, nuclear spectrcopy, Springer, Berlin*.
[13] Scholten , O., Iachello, F., & Arima, A. (1978). Interacting boson model collective nuclear states (the transition from SU(5) to SU(3)). *Ann. Phys., 115*, 325-360.
[14] Scholten, O., & Iachello, F. (1978). *Annals of Phys.*,, 115-325.
[15] Schreckebach, K., & et, a. (1982). *Phys. Lett. B*, 110-364
[16] Lopac, V. (1970). Semi-microscopic description of even tellurium isotopes. *Nucl. Phys., A155*, 513-525.
[17] Kanbe, M., & Kitao , K. (2001). Data Sheet for A = 128. *Nucl. Data Sheets*, 227-395.
[18] Katakura, J., & Kitao, K. (2002). Data Sheets for A = 126. *Nucl. Data Sheets*, 765-926.
[19] Katakura, J., & Wu., Z. D. (2008). Nucl.Data sheets fo A = 124. *Nucl. Data Sheets* , 1655-1877.
[20] Singh, B. (2001). Data Sheet for A =130. *Nucl. Data Sheets*, 33-242.
[21] Tamura, T. (2007). Nuclear data sheets for A = 122. *Nucl. Data Sheets*, 455-772.
[22] Lombard, R. J. (1998). Quasi particle description of 2^+ and 3^+ states of doubly even spherical nuclei. *Nucl. Phys., A636*, 449-462.
[23] Naqib, I. M., Christy, A., Hall, I., Nolan, M. F., & Thomas, D. J. (1977). Quadrupole moments of the first 2^+ states of doubly even nuclei in the Z = 50 region. *J. Phys G: Nucl. Phys., 3*, 507-517.
[24] Rao, S. S., & Rao, K. B. (1990). Low-energy excited states in ^{124}Te nucleus. *II Nuovo Cimento, 103*, 803-822.
[25] Robinson, S. J., Hamilton, W. D., & Snelling, D. M. (1983). Levels and transitions in ^{124}Te following thermal-neutron capture by ^{123}Te and the beta decay of oriented 124Sb . *J. Phys G: Nucl. Phys., 9*, 961-984.
[26] Rosel, F., Fries, H., Alder, K., & Pauli, H. (1978). Atomic Data and Nuclear Tables, 21. 205.
[27] Turkan, N., & Maras, I. (2011). Serach on results of IBM for region between 120 < A < 150: 120-^{128}Te and $^{122\text{-}134}$Xe nucleus . *Math. and computational Applications*, 467-476.
[28] Zamfir, N., & Casten, R. (2000). *J. Res. Natl. Inst. Stand. Technol., 105*, 147.
[29] Hossian Imam, Ahmed I. M. & Ahmed Saddon Taha (2012). Calculations of reduced transition probabilities B(E2) in $^{114\text{-}122}$Cd by IBM-1. *Armenian Journal of Physics, Vol.5, issue 3,* 101-104.

[30] Mohammed Ali, Amir A., Al-Attiah, Khalid Hussain Hattif & Al-Kafajy, Heiyam Najy (2018). A study of nuclear structure of $^{122\text{-}128}$Te Even-Even Isotopes by the interacting boson model-1. *Journal University of Kerbala, Vol. 16,* 418-424.