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Univariate Maintenance Model for a Deteriorating System under A – Series Process and Partial Square Sum Process

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This paper examines the maintenance model for a decaying system, taking into account the alpha-series process independently as well as successive running and restore times that follow a partial square sum process. Assume that the system's degeneration is stochastic and that it cannot be "as good as new" after restoration. In light of these presumptions, we employ a replacement method based on the system's failure quantity. An explicit formula for the long run mean cost for each unit time under Nstrategy is used to construct an analytically determined optimal replacement strategy N^* for minimising the long run mean cost for each unit time. There is also a numerical example provided.
Keywords: α – series process, geometric process, partial square sum process

1. Introduction

Let $\{X_n, n = 1,2,3,4\ \ldots$ be a sequence of independent non-negative random variables and let $G(x)$ be the distribution function of X_1. Then $\{X_{n+1}, n = 1,2,\ldots\}$ is known as a partial sum process, in the event that the distribution function of X_{n+1}, is $G\{2^{n-1}\,\beta_0^2\}$ where $\beta_0^2 > 0$ are constants with $\beta_n^2 = \beta_0^2 + \beta_1^2 + \beta_2^2 + \cdots + \beta_{n-1}^2$.

Lemma 1.1. For real β_n^2, $\beta_n^2 = 2^{n-1}\beta_0^2$.

Proof. When $n = 1$, $\beta_1^2 = \beta_0^2$, As a result, the result is correct for $n = 1$.

Assume that the result is correct for $n = k$.

$$\beta_{n+1}^2 = \left(\beta_0^2 + \beta_1^2 + \cdots + \beta_{k-1}^2\right) + \beta_k^2$$
$$= 2\beta_k^2$$

$$= 2(2^{k-1}\beta_0^2) \text{ (by induction assumption)}$$
$$= 2^{(k+1)-1}\beta_0^2$$

As, a result, the result is correct for $n = k + 1$ also .
Thus the distribution function of X_{n+1} is $G(2^{n-1}\beta_0^2)$ for $n = 1,2,3, ...$
Therefore the density function of X_{n+1} is

$$g_{n+1}(x) = \beta_n^2 \, g(\beta_n^2 x)$$

Lemma 1.2. Let $E(X_1) = \mu$ then for $n = 1,2,3, ...$ then $E(X_{n+1}) = \frac{\mu}{2^{n-1}\beta_0^2}$.

Proof.
$$E(X_{n+1}) = \int_{-\infty}^{\infty} x \, g_{n+1}(x)dx$$
$$= \int_{-\infty}^{\infty} x \, \beta_n^2 g(\beta_n^2 x)dx$$
$$\text{Put } y = \beta_n^2 x \quad \Rightarrow x = \frac{y}{\beta_n^2}$$
$$dx = \frac{1}{\beta_n^2}dy$$
$$E(X_{n+1}) = \int_{-\infty}^{\infty} \frac{y}{\beta_n^2} \, g(y)\beta_n^2 \frac{1}{\beta_n^2}dx$$
$$= \frac{1}{\beta_n^2}\int_{-\infty}^{\infty} y \, g(y)dy$$
$$= \frac{1}{\beta_n^2}E(Y)$$
$$= \frac{1}{\beta_n^2}E(X_1)$$
$$E(X_{n+1}) = \frac{\mu}{2^{n-1}\beta_0^2} \text{ for } n = 1,2, ...$$

Lemma 1.3. The partial sum process $\{X_n, n = 1,2, ...\}$ with parameter $\beta_0^2 > 0$ stochastically decreasing and hence it is a monotone process.

Proof. Note that for any $\delta \geq 0$,

$$G(\delta) \leq G(\beta_0^2\delta) \leq G(2\beta_0^2\delta) \leq \cdots \leq G(2^{n-1}\beta_0^2\delta)$$
$$P(X_1 > \delta) \geq P(X_2 \geq \delta) \geq P(X_3 > \delta) \ldots P(X_n > \delta)$$

This implies that $\{X_n, n = 1,2,3, ...\}$ is stochastically decreasing.

Given a sequence of non-negative random variables $\{Y_n, n = 1,2,3, ...\}$ if they are independent and the distribution function of Y_n is given by $F(n^\alpha y)$ for $n = 1,2,3, ...$ where α is a real number, then $\{Y_n,\ n = 1,2,3, ...\}$ is called an alpha-series process.

Result 1.4 Given a alpha-series process $\{Y_n, n = 1,2,3, ...\}$. If $\alpha < 0$, then $\{Y_n, n = 1,2,3, ...\}$ is stochastically increasing.

Result 1.5. Let $E(Y_1) = \lambda$ then $E(Y_n) = \frac{\lambda}{n^\alpha}$.

2 MODEL PRESUMPTIONS

A1. First, a new system is installed. It is possible to replace a malfunctioning system with an identical new one.

A2. Let X_1 be the running time before the 1st failure and let $G(X)$ be the distribution function of X_1. Let X_{n+1} be the running time after the n repair for $n = 1,2,3, ...$ Then the distribution function of X_{n+1} is $(\beta_n^2 x)$, where $\beta_n^2 > 0$ are constant. That is the successive running times $\{X_{n+1}, n = 1,2,3, \ldots\}$ after restore constitute a decreasing partial sum process. Also assume that $E(X_1) = \mu > 0$ and $\mu_{n+1} = E(X_{n+1}) = \frac{\mu}{2^{n-1}\beta_0^2}$.

A3. Let Y_1 be the restore time after the 1st failure and let $F(y)$ be the distribution function of Y_1. Let Y_n be the restore times after n failure. Then the distribution function of Y_n is $F(n^\alpha y)$ for $n = 1,2,3, \ldots$ where $\alpha < 0$ is a real number and $\lambda_n = E(Y_n) = \frac{\lambda}{n^\alpha}$. That is the consecutive restore times $\{Y_n, n = 1,2,3, \ldots\}$ form an increasing alpha-series process.

A4. The running times $\{X_n, n = 1,2,3, \ldots\}$ and restore times $\{Y_n, n = 1,2,3, \ldots\}$ are independent.

A5. Let R be the time of replacement with $E(R) = \psi$.

A6. The maintenance cost is ζ, the award rate is α and the replacement cost is ξ.

$$C(N) = \frac{The\ expected\ cost\ incurred\ in\ a\ cycle}{The\ expected\ length\ of\ a\ cycle}$$

$$= \frac{E(\zeta \sum_{n=1}^{N-1} Y_n + \xi - \tau \sum_{n=1}^{N} X_n)}{E(\sum_{n=1}^{N} X_n + \sum_{n=1}^{N-1} Y_n + R)}$$

$$= \frac{\zeta \sum_{n=1}^{N-1} E(Y_n) + \xi - \tau \sum_{n=1}^{N} E(X_n)}{\sum_{n=1}^{N} E(X_n) + \sum_{n=1}^{N-1} E(Y_n) + E(R)}$$

Thus

$$C(N) = \frac{\zeta \sum_{n=1}^{N-1} \lambda_n + \xi - \tau \sum_{n=1}^{N} \mu_n}{\sum_{n=1}^{N} \mu_n + \sum_{n=1}^{N-1} \lambda_n + \psi} \ldots (1)$$

3 OPTIMAL REPLACEMENT STRATEGY N^*

We will decide N^* for minimizing $C(N)$ under N strategy. From equation (1)

$$C(N) = \frac{(\zeta + \tau)\ \sum_{n=1}^{N-1} \frac{\lambda}{N^\alpha}\ +\ \xi\ +\ \tau\psi}{\sum_{n=1}^{N} \frac{\mu}{2^{n-1}\beta_0^2}\ +\ \sum_{n=1}^{N-1} \frac{\lambda}{N^\alpha}\ +\ \psi} - \tau \ldots (2)$$

$C(N+1) - C(N)$

$$= \left(\frac{(\zeta + \tau)\sum_{n=1}^{N} \lambda_n + \xi + \tau\psi}{\sum_{n=1}^{N+1} \mu_n + \sum_{n=1}^{N} \lambda_n + \psi} - \tau\right) - \left(\frac{(\zeta + \tau)\sum_{n=1}^{N-1} \lambda_n + \xi + \tau\psi}{\sum_{n=1}^{N} \mu_n + \sum_{n=1}^{N-1} \lambda_n + \psi} - \tau\right)$$

$$= \left(\frac{(\zeta + \tau)\sum_{n=1}^{N} \lambda_n + \xi + \tau\psi}{\sum_{n=1}^{N+1} \mu_n + \sum_{n=1}^{N} \lambda_n + \psi} - \frac{(\zeta + \tau)\sum_{n=1}^{N-1} \lambda_n + \xi + \tau\psi}{\sum_{n=1}^{N} \mu_n + \sum_{n=1}^{N-1} \lambda_n + \psi}\right)$$

After Simplifications, we get

$C(N+1) - C(N)$

$$= \frac{\left(\begin{array}{c}(\zeta + \tau)\lambda \left(2^{N-1}\beta_0^2\left(\mu + \sum_{n=2}^{N} \frac{\mu}{2^{n-2}\beta_0^2}\right) - N^\alpha \mu \sum_{n=1}^{N-1} \frac{1}{n^\alpha} + \psi 2^{N-1}\beta_0^2\right) - \\ (\xi + \tau\psi)\mu N^\alpha + \lambda\, 2^{N-1}\beta_0^2\end{array}\right)}{\left(2^{N-1}\beta_0^2\, N^\alpha\left(\left(\mu + \sum_{n=2}^{N+1} \frac{\mu}{2^{n-2}\beta_0^2}\right) + \sum_{n=1}^{N-1} \frac{\lambda}{n^\alpha} + \psi\right)\ \left(\mu + \sum_{n=2}^{N} \frac{\mu}{2^{n-2}\beta_0^2}\right) + \sum_{n=1}^{N-1} \frac{\lambda}{n^\alpha} + \psi\right)} \ldots (3)$$

Let, $B(N)$

$$= \frac{(\zeta+\tau)\lambda\left(2^{N-1}\beta_0^2\left(\mu+\sum_{n=2}^{N}\frac{\mu}{2^{n-2}\beta_0^2}\right)-N^{\alpha}\mu\sum_{n=1}^{N-1}\frac{1}{n^{\alpha}}+\psi 2^{N-1}\beta_0^2\right)}{(\xi+\tau\psi)\,(MN^{\alpha}+\lambda 2^{N-1}\beta_0^2)} \quad \ldots (4)$$

Now, we prove the following Lemma.

Lemma 3.1.

$C(N+1) > C(N) \Leftrightarrow B(N) > 1$

$C(N+1) = C(N) \Leftrightarrow B(N) = 1$

$C(N+1) < C(N) \Leftrightarrow B(N) < 1$

Proof. From Equation (3), the indication of $C(N+1)-C(N)$ is plainly equivalent to the indication of its numerator. Since the denominator of $C(N+1)-C(N)$ is always positive.

Lemma 3.2.$B(M)$ is non-diminishing in M.

Proof. From equation (4),

$$\text{Let } k(N) = \frac{(\zeta+\tau)\lambda}{(\xi+\tau\psi)\,(MN^{\alpha}+\lambda 2^{N-1}\beta_0^2)(M(N+1)^{\alpha}+\lambda 2^{N}\beta_0^2)}$$

$B(N+1)-B(N) =$

$$K(N)\begin{pmatrix}(\mu N^{\alpha}+\lambda 2^{N-1}\beta_0^2)\left(2^{N}\beta_0^2\left(\mu+\sum_{n=2}^{N+1}\frac{\mu}{2^{n-2}\beta_0^2}\right)-(N+1)^{\alpha}\mu\sum_{n=1}^{N}\frac{1}{N^{\alpha}}+\psi 2^{N}\beta_0^2\right)- \\ (\mu(N+1)^{\alpha}+\lambda 2^{N}\beta_0^2)\,(2^{N-1}\beta_0^2\left(\mu+\sum_{n=2}^{N}\frac{\mu}{2^{n-2}\beta_0^2}\right)-N^{\alpha}\mu\sum_{n=1}^{N-1}\frac{1}{N^{\alpha}}+\psi 2^{N-1}\beta_0^2\end{pmatrix}$$

$$= K(N)\begin{pmatrix}\left(\mu^2\beta_0^2+\lambda\beta_0^2\,\mu\sum_{n=1}^{N-1}\frac{1}{N^{\alpha}}\right)(N^{\alpha}2^{M}-(N+1)^{\alpha}2^{N-1})+ \\ \left(\beta_0^2\,\mu\sum_{n=2}^{N}\frac{\mu}{2^{n-2}\beta_0^2}+\beta_0^2\,\mu\,\psi\right)(M^{\alpha}2^{N}-(N+1)^{\alpha}2^{N-1})+ \\ \mu^2(2N^{\alpha}-(N+1)^{\alpha})+\lambda\beta_0^2\mu\,(2^{N}-2^{N-1}(N+1)^{\alpha}N^{\alpha})\end{pmatrix}$$

$$= K(N)\begin{pmatrix}(N^{\alpha}2^{N}-(N+1)^{\alpha}2^{N-1})+\left(\left(\mu^2\beta_0^2+\lambda\beta_0^2\,\mu\sum_{n=1}^{N-1}\frac{1}{N^{\alpha}}\right.\right. \\ \left.\left.+\left(\beta_0^2\mu\sum_{n=2}^{N}\frac{\mu}{2^{n-2}\beta_0^2}+\beta_0^2\,\mu\,\psi\right)\right)\right)+ \\ \mu^2(2N^{\alpha}-(N+1)^{\alpha})+\lambda\beta_0^2\mu\,(2^{N}-2^{N-1}(N+1)^{\alpha}N^{\alpha})\end{pmatrix}$$

After Simplifications, we get

$B(N+1)-B(N) =$

$$\frac{(\zeta+\tau)\lambda\begin{pmatrix}(N^{\alpha}2^{N}-(N+1)^{\alpha}2^{N-1})\left(\left(\mu^2\beta_0^2+\lambda\beta_0^2\,\mu\sum_{n=1}^{N-1}\frac{1}{N^{\alpha}}\right.\right. \\ \left.\left.+\left(\beta_0^2\mu\sum_{n=2}^{N}\frac{\mu}{2^{n-2}\beta_0^2}+\beta_0^2\,\mu\,\psi\right)\right)\right)+ \\ \mu^2(2N^{\alpha}-(N+1)^{\alpha})+\lambda\beta_0^2\mu\,(2^{N}-2^{N-1}(N+1)^{\alpha}N^{\alpha})\end{pmatrix}}{(\xi+\tau\psi)\,(MN^{\alpha}+\lambda 2^{N-1}\beta_0^2)(M(N+1)^{\alpha}+\lambda 2^{N}\beta_0^2)}$$

Theorem 3.3. The optimal replacement strategy N^* is chosen by
N^*= min $\{N|B(N) \geq 1\}$
Moreover, N^* is unique if and only if $B(N^*) > 1$
Proof. We note that $C(N)$ vhas a minimum value when $C(N+1) \geq C(N)$. Thus

$$N^* = min\{N|C(N+1) \geq C(N)\}$$
$$= min\{N|B(N) \geq 1\} \quad \text{(By lemma (3.1)).}$$

Since $B(N)$ is non- diminishing in N,there exist an integer N^* such that

$$B(N) \geq 1 \iff N \geq N^* \text{ and } B(N) < 1 \iff N < N^*$$

Thus N^* is unique if and only if $B(N^*) > 1$

4. Conclusion

An explicit formula for the long-term mean cost for each unit time under N is obtained by looking at a maintenance model for a decaying system with increasing running times. This is done by using the partial square sum process and successive restoration times that follow the alpha-series process. The best replacement plan, N^*, is determined analytically to minimise the long-term mean cost for each unit of time. There is, of course, general knowledge that the more developed a system is, the more experienced it is. This suggests that when the system malfunctions, we will fix it rather than replace it. An outline of the developed system is provided by a mathematical model.

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