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Copula Based Performance Analysis of a K-Out-Of-N Industrial System Model with Catastrophic Failure

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This study focusses on the reliability analysis of a k-out-of-n industrial system model comprising two interconnected subsystems series configuration. Subsystem-I is composed of three identical units in parallel configuration that are working under 1-out-of-3: G policy, while subsystem-II has two non-identical units that are working under 1-out-of-2: G: policy. In this both the subsystems are connected with controllers that may be perfect or imperfect at the time of requirement. A catastrophic failure due to frequent change in environmental conditions or man-made disturbance may be exist. Failure rates of units in both the subsystems are steady and undertook to follow exponential distribution, but their repair establishes two types of distributions such as general distribution and Gumbel-Hougaard family copula distribution. The system is analyzed by using the supplementary variable technique and Gumbel-Hougaard family of copula and obtain important reliability characteristics such as availability, reliability, Mean time to system failure and expected profit analysis. We highlight the use of copula repair, while identifying the factors for improvement and future directions of work.

Keywords: k-out-of-n: G system; availability; catastrophic failure; cost analysis; Gumbel-Hougaard family copula distribution.

1. Introduction

The purpose of reliability and availability is to assess measurement errors and offer solutions to improve such that errors are reduced. Errors have a significant impact on the quality of products and services. While reliability is defined as the probability of a system or component performing its intended functions for a specified period, availability is typically concerned with repairable systems and is defined as the probability of the system working at a specific time, despite previous crashes and repairs. Redundant systems that have been widely deployed in practice, such as space shuttles, communication satellites, dishwashers, hybrid cars, cargo ships, and combat planes, are regularly studied in the scientific literature. In general redundancy is used to improve system reliability and availability. We use it when we have same components connected in such a way that when one component stops working, the others will keep the system functioning. Standbys are of three types: (i) cold standby: In this inactive component have a zero-failure rate and cannot fail while in standby state; (ii) hot standby: In this standby unit has the same failure rate as when it is run with the operating unit; (iii) warm standby: In this state operating unit can fail, but its failure rate is less than that of operating unit. Furthermore, redundancy is an extremely cost-effective way to achieve a given level of system reliability. As a result, the k-out-of-n system structure, in which at least k out of n components must be operational for the system to work, is critical for improving dependability. A series system can be considered an n-out-of-n: G system, whereas a parallel system is a 1-out-of-n: G system. K-out-of-n warm standby systems have been used in a variety of industries, including medical diagnosis, redundant-system testing, network design, power production and transmission systems, and so on. Numerous writers have examined the system's availability, reliability, mean time to failure, and cost analysis for k-out-of-n: G redundant systems under a range of circumstances, including generalized multi-state system by Zuo and Tian [8], repairable systems by Kullstam [17], consecutive k-out-of-n using r repairman by Wu and Guan [31], single unit M|G|1 system model with helping unit by Kumar and Gupta [16], exact reliability formula for consecutive system by Liang, Xiong and Li [30], redundancy optimization under common cause failure by Bai, Yun and Chung [3], non-identical components considering shut-off rules using quasi-birth-death process by Moghaddass [18], with and without repair with three failure modes by Kumar and Sirohi [15], warm standby system with two category of units by Zhang, Xie and Horigome [20], a series Markov repairable system and presented availability indices as measure of reliability using time interval omission problem concept by Bao and Cui [28], real example of sliding window system by Levitin and Dai [4], generalized block replacement policy with respect to a threshold number of failed components and risk costs by Park and Pham [7], full system equipped with a single warm standby component by Erylmaz [19], delayed reporting of faults in a computer network by Poonia and Singh [10] and neglecting repair time by Jia et al. [29].

Initially, reliability models assumed that any complicated engineering system could only have one k-out-of-n system, in which all units were placed in parallel. However, there are many scenarios in which two or more k-out-of-n type systems can be configured in series, and the results are fantastic in terms of reliability. In such instances, we can partition the entire system into two subsystems in a series design. Many researchers have thoroughly investigated such series systems. Singh, Ram, and Rawal [23] summarized a unique approach for cost analysis of an engineering system consisting of two subsystems, subsystem-1 and subsystem-2, with controllers connected in series. Subsystem-1 operates on the k-out-of-n good policy, while subsystem-2 is made up of three similar units connected in series. The controllers control both subsystems, and the operator may purposefully fail the system. The system is analyzed using the supplementary variable technique and Laplace transforms. Lado and Singh [1] proposed a series system with two subsystems operated by human operator. In this, each subsystem has two identical units in parallel. The paper has studied via two types of repairs viz. copula repair and general repair and concluded that copula repair is more reliable compared to general repair. Sirohi et al. [2] and Singh et al. [27] examined a complex repairable system in series configuration with switch and catastrophic failure using copula repair and prove that copula repair is better policy than general repair.

Since downtime results in both tangible and intangible losses, creating a highly complex and sophisticated computer network design that can increase reliability and implement maintenance policies is now the most difficult challenge. Effective and precise reliability analysis techniques are required to comprehend the dependability of complicated computer networks in the event of catastrophic disasters. If the system is working with reduced efficiency, then we can repair the system via general repair, but if the system is incomplete and in shutdown mode, then we need to repair the system quickly and for this, we can use Gumbel-Hougaard copula distribution (This distribution used to restore completely failed system) by Nelson. Various authors including Gahlot et al. [6], Poonia [11], Singh et al. [24, 25], Singh and Poonia [22], and Poonia and Sirohi [9] etc. employed copula repair to analyze the system performance and state that the results obtained using copula repair are preferable compared to general repair. Yusuf [5] et al. examined a linear consecutive 2-out-of-4 system model with two subsystems under online and offline preventive maintenance and compared the results with copula repair using Kolmogorov technique. Singh et al. [26] studied a computer labs network of three labs joined via server under 2-out-of-3: G policy. Authors evaluated all the reliability characteristics and try to examine the future behavior of the system. As far as some real-life models are concerned, recently Nehra et al. [21] and Poonia [12] presented a paper on precision agriculture model using copula distribution and common cause breakdown. Using arbitrary parametric values, the authors assess availability, reliability, mean time to failure and profit analysis using copula distribution. The results were verified using a farming field. Furthermore, Poonia [14] developed a computer lab network and evaluated various reliability characteristics like availability, mean time to system failure, sensitivity and cost analysis.

Though, real-world circumstances are becoming increasingly complex, making simplistic engineering models ineffective. As a result, this research examines a complicated system made up of two subsystems connected in series. Subsystem-I is comprised of three identical units in parallel configuration that are working under 1-out-of-3: G policy, while subsystem-II has two dissimilar units that are working under 1-out-of-2: G: policy. In subsystem-II, priority in operation is given to the first unit whereas the second unit put into cold standby mode if not in use. As a result, the system continues to function until one of its subsystems fails totally. Based on the assumptions made above, the system could be modelled as a continuous-time stochastic process. Some reliability indicators, such as system reliability, availability, mean time to system failure, and cost analysis, are derived utilizing the supplementary variable technique, Laplace transforms, and copula repair. With the help of Maple-17, explicit expressions for reliability, availability, mean time to system failure , and cost analysis functions are obtained. Graphs present a comparative analysis of results. The transition state diagram of the designed model is shown in Fig-1.

2. Notation, assumption and description of states

2.1 NOTATIONS

$\lambda_1 / \mu_1(x)$	Failure rate / Repair rate of each unit in subsystem-1.

λ_2, λ_3 Failure rate of first / second unit in subsystem-2.

$\mu_2(x) / \mu_3(x)$ Repair rate of first / second unit in subsystem-2.

$\lambda_{S1} / \lambda_{S2}$ Failure rate of control device between units for subsystem-1/subsystem-2.

λ_C Failure rate related to catastrophic failure mode.

$\bar{P}(s)$ Laplace transformation of the state transition probability $P(t)$.

$P_i(x,t)$ The Probability that the system is in state S_i for $i = 1$ to 12 and the system is under repair with elapsed repair time is x,t. x is repaired variable and t is time variable.

$E_p(t)$ Expected profit in the interval $\left[0,t\right)$.

$\mu_0(x)$ Joint probability from failed state S_i to good state S_0 according to Gumbel-Hougaard family copula is given as $\mu_0(x) = C_\theta\{u_1(x), u_2(x)\} = \exp\left[x^\theta + \{\log\phi(x)\}^\theta\right]^{1/\theta}$ where $u_1(x) = \phi(x)$ and $u_2(x) = e^x$. Here θ is the parameter $1 < \theta < \infty$.

2.2 ASSUMPTION

The following assumptions have been made throughout the study of the model:

1. Initially the system is in state S_0, and all the units of subsystem-I and II are in good working conditions.
2. The subsystem-I works successfully as per 1-out-of-3: G policy.
3. The subsystem-II has two units, main unit and cold standby in parallel configuration. It works successfully if at least one unit is operating. Main unit is more efficient so preference in operation will be given to it as compared to cold standby unit.
4. Whenever there is a failure in two units of subsystem-I and main unit in subsystem-II, the system goes to unsafe state where system must stop functioning intentionally to avoid added failures with emergency failure rate λ_E.
5. One repairperson is available full-time with the system and may be called as soon as the system reaches a partially or completely failed state. After repair all the units in both the sub-systems becomes operational as good as new. No damage was reported due to repair of the system.
6. All failure rates are constant and follows the exponential distribution.
7. The failure rate and repair rate of each unit in subsystem-I is the same, while in subsystem-II, it is different for both the units.

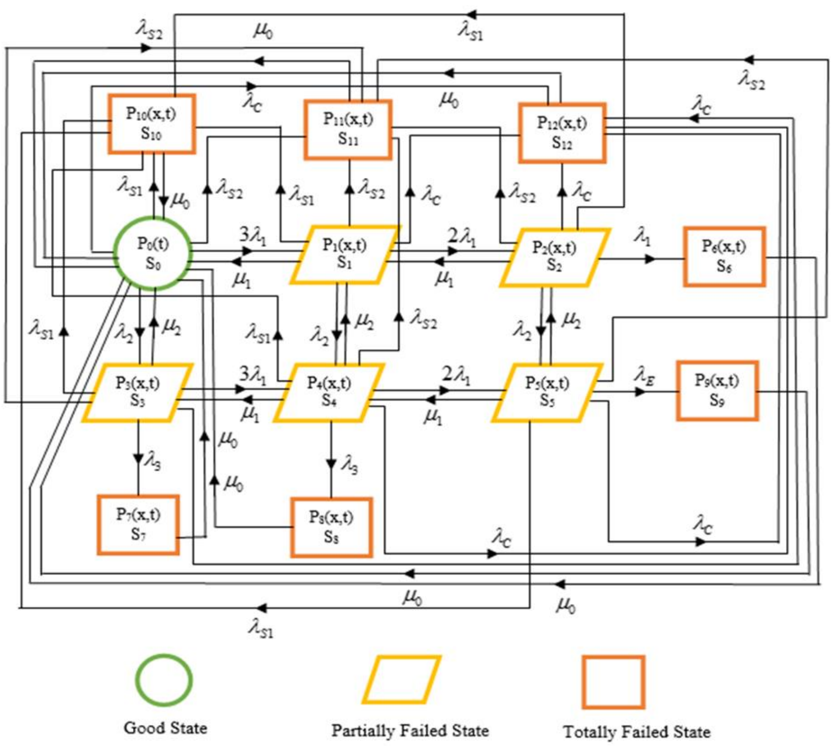
8. Both the subsystems are connected via controllers, which in the system is unreliable at the time of need, and the function of the switch is: “as long as the switch fails, the whole system fails immediately”.

9. The completely failed units need repair immediately using Gumbel-Hougaard, family of copula.

2.3 SYSTEM CONFIGURATION AND STATE TRANSITION DIAGRAM

System transition diagram in shown in Fig 1 below. In this, S_0 is perfect state, S_1, S_2, S_3, S_4 and S_5 partial failed/degraded and S_6, S_7, S_8, S_9, S_{10}, S_{11} and S_{12} are complete failed states. Due to failure in any unit in the subsystem 1 and in subsystem 2, the transitions approach to partially failed states S_1, S_2, S_3 S_4 and S_5 respectively. The state S_6, S_7 and S_8 are complete failed states due to failure of units in both the subsystems, while S_9 is completely failed state due to deliberate failure. The states S_{10} and S_{11} are complete failed states due to controller and S_{12} is due to catastrophic failure.

Figure 1 State transition diagram of the model



3. Formulation of mathematical model

By probability of considerations and continuity arguments, we obtain the set of difference-differential equations associated with the present mathematical model as follows

$$\left[\frac{\partial}{\partial t}+3\lambda_1+\lambda_2+\lambda_{S_1}+\lambda_{S_2}+\lambda_C\right]P_0(t)=\int_0^\infty \mu_1(x)P_1(x,t)\,dx+\int_0^\infty \mu_2(x)P_3(x,t)\,dx$$

$$+\sum_k \int_0^\infty \exp\left[x^\theta+\{\log\phi(x)\}^\theta\right]^{1/\theta} P_k(x,t)\,dx\{k=6,7,8,9,10,11,12\}$$

(1)

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+2\lambda_1+\lambda_2+\lambda_{S_1}+\lambda_{S_2}+\lambda_C+\mu_1(x)\right]P_1(x,t)=0$$

(2)

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_1+\lambda_2+\lambda_{S_1}+\lambda_{S_2}+\lambda_C+\mu_1(x)\right]P_2(x,t)=0$$

(3)

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+3\lambda_1+\lambda_3+\lambda_{S_1}+\lambda_{S_2}+\lambda_C+\mu_2(x)\right]P_3(x,t)=0$$

(4)

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+2\lambda_1+\lambda_3+\lambda_{S_1}+\lambda_{S_2}+\lambda_C+\mu_1(x)+\mu_2(x)\right]P_4(x,t)=0$$

(5)

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_E+\lambda_{S_1}+\lambda_{S_2}+\lambda_C+\mu_1(x)+\mu_2(x)\right]P_5(x,t)=0$$

(6)

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\exp\left[x^\theta+\{\log\phi(x)\}^\theta\right]^{1/\theta}\right]P_k(x,t)=0\ \ \{k=6,7,8,9,10,11,12\}$$

(7)

Boundary conditions

$$P_1(0,t)=3\lambda_1P_0(t)$$

(8)

$$P_2(0,t)=2\lambda_1P_1(0,t)=6\lambda_1^2P_0(t)$$

(9)

$$P_3(0,t)=\lambda_2P_0(t)$$

(10)

$$P_4(0,t)=3\lambda_1P_3(0,t)+\lambda_2P_1(0,t)=6\lambda_1\lambda_2P_0(t)$$

(11)

$$P_5(0,t) = 2\lambda_1 P_4(0,t) + \lambda_2 P_2(0,t) = 18\lambda_1^2 \lambda_2 P_0(t)$$

(12)

$$P_6(0,t) = \lambda_1 P_2(0,t) = 6\lambda_1^3 P_0(t)$$

(13)

$$P_7(0,t) = \lambda_3 P_3(0,t) = \lambda_2 \lambda_3 P_0(t)$$

(14)

$$P_8(0,t) = \lambda_3 P_4(0,t) = 6\lambda_1 \lambda_2 \lambda_3 P_0(t)$$

(15)

$$P_9(0,t) = \lambda_E P_5(0,t) = 18\lambda_1^2 \lambda_2 \lambda_E P_0(t)$$

(16)

$$P_{10}(0,t) = \lambda_{S_1}\left[P_0(t) + P_1(0,t) + P_2(0,t) + P_3(0,t) + P_4(0,t) + P_5(0,t)\right]$$

(17)

$$P_{11}(0,t) = \lambda_{S_2}\left[P_0(t) + P_1(0,t) + P_2(0,t) + P_3(0,t) + P_4(0,t) + P_5(0,t)\right]$$

(18)

$$P_{12}(0,t) = \lambda_C\left[P_0(t) + P_1(0,t) + P_2(0,t) + P_3(0,t) + P_4(0,t) + P_5(0,t)\right]$$

(19)

Initials conditions

$P_0(0) = 1$, and other state probabilities are zero at $t = 0$

(20)

Laplace transformation of boundary conditions after repair.

$$\overline{P}_1(0,s) = 3\lambda_1 \overline{P}_0(s) + \int_0^\infty \mu_1(x)\overline{P}_2(x,s)dx + \int_0^\infty \mu_2(x)\overline{P}_4(x,s)dx$$

(21)

$$\overline{P}_2(0,s) = 2\lambda_1 \overline{P}_1(x,s) + \int_0^\infty \mu_2(x)\overline{P}_2(x,s)dx$$

(22)

$$\overline{P}_3(0,s) = \lambda_2 \overline{P}_0(s) + \int_0^\infty \mu_1(x)\overline{P}_4(x,s)dx$$

(23)

$$\bar{P}_4(0,s) = 3\lambda_1\bar{P}_3(x,s) + \lambda_2\bar{P}_1(x,s) + \int_0^\infty \mu_1(x)\bar{P}_5(x,s)\,dx \tag{24}$$

Now solving all the equations with the boundary conditions, one may get

$$\bar{P}_0(s) = \frac{1}{D(s)} \tag{25}$$

$$\bar{P}_1(s) = \frac{3\lambda_1}{D(s)}\frac{1-\bar{S}_{\mu_1}\left(s+2\lambda_1+\lambda_2+\lambda_{S_1}+\lambda_{S_2}+\lambda_C\right)}{\left(s+2\lambda_1+\lambda_2+\lambda_{S_1}+\lambda_{S_2}+\lambda_C\right)} = \frac{3\lambda_1}{D(s)}\frac{1-P}{\left(s+2\lambda_1+\lambda_2+\lambda_{S_1}+\lambda_{S_2}+\lambda_C\right)} \tag{26}$$

$$\bar{P}_2(s) = \frac{6\lambda_1^2}{D(s)}\frac{1-\bar{S}_{\mu_1}\left(s+\lambda_1+\lambda_2+\lambda_{S_1}+\lambda_{S_2}+\lambda_C\right)}{\left(s+\lambda_1+\lambda_2+\lambda_{S_1}+\lambda_{S_2}+\lambda_C\right)} \tag{27}$$

$$\bar{P}_3(s) = \frac{\lambda_2}{D(s)}\frac{1-\bar{S}_{\mu_2}\left(s+3\lambda_1+\lambda_3+\lambda_{S_1}+\lambda_{S_2}+\lambda_C\right)}{s+3\lambda_1+\lambda_3+\lambda_{S_1}+\lambda_{S_2}+\lambda_C} = \frac{\lambda_2}{D(s)}\frac{1-Q}{s+3\lambda_1+\lambda_3+\lambda_{S_1}+\lambda_{S_2}+\lambda_C} \tag{28}$$

$$\bar{P}_4(s) = \frac{6\lambda_1\lambda_2}{D(s)}\frac{1-\bar{S}_{\mu_3}\left(s+2\lambda_1+\lambda_3+\lambda_{S_1}+\lambda_{S_2}+\lambda_C\right)}{s+2\lambda_1+\lambda_3+\lambda_{S_1}+\lambda_{S_2}+\lambda_C} \tag{29}$$

$$\bar{P}_5(s) = \frac{18\lambda_1^2\lambda_2}{D(s)}\frac{1-\bar{S}_{\mu_3}\left(s+\lambda_E+\lambda_{S_1}+\lambda_{S_2}+\lambda_C\right)}{s+\lambda_E+\lambda_{S_1}+\lambda_{S_2}+\lambda_C} \tag{30}$$

$$\bar{P}_6(s) = \frac{6\lambda_1^3}{D(s)}\frac{1-\bar{S}_{\mu_0}(s)}{s} \tag{31}$$

$$\bar{P}_7(s) = \frac{\lambda_2\lambda_3}{D(s)}\frac{1-\bar{S}_{\mu_0}(s)}{s} \tag{32}$$

$$\bar{P}_8(s)=\frac{6\lambda_1\lambda_2\lambda_3}{D(s)}\frac{1-\bar{S}_{\mu_0}(s)}{s}$$

(33)

$$\bar{P}_9(s)=\frac{18\lambda_1^2\lambda_2\lambda_E}{D(s)}\frac{1-\bar{S}_{\mu_0}(s)}{s}$$

(34)

$$\bar{P}_{10}(s)=\frac{\lambda_{S_1}}{D(s)}\left[1+3\lambda_1+\lambda_2+6\lambda_1\lambda_2+6\lambda_1^2+18\lambda_1^2\lambda_2\right]\frac{1-R}{s}=\frac{\lambda_{S_1}}{D(s)}\frac{U(1-R)}{s}$$

(35)

$$\bar{P}_{11}(s)=\frac{\lambda_{S_2}}{D(s)}\left[1+3\lambda_1+\lambda_2+6\lambda_1\lambda_2+6\lambda_1^2+18\lambda_1^2\lambda_2\right]\frac{1-R}{s}=\frac{\lambda_{S_2}}{D(s)}\frac{U(1-R)}{s}$$

(36)

$$\bar{P}_{12}(s)=\frac{\lambda_{S_2}}{D(s)}\left[1+3\lambda_1+\lambda_2+6\lambda_1\lambda_2+6\lambda_1^2+18\lambda_1^2\lambda_2\right]\frac{1-R}{s}=\frac{\lambda_{S_2}}{D(s)}\frac{U(1-R)}{s}$$

(37)

where $D(s)=s+3\lambda_1+\lambda_2+\lambda_{S_1}+\lambda_{S_2}+\lambda_C-3\lambda_1P-\lambda_2Q-RU$

$$P=\bar{S}_{\mu_1}\left(s+2\lambda_1+\lambda_2+\lambda_{S_1}+\lambda_{S_2}+\lambda_C\right)=\frac{\mu_1}{s+2\lambda_1+\lambda_2+\lambda_{S_1}+\lambda_{S_2}+\lambda_C+\mu_1}$$

$$Q=\bar{S}_{\mu_2}\left(s+3\lambda_1+\lambda_3+\lambda_{S_1}+\lambda_{S_2}+\lambda_C\right)=\frac{\mu_2}{s+3\lambda_1+\lambda_3+\lambda_{S_1}+\lambda_{S_2}+\lambda_C+\mu_2}$$

$$R=\bar{S}_{\mu_0}(s)=\frac{\mu_0}{s+\mu_0}$$

and

$$U=6\lambda_1^3+\lambda_2\lambda_3+6\lambda_1\lambda_2\lambda_3+18\lambda_1^2\lambda_2\lambda_E+\left(1+3\lambda_1+\lambda_2+6\lambda_1\lambda_2+6\lambda_1^2+18\lambda_1^2\lambda_2\right)\left(\lambda_{S_1}+\lambda_{S_2}+\lambda_C\right)$$

Sum of Laplace transformations of the state transitions, where the system is in operational mode and failed state at any time, is as follows

$$\bar{P}_{up}(s)=\bar{P}_0(s)+\bar{P}_1(s)+\bar{P}_2(s)+\bar{P}_3(s)+\bar{P}_4(s)+\bar{P}_5(s)$$

(38)

$$\bar{P}_{down}(s)=1-\bar{P}_{up}(s)$$

(39)

4. Analytical study

4.1 AVAILABILITY ANALYSIS

When repair follows general and Gumbel-Hougaard family copula distribution, we have

$$\bar{S}_{\mu_0}(s) = \bar{S}_{\exp\left[x^{\theta}+\{\log\phi(x)\}^{\theta}\right]^{1/\theta}}(s) = \frac{\exp\left[x^{\theta}+\{\log\phi(x)\}^{\theta}\right]^{1/\theta}}{s+\exp\left[x^{\theta}+\{\log\phi(x)\}^{\theta}\right]^{1/\theta}} = \frac{\mu_0(x)}{s+\mu_0(x)}$$

setting $\bar{S}_{\alpha_i}(s) = \dfrac{\alpha_i}{s+\alpha_i}, i = 1,2,3$ and $\bar{S}_{\phi}(s) = \dfrac{\phi}{s+\phi}$. Taking the values of different parameters as $\lambda_1 = 0.020, \lambda_2 = 0.025, \lambda_3 = 0.030, \lambda_E = 0.40, \lambda_{S_1} = 0.011, \lambda_{S_2} = 0.012, \lambda_C = 0.025, \theta = 1, x = 1, \mu_i = 1$ $(i = 1,2,3)$ in (38), then taking inverse Laplace transform, we obtain the availability of the system. To evaluate the availability of the system we discussed here following cases:

(a) When repair follows Gumbel–Hougaard family copula distribution:

$$P_{up}(t) = 0.000401e^{-1.1780t} + 0.033716e^{-2.8150t} - 0.025111e^{-1.2853t} - 0.001537e^{-1.1956t} + 0.993857e^{-0.0060t} - 0.001165e^{-1.1430t} - 0.000161e^{-1.1280t}$$

(40)

(b) When same failure rates for both the units in subsystem 2:

$$P_{up}(t) = -0.000781e^{-1.1780t} - 0.002097e^{-1.1730t} + 0.035638e^{-2.8219t} - 0.035185e^{-1.3309t} - 0.000028e^{-1.2051t} + 1.002818e^{-0.0089t} - 0.000364e^{-1.1280t}$$

(41)

(c) When all types of failure rates are same:

$$P_{up}(t) = 0.034954e^{-2.8195t} - 0.032330e^{-1.3161t} + 1.000332e^{-0.0082t} - 0.002654e^{-1.1680t} - 0.000303e^{-1.128t}$$

(42)

(d) No controller in the subsystem-2:

$$P_{up}(t) = 0.024749e^{-2.7883t} - 0.023318e^{-1.2669t} - 0.001408e^{-1.1737t} + 1.000917e^{-0.0069t} + 0.000400e^{-1.1560t} - 0.000136e^{-1.1060t} - 0.001176e^{-1.1210t}$$

(43)

(e) No controller in the both the subsystems:

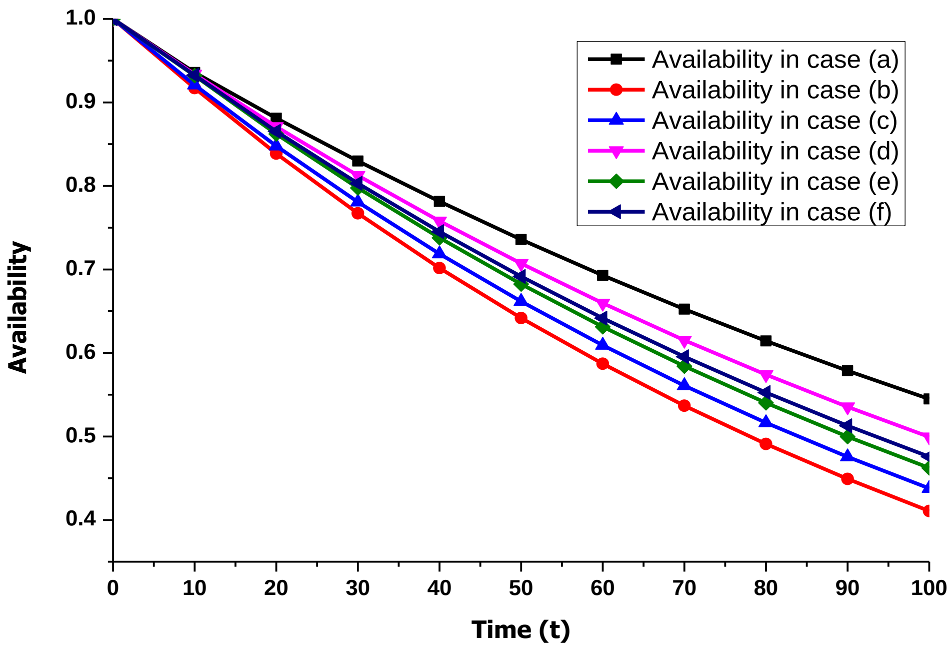
$$P_{up}(t) = 0.000399e^{-1.1350t} - 0.000165e^{-0.0850t} - 0.001187e^{-1.1000t} + 0.015948e^{-2.7628t}$$
$$-0.021333e^{-1.2495t} + 0.001289e^{-1.1528t} - 1.007629e^{-0.0078t} \quad (44)$$

(f) When catastrophic failure is ignored:

$$P_{up}(t) = -0.000164e^{-1.0930t} - 0.001183e^{-1.1080t} + 0.000399e^{-1.1430t} + 0.019329e^{-2.7725t}$$
$$-0.022123e^{-1.2561t} - 0.001334e^{-1.1608t} + 1.005076e^{-0.0075t} \quad (45)$$

For different values of time variable $t = 0, 10, 20, 30, 40, 50, 60, 70, 80, 90$ and 100 units of time, one may get different values of $P_{up}(t)$ with the help of (40-45) as shown in Fig-2.

Figure 2 Availability as a function of time



4.2 RELIABILITY ANALYSIS

Taking all repair rates equal to zero and obtain inverse Laplace transform in (38), we get an expression for the reliability of the system after taking the failure rates as $\lambda_1 = 0.020, \lambda_2 = 0.025, \lambda_3 = 0.030, \lambda_E = 0.40, \lambda_{S_1} = 0.011, \lambda_{S_2} = 0.012, \lambda_C = 0.025$. Here we have considered only two cases as rest cases are giving almost same output:

(a) Reliability of the system when failure rates follow exponential distribution:

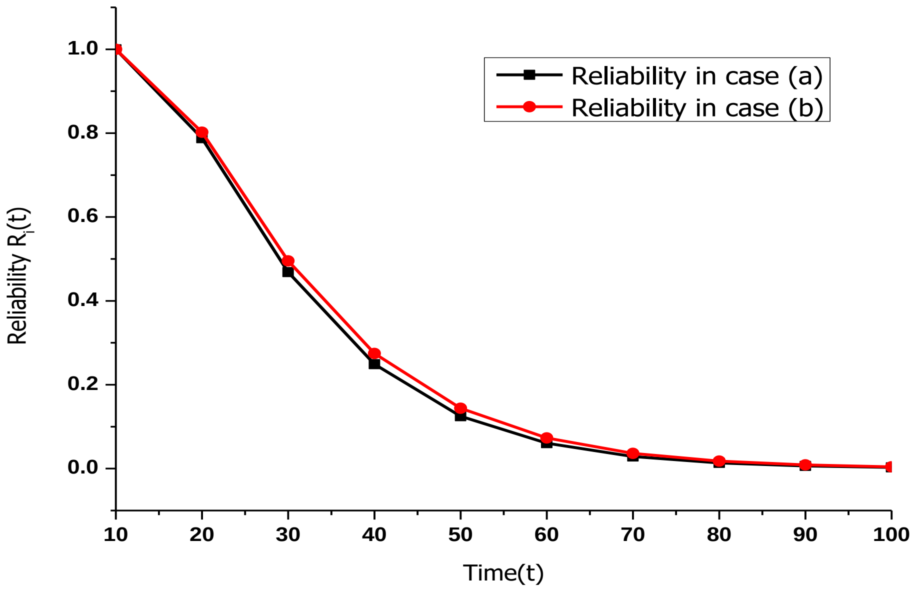
$$P_{up}(t) = 0.018381e^{-2.17692t} - 0.519446e^{-0.1260t} - 1.885541e^{-0.1100t} + 3.689836e^{-0.0788t}$$
$$-0.108105e^{-0.1060t} + 0.014969e^{-0.0670t} - 0.210095e^{-0.0900t} \quad (46)$$

(b) Reliability of the system when all types of failure rates are same and follow exponential distribution:

$$P_{up}(t) = 0.015210e^{-0.0650t} + 0.018218e^{-2.7687t} - 0.386880e^{-0.1250t} - 1.992909e^{-0.1050t}$$
$$+3.566179e^{-0.0742t} - 0.219818e^{-0.0850t} \quad (47)$$

For different values of time variable $t = 0, 10, 20, 30, 40, 50, 60, 70, 80, 90$ and 100 units of time, one may get different values of reliability $R(t)$ with the help of (46-47) as shown in Fig-3.

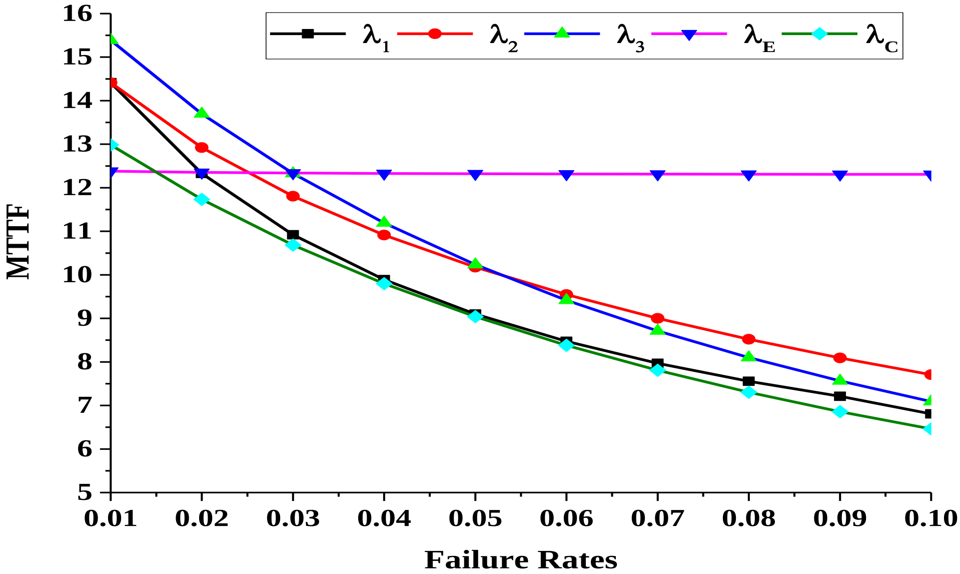
Figure 3 Reliability as a function of time



4.3 Mean Time to Failure (MTTF)

If $R_i(t)$ is the reliability function obtained by taking inverse Laplace transformation of $\bar{P}_{up}(s)$ then average time to system failure for a continuous valued function is: $\text{MTTF} = \int_0^\infty R(t)dt = \lim_{s\to 0} R(s)$. Taking all repair rate to zero and the limit as s tends to zero in (38) for the exponential distribution; we can obtain the MTTF. Now taking the values of different parameters as $\lambda_1 = 0.020, \lambda_2 = 0.025, \lambda_C = 0.030, \lambda_E = 0.040$ and $\lambda_C = 0.025$ and varying $\lambda_1, \lambda_2, \lambda_3, \lambda_E$ and λ_C one by one respectively as $0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.10$, the variation of MTTF, with respect to failure rates can be obtained as Fig-4.

Figure 4 MTTF as a function of failure rates



4.4 COST ANALYSIS

Let the service facility be always available, then expected profit during the interval $[0, t)$ is

$$E_p(t) = K_1 \int_0^t P_{up}(t) dt - K_2 t \tag{48}$$

Where K_1 and K_2 are the revenue generation and service cost in unit time. For same set of parameters defined in (48), one can obtain expression for incurred profit as a function of time for copula repair and general repair respectively as:

$$E_p(t) = K_1 \{ 0.014770e^{-1.2268t} + 0.000024e^{-1.1322t} - 0.007368e^{-2.7672t} + 0.000909e^{-1.1100t} + 0.000291e^{-1.1150t} + 0.000142e^{-1.0550t} - 269.039471e^{-0.0037t} + 269.030703 \} - K_2 t \tag{49}$$

$$E_p(t) = K_1 \{ -7.8255\ 10^{-7} e^{-1.1322t} - 0.026376e^{-1.0279t} - 0.001007e^{-1.1100t} - 0.004304e^{-1.2562t} + 0.000315e^{-1.1150t} + 0.000254e^{-1.0550t} - 269.001600e^{-0.0036t} + 269.030703 \} - K_2 t \tag{50}$$

Setting $K_1 = 1$ and $K_2 = 0.6, 0.5, 0.4, 0.3, 0.2$ and 0.1 respectively and varying $t = 0, 10, 20, 30, 40, 50, 60, 70, 80, 90$ and 100 units of time, the results for expected profit can be obtain as per Fig-5 and Fig-6.

Figure 5 Expected profit as a function of time for Copula repair

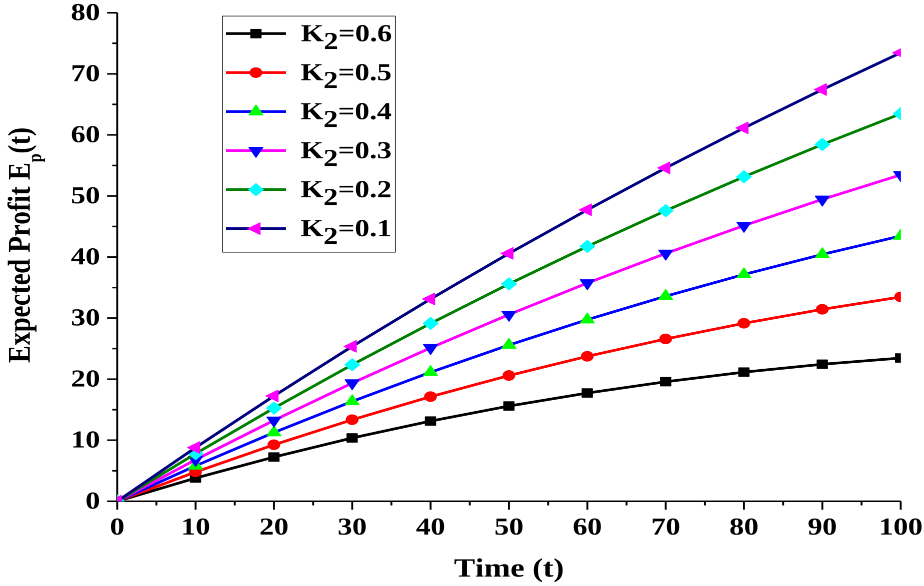
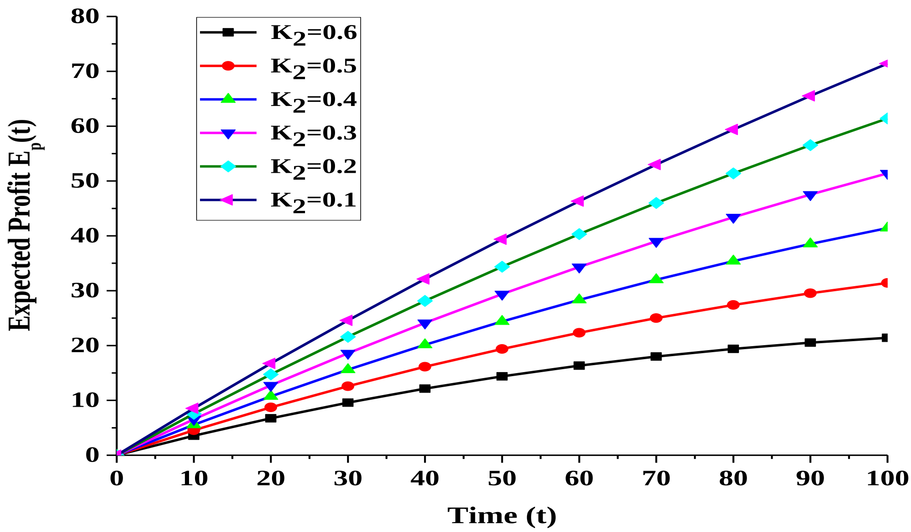


Figure 6 Expected profit as a function of time for general repair



5. Conclusion

This study focusses on the reliability analysis of a k-out-of-n industrial system model comprising two interconnected subsystems series configuration. Both the sub-systems are working under k-out-of-n: G policy under definite conditions. Explicit expressions have been derived using supplementary variable technique. Warm/cold-standby redundancy has been used as an effective technique for improving reliability of system design. The following conclusions may be drawn based on the study conducted in this paper:

1. Fig-2 gives the availability of the system under six different cases. One can clearly observe that availability of the system decreases as the value of time t increases.

2. Fig-3 gives reliability of the system at different values of time. We discussed reliability in two cases. The graph showing a steep fall in reliability from top to lowermost in a very short period in both the cases based on failure rate of units.

3. From Fig-2 and Fig-3, one can observe that availability is better than the reliability, which highlights the requirement of systematic repair for any complex systems for healthier performance.

4. Fif-4 yields the MTTF of the system with respect to variation in failure rate, respectively, when other parameters have been kept constant. MTTF of the system is decreasing concerning different failure rates.

5. Fig-5 and Fig-6 show that expected profit increases as service cost K_2 decreases, while the revenue cost per unit time is fixed at K_1=1 under copula repair and general repair respectively. The calculated expected profit is maximum for K_2= 0.1 and minimum for K_2=0.6. We observe that as service cost decreases, profit increase with variation of time. In general, for low service costs, the expected profit is high in comparison to high service costs. Furthermore, the copula repair is more profitable as compared to general repair.

The model developed in this paper was found to be highly advantageous in proper maintenance analysis, decision, and evaluation of performances. Another possible future work is to evaluate maximum reliability and availability of the investigated system.

CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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