Nanotechnology Perceptions
ISSN 1660-6795
www.nano-ntp.com

Decomposition Theorems and KMFS, FKMS Algorithms on Multi Fuzzy Sets & Fuzzy Multi Sets

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Abstract: The decomposition theorem using alpha cuts allows the same extension that can be performed in a set-valued manner. The decomposition theorem helps to travel in both theory and practical applications with regard to stage-by-stage decision making under uncertainty through alpha cuts. Through union of special fuzzy sets, we get back the original fuzzy set. Through these, decomposition theorems are developed in fuzzy sets. In this article, we developed decomposition theorems on Multi fuzzy sets (MFS) and Fuzzy Multi sets (FMS) in the same way. Numerical examples with suitable scenarios for decomposition theorem of MFS and FMS are provided. We proposed new algorithms named as KMFS algorithm in MFS and FKMS algorithm in FMS after defining Hamacher product of Tr-MFN and normalized hamming distance between two triangular multi fuzzy numbers A and B for approaching MCDM applications. Further interpretation for the above proposed algorithms is conferred using suitable real-life applications.
Keywords: MFS, FMS, Decomposition Theorems, Hamacher product of Tr-MFN, Normalized Hamming distance, KMFS algorithm, FKMS algorithm.

1.INTRODUCTION

The fuzzy set theory, developed by Lotfi A. Zadeh [2], extends classical set theory by accommodating uncertainty through degrees of membership ranging from 0 to 1, in contrast to binary membership. This framework finds applications in various domains, including artificial intelligence, control systems, and decision making, where uncertainty is a critical factor. Multi fuzzy sets [4], introduced by Yager, handle multiple criteria simultaneously, allowing for a more nuanced representation of complex uncertainties across different dimensions. These sets are particularly useful in decision support systems, pattern recognition, and modeling systems with diverse sources of imprecision. Fuzzy multisets [5], on the other hand, combine fuzzy sets and multisets, enabling elements to have varying degrees of membership and multiple occurrences with different memberships. The emphasis is on handling both imprecision and repetition in data, making it useful for scenarios where elements may exist in multiple instances with different strengths of belonging. A multi fuzzy set and fuzzy multiset are not the same. While they both involve the concept of fuzzy sets, they address different aspects of uncertainty. They are well-suited for representing imprecise and repetitive information in scenarios involving uncertain data. Additionally, the decomposition theorem in fuzzy set theory[1] provides a method for representing a fuzzy relation as a combination of simpler fuzzy relations, aiding in the modular representation and understanding of relationships between elements. This theorem is particularly valuable in applications such as fuzzy logic, decision making, and pattern recognition, where it helps in dissecting complex relationships into more manageable components. We gathered some details for literature survey from some papers. Sabu Sabastian[4], proposed the concept of multi fuzzy mapping and Atanssov intuitionistic fuzzy sets generating maps in multi fuzzy set theory. In [8], authors introduced the concept of multi fuzzy set over type-2 fuzzy set and they defined some operations and algebraic properties about T2MFS. Then they defined two measures over the sets. In [9], authors developed the notion of strong (α, β) cut on IFS and their produced decomposition theorem of an IFS. In [10], authors developed decomposition theorems on vague sets and they are established the relationship among vague sets. They are divided into two fuzzy set A^T, AR. Because AT and AR can be converted into classic sets by using the decomposition theorems on fuzzy sets. In this paper, we defined decomposition theorems and multi criteria decision making on multi fuzzy set and fuzzy multi set. Then we proposed Hamacher's product and hamming distance in algorithms on multi fuzzy sets and fuzzy multi set and we developed application of best smartphone selection using MCDM methods on multi fuzzy set and fuzzy multiset.

2.PRELIMINARIES

Definition 2.1[2]: Assuming that V is a number of objects declared generally by v, next a fuzzy set B in V is a set of ordered pairs $B = \{(v, \mu_B(x)) | v \in V\}$.$\mu_B(v)$ismustered the membership function which maps V to the membership space M. The range of the membership function is a subset of the non-negative real numbers whose supremum is finite.

Definition 2.2[1]: Assume B beeach fuzzy set in V. Next for each$\alpha \in [0,1]$, α cut of B, declared by $\alpha_{B,}$ is correct as $\alpha_{B,} = \{v: v \in V \text{ such that } \mu_B(v) \geq \alpha\}$ and the strong α cut of B, denoted by $\alpha +_{B,}$ is correct as $\alpha +_{B,} = \{v: v \in V \text{ such that } \mu_B(v) > \alpha\}$.

Definition 2.3[1]:(Special fuzzy set)Assume B be each fuzzy set in V, next claimα^B by $\alpha^B(v) = \alpha.^{\alpha} B(v)$. Also $\alpha +^B (v) = \alpha.^{\alpha+} B(v)$; where α_B and $\alpha +_B$ are fuzzy sets.

Definition 2.4[1]: Assume B beeach fuzzy set in V. Next the level set of B declared by $\Lambda(B)$, is described as

$\Lambda(B) = \{\alpha | B(v) = \alpha: v \in V\}$.

Theorem 2.1[1]: (First Decomposition Theorem of fuzzy sets)

For each fuzzy set B, $B = \cup_{A \in [0,1]} {}_A B$.

Theorem 2.2[1]: (Second Decomposition Theorem of fuzzy sets)

For each fuzzy set B, $B = \cup_{A \in [0,1]} {}_{A^+} B$.

Theorem 2.3[1]: (Third Decomposition Theorem of fuzzy sets)

For each fuzzy set B, $B = \cup_{\alpha \in \Lambda(B)} {}_{\alpha} B$. Where $\Lambda(\mathbf{B})$is the level set ofB.

Definition 2.5[4]:A Multiset(MS)Dhauled from the set V is expressed by a count function $C_D: V \to N$, where N expressed the set of non-negative integers. $C_D(v)$is the count of presence of the element x in the multiset D. The multiset(MS)Dhauled from $V = \{v_1, v_2, \dots v_n\}$ will be expressed by $D = \{\frac{v_1}{d_1}, \frac{v_2}{d_2}, \dots, \frac{v_n}{d_n}\}$ where d_i is the count of presence of the element v_i, (i=1,2,..n) in the multisetD.

Definition 2.6[4]: Assume $V \neq \emptyset$, N the set of all natural numbers and $\{P_i: i \in N\}$a family of complete lattices. A multi fuzzy set(MFS)B in V is a set of ordered sequences:$B = \{\{< v, (\mu_1(v), \mu_2(v), \dots, \mu_n(v), ..>\}; v \in V\}$Where$\mu_i \in P_i^V$, for i∈N.

Example 2.1[4]: Provide us examine fuzzy set(FS) A as pursue R= {0.6/ p, 0.7/ p, 0.3/ p, 0.9/q, 0.5/q, 0.4/q, 1.0/q, 0.4/s} of the universal setV={p,q,s}. From this fuzzy set(FS), we see that the element pensue three times with membership values 0.6, 0.7, 0.3 respectively; the element qensue four times with membership values 0.9, 0.5, 0.4 and 1.0 respectively and the element sensue once with a membership value 0.4. thus, the set can be altered in the form as R= {(0.6,0.7,0.3)/p, (0.9,0.5,0.4,1.0)/q, 0.4/s} which is essentially is a multi- fuzzy set.

Definition 2.7[5]: Let s be a positive integer and U be a universal set. A fuzzy multi set B over U is a set of ordered sequences$B = \{v/(\mu_B^1, \mu_B^2, \dots, \mu_B^i)): v \in U\}$ where$\mu_B^i \in P(U), i = 1,2, .. s$. The function $\mu_B = (\mu_B^1, \mu_B^2, \dots, \mu_B^i)$ called a multi-membership function of fuzzy multi set B, and s is mustered the dimension of fuzzy multi set B.

Example 2.2[5]: Assume a fuzzy multisetsB={(p,0.3), (p,0.4), (q,0.9), (q,0.6), (q,0.6)} of V= {p,q,r,s}, which means that p with the membership 0.3, p with 0.4, q with the membership 0.9, and two y's with 0.6 are contained in B. We may write B={{0.3,0.4}/p,{0.9,0.6,0.6}/q} in which the multisets of membership {0.3,0.4} and {0.9,0.6,0.6} corresponds to p and q, respectively.

Definition 2.8[5]: (Operations on fuzzy multi sets)

Let $M, N \in FM(V)$, where $FM(V)$ refers to the set of all fuzzy multisets(FS) over$V \neq \emptyset$. The succeeding are primitive relations and operations for fuzzy multisets(FMS):

(1) Inclusion: $M \subseteq N \Leftrightarrow \mu_M^j(v) \leq \mu_N^j(v), j = 1,..P(v) \forall v \in V$.

(2) Equality: $M = N \Leftrightarrow \mu_M^j(v) = \mu_N^j(v), j = 1,..P(v) \forall v \in V$.

(3) Union: $\mu_{M\cup N}^j(v) \Leftrightarrow \mu_M^j(v) \vee \mu_N^j(v), j = 1,..P(v) \forall v \in V$, where $\vee$ is the maximum operation.

(4) Intersection: $\mu_{M\cap N}^j(v) \Leftrightarrow \mu_M^j(v) \wedge \mu_N^j(v), j = 1,..P(v) \forall v \in V$, where $\wedge$ is the minimum operation.

Definition 2.9[12]:T-norms are associative, monotonic and commutative two valued functions t that map from $[0,1] \times [0,1]$into $[0,1]$. These properties are formulated with the following conditions:

1. $t(0,0) = 0$ and $t(\mu_{X_1}(x), 1) = t(1, \mu_{X_1}(x)) = \mu_{X_1}(x)$,
2. if $\mu_{X_1}(x) \leq \mu_{X_3}(x)$ and $\mu_{X_2}(x) \leq \mu_{X_4}(x)$, then $t(\mu_{X_1}(x), \mu_{X_2}(x)) \leq t(\mu_{X_3}(x), \mu_{X_4}(x))$,
3. $t(\mu_{X_1}(x), \mu_{X_2}(x)) = t(\mu_{X_2}(x), \mu_{X_1}(x))$,
4. $t(\mu_{X_1}(x), t(\mu_{X_2}(x), \mu_{X_3}(x))) = t(t(\mu_{X_1}(x), \mu_{X_2}(x)), \mu_{X_3}(x))$

the T-norm which are rational functions are the Hamacher T-norm is defined by

$$t_{2.5}(\mu_{X_1}(x), \mu_{X_2}(x)) = \frac{\mu_{X_1}(x).\mu_{X_2}(x)}{\mu_{X_1}(x)+\mu_{X_2}(x)-\mu_{X_1}(x).\mu_{X_2}(x)}$$

Definition 2.10[12]: S-norms are associative, monotonic and commutative two valued functions s that map from $[0,1] \times [0,1]$into $[0,1]$. These properties are formulated with the following conditions:

1. $s(1,1) = 0$ and $t(\mu_{X_1}(x), 0) = t(0, \mu_{X_1}(x)) = \mu_{X_1}(x)$,
2. if $\mu_{X_1}(x) \leq \mu_{X_3}(x)$ and $\mu_{X_2}(x) \leq \mu_{X_4}(x)$, then $s(\mu_{X_1}(x), \mu_{X_2}(x)) \leq s(\mu_{X_3}(x), \mu_{X_4}(x))$,
3. $s(\mu_{X_1}(x), \mu_{X_2}(x)) = s(\mu_{X_2}(x), \mu_{X_1}(x))$,
4. $s(\mu_{X_1}(x), s(\mu_{X_2}(x), \mu_{X_3}(x))) = s(s(\mu_{X_1}(x), \mu_{X_2}(x)), \mu_{X_3}(x))$

the S-norm which are rational functions are the Hamacher S-norm is defined by

$$s_{2.5}(\mu_{X_1}(x), \mu_{X_2}(x)) = \frac{\mu_{X_1}(x)+\mu_{X_2}(x)-2.\mu_{X_1}(x).\mu_{X_2}(x)}{1-\mu_{X_1}(x).\mu_{X_2}(x)}$$

Definition 2.11[12]: Let $w_{\tilde{I}} \in [0,1]$ and a, b, c$\in$ R such that $a \leq b \leq c$. Then, a generalized triangular fuzzy number $\tilde{a} = \langle (a,b,c); w_{\tilde{a}} \rangle$ is a special fuzzy set on the real number set R, whose membership function is defined as;

$$\mu_{\tilde{a}}(x) = \begin{cases} \dfrac{(x-a)\mu_{\alpha i}}{b-a} & (a \leq x < b) \\ \mu_{\alpha_i} & x = b \\ \dfrac{(c-x)w_{\tilde{a}}}{(c-b)} & (b < x \leq c) \\ 0 & \text{otherwise} \end{cases}$$

Definition 2.12[12]: Let $\tilde{a} = \langle (a_1, b_1, c_1); w_{\tilde{a}} \rangle$, $\tilde{a} = \langle (a_2, b_2, c_2); w_{\tilde{b}} \rangle$ be two generalized triangular fuzzy numbers and $\gamma \neq 0$ be any real number. Then,

1. $\tilde{a} + \tilde{b} = \langle (a_1 + a_2, b_1 + b_2, c_1 + c_2); w_1 \wedge w_2 \rangle$
2. $\tilde{a} - \tilde{b} = \langle (a_1 - a_2, b_1 - b_2, c_1 - c_2,); w_1 \wedge w_2 \rangle$
3. $\tilde{a}\tilde{b} = \langle (a_1a_2, b_1b_2, c_1c_2); w_1 \wedge w_2 \rangle$
4. $\tilde{a}/\tilde{b} = \langle (a_1/a_2, b_1/b_2, c_1/c_2); w_1 \wedge w_2 \rangle$
5. $\gamma\tilde{a} = \begin{cases} \langle (\gamma a_1, \gamma b_1, \gamma c_1); w_1 \rangle & (\gamma > 0) \\ \langle (\gamma c_1, \gamma b_1, \gamma a_1); w_1 \rangle & (\gamma < 0) \end{cases}$

Definition 2.13[12]: Let $A = \langle (a_1, b_1, c_1); \eta_A{}^1, \eta_A{}^2, \ldots\ldots, \eta_A{}^p \rangle$,

$B = \langle (a_2, b_2, c_2); \eta_B{}^1, \eta_B{}^2, \ldots\ldots, \eta_B{}^p \rangle \in \Lambda$ and $\gamma \neq 0$ be any real number. Then

1. A+B= $\langle (a_1 + a_2, b_1 + b_2, c_1 + c_2); s(\eta_A{}^1, \eta_B{}^1), s(\eta_A{}^2, \eta_B{}^2), \ldots\ldots, s(\eta_A{}^p, \eta_B{}^p) \rangle$

2. A-B= $\langle (a_1 - a_2, b_1 - b_2, c_1 - c_2); s(\eta_A{}^1, \eta_B{}^1), s(\eta_A{}^2, \eta_B{}^2), \dots\dots, s(\eta_A{}^p, \eta_B{}^p)\rangle$
3. A.B= $\begin{cases} \langle (a_1a_2, b_1b_2, c_1c_2); t(\eta_A{}^1, \eta_B{}^1), t(\eta_A{}^2, \eta_B{}^2), \dots\dots, t(\eta_A{}^p, \eta_B{}^p)\rangle; (c_1 > 0, c_2 > 0) \\ \langle (a_1d_2, b_1c_2, c_1b_2); t(\eta_A{}^1, \eta_B{}^1), t(\eta_A{}^2, \eta_B{}^2), \dots\dots, t(\eta_A{}^p, \eta_B{}^p)\rangle; (c_1 < 0, c_2 > 0) \\ \langle (c_1c_2, b_1b_2, a_1a_2); t(\eta_A{}^1, \eta_B{}^1), t(\eta_A{}^2, \eta_B{}^2), \dots\dots, t(\eta_A{}^p, \eta_B{}^p)\rangle; (c_1 < 0, c_2 < 0) \end{cases}$
4. A.B= $\begin{cases} \langle (a_1/c_2, b_1/c_1, b_2/a_2); t(\eta_A{}^1, \eta_B{}^1), t(\eta_A{}^2, \eta_B{}^2), \dots\dots, t(\eta_A{}^p, \eta_B{}^p)\rangle; (c_1 > 0, c_2 > 0) \\ \langle (c_1/c_2, b_1/b_2, a_1/a_2); t(\eta_A{}^1, \eta_B{}^1), t(\eta_A{}^2, \eta_B{}^2), \dots\dots, t(\eta_A{}^p, \eta_B{}^p)\rangle; (c_1 < 0, c_2 > 0) \\ \langle (c_1/a_2, c_1/b_1, b_2/a_2); t(\eta_A{}^1, \eta_B{}^1), t(\eta_A{}^2, \eta_B{}^2), \dots\dots, t(\eta_A{}^p, \eta_B{}^p)\rangle; (c_1 < 0, c_2 < 0) \end{cases}$
5. $\gamma A = \langle (\gamma a_1, \gamma b_1, \gamma c_1); 1 - (1 - \eta_A{}^1)^\gamma, 1 - (1 - \eta_A{}^2)^\gamma, \dots\dots.., 1 - (1 - \eta_A{}^p)^\gamma\rangle\ (\gamma \le 0)$
6. $A^\gamma = \langle (a_1{}^\gamma, b_1{}^\gamma, c_1{}^\gamma); (\eta_A{}^1)^\gamma, (\eta_A{}^2)^\gamma, (\eta_A{}^3)^\gamma, \dots\dots, (\eta_A{}^p)^\gamma\rangle (\gamma \le 0)$

Definition 2.14[12]: Let A= $\langle (a_1, b_1, c_1); \eta_A{}^1, \eta_A{}^2, \dots\dots, \eta_A{}^p\rangle \in \Lambda$. Then the normalized TFM-number of A is given by

$$\overline{A} = \langle\left(\frac{a_1}{a_1 + b_1 + c_1}, \frac{b_1}{a_1 + b_1 + c_1}, \frac{c_1}{a_1 + b_1 + c_1}\right); \eta_A{}^1, \eta_A{}^2, \dots\dots, \eta_A{}^p\rangle$$

Definition 2.15(12):
Let $\overline{A} = \langle (a_1, b_1, c_1); \eta_{A_1}, \eta_{A_2}, \dots\dots, \eta_{A_p}\rangle$, $\overline{B} = \langle (a_2, b_2, c_2); \eta_{A_1}, \eta_{A_2}, \dots\dots, \eta_{A_p}\rangle \in \Lambda$. Then to compare $\overline{A}$and $\overline{B}$ the TFM-number positive ideal solution and negative ideal solution are defined as

$$f_A{}^+ = \langle (a^+, b^+. c^+); (\eta_{A_1})^+, (\eta_{A_2})^+, \dots\dots, (\eta_{A_p})^+\rangle$$

$= \langle (1,1,1); 1,1,1, \dots.,1\rangle,$

$$f_A{}^- = \langle (a^-, b^-. c^-); (\eta_{A_1})^-, (\eta_{A_2})^-, \dots\dots, (\eta_{A_p})^-\rangle$$

$= \langle (0,0,0); 0,0,0,0, \dots,0\rangle$. respectively.

3. DECOMPOSITION THEOREMS AND KMFS ALGORITHM IN MULTI FUZZY SET

3.1First Decomposition Theorem on MFS:

Assume V be a non-empty set and B be a Multi fuzzy subset in V. If $B = (\mu_{B_1}, \mu_{B_2}, \dots, \mu_{B_k}) \in V$, Then

$B = (\cup_{\alpha\epsilon[0,1]}\ {}_\alpha(\mu_{B_1}, \mu_{B_2}, \dots, \mu_{B_k})$, where U is the standard fuzzy union.

Proof:

Assume v be an any element in V and let $(\mu_{B_1}, \mu_{B_2}, \dots, \mu_{B_k})(v) = a$

Then $(\cup_{\alpha\epsilon[0,1]}\ {}_\alpha(\mu_{B_1}, \mu_{B_2}, \dots, \mu_{B_k}) = \sup_{\alpha\epsilon[0,1]} \alpha^{(\mu_{B_1}, \mu_{B_2}, \dots, \mu_{B_k})(v)}$

$$= \left(\max\left[\sup_{\alpha\epsilon[0,b]} \alpha^{(\mu_{B_1}, \mu_{B_2}, \dots, \mu_{B_k})(v)}, \sup_{\alpha\epsilon(b,1]} \alpha^{(\mu_{B_1}, \mu_{B_2}, \dots, \mu_{B_k})(v)}\right]\right)$$

For each $\alpha\epsilon(b, 1]$ we have $\alpha > b$, So,

$(\mu_{B_1}, \mu_{B_2}, \dots, \mu_{B_k})(v) = b < \alpha$,

$(\mu_{B_1}, \mu_{B_2}, \dots, \mu_{B_k})(v) < \alpha$and, therefore,

$\alpha^{(\mu_{B_1}, \mu_{B_2}, \dots, \mu_{B_k})}(v) = 0.$

On the other hand, for each$\alpha\epsilon[0,b]$,wehave $B(v)=b \ge \alpha$, therefore,

$$\alpha^{(\mu_{B_1}, \mu_{B_2}, \dots, \mu_{B_k})}(v) = b.$$

Hence,

$\left(\cup_{\alpha\epsilon[0,1]} \alpha^{(\mu_{B_1}, \mu_{B_2}, \dots, \mu_{B_k})}\right)(v) = \sup_{\alpha\epsilon[0,b]} \alpha = b = B(v).$

3.2 Second Decomposition Theorem on MFS:
Assume V be a non-empty set and B be a Multi fuzzy subset in V. If $B = (\mu_{B_1}, \mu_{B}, \ldots, \mu_{B_k}) \in V$,then

$B = (\cup_{\alpha\epsilon[0,1]}\,{}_{\alpha^+}(\mu_{B_1}, \mu_{B_2}, \ldots, \mu_{B_k})$where U is the standard fuzzy union.

Proof:
Assume x be an any element in V and let $\left(\mu_{B_1}, \mu_{B_2}, \ldots, \mu_{B_k}\right)(v) = b$
Then

$$(\cup_{\alpha\epsilon[0,1]}\,{}_{\alpha^+}(\mu_{B_1}, \mu_{B_2}, \ldots, \mu_{B_k}) = \sup_{\alpha\epsilon[0,1]} \alpha^{+\left(\mu_{B_1},\mu_{B_2},\ldots,\mu_{B_k}\right)(v)}$$

$$= \left(\max\left[\sup_{\alpha\epsilon[0,b]} \alpha^{+\left(\mu_{B_1},\mu_{B_2},\ldots,\mu_{B_k}\right)(v)}, \sup_{\alpha\epsilon(b,1]} \alpha^{+\left(\mu_{B_1},\mu_{B_2},\ldots,\mu_{B_k}\right)(v)}\right]\right)$$

For each $\alpha\epsilon(b, 1]$ we have $\alpha > b$, So,
$\left(\mu_{B_1}, \mu_{B_2}, \ldots, \mu_{B_k}\right)(v) = b < \alpha$,

$$\left(\mu_{B_1}, \mu_{B_2}, \ldots, \mu_{B_k}\right)(v) < \alpha$$

and, therefore,
${}_{\alpha^+}(\mu_{B_1}, \mu_{B_2}, \ldots, \mu_{B_k})\,(v) = 0$.
On the other hand, for each $\alpha\epsilon\,[0, b]$, we have $A(v) = a \geq \alpha$, therefore,
${}_{\alpha^+}(\mu_{B_1}, \mu_{B_2}, \ldots, \mu_{B_k})(v) = b$.
Hence,

$$\left(\Big(\bigcup_{\alpha\epsilon[0,1]} {}_{\alpha^+}(\mu_{B_1}, \mu_{B_2}, \ldots, \mu_{B_k})\right)(v) = \sup_{\alpha\epsilon[0,b]} \alpha = b = B(v)$$

3.3 Third Decomposition Theorem on MFS:
Assume V be a non-empty set and B be a Multi fuzzy subset in V. If $B = (\mu_{B_1}, \mu_{B}, \ldots, \mu_{B_k}) \in V$, then

$B = (\cup_{\alpha\epsilon L_{\mu_B}}\, \alpha(\mu_{B_1}, \mu_{B_2}, \ldots, \mu_{B_k})$, where U is the standard fuzzy union.

Proof:
The verificationregarding this theorem is akin to the second decomposition theorem based on an MFS.

3.4. Numerical example for Decomposition theoremon MFS:
In this scenario, let's Assume V be a non-empty set and B be a Multi fuzzy subset in V. If $B = (\mu_{B_1}, \mu_{B_2}, \ldots, \mu_{B_k}) \in V$. hereB representing four babies are b_1, b_2, b_3, and b_4, each with four membership values are $(\mu_{B_1}, \mu_{B_2}, \mu_{B_3}, \mu_{B_4})$. These values correspond to different factors influencing the growth of each baby.For a baby growth, sleep pattern is first important factor, next nutrition is second important factor, genetic factor is third tool for baby growth, final tool is health check-up (sleep pattern > nutrition > genetic factor> health check-up). The membership value μ_{B_1} represents the importance of health checkups for a baby's growth, indicating how regularly they are monitored by healthcare professionals. The membership value μ_{B_2}signifies the role of genetic factors in the baby's growth and development, considering traits inherited from their parents. Membership value μ_{B_3} reflects the significance of nutrition in the baby's diet for growth. Finally, membership value μ_{B_4}indicates the impact of sleep patterns on the baby's growth, highlighting the importance of sufficient and regular sleep for their development. By assigning these membership values to the babies in set B, here we are listed in increasing order about each baby growth.

$$B=[(b_1, (0.25,0.32,0.56,0.61)), (b_2, (0.31,0.43,0.65,0.77)), (b_3, (0.48,0.52,0.76,0.82)), (b_4, (0.58,0.62,0.81,0.98))]$$

Provide us denote B for convenience as

$$B = \left(\frac{(0.25,0.32,0.56,0.61)}{b_1} + \frac{(0.31,0.43,0.65,0.77)}{b_2} + \frac{(0.48,0.52,0.76,0.82)}{b_3} + \frac{(0.58,0.62,0.81,0.98)}{b_4} \right)$$

$$(0.25,0.32,0.56,0.61)_{\mu_B} = \frac{1}{b_1} + \frac{1}{b_2} + \frac{1}{b_3} + \frac{1}{b_4}$$

$$(0.31,0.43,0.65,0.77)_{\mu_B} = \frac{0}{b_1} + \frac{1}{b_2} + \frac{1}{b_3} + \frac{1}{b_4}$$

$$(0.48,0.52,0.76,0.82)_{\mu_B} = \frac{0}{b_1} + \frac{0}{b_2} + \frac{1}{b_3} + \frac{1}{b_4}$$

$$(0.58,0.62,0.81,0.98)_{\mu_B} = \frac{0}{b_1} + \frac{0}{b_2} + \frac{0}{b_3} + \frac{1}{b_4}$$

$$\alpha^{B(v)} = \alpha^{\alpha}B(v)$$

$$(0.25,0.32,0.56,0.61)\alpha_{B(v)} = \left(\frac{(0.25,0.32,0.56,0.61)}{b_1} + \frac{(0.25,0.32,0.56,0.61)}{b_2} + \frac{(0.25,0.32,0.56,0.61)}{b_3} + \frac{(0.25,0.32,0.56,0.61)}{b_4}\right)$$

$$(0.31,0.43,0.65,0.77)\alpha_{B(v)} = \left(\frac{0}{b_1} + \frac{(0.31,0.43,0.65,0.77)}{b_2} + \frac{(0.31,0.43,0.65,0.77)}{b_3} + \frac{(0.31,0.43,0.65,0.77)}{b_4}\right)$$

$$(0.48,0.52,0.76,0.82)\alpha_{B(v)} = \left(\frac{0}{b_1} + \frac{0}{b_2} + \frac{(0.48,0.52,0.76,0.82)}{b_3} + \frac{(0.48,0.52,0.76,0.82)}{b_4}\right)$$

$$(0.58,0.62,0.81,0.98)\alpha_{B(v)} = \left(\frac{0}{b_1} + \frac{0}{b_2} + \frac{0}{b_3} + \frac{(0.58,0.62,0.81,0.98)}{b_4}\right)$$

$$(\mu_{B_1}, \mu_{B_2}, \mu_{B_3}, \mu_{B_4}) = \cup_{\alpha\in[0,1]}\left((0.25,0.32,0.56,0.61), (0.31,0.43,0.65,0.77), (0.48,0.52,0.76,0.82), (0.58,0.62,0.81,0.98)\right)$$

$(\mu_{B_1}, \mu_{B_2}, \mu_{B_3}, \mu_{B_4}) =$ B.Hence proved.

We require the following new definitions to propose KMFS algorithm in MFS.

Definition 3.5: Consider $A=\langle[a_1a_2a_3]; \left(\mu_{A_1}, \mu_{A_2}, \dots, \mu_{A_p}\right)\rangle$, $B=\langle[b_1b_2b_3]; \left(\mu_{B_1}, \mu_{B_2}, \dots, \mu_{B_p}\right)\rangle$ and $C = \langle[c_1c_2\ c_3]; (\mu_{C_1}, \mu_{C_2}, \dots, \mu_{C_p})\rangle$ in V then Hamacher product of Tr-MFN is,

$$t = \left([a_1a_2a_3, b_1b_2b_3, c_1c_2c_3]; \left\{\frac{\mu_{a_1}\mu_{a_2}\mu_{a_3}}{(\mu_{a_1}+\mu_{a_2}+\mu_{a_3})-(\mu_{a_1}\mu_{a_2}\mu_{a_3})}, \frac{\mu_{b_1}\mu_{b_2}\mu_{b_3}}{(\mu_{b_1}+\mu_{b_2}+\mu_{b_3})-(\mu_{b_1}\mu_{b_2}\mu_{b_3})}, \frac{\mu_{c_1}\mu_{c_2}\mu_{c_3}}{(\mu_{c_1}+\mu_{c_2}+\mu_{c_3})-(\mu_{c_1}\mu_{c_2}\mu_{c_3})}\right\}\right)$$

Definition 3.6: Let $A=\langle[a_1a_2a_3]; \left(\mu_{A_1}, \mu_{A_2}, \dots, \mu_{A_p}\right)\rangle$ and $B=\langle[b_1b_2b_3]; \left(\mu_{B_1}, \mu_{B_2}, \dots, \mu_{B_p}\right)\rangle$ be two triangular multi fuzzy numbers, then the normalized hamming distance between A and B is defined as

$$D(f_i,f_i+) = \frac{1}{8P}\Big[\left|\left(1+\mu_{a_1}\right)a_1 - \left(1+\mu_{b_1}\right)b_1\right| + \left|\left(1+\mu_{a_1}\right)a_2 - \left(1+\mu_{b_1}\right)b_2\right| + \left|\left(1+\mu_{a_1}\right)a_3 - \left(1+\mu_{b_1}\right)b_3\right| + \left|\left(1+\mu_{a_2}\right)a_1 - \left(1+\mu_{b_2}\right)b_1\right| + \left|\left(1+\mu_{a_2}\right)a_2 - \left(1+\mu_{b_2}\right)b_2\right| + \left|\left(1+\mu_{a_2}\right)a_3 - \left(1+\mu_{b_2}\right)b_3\right| + \left|\left(1+\mu_{a_3}\right)a_1 - \left(1+\mu_{b_3}\right)b_1\right| + \left|\left(1+\mu_{a_3}\right)a_2 - \left(1+\mu_{b_3}\right)b_2\right| + \left|\left(1+\mu_{a_3}\right)a_3 - \left(1+\mu_{b_3}\right)b_3\right|\Big].$$

3.7. KMFS Algorithm in MFS:

Step 1: Construct the Tr-MF numbers multi-criteria decision matrix $A=\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m1} & \dots & a_{mn} \end{bmatrix}$ for decision.

Step 2: Compute overall values f_i= Tr-MFG(a_{i1}, a_{i2}, a_{i3}); note that iff$_i$ for all $i \in I_m$ is not normalized Tr-MF numbers, then we define the normalized Tr-MF numbers using Hamacher's product.

Step 3: Calculate the distance between collective overall values$f_i = \langle [a_i, b_i, c_i]; (\mu_{i_1}, \mu_{i_2}, \ldots, \mu_{i_p}) \rangle$ and positive ideal solution f_i^+($f_i^+ = \langle (1,1,1); 1,1, \ldots 1 \rangle$)

Step 4: Rank all the attributes $A_i (i = 1,2,3, \ldots, m)$

Step 5: Select the best in accordance with D(f_i, f_i^+), the better are the alternatives, A_i.

3.8. Applicationto address KMFS algorithm:

Consider collection of five smartphones S = {SP1, SP2, SP3, SP4, SP5} analysis of trending smartphone list. we selected three key features of the smartphone is denoted as linguistic variables are price(ft1), internal storage(ft2), and processor speed(ft3) for each smartphone. These variables serve as the foundation for categorizing and evaluating different aspects of smartphones. For each linguistic variable, we have established three linguistic values to represent varying degrees or levels. For instance, in considering price, we have delineated high price(pr1), average price(pr2), and low-price(pr3) categories. Then considering internal storage, we have delineated low capacity(in1), medium capacity(in2) and high capacity(in3). Then considering processor speed, we have delineated high speed(ps1), medium speed(ps2) and low speed(ps3) are three distinct linguistic values each to capture the range of options available in the smartphone market. To quantify and manage the inherent uncertainty or fuzziness in the data, we have opted for triangular multi-fuzzy numbers as a mathematical framework. These numbers comprise three values $[a_1, a_2, a_3]$ representing the expected outcomes for linguistic variable of price for SP1 and consider for each linguistic variable. In applying this method to analyze smartphone sales outcomes, we compare the expected outcomes represented by the triangular multi-fuzzy numbers with the actual sales data. The membership values for a_1, a_2, a_3serve as indicators of the degree to which the expected outcomes align with the observed sales patterns. By assessing the correspondence between expected and actual outcomes across different linguistic values for price, internal storage, and processor speed, we gain insights into consumer preferences and market dynamics within the smartphone industry. This approach enables a comprehensive examination of the relationship between linguistic variables and sales outcomes, facilitating informed decision-making and strategic planning in the competitive smartphone market landscape.

Now, manipulating the KMFS algorithm to the above stated scenario.

Step 1:Consider the input data of the Tr-MF numbers multi-criteria decision matrix

$$A=\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$ for decision.

	sp1	sp2	sp3	sp4	sp5
ft1	[0.6,0.6,0.8]; (0.84,0.05,0.05)	[0.1,0.5,0.8]; (0.22,0.76,0.72)	[0.5,0.6,0.3]; (0.88,0.89,0.75)	[0.5,0.4,0.6]; (0.89,0.80,0.12)	[0.2,0.8,0.9]; (0.12,0.85,0.79)
ft2	[0.8,0.5,0.6]; (0.34,0.73,0.77)	[0.2,0.5,0.5]; (0.15,0.75,0.63)	[0.1, 0.5,0.8]; (0.20,0.41,0.68)	[0.4,0.5,0.6]; (0.80,0.60,0.22)	[0.5,0.6,0.3]; (0.23,0.72,0.81)
ft3	[0.5,0.6,0.3]; (0.28,0.52,0.90)	[0.8,0.5,0.1]; (0.62,0.41,0.08)	[0.5,0.2,0.4]; (0.12,0.54,0.68)	[0.2,0.8,0.3]; (0.40,0.62,0.53)	[0.3,0.6,0.8]; (0.35,0.25,0.11)

Step 2: Applying the Tr-MFG operator to derive the collective overall preference triangular multi fuzzy set f_i:

$SP_1 = [0.288,0.192,0.090]; (0.0022,0.1313,0.0835)$

$SP_2 = [0.04,0.035,0.12]; (0.0762,0.0200,0.0186)$

$SP_3 = [0.09,0.03,0.048]; (0.3039,0.1987,0.0340)$

$SP_4 = [0.04,0.03,0.048]; (0.0495,0.0867,0.0926)$

$SP_5 = [0.048,0.0305,0.0137]; (0.0480,0.0305,0.0137)$

Step 3:Calculate the distances between collective overall$f_i$and Tr-MF positive ideal solution f+:

$D(SP_1, f^+) = 0.5665$
$D(SP_2, f^+) = 0.7112$
$D(SP_3, f^+) = 0.7264$
$D(SP_4, f^+) = 0.7322$
$D(SP_5, f^+) = 0.6923$
Step 4:Rank all the alternatives A_i in accordance with descending order

$$SP_4 > SP_3 > SP_2 > SP_5 > SP_1$$

Step 5:The most desirable alternative is SP_4.

4. DECOMPOSITION THEOREMS AND FKMS ALGORITHM IN FUZZY-MULTI SET

4.1.First Decomposition Theorem on FMS:

Assume V be a non-empty set and B be a fuzzy multi subset in V. If B= $(\mu_B^1, \mu_B^2, \dots, \mu_B^k) \in V$, then $B = (\cup_{\alpha\epsilon[0,1]} \alpha^{(\mu_B^1, \mu_B^2, \dots, \mu_B^k)})$,where U is the standard fuzzy union.

Proof:

Assume v be anany element in V and assume $(\mu_B^1, \mu_B^2, \dots, \mu_B^k)(v) = a$

Then $(\cup_{\alpha\epsilon[0,1]} \alpha^{(\mu_B^1, \mu_B^2, \dots, \mu_B^k)(v)}) = \sup_{\alpha\epsilon[0,1]} \alpha^{(\mu_B^1, \mu_B^2, \dots, \mu_B^k)(v)}$

$$= \left(\max\left[\sup_{\alpha\epsilon[0,b]} \alpha^{(\mu_B^1, \mu_B^2, \dots, \mu_B^k)(v)}, \sup_{\alpha\epsilon(b,1]} \alpha^{(\mu_B^1, \mu_B^2, \dots, \mu_B^k)(v)}\right]\right)$$

For each $\alpha\epsilon(b, 1]$ we have $\alpha > b$, So,
$(\mu_B^1, \mu_B^2, \dots, \mu_B^k)(v) = b < \alpha$, $(\mu_B^1, \mu_B^2, \dots, \mu_B^k)(v) < \alpha$
and therefore, $\alpha^{(\mu_B^1, \mu_B^2, \dots, \mu_B^k)}(v) = 0$.
On the other hand, for each $\alpha\epsilon$ [0, b], we have B(v) = b ≥ α,
therefore, $\alpha^{(\mu_B^1, \mu_B^2, \dots, \mu_B^k)}(v) = b$.
Hence, $\left(\cup_{\alpha\epsilon[0,1]} \alpha^{(\mu_B^1, \mu_B^2, \dots, \mu_B^k)}\right)(v) = \sup_{\alpha\epsilon[0,a]} \alpha = b = B(v)$.

4.2.Second Decomposition Theorem on FMS:

Assume V be a non-empty set and B be a fuzzy multi subset in V. If B= $(\mu_B^1, \mu_B^2, \dots, \mu_B^k) \in V$, then $B = (\cup_{\alpha\epsilon[0,1]} \alpha +^{(\mu_B^1, \mu_B^2, \dots, \mu_B^k)})$, where U is the standard fuzzy union.

Proof:

Assume v bean any element in V and assume $(\mu_B^1, \mu_B^2, \dots, \mu_V^k)(v) = b$

Then$(\cup_{\alpha\epsilon[0,1]} \alpha^{+(\mu_B^1, \mu_B^2, \dots, \mu_B^k)(v)}) = \sup_{\alpha\epsilon[0,1]} \alpha^{+(\mu_B^1, \mu_B^2, \dots, \mu_B^k)(v)}$

$$= \left(\max\left[\sup_{\alpha\epsilon[0,a]} \alpha^{+(\mu_B^1, \mu_B^2, \dots, \mu_B^k)(v)}, \sup_{\alpha\epsilon(a,1]} \alpha^{+(\mu_B^1, \mu_B^2, \dots, \mu_B^k)(v)}\right]\right)$$

For each $\alpha\epsilon(b, 1]$ we have $\alpha > b$,So,
$(\mu_B^1, \mu_B^2, \dots, \mu_B^k)(v) = b < \alpha$,

$$(\mu_B^1, \mu_B^2, \dots, \mu_B^k)(v) < \alpha$$

and, therefore,
$\alpha^{+(\mu_B^1, \mu_B^2, \dots, \mu_B^k)}(v) = 0$.
On the other hand, for each $\alpha\epsilon$ [0, b], we have B(v) = b ≥ α, therefore,
$\alpha^{+(\mu_B^1, \mu_B^2, \dots, \mu_B^k)}(v) = b$.
Hence,$\left(\cup_{\alpha\epsilon[0,1]} \alpha^{+(\mu_B^1, \mu_B^2, \dots, \mu_B^k)}\right)(v) = \sup_{\alpha\epsilon[0,b]} \alpha = b = B(v)$.

4.3Third Decomposition Theorem on FMS:

Assume V be a non-empty set and B be a fuzzy multi subset in V. If B= $(\mu_B^1, \mu_B^2, \dots, \mu_B^k) \in V$, then $B = (\cup_{\alpha\epsilon L_{\mu_B}} \alpha^{(\mu_B^1, \mu_B^2, \dots, \mu_B^k)})$, where U is the standard fuzzy union.

Proof:

The verification regarding this theorem is akin to the second decomposition theorem based on an FMS.

4.4. Numerical example for Decomposition theorem on FMS:

In this scenario, let's Assume V be a non-empty set and B be a fuzzy multi subset in V. If B= $(\mu_B^1, \mu_B^2, \dots, \mu_B^k) \in V$. Here B representing four babies are b_1, b_2, b_3, and b_4, each baby with four membership valuesare$(\mu_B^1, \mu_B^2, \mu_B^3, \mu_B^4)$. These values correspond to different months influencing the growth of each baby. The membership value μ_B^1 represents the third month baby's growth. The membership value μ_B^2 represents the sixth month baby's growth. Membership value μ_B^3 represents the nineth month baby's growth. Finally, membership value μ_B^4 represents the one year baby's growth for each baby. By assigning these membership values to the babies in set B, here we are listed in increasing order about each baby growth.

$$B=[(b_1,(0.15,0.26,0.37,0.42)),(b_2,(0.24,0.37,0.48,0.59)),(b_3,(0.32,0.41,0.51,0.65)),(b_4,(0.43,0.56,0.62,0.77))]$$

Provide us denote B for convenience as

$$B=\left(\frac{(0.15,0.26,0.37,0.42)}{b_1}+\frac{(0.24,0.37,0.48,0.59)}{b_2}+\frac{(0.32,0.41,0.51,0.65)}{b_3}+\frac{(0.4,0.5,0.6,0.7)}{b_4}\right)$$

$$(0.15,0.26,0.37,0.42)_{\mu_B}=\frac{1}{b_1}+\frac{1}{b_2}+\frac{1}{b_3}+\frac{1}{b_4}$$

$$(0.24,0.37,0.48,0.59)_{\mu_B}=\frac{0}{b_1}+\frac{1}{b_2}+\frac{1}{b_3}+\frac{1}{b_4}$$

$$(0.32,0.41,0.51,0.65)_{\mu_B}=\frac{0}{b_1}+\frac{0}{b_2}+\frac{1}{b_3}+\frac{1}{b_4}$$

$$(0.43,0.56,0.62,0.77)_{\mu_B}=\frac{0}{b_1}+\frac{0}{b_2}+\frac{0}{b_3}+\frac{1}{b_4}$$

$$\alpha^{B(v)}=\alpha^{\alpha}B(v)$$

$$(0.15,0.26,0.37,0.42)\alpha_{B(v)}=\left(\frac{(0.15,0.26,0.37,0.42)}{b_1}+\frac{(0.15,0.26,0.37,0.42)}{b_2}+\frac{(0.15,0.26,0.37,0.42)}{b_3}+\frac{(0.15,0.26,0.37,0.42)}{b_4}\right)$$

$$(0.24,0.37,0.48,0.59)\alpha_{B(v)}=\left(\frac{0}{b_1}+\frac{(0.24,0.37,0.48,0.59)}{b_2}+\frac{(0.24,0.37,0.48,0.59)}{b_3}+\frac{(0.24,0.37,0.48,0.59)}{b_4}\right)$$

$$(0.32,0.41,0.51,0.65)\alpha_{B(v)}=\left(\frac{0}{b_1}+\frac{0}{b_2}+\frac{(0.32,0.41,0.51,0.65)}{b_3}+\frac{(0.32,0.41,0.51,0.65)}{b_4}\right)$$

$$(0.43,0.56,0.62,0.77)\alpha_{B(v)}=\left(\frac{0}{b_1}+\frac{0}{b_2}+\frac{0}{b_3}+\frac{(0.43,0.56,0.62,0.77)}{b_4}\right)$$

$$(\mu_B^1,\mu_B^2,\mu_B^3,\mu_B^4)=\bigcup_{\alpha\in[0,1]}((0.15,0.26,0.37,0.42),(0.24,0.37,0.48,0.59),(0.32,0.41,0.51,0.65),(0.43,0.56,0.62,0.77))$$

$(\mu_B^1,\mu_B^2,\mu_B^3,\mu_B^4)=$ B.Hence proved.

We require the following new definitions to propose FKMS algorithm in FMS.

Definition 4.5: Let A=$\langle[a^1b^1c^1];(\mu_{A^1},\mu_{A^2},\dots,\mu_{A^P})\rangle$, B=$\langle[a^2b^2c^2];(\mu_{B^1},\mu_{B^2},\dots,\mu_{B^P})\rangle$ and $C=\langle[a^3b^3c^3];(\mu_{C^1},\mu_{C^2},\dots,\mu_{C^P})\rangle$in V then Hamacher product of Tr-FMN is,

t = ([(a¹b¹c¹), (a²b²c²), (a³b³c³)];

$$\left\{\frac{\mu_{a^1}\mu_{b^1}\mu_{c^1}}{(\mu_{a^1}+\mu_{b^1}+\mu_{c^1})-(\mu_{a^1}\mu_{b^1}\mu_{c^1})},\frac{\mu_{a^2}\mu_{b^2}\mu_{c^2}}{(\mu_{a^2}+\mu_{b^2}+\mu_{c^2})-(\mu_{a^2}\mu_{b^2}\mu_{c^2})},\frac{\mu_{a^3}\mu_{b^3}\mu_{c^3}}{(\mu_{a^3}+\mu_{b^3}+\mu_{c^3})-(\mu_{a^3}\mu_{b^3}\mu_{c^3})}\right\})$$

Definition 4.6: Let A=$\langle[a^1b^1c^1];(\mu_{A^1},\mu_{A^2},\dots,\mu_{A^P})\rangle$ and B=$\langle[a^2b^2c^2];(\mu_{B^1},\mu_{B^2},\dots,\mu_{B^P})\rangle$ be two triangular fuzzy multi numbers, then the normalized hamming distance between A and B is defined as

$D(f_i, f_i+)= \frac{1}{8P} [|(1+\mu_{a^1})a_1-(1+\mu_{b^1})a_2|+|(1+\mu_{a^1})b_1-(1+\mu_{b^1})b_2|+|(1+\mu_{a^1})c_1-(1+\mu_{b^1})c_2|+|(1+\mu_{a^2})a_1-(1+\mu_{b^2})a_2|+|(1+\mu_{a^2})b_1-(1+\mu_{b^2})b_2|+|(1+\mu_{a^2})c_1-(1+\mu_{b^2})c_2|+|(1+\mu_{a^3})a_1-(1+\mu_{b^3})a_2|+|(1+\mu_{a^3})b_1-(1+\mu_{b^3})b_2|+|(1+\mu_{a^3})c_1-(1+\mu_{b^3})c_2|]$

4.7. FKMS Algorithm in Fuzzy Multi Set:

Step 1:Construct the Tr-FM numbers multi-criteria decision matrix $A=\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m1} & \dots & a_{mn} \end{bmatrix}$ for decision.

Step 2:Compute overall values f_i= Tr-FMG(a_{i1}, a_{i2}, a_{i3}); note that iff$_i$ for all $i \in I_m$ is not normalized Tr-FM numbers, then we define the normalized Tr-FM numbers using Hamacher's product on Tr-FM numbers

Step 3: Calculate the distance between collective overall values $f_i = \langle [a^i, b^i, c^i]; (\mu_i^1, \mu_i^2, \dots, \mu_i^p) \rangle$ and positive ideal solution fi+ ($f_i^+ = \langle (1,1,1); 1,1, \dots 1 \rangle$)

Step 4: Rank all the attributes $A_i (i = 1,2,3, \dots, m)$

Step 5: Select the best in accordance with D (f_i, f_i^+), the better are the alternatives,A_i

4.8. Application to address FKMS algorithm:

Consider in our methodology for analyzing smartphone data, we have identified three linguistic variables are price range, internal storage capacity, and processor speed. Each variable is categorized using three linguistic values corresponding to different years are 2018, 2020, and 2022. This categorization likely reflects the evolution of smartphone technology and features over time. Employing triangular fuzzy multi-numbers, we represent the data with three values (a, b, c), where a, b and c denote the range of the expected outcomes, and each is associated with membership values reflecting the degree of correspondence with the linguistic value it represents. These values provide insights into the expected trends and characteristics of smartphones from each year. By comparing these expected outcomes with the actual sales data represented by the membership values for a, b and c, you gain a comprehensive understanding of how well the expected trends align with real-world market dynamics. This approach enables a nuanced analysis of the relationships between linguistic variables and sales outcomes, offering valuable insights for decision-making in the competitive smartphone industry.

Now, manipulating the FKMS algorithm to the above stated scenario.

Step 1: Input data of the Tr-FM numbers multi-criteria decision matrix $A=\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$ for decision:

	sp1	sp2	sp3	sp4	sp5
ft1	[0.6,0.6,0.8]; (0.84,0.05,0.05)	[0.1,0.5,0.8]; (0.22,0.76,0.72)	[0.5,0.6,0.3]; (0.88,0.89,0.75)	[0.5,0.4,0.6]; (0.89,0.80,0.12)	[0.2,0.8,0.9]; (0.12,0.85,0.79)
ft2	[0.8,0.5,0.6]; (0.34,0.73,0.77)	[0.2,0.5,0.5]; (0.15,0.75,0.63)	[0.1, 0.5,0.8]; (0.20,0.41,0.68)	[0.4,0.5,0.6]; (0.80,0.60,0.22)	[0.5,0.6,0.3]; (0.23,0.72,0.81)
ft3	[0.5,0.6,0.3]; (0.28,0.52,0.90)	[0.8,0.5,0.1]; (0.62,0.41,0.08)	[0.5,0.2,0.4]; (0.12,0.54,0.68)	[0.2,0.8,0.3]; (0.40,0.62,0.53)	[0.3,0.6,0.8]; (0.35,0.25,0.11)

Step 2: Applying the Tr-FMG operator to derive the collective overall preference triangular fuzzy multiset f_i:

$$SP_1 = [0.18,0.288,0.096]; (0.0711,0.0154,0.0179)$$
$$SP_2 = [0.056,0.125,0.024]; (0.0564,0.0717,0.0070)$$

$$SP_3 = [0.175,0.096,0.036]; (0.0526,0.1939,0.1650)$$
$$SP_4 = [0.06,0.16,0.006]; (0.072,0.1483,0.0385)$$
$$SP_5 = [0.03,0.192,0.576]; (0.0199,0.0732,0.0125)$$

Step 3: Calculate the distances between collective overall f_i and Tr-FM positive ideal solution f^+:

$D(SP_1, f^+) = 0.6598$
$D(SP_2, f^+) = 0.6863$
$D(SP_3, f^+) = 0.7064$
$D(SP_4, f^+) = 0.6782$
$D(SP_5, f^+) = 0.6467$

Step 4: Rank all the alternatives Ai in accordance with descending order

$$SP_3 > SP_2 > SP_4 > SP_1 > SP_5$$

Step 5: The most desirable alternative isSP_3 .

5.CONCLUSION

In this paper, we defined decomposition theorems on multi fuzzy set and fuzzy multiset. A multi fuzzy set and fuzzy multiset are not the same. While they both involve the concept of fuzzy sets, they address different aspects of uncertainty. They are well-suited for representing imprecise and repetitive information in scenarios involving uncertain data.Then we developed Hamacher's product for triangular multi fuzzy number and triangular fuzzy multi number through T-norm. Then we developed hamming distance formula for triangular multi fuzzy number and triangular fuzzy multi number. Next, we developed application of best smartphone using multi criteria decision making on MFS and FMS.

6. FUTURE WORK

Herein, we developed and entrenched decomposition theorems of "Intuitionistic multi-fuzzy sets and Intuitionistic fuzzy multi-set". Next, we are interested in using T-norm and T-conorm and ranking method at "Intuitionistic multi-fuzzy set and Intuitionistic fuzzy multi-set". In this paper, we took assumption data, in future, we will take real data may be extensively enforced in multi-criteria decision-making model (MCDM) and fuzzy system for microarray [11].

7.REFERENCES

1. Klir, G.J., Yuan, B., Fuzzy Sets and Fuzzy Logic: Theory and Applications, Prentice Hall.1995.
2. Zadeh,L.A., Fuzzy sets. Information and control, 8(3)(1965),338-356.
3. LakshmanaGomathiNayagam, V., Jeevaraj, S., Geetha Sivaraman., Total Ordering Defined on the set of all Intuitionistic Fuzzy Numbers, 1(2015) 1-13
4. Sabu Sebastian., Multi Fuzzy Sets,2010.
5. Onasanya, B.O., Sholabomi, A.S., Fuzzy Multiset operations in Application,2019.
6. Kameswari, M., Mohiuddine, S.A., LakshmanaGomathiNayagam. V., AnbazhaganNeelamegam., On level spaces of fuzzy bitopological spaces, 26(2)(2022).
7. Sadaaki Miyamoto, Fuzzy Multisets and Their Generalizations, (2000) 305-8573.
8. Mohuya B. kar, Bikashkoli Roy, Samarjit Kar, SaibalMajumdar, DraganPamcar, Type-2 Multi fuzzy sets and their applications in Decision Making,11 (2019), 170.
9. Shiny Jose, Sunny Kuriakose, Decomposition theorems of an intuitionistic fuzzy set, 2(2012),31-36.
10. Lijun Yang, Lin Wang, jiankun Yu, Decomposition Theorems on Vague Sets, 65(2010),3201-3209.
11. Ganesh Kumar, P, Aruldoss Albert Victoire. T, Renugadevi. P, Devaraj. D, Design of fuzzy expert system for microarray data classification using a novel Genetic Swarm Algorithm,39(2012), 1811-1821
12. VakkasUlucay, Irfan Deli, Mehmet Sahin, Trapezoidal fuzzy multi-number and its application to multi-criteria decision-making problems, doi:10.1007/s00521-016-2760-3.