Nanotechnology Perceptions
ISSN 1660-6795
www.nano-ntp.com

Inverse 2-Equitable Domination in Fuzzy Graphs

T. Rabeeh Ahamed[1], S. Ismail Mohideen[2], R. Jahir Hussain[3], S. Muthupandiyan[4]

[1]Research Scholar, Department of Mathematics, Jamal Mohamed College (Autonomous), (Affiliated to Bharathidasan University), India.
[2]Associate Professor, Research Scholar Department of Mathematics, Jamal Mohamed College (Autonomous), (Affiliated to Bharathidasan University), India.
[3]Associate Professor, Research Scholar Department of Mathematics, Jamal Mohamed College (Autonomous), (Affiliated to Bharathidasan University), India.
[4]Assistant Professor, Department of Mathematics, School of Engineering and Technology, Dhanalakshmi Srinivasan University, India.
Email: rabeehahamed@gmail.com

A sub set $D \subseteq V(G)$ of a fuzzy graph $G(\sigma, \mu)$ is said to be equitable dominating set if each $v \in V - D$, there exists a vertex $u \in D$ such that $uv \in E(G)$ and $|\deg_s(u) - \deg_s(v)| \leq 1$. An equitable dominating set $D$of $V(G)$ is said to be 2-equitable dominating set in a fuzzy graphs $G(\sigma, \mu)$, if every vertex $v \in V - D$ there exists a vertex $u \in D$ or v is equitable dominated by at least two vertices in D. If $V - D$ contains a 2 - equitable dominating set D' with respect to D then D' is said to be an inverse 2-equitable dominating set of a fuzzy graphs $G(\sigma, \mu)$. In this study, inverse 2-equitable equitable dominating set, its number in fuzzy graphs are introduced. Bounds and some theorems related to inverse 2-equitable equitable domination numbers are stated and proved.

Keywords: Fuzzy graph, equitable dominating set, equitable domination number, 2 - equitable dominating set, inverse equitable dominating set and its number, inverse 2 - equitable dominating set, inverse 2 - equitable dominating number.

1. Introduction

Applications of fuzzy graph are include data mining, clustering, image capturing, networking, communications, planning, etc., L.A Zadeh [1] introduced fuzzy sets in 1965. Fuzzy graph theory was initiated by A. Rosenfeld [2] in 1975. C Gurubaran et.al., [3] introduced the concept inverse equitable domination number in fuzzy graphs in 2018. Gurubaran et.,all[4] initiated the concept 2- equitable domination in fuzzy graphs in 2018. Complementary nil g-eccentric domination fuzzy graphs concepts introduced by Mohamed Ismayil and Muthupandiyan[5] in 2020. S. Muthupandiyan and A. Mohamed Ismayil [6] introduced the concept perfect g-eccentric domination in fuzzy graph in 2021.

S. Muthupandiyan and A. Mohamed Ismayil [7] introduced the concept isolate g-eccentric domination in fuzzy graph in 2023. John JC, Xavier P, Priyanka GB.[8] Divisor 2-equitable domination in fuzzy graphs in 2023. Muthupandiyan S, Ismayil AM[9] introduced the concept of inverse g-eccentric domination in fuzzy graphs in 2022. Rabeeh Ahamed et.al.,[10] stated the concept complementary nil equitable domination in fuzzy graphs in 2024.

2. Basic Definitions

Definition 2.1[5]: A fuzzy graph $G = (\sigma, \mu)$is characterized with two functions ρ on V and μ on $E \subseteq V \times V$, where $\sigma: V \to [0,1]$ and $\mu: E \to [0,1]$ such that $\mu(x, y) \leq \rho(x) \wedge \rho(y)\ \forall x, y \in V$. We expect that V is finite and non-empty, μ is reflexive and symmetric. We indicate the crisp graph $G^* = (\sigma^*, \mu^*)$ of the fuzzy graph $G(\sigma, \mu)$ where $\sigma^* = \{x \in V: \rho(x) > 0\}$ and $\mu^* = \{(x, y) \in E: \mu(x, y) > 0\}$. The fuzzy graph $G = (\sigma, \mu)$ is called trivial in this case $|\rho *| = 1$.

Definition 2.2[5]: A path P of length n is a sequence of distinct nodes $u_0, u_1, \ldots, u_n$ such that $\mu(u_{i-1}, u_i) > 0, i = 1,2, \ldots, n$ and the degree of membership of a weakest arc is defined as its strength.

Definition 2.3[5]: An edge is said to be strong if its weight is equal to the strength of connectedness of its end nodes. Symbolically, $\mu(u, v) \geq CONN_{G-(u,v)}(u, v)$.

Definition 2.4[5]: The order and size of a fuzzy graph $G(\sigma, \mu)$ are defined by $p = \sum_{u \in V} \sigma(u)$ and $q = \sum_{uv \in E} \mu(u, v)$ respectively.

Definition 2.5[5]: Let $G(\sigma, \mu)$ be a fuzzy graph. The strong degree of a vertex $v \in \sigma^*$ is defined as the sum of membership values of all strong arcs incident at v and it is denoted by $d_s(v)$. Also, it is defined by $d_s(v) = \sum_{u \in N_s(v)} \mu(u, v)$ where $N_s(v)$ denotes the set of all strong neighbors of v.

Definition 2.6[6]: A fuzzy graph $G(\sigma, \mu)$ is connected if $CONNG(u, v) > 0$ where $CoNNG(u, v)$ is strength of connectedness between two vertices u, v in $G(\sigma, \mu)$.

Definition 2.7[5]: In a fuzzy graph $G(\sigma, \mu)$, strength of connectedness between two vertices $u, v \in V(G)$ is maximum strength of all paths between u, v in $V(G)$.

Definition 2.8[5]: A subset D of V is called a dominating set (DS) in G if for every $v \notin D$ there exist $u \in D$ such that u dominates v. The minimum scalar cardinality taken over all dominating set is called domination number and is denoted by the symbol γ. The maximum scalar cardinality of a minimal dominating set is called upper domination number and is denoted by the symbol Γ.

Definition 2.9[4]: A sub set $D \subseteq V(G)$ of a fuzzy graphs $G(\sigma, \mu)$ is said to be equitable dominating set (EDS) if each $v \in V - D$, there exists a vertex $u \in D$ such that $uv \in E(G)$ and

$|\deg_s(u) - \deg_s(v)| \leq 1$.

Definition 2.10[4]: An equitable dominating set $D \subseteq V$ of a fuzzy graph $G = (\sigma, \mu)$ is called 2 – equitable dominating set if for every vertex $v \in V - D$ there exist $v \in D$ or v is equitable dominated by at least two vertices in D.

Definition 2.11[3]: Let $D \subseteq V(G)$ of a fuzzy graph is an equitable dominating set. If $V - D$ contains an equitable dominating set D' with respect to D, then D' is an inverse equitable dominating set of a fuzzy graph G.

3. Main Results

Inverse 2- Equitable Domination in Fuzzy Graphs

In this section discuss about inverse 2- equitable dominating set and its number in fuzzy graphs. Bound and theorem related to inverse 2- equitable domination number in fuzzy graphs are stated and proved.

Definition 3.1:

If $V - D$ contains an 2.- .equitable dominating set D' with respect to D, then D' is an inverse 2..-equitable dominating set of a fuzzy graph $G(\sigma, \mu)$. An inverse 2 – equitable dominating set D' is said to be minimal if no proper subset of D'' is an inverse 2 – equitable dominating set. The minimum scalar cardinality of a minimal inverse 2 – equitable dominating set of G is called the inverse 2 – equitable dominating number of G and is denoted by $\gamma_{2eqd}^{-1}(G)$. The maximum scalar cardinality of a minimal inverse 2 – equitable dominating set of G is called the upper inverse 2 – equitable dominating number of G and is denoted by $\Gamma_{2eqd}^{-1}(G)$.

Note 3.1: The minimum inverse 2 - equitable dominating set is denoted by γ_{2eqd}^{-1}-set.

Example 3.1: Consider the fuzzy graph $G(\sigma, \mu)$.

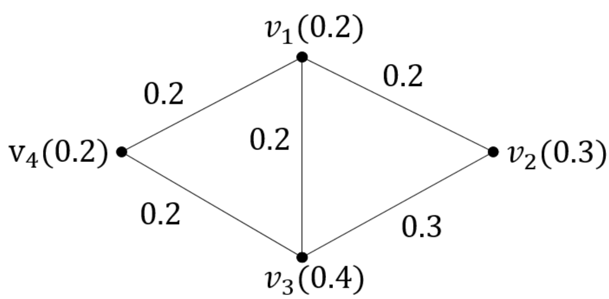

Figure: Inverse 2-Equitable Dominating Set in a Fuzzy Graph

From the fuzzy graph given in example 3.1, the followings are observed.

1. The minimum 2- equitable dominating set is $D_1 = \{ v_2, v_4, \}$, then $\gamma_{2eqd}(G) = 0.5$.

2. The inverse 2- equitable dominating set is, $D'_1 = \{ v_1, v_3 \}$, then $\gamma^{-1}_{2eqd}(G) = \Gamma^{-1}_{2eqd}(G) = 0.6$.

Observation 3.1: For any connected fuzzy graphs $G(\sigma, \mu)$

1. $\gamma(G) \leq \gamma_{eqd}(G) \leq \gamma_{2eqd}(G) \leq \gamma^{-1}_{2eqd}(G)$
2. $\gamma^{-1}_{2eqd}(G) \leq \Gamma^{-1}_{2eqd}(G)$.
3. Obviously any minimum 2-equitable dominating set is also minimal but the converse is not true.
4. For any fuzzy graph G, $\gamma_{2eqd}(G) + \gamma^{-1}_{2eqd}(G) \leq p$.

Observation 3.2: For any FG $G = (\sigma, \mu)$,

1. Every super set of an inverse 2-equitable dominating set is also an inverse 2-equitable dominating set.
2. Complement of an inverse 2-equitable dominating set is not a need an inverse 2-equitable dominating set
3. γ^{-1}_{2neqd} -set need not be unique.

Proposition 3.1: For any fuzzy graph G with order p, then $\sum_{\substack{v_i, v_j \in G \\ v_i \neq v_j}} \min\left(\sigma(v_i), \sigma(v_j)\right) \leq \gamma^{-1}_{2eqd}(G) \leq p$.

Proof:
Let D' be a inverse dominating set of a fuzzy graph G having atleast two vertices has minimum of V which is a sum of minimum value of vertices $v_i, v_j \in D', \gamma^{-1}_{2eqd}(G) \leq p$ it is obviously true.

Theorem 3.1: Let G be a fuzzy graph, $\gamma^{-1}_{2eqd}(G) = p$ iff the fuzzy graph G has adjacent to less than two vertices.

Proof:
Let G be a fuzzy graph then $\gamma^{-1}_{2eqd}(G) = p$ then definition of fuzzy graph has all vertices in dominating set D'. which shows that every vertex in G has adjacent to less than two vertices. Conversely, G be a fuzzy graph has adjacent to less than two vertices then every vertex is in regular dominating set. Which is $\gamma^{-1}_{2\varepsilon qd}(G) = p$

Theorem 3.2: Let D is a minimal 2 - equitable dominating set, then $V - S$ contains minimal inverse 2 - equitable dominating set if every vertex of V in a fuzzy graph G adjacent to more than two vertices in V.

Proof:
Let D be a minimal 2 - equitable set of G suppose that $V - D$ is not a 2 - equitable dominating set, then there exists at least one vertex $v \in D$ which is not an 2 equitable adjacent to any vertex

in $V - D$. Therefore $V - D$ is 2 - equitable adjacent to at least two vertices in D then $D - \{v\}$ is a 2 - equitable dominating set which is a contradiction. Hence every vertex in D must be equitable adjacent to at least two vertices in $V - D$. Hence $V - D$ is an inverse 2 - equitable dominating set which contains minimal inverse 2 - equitable dominating set.

Theorem 3.3: Let G be a connected fuzzy graph has no non - equitable edge and H is spanning subgraph of G then $\gamma^{-1}_{in2eqd}(G) \leq \gamma^{-1}_{in2eqd}(H)$.

Proof:
Let G be a connected fuzzy graph and H is the spanning subgraph of H. consider D' is minimum inverse 2 - equitable dominating set of G, D' also a inverse 2 - equitable dominate all the vertices in $V(H) - D$ that is D' is an inverse 2 - equitable dominating set in H. Hence $\gamma^{-1}_{2eqd}(G) \leq \gamma^{-1}_{2eqd}(H)$.

Theorem 3.4: For any fuzzy graph $G, \gamma_{2eqd} + \min\sigma(v_i) \leq \gamma^{-1}_{2eqd}(G)$, for $v_i \notin D'$.

Proof:
Let D' be inverse 2 - equitable dominating set with minimum cardinality γ^{-1}_{2eqd}. for any vertex $v_i \in D', D' - \{v_i\}$ is 2 - equitable dominating set. Hence $\gamma_{2eqd} + \min\sigma(v_i) \leq \gamma^{-1}_{2eqd}(G)$.

Theorem 3.5: Let G be a fuzzy graph without isolated vertices. Then $\gamma^{-1}(G) \leqslant \gamma^{-1}_{2eqd}(G)$

Proof:

Every inverse 2- equitable dominating set is a inverse dominating set. Thus $\gamma^{-1}(G) \leqslant \gamma^{-1}_{2eqd}(G)$.

Theorem 3.6 An inverse 2 - equitable dominating set exists for any strong fuzzy graph G .

Proof.

Let $G = (\sigma, \mu)$ be a regular fuzzy graph. Suppose a strong fuzzy graph G has a 2- equitable dominating set, obviously it contains a 2 equitable dominating set D'. Therefore every strong fuzzy graph has an inverse 2 - equitable dominating set and it exists for strong fuzzy graph.

Theorem 3.7 For any fuzzy graph G $\gamma_{2eqd}(G) \leq \gamma^{-1}_{2eqd}(G)$.

Proof.

It is clear that every inverse 2- equitable dominating set is a 2-equitable dominating set. we get $\gamma_{2eqd}(G) \leq \gamma^{-1}_{2eqd}(G)$.

Theorem 3.8 For a fuzzy graph $G = (\sigma, \mu)$ if γ^{-1}_{2eqd} is a inverse 2-equitable dominating set then $V - D$ is a dominating set of a fuzzy graph G.

Proof

Let v be any vertex in D', D' is an inverse 2-equitable set in G . Since G has no isolated vertex $v \in N_s(u)$.It is clearly every inverse 2-equitable dominating set is a equitable dominating set such that $v \in V - S$. Hence every vertex of D' dominates some of the vertices in $V - S$. Therefore, $V - D$ is a dominating set of fuzzy graph G.

Theorem 3.9

An inverse 2-equitable dominating set D_1 of a fuzzy graph G is minimal if and only if for every vertex $u \in D_1$ one of the following conditions holds

(i) there exists vertex $v \in V - D_1$ such that $N_s(v) \cap D_1 = \{u\}$

(ii) $N_s(u) \cap D_1 = \emptyset$

Proof

Suppose that D_1 is an inverse 2-equitable dominating set of a fuzzy graph G and (i) & (ii) not hold. Then for some vertex $u \in D_1{}'$ there exists $v \in N_s(u) \cap D_1$. Therefore $D_1 - \{u\}$ is an equitable dominating set of G, a contradiction with the minimality of D_1. Conversely, let for every $u \in D_1$ one of the conditions (i) or (ii) holds. Suppose that D_1 is not minimal, there exists $u \in D_1$ such that $D_1 - \{u\}$ is an equitable dominating set of a fuzzy graph G. This means there exists $v \in D - \{u\}$ which is equitable adjacent to u. Hence (ii) does not satisfy.

Theorem 3.10

For any fuzzy graph G without equitable isolated nodes, $\gamma(G) \leq \min\{\gamma^{-1}(G), \gamma_{2ed}(G), \gamma_{2ed}^{-1}(G)\}$

Proof

Every inverse 2-equitable dominating set is an inverse dominating sets of G and every inverse dominating set is dominating set, similarly every 2-equitable dominating set is dominating set. Hence $\gamma(G) \leq \min\{\gamma^{-1}(G), \gamma_{2ed}(G), \gamma_{2ed}^{-1}(G)\}$.

4. Conclusion

In this article, an inverse 2 - equitable dominating set, its number in fuzzy graphs are obtained. Theorems related to an inverse 2 - equitable dominating set and number in a fuzzy graphs are stated and proved. Bounds and some points related an inverse 2-equitable domination number are observed and discussed.

References

1. Zadeh LA. Fuzzy sets. Information and control, 1965;8(3):338-353. Available at https://doi.org/10.1016/S0019-9958(65)90241-X
2. Rosenfeld A. Fuzzy graphs. InFuzzy sets and their applications to cognitive and decision Processes, Academic press, 1975 ;77-95.
3. Available at https://doi.org/10.1016/B978-0-12-775260-0.50008-6
4. C Gurubaran, A. Prasanna2 and S. Ismail Mohideen, Inverse equitable domination in fuzzy graphs. Journal of Computer and Mathematical Sciences 2018;9(10):1574-1584.
5. Available at bulletin_imvi_6_1_2015_49_54.pdf (imvibl.org)
6. C. Gurubaran, A. prasanna and A. Mohamed Ismayil, 2- Equitable domination in Fuzzy
7. Graphs, International Journal of Applied Engineering Research, 2018;13(16), 12531-12535.
8. Available at ijaerv13n16_18.pdf (ripublication.com)
9. Ismayil AM, Muthupandiyan S. Complementary nil g-eccentric domination in fuzzy graphs.

Advances in Mathematics: Scientific Journal. 2020;9(4):1719-1728. .
10. Available at https://doi.org/10.37418/amsj.9.4.28
11. Muthupandiyan S, Ismayil AM. Perfect g-eccentric domination in fuzzy graph. Journal of
12. Mathematical Control Science and Applications Vol. 7 No. 2 (July-December, 2021
13. Available at 32_S_Muthupandiyan_July_December_2021.pdf (mukpublications.com)
14. Muthupandiyan S, Ismayil AM. Isolate g-eccentric domination in fuzzy graph. Ratio Mathematica. 2023;50.
15. Available at http://dx.doi.org/10.23755/rm.v50i0.1548
16. John JC, Xavier P, Priyanka GB. Divisor 2-Equitable Domination in Fuzzy Graphs. Discrete
17. Mathematics and Mathematical Modelling in the Digital Era: ICDM3DE-2023, Gandhigram, India, March 23–25.:219.
18. Muthupandiyan S, Ismayil AM. Connected g-Eccentric Domination in Fuzzy Graph. Journal of Algebraic Statistics. 2022;13(2):3377-3382.
19. Available at Connected g-Eccentric Domination in Fuzzy Graph | JOURNAL OF ALGEBRAIC STATISTICS
20. Ahamed TR, Mohideen SI, Hussain RJ, Complementary Nil Equitable Domination in Fuzzy Graphs. Indian Journal of Science and Technology, 2024:17(38): 3942-3946.
21. Available at https://doi.org/10.17485/IJST/v17i38.2819