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Controller Design Using Grey Wolf Optimization Technique for Reduced Order System

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The primary goal of this study is to design a controller for the reduced order sys-tem obtained using Grey Wolf Optimization technique through direct approach. The objective of model order reduction is to maintain the characteristics of the original higher-order system into reduced order model. The method of model order reduction involves the conversion of the critical attributes of the full-scale model into a reduced-order system. The performance of the proposed model or-der reduction method using Grey Wolf optimization is effectively evaluated, and two standard numerical examples are used to demonstrate the unique features of this approach. Using power series expansion, the controller is designed for origi-nal high order system and reduced order model obtained using GWO technique. The response of closed loop high order system response and closed loop reduced order model response are compared with reference model to prove its efficiency. Comparisons are also made between the models generated by this innovative approach and those derived from present and frequently employed model reduc-tion techniques. The recommended approach's efficacy is assessed using perfor-mance error metrics and transient response parameters.

Keywords: Model reduction technique, Step response, Reduced-order system, Grey Wolf optimization algorithm, controller design, ISE.

1. Introduction

In today's technological era, systems across various fields such as engineering, societal processes, and environmental systems are often characterized by high dimensionality and complexity. Studying, modeling, controlling, designing, and computing large-scale systems is difficult using traditional methods. To deal with this, it's important to create smaller systems that work the same as the big ones. This approach, known as Model Order Reduction (MOR), keeps the key aspects of the original system in a simplified version and is extensively applied in numerous engineering and science sectors to simplify high-dimensional real-time systems [1-5].

Originally, model reduction was suggested in the frequency domain to lower the linear dynamic system's transfer function's order [6]. Notable techniques in this domain include Pade approximation [7], Routh stability [9], the stability equation [10], the Routh approximation [8], and pole clustering [10] are valuable methods, but they come with some drawbacks. For instance, the Pade approximation may yield an unstable lower-order system despite the stability of the actual system [11]. The Routh approximation method is ineffective for non-strictly proper transfer functions [8], and The Routh stability method fails to maintain dominant poles in non-minimum phase systems [12].However, these approaches may not consistently simplify the complexity of large-scale systems, and can sometimes create unstable micro-models, even when the original systems are stable.

To overcome the drawbacks of classical techniques some hybrid techniques were developed in literature. [13] presented an enhancement to the Routh-Padé approximation techniques, refining the method for more accurate system approximations.[14] developed a new algorithm aimed at model order reduction specifically for interval systems, enhancing the process of simplifying such models. [15] applied Routh approximation and factor division methods to reduced-order modeling, while [16] proposed an improved Routh approximation method for order reduction of linear dynamic systems. [17]proposed a technique for linear time-invariant systems, offering an innovative approach. Lastly, [18] utilized a clustering method combined with Padé approximation to effectively reduce the order of linear systems. Nevertheless, it involves additional simulation time and mathematical calculations for fine-tuning and adjusting gain factors. Paper [20] details the Fast-Dynamic Grey Wolf Optimizer (GWO). This method utilizes a multi-moment matching technique to enhance the efficiency and accuracy of the order reduction process. [21] presents the GWO, highlighting its effectiveness in solving complex optimization problems. GWO performs competitively against established algorithms in terms of accuracy and convergence speed, demonstrating robust performance with minimal parameter sensitivity. Its efficiency and versatility make it a valuable tool for various engineering and computational tasks. [19] explore the use of GWO for order reduction with Factor Division in large-scale Linear Time-Invariant system, demonstrating its effectiveness in simplifying complex models while maintaining stability and accuracy by minimising ISE. [23] utilizes a simplified Grey Wolf Optimizer (GWO) approach to create an adaptive fuzzy PID controller for managing frequency in distributed power generation systems, which enhances the controller's performance and adaptability.[24] present an enhanced GWO algorithm, highlighting its improved convergence and accuracy across various applications, including sensor networks. These studies collectively showcase GWO's versatility and effectiveness in optimization tasks.

The method uses an advanced Grey Wolf Optimization technique to calculate the transfer function, resulting in better model reduction with greater accuracy and efficiency. The objective is to develop a low-order controller that effectively manages the original high-order system, making the overall system easier to understand and control. The method ensures that the stability and initial time moments of the original system are preserved in the reduced-order model. The quality of the reduced-order model is evaluated by its ability to represent the desired system characteristics. Model reduction focuses on open-loop considerations, but closed-loop stability performance is also important in controller design.

For designing Proportional-Integral-Derivative (PID) controllers many methods are available. In this research paper, an algebraic scheme for a PID controller for Linear Time Invariant

(LTI) Continuous Systems is designed. The closed-loop transfer functions of reduced-order models with PID controllers with the reference model transfer function in the frequency domain are compared. The analysis and development of large-scale models are complex and require ongoing efforts to simplify higher-order models. These higher-dimensional systems are prevalent across various engineering and scientific disciplines, including aeronautics, control systems, power systems, thermodynamics, and more. Model reduction of complicated systems is a prevailing focus in biological systems, control systems, electromagnetic fields, mechanical engineering, power systems, chemical engineering, and other specialized domains.

Large-scale system modeling and controller construction are time-consuming and complex processes. The more complicated the dynamic system, the longer the simulation takes and the more expensive the controller is to build. A "good" approximation model for a complex model can be created in order to overcome these limitations, and the controller is built around it. Creating a reduced order controller (ROC) that makes it simple to regulate the original system so that it's overall structure is low order and understandable, is one of the main objectives of order reductions. [30-35]. Therefore, using MOR methods, it is possible to decrease the high-order controller to a low-order controller without creating any further mistakes. While the stability of the closed-loop is a crucial concern for the reduction of controllers, the reduction of the model is based on open-loop factors.

A new system reduction technique utilizes Grey Wolf optimization to simplify and design a controller for higher-order systems, aiming to minimize the error between the original system and the reduced-order mode..Two examples are used to test and compare the effectiveness of the proposed model. This model reduction technique is simple and ensures that the reduced-order system remains stable if the original system is stable. The paper is organized as follows: Section 2 outlines the problem of system reduction, Section 3 explains the main steps of the proposed technique, Section 4 describes the new method for designing a controller for large-scale systems, and Section 5 validates the proposed algorithms using two well-known numerical examples from the literature. The paper concludes in Section 6.

2. Problem Statement

For n^{th}-order linear dynamic SISO continuous system can be illustrated as:

$$H(s) = \frac{N(s)}{D(s)} = \frac{u_{n-1}s^{n-1}+\cdots+u_1 s+u_0}{v_n s^n+\cdots+v_1 s+v_0} \quad (1)$$

where $u_i; 0 \le i \le n - 1$ and $v_i; 0 \le i \le n$ represents the scalar constants.

The goal is to derive an r^{th} order model, where $r < n$ in the form described by equation (1), so that $H_r(s)$closely approximates $H(s)$ as specified in [19,22].

$$H_r(s) = \frac{N_r(s)}{M_r(s)} = \frac{k_{r-1}s^{r-1}+\cdots+k_1 s+k_0}{m_r s^r+\cdots+m_1 s+m_0} \quad (2)$$

Where $m_i; 0 \le i \le r - 1$ and $k_i; 0 \le i \le r$ are unknown scalar constants.

3. Grey Wolf Optimization Algorithm

GWO is a nature-inspired algorithm that can be effectively utilized for model order reduction [19-24]. The process begins with the initialization of a population of "wolves," each representing a potential model of varying order. In the optimization process, a group of wolves is dispersed randomly throughout the solution space. Their ability to find the best solutions is measured using an objective function that assesses their fitness. This is typically demonstrated by comparing the accuracy of the reference model with that of its reduced version. Subsequently, based on their fitness, the wolves are ranked. The top three wolves, referred to as α, β, and δ, are identified as the most promising solutions, while all others are represented as Omega (ω) wolves.

The positions of the wolves are updated by using the positions of α, β, and δ, reflecting the social hierarchy and hunting behaviour. This involves mathematical updates to the model orders based on the best-found solutions, ensuring that the positions remain within feasible bounds. Convergence is determined by evaluating whether the improvements in the objective function have stabilized or if iterations(maximum) has been reached. If convergence criteria are met, the best-found model order is selected as the reduced model. Otherwise, the process iterates from the evaluation step until a satisfactory solution is achieved.

In summary, the Grey Wolf Optimizer for model order reduction begins by initializing a population of potential modelbased on fitness, and then updates their positions according to the best solutions found. The algorithm iterates until convergence, efficiently balancing accuracy and complexity by mimicking the social behaviour of Greywolves [19, 22]. The basic equations are illustrated in [21]

Key actions in a search for grey wolves are:

- Following, tracking, and getting near to the target.
- When the target stops moving, it follow, encircle and harass it.
- Take direct aim at the goal.

Equations (3) & (4) provide a mathematical description of the encircling process:

$$\vec{U} = \left|\vec{V}\vec{N_1}(t) - N(t)\right| \quad (3)$$

$$\vec{N}(t+1) = \vec{N_1}(t) - \vec{W}.\vec{X} \quad (4)$$

Coefficient vectors are represented by $\vec{W}$ and $\vec{V}$ and prey are $\vec{N}$ and $\vec{N_1}$ represents the position vectors, t is the current iteration given by equation (3) and (4). Equations (5) and (6) are used to find the coefficient vectors $\vec{W}$ and $\vec{V}$:

$$\vec{W} = 2\vec{b}.r_1 - \vec{b} \quad (5)$$

$$\vec{V} = 2.r_2 \quad (6)$$

where r_1 and r_2 are random vectors and b is a variable changing from 2 to 0

When grey wolves identify potential prey, they encircle it to close in. During this process, the

Betas and Deltas assist the Alphas in steering the hunt. The optimal solution is directed by the three leading positions—those of the Alphas, Betas, and Deltas—while the remaining solutions, including the Omegas, are adjusted. Equations (7) to (12) describe how the positions of the α, β, and δ wolves are updated as they encircle the prey.

$$\overrightarrow{U_\alpha} = \left|\overrightarrow{V_1 N_\alpha} - \vec{N}\right| \quad (7)$$

$$\overrightarrow{U_\beta} = \left|\overrightarrow{V_2 N_\beta} - \vec{N}\right| \quad (8)$$

$$\overrightarrow{U_\delta} = \left|\overrightarrow{V_3 N_\delta} - \vec{N}\right| \quad (9)$$

Thus using equation (4)

$$\overrightarrow{N_1} = \overrightarrow{N_\alpha} - \overrightarrow{W_1}.\left(\overrightarrow{U_\alpha}\right) \quad (10)$$

$$= \overrightarrow{N_\beta} - \overrightarrow{W_{2.}}\left(\overrightarrow{U_\beta}\right) \quad (11)$$

$$\overrightarrow{N_3} = \overrightarrow{N_\delta} - \overrightarrow{W_3.(U_\delta)} \quad (12)$$

Equation (13)is used to update the position of the current search agent

$$\vec{N}(t+1) = \frac{\left(\overrightarrow{N_1} + \overrightarrow{N_2} + \overrightarrow{N_3}\right)}{3} \quad (13)$$

One of the advantages of GWO is that there are very less parameters to adjust. In model order reduction, GWO provides fast convergence, robust performance, minimal parameter tuning, and effective handling of complex systems, making it a powerful tool for simplifying large-scale models efficiently. The flowchart of the GWO is depicted in Figure 1. To assess the effectiveness of the technique for MOR (Model Order Reduction), a continuous time SISO system taken from the literature is used. The performance index, ISE, is reduced through the use of the GWO technique, that's given as:

$$\mathrm{ISE} = \int_0^\infty [\emptyset(t) - \emptyset_r(t)]^2 dt \quad (14)$$

The step response of the lower-order system is given by $\emptyset_r(t)$, $\emptyset(t)$represents the step response of the original system.

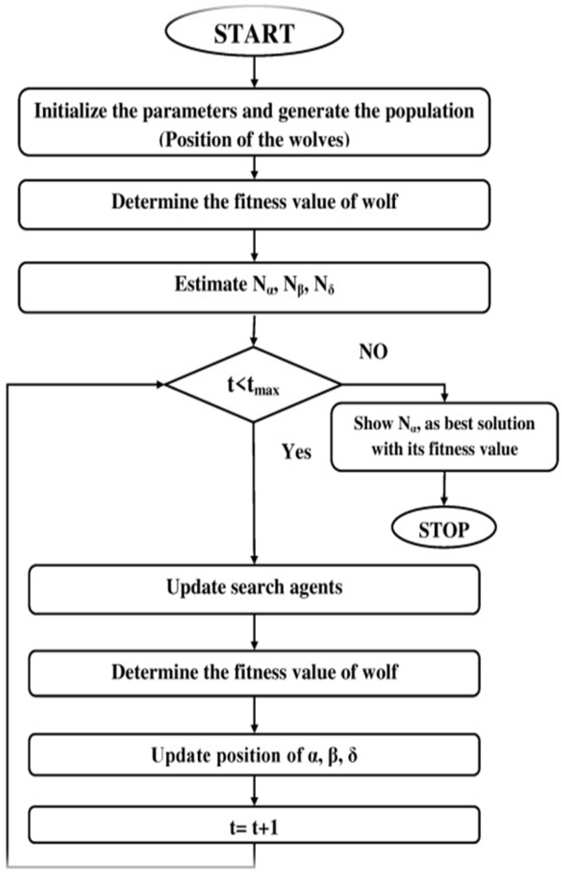

Figure.1. Flow chart of GWO

4. Controller Design

The challenge lies in creating a controller $\boldsymbol{G_c(s)}$ for an uncontrolled plant $\boldsymbol{G_p(s)}$ that has a suitable fast response and with unity feedback stable closed-loop response can be analyzed. One way to phrase the design issue is through a direct approach. In this approach controller is obtained on the basis of reduced order model called process reduction. Another approach is to obtain a controller for full order system and then to reduce closed loop response of high order controller and original system with unity feedback called controller reduction.

The high-order plant $\boldsymbol{G_p(s)}$ exhibits poor dynamic characteristics, a controller $\boldsymbol{G_c(s)}$ must be found so that the controlled system's frequency and time responses closely resemble those of the reference model.

Step 1: Given a plant with transfer function $\boldsymbol{G_p(s)}$, it is essential to construct a reference model M(s) to ensure that the closed-loop response of the controlled system with unity feedback closely matches the response of the reference model.

The transfer functions are given as:

$$\boldsymbol{G_p(s)} = \frac{a_0+a_1s+\cdots+a_ms^m}{b_0+b_1s+\cdots+b_ns^n} \quad (15)$$

$$\boldsymbol{M(s)} = \frac{g_0+g_1s+\cdots+g_ms^u}{h_0+h_1s+\cdots+h_ns^v} \quad (16)$$

Step 2: The equivalent open-loop model transfer function is given by

$$\widetilde{M}(s) = \frac{M(s)}{1-M(s)} \quad (17)$$

Step 3:Controller's structure $G_c(s)$ is given by

$$G_c(s) = \frac{p_0+p_1s+\cdots+p_ks^k}{q_0+q_1s+\cdots+b_ls^l} \quad (18)$$

Step 4: Response of the closed-loop system with that of the reference model, can be obtained as

$$G_c(s)G_p(s) = \widetilde{M}(s) \quad (19)$$

$$G_c(s) = \frac{\widetilde{M}(s)}{G_p(s)} = \sum_{i=0}^{\infty} e_i s^i \quad (20)$$

e_i' denotes the power series expansion coefficients, where$p_i$and q_i are obtained by the help of equation (20) and (18) at s=0,

$$p_0 = q_0e_0$$

$$p_1 = q_0e_1 + q_1e_0$$

$$p_2 = q_0e_2 + q_1e_1 + q_2e_0$$

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$$p_i = q_0e_i + q_1e_{i-1} + \cdots + q_ie_0$$

$$o = q_0e_{i+1} + q_ie_i + \cdots + q_{i+1}e_0$$

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$$o = q_0e_{i+j} + q_ie_{i+j-1} + \cdots + q_je_i \quad (21)$$

By solving the above linear equations, obtain the controller's desired structure.

Step 5: The closed loop transfer function with controller $G_c(s)$ and original plant $G_p(s)$ can be obtained as

$$G_{cl}(s) = \frac{G_c(s)G_p(s)}{1+G_c(s)G_p(s)} \quad (22)$$

Step 6: Reduce the plant $G_p(s)$ to $R_p(s)$ using the GWO technique and repeat step 3 & 4 to obtain $R_c(s)$ and then obtain closed loop transfer function for reduced order plant as

$$R_{cl}(s) = \frac{R_c(s)R_p(s)}{1+R_c(s)R_p(s)} \quad (23)$$

Step 7: Compare the response of $M(s), G_{cl}(s)$ *and* $R_{cl}(s)$ for validation.

5. Simulation Result and Comparison

To ensure a comprehensive evaluation it is compared with recently introduced system order reduction methods. This comparison involves computing the performance error index (ISE) using GWO. ISE is reduced using Grey Wolf Optimization.

$$ISE = \int_0^\infty [\emptyset(t) - \emptyset_r(t)]^2 dt$$

The step response is given by $\emptyset(t) \rightarrow$ Higher Order System and $\emptyset_r(t) \rightarrow$ Lower Order System

Example 1: It refers to the fuel control system from reference [27]

$$G_p(s) = \frac{0.4299s^2 + 0.601s + 0.1069}{s^3 + 0.7026s^2 + 0.8746s + 0.1107}$$

$$M(s) = \frac{0.16}{s^2 + s + 0.16}$$

$G_p(s)$ represents the transfer function of fuel control plant and $M(s)$ is reference model for this plant.

The transfer function (open loop) is obtained using equation (17)

$$\widetilde{M}(s) = \frac{0.16s^2 + 0.16s + 0.0256}{s^4 + 2s^3 + 1.16s^2 + 0.16s}$$

The controller structure is given by

$$G_c(s) = \frac{K(1 + K_1 s)}{s(1 + K_2 s)}$$

To ensure that the closed-loop system response precisely matches that of the reference model, the required controller is determined using equation (20).

$$G_c(s) = \frac{\widetilde{M}(s)}{G_p(s)}$$
$$= \frac{0.16s^5 + 0.2724s^4 + 0.278s^3 + 0.1756s^2 + 0.0401s + 0.002834}{0.4299s^6 + 1.461s^5 + 1.808s^4 + 0.9797s^3 + 0.2202s^2 + 0.0171s}$$

$$= \frac{1}{s}(0.1657 + 0.2118s - 1.9490s^2 + 11.6943s^3 - 59.496s^4 + \cdots)$$

Power series expansion coefficients and $G_c(s)$ for the original plant is given by:

$$K = 0.1657 \qquad K_1 = 10.479 \qquad K_2 = 9.2$$

$$G_c(s) = \frac{0.1657(1 + 10.479s)}{s(1 + 9.2s)}$$

$$= \frac{1.736s + 0.1657}{9.2s^2 + s}$$

Closed-loop transfer function $G_{cl}(s)$ can be calculated as

$$G_{cl}(s) = \frac{0.7463s^3 + 1.115s^2 + 0.2852s + 0.01771}{9.2s^5 + 7.464s^4 + 9.495s^3 + 3.008s^2 + 0.3959s + 0.01771}$$

Now, similarly, the controller is designed for a reduced order model of the original system. The original system $G_p(s)$ is reduced to second order model using GWO

$$R_p(s) = \frac{0.14s + 0.7503}{0.6289s^2 + 0.6945s + 0.8107}$$

The reduced order controller is given by

$$R_c(s) = \frac{\widetilde{M}(s)}{R_p(s)}$$

Using steps 3, 4 and 5 calculate the $R_c(s)$

$$R_c(s) = \frac{0.456818214s + 0.172880181}{2.972322723s^2 + s}$$

The closed loop transfer function is obtained using equation (23)

$$R_{cl}(s) = \frac{0.0640s^2 + 0.3670s + 0.1297}{1.8693s^4 + 2.6932s^3 + 3.1681s^2 + 1.1777s + 0.1297}$$

The step response of the lower order and higher order systems are contrasted in Figure 2. When compared to some other conventional ways, it is evident that the response of the reduced model calculated by the suggested scheme is substantially closer to the response of the provided reference model.

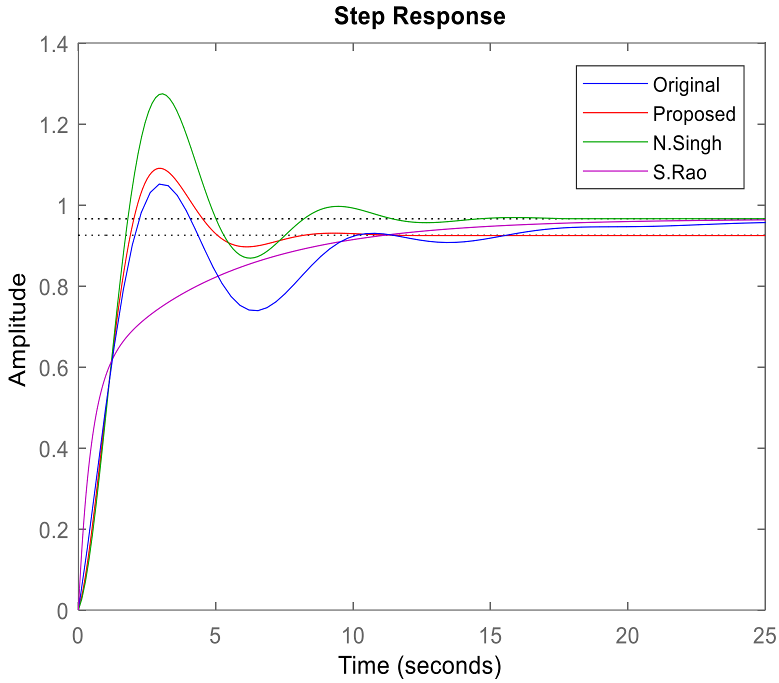

Figure 2. Step response of reduced order model

Figure 3 represents the comparison of time responses of the closed loop original plant with controllers. These controllers are computed with the help of the original system and lower order systems. Simulation result shows that the obtained controllers perform well under both steady state and transient responses. It is also clear from the frequency response that the response of proposed controller matches with the reference model at lower as well as higher frequencies, as shown in Figure 4.

The step response graph for closed loop system is illustrated in Figure 3, showing the performance of different reduced models compared to the original and reference systems. The response curves for six models—Original, Proposed, Reference, N. Singh, S. Rao, and A. Narwal—are displayed, each representing how the system responds to a step input over time.

The Proposed reduced order model (black line) closely follows the reference model (red line), demonstrating a well-balanced response with a quick rise time and no overshoot. This suggests that the proposed reduced order model effectively replicates the desirable characteristics of the reference system, including stability and speed. The Original model (blue line), which serves as a baseline, shows almost similar initial rise, settling time and overshoot, compared to the proposed and reference models. The N. Singh model (green line) also performs well, with a response curve nearly identical to the reference, indicating its effectiveness in achieving a stable system with a quick rise time. The S. Rao model (purple line), while maintaining stability, exhibits a slightly slower response, which is evident from its delayed rise and peak times. Lastly, the A. Narwal model (yellow line) aligns closely with the other models, showing consistent performance with a stable and smooth response. Overall, the proposed model stands out as it mirrors the reference model's performance most closely, making it an optimal choice for balancing system stability and response speed.

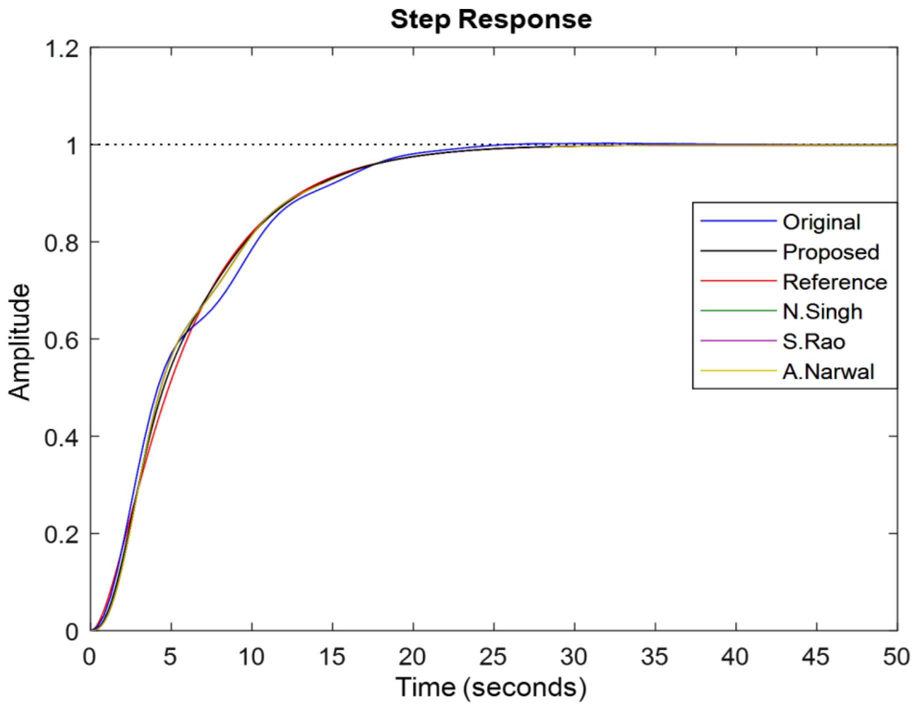

Figure 3. Comparison of Step Response closed loop performance (Example 1).

It is also clear from the frequency response that the response of proposed controller matches with the reference model at lower as well as higher frequencies, as shown in Figure 4.

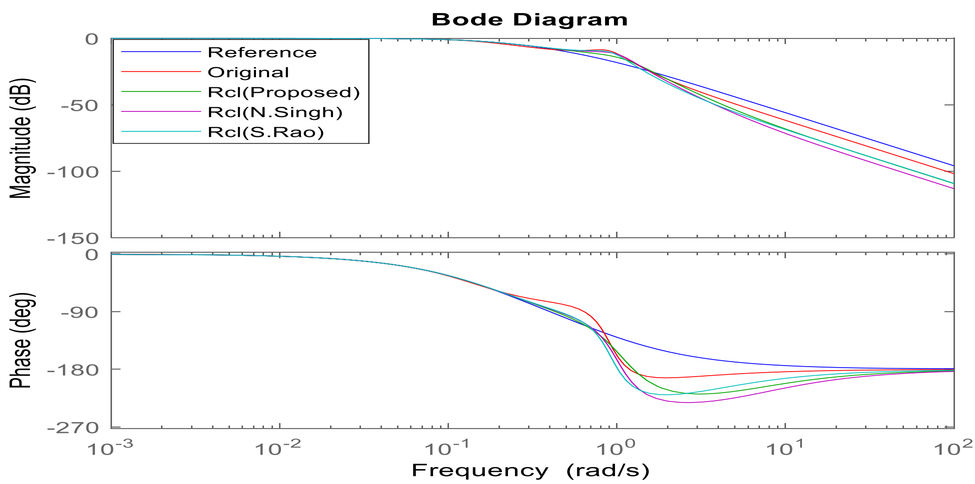

Figure 4 Frequency response of closed loop system

Table 1 shows the time domain specifications of the closed loop systems with controllers. From this table, it can be seen that the time domain specifications of the closed loop system with the controller calculated by using lower order systems are approximately matched with the specifications of the closed loop models with controllers design by using the original system. The controller design by using lower order system is comparatively easier than the design of controller by using the higher dimensional system. This table also indicates that the time domain specifications of the closed loop models with the controllers are approximately the same as the specification of the required reference system. Hence, the proposed method can be used for the design of controller for obtaining the required performances of the dynamical systems

Table 1 compares various Model Order Reduction (MOR) methods based on their impact on key system performance metrics: rise time, settling time and overshoot. These metrics evaluate how well each method preserves the original system's dynamic response, highlighting the trade-offs and effectiveness of each approach.

Table 1: Comparison of various MOR methods

Reduced Techniques	Reduced Model	PID Controller (K_1,K_2,K_3)	Rise Time (sec)	Settling Time (sec)	Overshoot (sec)
-	Reference	-	11.560	20.998	0
-	Original	$\mathbf{0.1657,\ 10.479, 9.2}$	12.186	19.816	0
Proposed	$\frac{\mathbf{0.14s+0.7503}}{\mathbf{0.6289s^2+0.6945s+0.81}}$	$\mathbf{0.17288,\ 2.6423,\ 2.972322}$	11.475	21.069	0
N.Singh[28-29]	$\frac{\mathbf{0.1634s+1.053}}{\mathbf{s^2+0.7206s+1.09}}$	$\mathbf{0.1657,\ 2.2021,\ 2.6965}$	11.358	21.088	0
S.Rao[30]	$\frac{\mathbf{0.958439s+0.32069}}{\mathbf{0.7026s^2+1.7492s+0.33}}$	$\mathbf{0.1657,\ 5.949868,\ 4.6708013}$	11.8	22.01	0

The Table 1 compares the performance of various reduced techniques applied to the original higher order system, highlighting key metrics such as rise time, settling time and overshoot. Among the models, the proposed model stands out for its balanced performance. The proposed model's and its PID controller gains are (K1 = 0.17288), (K2 = 2.6423), and (K3 = 2.972322). This configuration achieves a slightly improved rise time of 11.475 seconds compared to the original model, while maintaining a settling time of 21.069 seconds and no overshoot, closely aligning with the reference system's performance.

Example 2: Consider the regulator problem as per [31].

$$\boldsymbol{G_p(s) = \frac{s^5+8s^4+20s^3+16s^2+3s+2}{s^6+18.3s^5+102.4s^4+209.5s^3+155.9s^2+33.6s+2}}$$

$$\boldsymbol{M(s) = \frac{0.023s+0.0121}{s^2+0.21s+0.0121}}$$

The transfer function of the open loop is illustrated below

$$\widetilde{M}(s) = \frac{0.023s^3 + 0.01693s^2 + 0.002819s + 0.0001464}{s^4 + 0.397s^3 + 0.05137s^2 + 0.002263s}$$

The desired controller structure is given by

$$G_c(s) = \frac{\widetilde{M}(s)}{G_p(s)}$$

$$= \frac{0.0223s^9 + 0.4378s^8 + 2.668s^7 + 6.603s^6 + 7.424s^5 + 4.018s^4 + 1.085s^3 + 0.1514s^2 + 0.01056s + 0.0002928}{s^9 + 8.397s^8 + 23.23s^7 + 24.35s^6 + 10.4s^5 + 4.058s^4 + 0.9843s^3 + 0.1095s^2 + 0.004525s}$$

$$= \frac{1}{s}(0.0647 + 0.767s + 0.8228s^2 - 4.971s^3 - 7.1488s^4 + \cdots)$$

Taking a PID controller structure as

$$G_c(s) = K_1 + \frac{K_2}{s} + K_3 s$$

The parametersK_1, K_2,and $K_3$of the controller are derived by matching coefficients with the power series expansion, leading to the formulation of the PID controller as follows:

$$G_c(s) = 0.7669 + \frac{0.0647}{s} + 0.8228s$$

The corresponding closed loop transfer function is

$$G_{cl}(s) = \frac{0.8228s^7 + 7.349s^6 + 22.66s^5 + 29.02s^4 + 16.03s^3 + 4.981s^2 + 1.728s + 0.1294}{1.823s^7 + 25.65s^6 + 125.1s^5 + 238.5s^4 + 172s^3 + 38.58s^2 + 3.728s + 0.1294}$$

The reference model and equivalent open loop model are the same as that for the original system. The original system is reduced to second order model using GWO

$$R_p(s) = \frac{-0.1985s + 0.277}{1.065s^2 + 4.997s + 0.2356}$$

The controller structure is obtained as

$$R_c(s) = \frac{\widetilde{M}(s)}{R_p(s)} = \frac{0.024495s^5 + 0.132961s^4 + 0.09302s^3 + 0.0182299s^2 + 0.0013957s + 0.000034491}{-0.1985s^5 + 0.1982s^4 + 0.099779s^3 + 0.0137798s^2 + 0.00062685s}$$

$$= \frac{1}{s}(0.0550227 + 1.01698652s - 2.032511765s^2 + \cdots)$$

Taking a PID controller structure as

$$R_c(s) = K_1 + \frac{K_2}{s} + K_3 s$$

By matching the coefficients, the parameters K_p, K_i, and K_d of the controller are derived, resulting in the following PID controller:

$$R_c(s) = 1.01698652 + \frac{0.0550227}{s} - 2.032511765s$$

The corresponding closed loop transfer function is

$$R_{cl}(s) = \frac{0.4035s^3 + 0.7649s^2 + 0.2708s + 0.0152}{1.4685s^3 + 4.2321s^2 + 0.5064s + 0.0152}$$

The step reactions of the lower order and higher order systems are contrasted in Figure 5. When compared to some other conventional ways, it is evident that the response of the reduced model calculated by the suggested scheme is substantially closer to the response of the provided reference model.

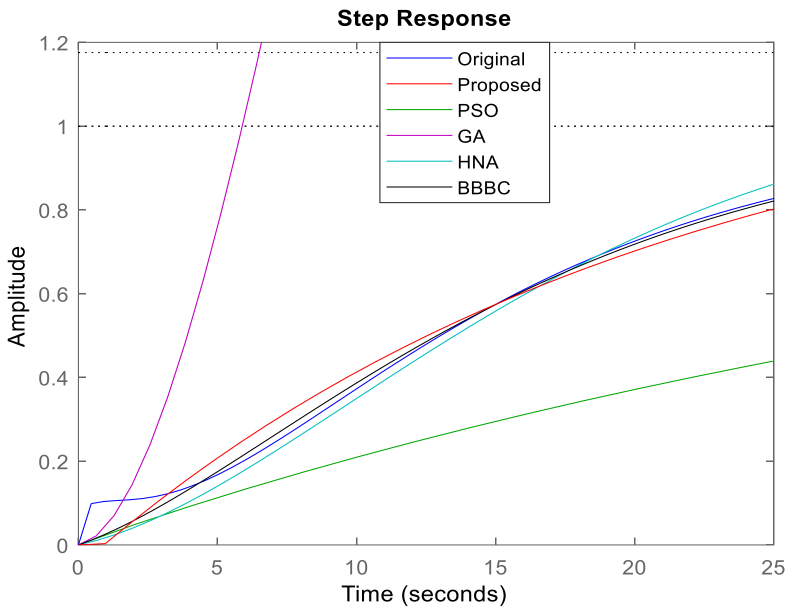

Figure 5. Step response of reduced order model

The plot in Figure 6 shows how a system's response changes over time using different modeling and optimization techniques. The x-axis represents time in seconds, and the y-axis shows the system's response strength. The plot compares the original system's response with other methods, including a reduced model with a reference model, and models are optimized using different algorithms like PSO, GA, HNA, and BBBC. All the responses eventually converge to a steady-state value of 1, but they exhibit slight differences in the transient response phase, where the system is adjusting to the step input. The original and reference models serve as benchmarks, with the optimized models aiming to closely match or improve upon these responses. Each method shows variations in overshoot, settling time, and overall performance, highlighting the differences in how these optimization techniques influence the system's behaviour.

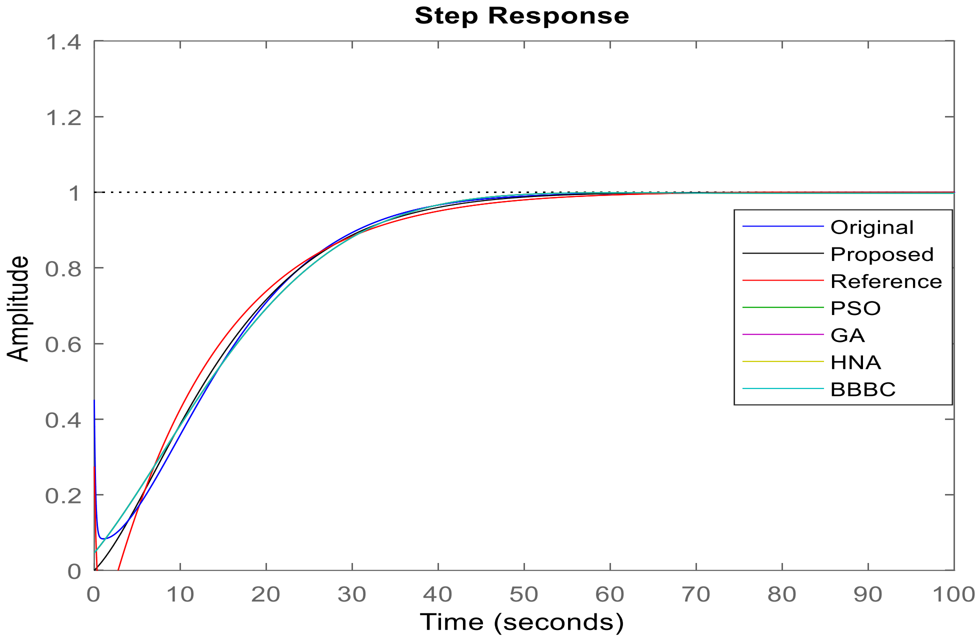

Figure 6 Comparison of Step response of close loop performance (Example 2)

It is also observed through the frequency response characteristic that at lower frequency the response of proposed controller matches with the reference model response while for higher frequency the response is near to response of original closed loop system as shown in Figure 7

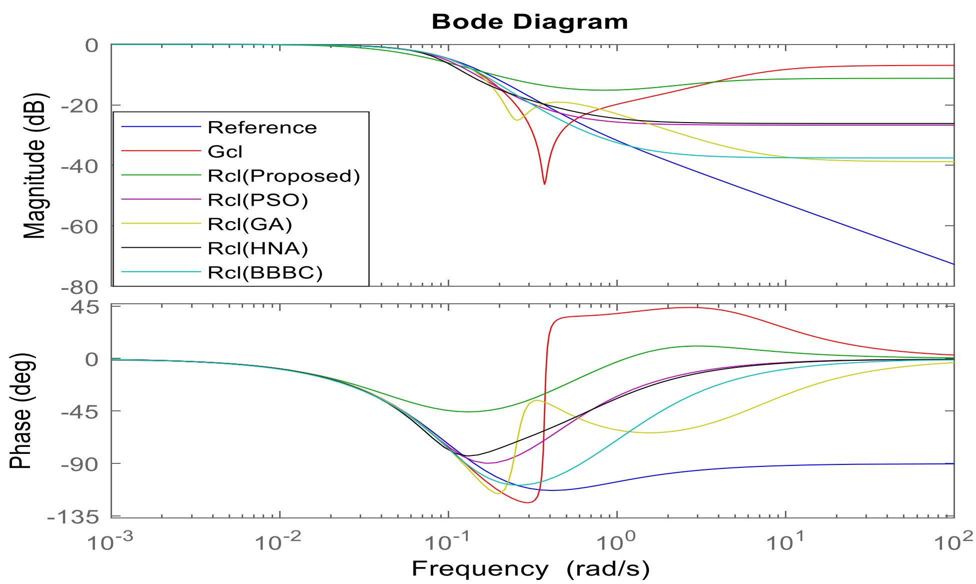

Figure 7. Frequency response of closed loop system

Table 2 give the comparison between the reduced model, the original system, and various optimization techniques such as PSO, GA, HNA, and BBBC reveals key differences in performance. The Original system offers a 20.36% faster rise time and slightly better settling time by 2.38%, with no overshoot. The Proposed model provides a balanced performance with a 2.43% faster rise time, zero overshoot, but a 5.96% slower settling time. The PSO technique stands out with a 5.63% faster settling time, though it has a 2.86% slower rise time. The GA method eliminates overshoot with a 1.95% faster settling time. The HNA technique improves settling time by 8.88% but introduces significant overshoot, with a 7.14% slower rise time. The BBBC approach shows minimal variation from the reference, with a slight slowdown in rise and settling times but eliminates overshoot. The Proposed model and original system effectively eliminate overshoot but vary in other performance aspects.

Table2. Comparison of various MOR methods.

Reduced Techniques	Reduced Model	PID Controller (K_1,K_2,K_3)	Rise Time (sec)	Settling Time (sec)	Overshoot (sec)
-	Reference	-	28	46.2	0
-	Original	0.767859, 0.064707, 0.801795	22.3	45.1	0
Proposed	$\frac{-0.1985s+0.277}{1.065s^2+4.997s+0.2356}$	1.01698652,0.0550227, 2.032511765	27.32	48.9580	0
PSO[33]	$\frac{0.02555s+0.01036}{s^2+0.4756s+0.01036}$	0.714036, 0.06471, 1.89926	28.8	43.6	0
GA [34]	$\frac{0.0113s+0.0736}{s^2+0.1436s+0.009369}$	0.714036, 0.06471, 1.89926	28	45.3	0
HNA [28-29]	$\frac{0.01414s+0.009369}{s^2+0.1436s+0.009369}$	0.671, 0.0647, 3.668	30	42.1	0.95
BBBC [35]	$\frac{0.0233s+0.01176}{s^2+0.2035s+0.01176}$	0.06191, 0.7625, 0.5764	28.2	47.5	0

6. Conclusion

In this paper, the task of designing PID controllers has been accomplished successfully for reduced order model. The direct method of controller design is considered in this paper. The reduction of the closed loop system is performed using GWO by reducing the error between the reference model and the reduced model. Later, the unknown controller parameters have been found using pade approximation technique. Illustrative examples from the accessible literature have been solved to support the appropriateness of the suggested approaches. The step replies demonstrate how effective the suggested GWO approach is at reducing model order.

There are two numerical examples that illustrate the suggested algorithm. The comparison of step response in the first example shows that the suggested approach provides a closer approximation to the large scale model. At lower frequencies, the suggested controller's response is comparable to that of the reference model, while at higher frequencies, it approaches that of the original closed loop system. In second example comparison of step response and frequency response shows that the proposed method gives closer approximation to the large scale model at all frequencies. A comparison of the step responses of the reference model and close loop reduced order system after including PID controller in loop is shown in figure 2 and figure 5. Table-1 and Table-2 gives the comparison on the basis of various parameters of original plant and second order reduced model and closed loop responses. This method is simple, proficient and takes little computational time.

Data Availability: The author confirm that the data supporting the findings of this study are available within the article [and/are] its supplementary materials.

References

1. Gautam RK, Singh N, Choudhary NK and Narain A (2018) Model order reduction using factor division algorithm and fuzzy c-means clustering technique. Transactions of the Institute of Measurement and Control 41(2): 468–475.
2. Haider S, Ghafoor A, Imran M and Fahad MM (2018) Time-limited Gramians-based model order reduction for second-order form sys-tems. Transactions of the Institute of Measurement and Control 41(8): 2310–2318.
3. Narain A, Chandra D and Singh RK (2014) Model order reduction using Fuzzy C-Means clustering. Transactions of the Institute of Measurement and Control 36(8): 992–998.
4. Paul S and Chang J (2018) Fast numerical analysis of electric motor using nonlinear model order reduction. IEEE Transactions on Mag-netics 54(3): 1–4.
5. Rosenzweig P, Kater A and Meurer T (2018) Model predictive con-trol of piezo-actuated structures using reduced order models. Control Engineering Practice 80: 83–93.
6. Davison EJ (1966) A method for simplifying linear dynamic sys-tems. IEEE Transactions on Automatic Control 11(1): 93–101.
7. Shamash Y (1974) Stable reduced-order models using Pade´-type ap-proximations. IEEE Transactions on Automatic Control 19(5): 615–616.
8. Langholz G and Feinmesser D (1978) Model order reduction by Routh approximations. International Journal of Systems Science
a. 9(5): 493–496.
9. Krishnamurthy V and Seshadri V (1978) Model reduction using the Routh stability criterion. IEEE Transactions on Automatic Control 23(3): 729–731.
10. Sinha AK and Pal J (1990) Simulation based reduced order model-ling using a clustering technique. Computers and Electrical Engi-neering 16(3): 159–169.
11. Prajapati AK and Prasad R (2018a) Failure of Pade´ approximation and time moment matching techniques in reduced order modelling. In: Proceedings of the 3rd IEEE International Conference 12 Trans-actions of the Institute of Measurement and Control 00(0) for Con-vergence in Technology (I2CT -2018), Pune, India, 6–8 April 2018, pp. 1–4. IEEE Xplore.
12. Shamash Y (1980) Failure of the Routh-Hurwitz method of reduc-tion. IEEE Transactions on Automatic Control 25(2): 313–314.
13. Antonio L and Umberto V (1982) An improvement in the Routh-Pade´ approximation techniques. International Journal of Control 36(4): 643–661.
14. Kumar DK, Nagar SK and Tiwari JP (2013) A new algorithm for model order reduction of interval systems. Bonfring International Journal of Data Mining 3(1): 6–11.
15. Prajapati AK and Prasad R (2018b) Reduced order modelling of LTI systems by using Routh approximation and factor division methods. Circuit, System and Signal Processing 38(7): 3340–3355.
16. Prajapati AK and Prasad R (2018c) Order reduction of linear dynam-ic systems by improved Routh approximation method IETE Journal of Research: 1–14.
17. Sikander A and Prasad R (2017) A new technique for reduced-order modelling of linear time-invariant system. IETE Journal of
a. Research 63(3): 316–324.
18. Vishwakarma CB and Prasad R (2008) Clustering method for reduc-ing order of linear system using Pade´

approximation. IETE Journal of Research 54(5): 326–330.
19. P. Bhadauria, N. S. Pal and J. Singh, "Model Order Reduction using Grey Wolf Optimization and Factor Division Method," 2019 2nd In-ternational Conference on Power Energy, Environment and Intelli-gent Control (PEEIC), Greater Noida, India, 2019, pp. 508-512, doi: 10.1109/PEEIC47157.2019.8976654. keywords: {Order reduc-tion;Integral square error;Grey Wolf Optimiza-tion;FactorDivision;Stability;Linear Time-Invariant System},
20. Soloklo HN, Bigdeli N. Fast-Dynamic Grey Wolf Optimizer for solv-ing model order reduction of bilinear systems based on multi-moment matching technique. Applied Soft Computing 2022;130:109730. https://doi.org/10.1016/j.asoc.2022.109730.
21. S. Mirjalili, S. M. Mirjalili and A. Lewis, "Grey Wolf Optimizer" in Adv. Eng. Softw., ELSEVIER, vol. 69, pp. 46-61, 2014.
22. Bhatnagar U, Gupta A. Application of grey wolf optimization in or-der reduction of large scale LTI systems. 2017 4th IEEE Uttar Pra-desh Section International Conference on Electrical, Computer and Electronics (UPCON) 2017. https://doi.org/10.1109/upcon.2017.8251132.
23. Padhy, S., Panda, S. Application of a simplified Grey Wolf optimiza-tion technique for adaptive fuzzy PID controller design for frequency regulation of a distributed power generation system. Prot Control Mod Power Syst 6, 2 (2021). https://doi.org/10.1186/s41601-021-00180-4
24. Hou, Y.; Gao, H.; Wang, Z.; Du, C. Improved Grey Wolf Optimiza-tion Algorithm and Application. Sensors 2022, 22, 3810. https://doi.org/10.3390/s22103810
25. LAM, J. (1993). Model reduction of delay systems using Pade ap-proximants. International Journal of Control, 57(2), 377–391. https://doi.org/10.1080/00207179308934394
26. V. D. Hajare and B. M. Patre, "Design of PID controller based on re-duced order model and Characteristic Ratio Assignment method," 2013 IEEE International Conference on Control Applications (CCA), Hyderabad, India, 2013, pp. 1270-1274, doi: 10.1109/CCA.2013.6662927. keywords: {Mathematical model; Con-trolsys-tems;Tuning;Computationalmodeling;Uncertainty;Approximationmethods;Robustness}
27. L.A. Aguirre, "New algorithm for closed loop model matching", Electronics Letter, Volume 27, Issue 24, 21 November 1991, p. 2260 – 2262
28. N. Singh, "Reduce order Modeling and Control Design", Ph.D. The-sis, IIT Roorkee, Rorkee, India, 2007.
29. Nidhi Singh, "Controller Design & Reduction using Modified Hankel Norm Approximation Method", National Conference on Re-cent Trends in Energy, Systems and Control, NIT Kurukshetra, March15-16, 2013.
30. S. Rao Potturu, "Reduce order Modeling for Control System De-sign", Ph.D. Thesis, IIT Roorkee, Rorkee, India, 2019.
31. S. R. Desai, Rajendra Prasad . PID Controller Design using BB-BCOA Optimized Reduced Order Model. Advanced Computing and Communication Technologies for HPC Applications. ACCTHPCA, 5 (July 2012), 32-37.
32. Mahalanabis, A.K. (1984), Large scale systems modelling and con-trol, M. Jamshidi, North Holland, 1983. Optim. Control Appl. Meth., 5: 367-367. https://doi.org/10.1002/oca.4660050410
33. S. R. Desai, and R. Prasad, "Design of PID Controller using particle swarm optimized reduced order model," presented at the Interna-tional Conference on Control Instrumentation System Conference, Manipal Institute of Technology,Manipal University,Manipal,India, 3-6 Nov, 2011.
34. S. R. Desai, and R. Prasad, "Genetically optimized model order re-duction for PID controller," presented at the International Confer-ence on System Dynamics and Control, Manipal Institute of Tech-nology,Manipal University,Manipal,India,19-20 Aug., 2010.
35. S. R. Desai, "Reduce order Modelling in Control System", Ph.D. Thesis, IIT Roorkee, Rorkee, India, 2013.
36. Suman, SK, "A New Scheme for the Approximation of Linear Dy-namical Systems and Its Application to Controller Design", Circuits Systems and Signal Processing, Volume 43, Issue 2, Page 766-794,2024 DOI 10.1007/s00034-023-02503-2.
37. Banerjee, R, Biswas, A , Bera, J, "A novel integrated differential-Routh approach to develop reduced order controller with improved performance", Electrical Engineering, Volume 106, Issue 3, Page 3001-3015,2024 DOI 10.1007/s00202-023-02123-8