Assessment of the Dynamic Impact of a Truck on the Bridge Pavement Based on the Proposed Mathematical Model of Vehicle Movement

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The rapid development of road transportation, particularly the increase in weight and dimensions of freight vehicles, presents a critical challenge for Ukraine's road network, especially in the context of active military operations. The suspension of air travel and disruption of rail transportation have led to a surge in road traffic, including on bridges. This study focuses on the assessment of the dynamic impact of trucks on bridge pavements using a proposed mathematical model of vehicle movement. Prior to the full-scale invasion by Russia, Ukraine had over 16,000 bridges on public roads. Studies on the technical condition of these bridges have exposed a significant disparity between the regulated service life and their actual lifespan. Most reinforced concrete bridge structures exhibit defects, which indicates the need for extensive repairs or reconstruction. The existing guidelines for calculating bridge load capacity, based on outdated load models during the design approval stage, do not align with the projected traffic flow development. This research addresses the dynamic load on road surfaces, which intensifies with truck traffic, leading to significant deformations. Such deformations, along with other factors, contribute to pavement deterioration and impact the integrity of bridge spans. The study considers the influence of elastic tire deformation on vehicle stability and controllability, emphasizing the importance of accounting for these factors during the design and construction phases. In the context of martial law, determining the actual bearing and carrying capacity of bridges is crucial to ensure the country's defense capability. The results of this study provide insights into the impact of truck dynamics on bridge pavements and contribute to enhancing bridge design, maintenance, and operational strategies. By emphasizing the need for updated load models and considering the dynamic effects of truck traffic, this research offers practical implications for improving the safety and sustainability of bridge infrastructure based in Ukraine.

Keywords: Dynamic Load, Road Bridge, Load Capacity, Truck, Load Model, Car Wheel, Road Surface, Road Surface Deformation, Pothole.

1. Introduction

Bridge crossings are one of the most crucial components of the state road transportation system. The experience of developed countries indicates that the maintenance of transportation infrastructure accounts for up to 50 percent of expenditures in the road sector. It is imperative to ensure that Ukraine's bridges can accommodate modern trucks.

The rapid development of road transport, particularly during times of martial law, aims to increase the weight and dimensions of trucks, which poses a critical problem for Ukraine's outdated road network. According to the Ministry of Communities and Territories Development of Ukraine, a total of 322.6 million tons of freight were transported. Over the past 60 years, the intensity of traffic flow, as well as the size and weight of road trains, have significantly increased, greatly impacting the technical condition of bridges.

Current methods for determining the load capacity of bridges are based on theoretical load models, which have been revised five times since 1957. Calculating the load capacity based on these theoretical models is impractical due to the absence of an actual vehicle analog to the AK load models.

These calculation methods are derived from the load models in effect at the time of project approval. However, this approach implies a loss of the original load-bearing capacity rather than an accurate assessment of the actual load capacity in the current traffic flow. During the operation of bridge crossings, dynamic loads arise from vehicle traffic, resulting in pavement defects that vary in their geometric parameters.

Bridge engineers assess the technical condition of the road surface during bridge inspections, using a five-point scale. Deteriorating bridge deck conditions lead to several changes, including increased levels of vehicle vibration, greater energy consumption for vibrations (efficiency), reduced average speed (efficiency), increased overhaul mileage (reliability), and diminished driving comfort. These factors contribute to elevated dynamic forces acting on both the vehicle suspension and the contact patch between the vehicle wheel and the bridge pavement.

This paper will present a mathematical model of truck movement that considers the deformations of the road bridge pavement. This model will provide a means to determine the dynamic load acting at the contact point between the car wheel and the bridge pavement of the crossing. The application of this model will enhance the methodology for calculating bridge load capacity, providing clearer insights into its technical condition and service life.

By studying the dynamic impact on bridge-bearing structures resulting from defects (irregularities) in the approaches and bridge deck pavement, it will be possible to predict the degradation rate of individual bridge elements and propose preventive maintenance measures. The outcome of this dynamic impact study will be a dynamic coefficient for the proposed load model, which can be applied based on the condition of the existing pavement and the smoothness of the bridge's longitudinal profile.

1.1. Literature Review and Problem Statement

Most of the bridges in Ukraine were constructed according to outdated standards from the previous century [1,2]. Consequently, there is an urgent problem regarding the assessment of the actual load capacity of these bridges and determining their current condition under the influence of modern traffic flow [7,10]. The transport industry of Ukraine needs to update its regulatory and technical framework to align with the requirements of developed countries, particularly the European Union, North America, and Asia, where constant monitoring of its relevance is undertaken, and investments in the latest developments are made to enhance the reliability and durability of bridge crossings and the overall transport network [5,6,9]. The issue of assessing the load capacity and bearing capacity of bridges, including their reliability and service life, is explored in several studies [3,4,8], where the socioeconomic significance of such research is emphasized.

The main focus of the scientific investigations presented [1,2] is to examine the influence of defects in existing bridges on their bearing capacity and the changes in the stress-strain state of monolithic span structures after prolonged operation. Bodnar et al. [1] discuss the modern concept of the bearing capacity of road bridges, including its key elements and types depending on the achievement of limit states of groups 1 and 2. Their research provides examples of the loss of bearing capacity in bridge structures due to reaching their limit states and identifies the causes of structural failure. The study of Rozdorozhniuk et al. [2], based on bridge testing, determined the parameters of spatial operation and the stress-strain state of the span structure by measuring absolute deformations (deflections) under static load (Figure 1).

Figure 1. Part A and Part B – General View of the Test Load [2]

The findings from previous studies [1, 2] demonstrate that determining the load capacity of bridges based on modern traffic flow, while considering the impact of defects, is a crucial indicator of reliability and durability.

In a research conducted by Skokandi et al. [3], an overview of traffic load models and recommendations for utilizing weighing data in motion when assessing the condition of existing road bridges were provided. Additionally, de Souza Mendes et al. [4] examined the parameters of trucks in motion on a motorway section with a bridge. The objective of this study was to reduce fuel consumption and harmful gas emissions while increasing the bridge's capacity.

Wang et al. [5] presented the relationship between elastic deformation of bridge load-bearing elements over time, taking into account the impact of structural degradation and moving temporary loads. This was done to determine reliability. The scientific article of Gocal et al. [6] builds upon the analytical study presented [5] and compares the effects of different transport model loads.

A research team from Rutgers University has developed cost-effective, portable, non-destructive equipment for testing bridge capacity [7]. In addition, the method of testing a moving load was proposed based on constructing a line of influence to assess the bearing capacity of a bridge [8]. The research paper of Putra et al. [9] described the usage of wireless sensors of the WIM (weighing in motion) complex in the system for measuring the load capacity of a structure.

Ling et al. [10] analyze the impact of the Automated Truck Platooning (ATP) system on the durability of bridges on public roads in the People's Republic of China. The ATP system consists of a convoy of trucks designed to reduce the impact of the load on bridges and the environment.

A method for determining the road profile based on the analysis of vehicle accelerations in motion is presented by Keenahan et al. [11]. The proposed direct integration algorithm is based on a specific response of the vehicle acceleration change, which is determined from the Half-Car and road models. This approach, combined with the entropy method, considers the concept of fleet monitoring for subsequent inspections of roads and bridges. It enables the determination of the road profile, considering predefined vehicle design features and using sensors integrated into the roadway.

The analytical method for calculating the road profile based on specified or measured vehicle accelerations is proposed by Ngwangwa & Galatioto [12]. This research was conducted using a Quarter-car model. The developed approach yields positive results, primarily at lower vehicle speeds, which may not fully meet the requirements for the operation of high-category roads. However, it is important to note that a truck traveling at higher speeds has a more destructive effect on the road surface.

The study discussed by Gong & Chen [13] focuses on the investigation of fatigue crack development on bridge pavements under the influence of vehicle load. The authors developed a three-dimensional finite element model to simulate the interaction between a car wheel and a bridge pavement. The proposed model considers multiple factors, including radius of curvature, transverse and longitudinal slopes, driving speed, and load from the tire-bridge interaction.

The influence of road and bridge pavement conditions on vehicle dynamics was examined [14]. An empirical model of vehicle tire rolling was utilized, considering the effect of interference filtering. This allowed for the detection of asymmetry in wheel oscillations and determination of the residual impact on the pavement. However, the paper does not incorporate suspension workflows, which significantly impact how a car wheel handles road surface defects.

Malekjafarian et al. [15] presented a two-stage approach for detecting bridge pavement damage based on vehicle response and measurement of the spectrum of vertical acceleration was proposed. Vehicle response was predicted based on its speed during multiple passages over the bridge, serving as an indicator for assessing pavement deformations. However, it does not consider the damping coefficient of a car wheel tire.

A practical methodology for modeling vehicle oscillations when driving on soft off-road ground is presented in the article of Taghavifa et al. [16]. An analysis of the off-road vehicle model was conducted to determine optimal parameters for the vehicle suspension design. The study considers reactions to suspension mass oscillations and changes in dynamic load acting on the tire in contact with the soft ground.

In the study of Rys [17], a methodology for considering dynamic loads from a vehicle for pavement design is presented. The authors calculated dynamic axle load spectra for pavements with varying flatness and vehicle speeds. It is established that the detrimental impact of dynamic loads on the axle increases significantly with pavement unevenness deterioration, particularly at high vehicle speeds.

Additionally, an increase in the elastic modulus of non-rigid pavements does not compensate for the adverse dynamic effects. However, the paper does not provide mathematical dependencies for determining the dynamic load of a vehicle on the road surface.

The objective of Papaioannou et al. [18] is to determine the ability of active and semi-active suspensions to reduce road train tire wear. The developed mathematical model includes a tread study to investigate tire wear during road curves. A novel design of active suspension based on the H-approach is proposed and compared with passive, semi-active, and other active suspension systems. The suspensions are compared in terms of tire wear levels, as well as other factors (comfort, road stability, etc.) that affect the efficient operation of the vehicle.

The study of Razboynikov [19] was conducted to enhance the directional stability of a passenger car when driving on uneven roads through the control of active suspension processes. A calculation scheme and mathematical model of car movement on uneven roads were developed. Theoretical studies identified the causes of directional stability loss in such conditions. Requirements were formulated, and an algorithm for the operation of the car's active suspension was developed. The results of theoretical and experimental studies validate its effectiveness.

The method for modeling the load impact of vehicles on bridges based on the data from Weigh-in-Motion (WIM) systems is presented in the research of Sun et al. [20]. This proposed approach enables a more accurate modeling of load variables by incorporating actual vehicle parameters.

Oluwafemi Oguntayo et al. [21] described the determination of the detrimental impact of trucks on the road surface. The authors considered the distribution of axle loads, tire air pressure, and contact area with the road surface for four classes of trucks with different axle configurations. However, it is important to note that the study only involved static measurements.

Moreover, the road profile is modeled by solely altering the vertical coordinate, while the longitudinal coordinate of the road profile corresponds to the longitudinal coordinate of the car wheel's center [11, 12, 15-18]. The changes in the inclination angle of the tire contact patch with a pavement defect are not considered in these works. Additionally, in the mathematical modeling of suspension work processes [11-15], the coefficient of shock absorber resistance during the rebound and compression stroke is assumed to be the same. However, in reality, the coefficient of resistance of the shock absorber during the compression stroke is typically four times less than during the rebound stroke. Furthermore, authors [16, 18, 19, 20] neglect the movement of trucks on roads with pavement defects, which leads to dynamic loading on the pavement and bridge structures. This dynamic loading has a significant impact on maintenance, reliability analysis, and safety assessment of bridge structures.

Therefore, there is a pressing need to research the determination of the dynamic load from a truck wheel on a road surface with a defect, particularly in the case of a pothole. Additionally, the effect of changing the geometric parameters of the pothole on the dynamic load from the truck should be considered. It is evident that the dynamic load will be greater at higher speeds.

Consequently, investigating truck movement at the maximum speed limited by traffic rules in Ukrainian cities is essential. This is the objective undertaken by the authors in this paper.

1.2. The Purpose and Objectives of the Research

The purpose of this research is to evaluate the dynamic impact of a truck on a bridge deck with an increasing pothole using a mathematical model.

To accomplish the established goal, the following tasks were addressed in this study:

- Analysis of the factors determining the dynamic impact of a truck on a bridge deck with a pothole.
- Justification of the initial data and conditions for a mathematical model representing the maneuver of a truck wheel when encountering a pothole on the bridge pavement.
- Development of a mathematical model for simulating the maneuver of a truck wheel when encountering a pothole on the bridge pavement.
- Conducting theoretical research and analysis of the dynamic impact of a truck on a bridge deck with an increasing pothole.

The underlying concept of this research is that considering the dynamic load resulting from a truck traversing a pothole on the road surface will enable a more accurate determination of the actual load-bearing capacity of bridge crossings. Consequently, this can contribute to enhancing their reliability and durability.

2. Materials and Methods

The object of the scientific research is the dynamic load generated by a truck on a bridge deck. The subject of the research is the assessment of the impact variation of a vehicle on a road surface with an increasing pothole.

2.1. Analysis of Factors Affecting the Dynamic Impact of a Truck on a Bridge Deck with a Pothole

Overcoming a pothole on the road surface of a bridge by a truck wheel results in a change in the dynamic response of the supporting surface. This reaction is transmitted to the sprung mass through the elastic tire and suspension of the truck.

Consequently, both forced (during the pothole impact) and free (after the impact) oscillations of the sprung and unsprung masses of the truck occur. Hence, even after successfully navigating a pothole on the road surface of a bridge, the truck's movement will still be accompanied by a change in the dynamic reactions of its wheels. According to Newton's third law, the dynamic load occurring at the tire's contact with the bridge's supporting surface will have an equal magnitude but opposite direction to the reaction from the supporting surface.

Therefore, the dynamic load generated by a truck depends on various parameters, such as the vehicle's characteristics (driving mode, mass parameters, geometric parameters, suspension stiffness, shock absorber characteristics, as well as the elastic and damping properties of its tires) and the geometric parameters of the pothole on the road surface (profile shape, depth, length, etc.).

2.2. Selection and Justification of Initial Data and Conditions for Mathematical Modeling of Overcoming a Pothole on the Pavement of a Bridge Crossing by a Truck Wheel

The dynamic load experienced during the overcoming of a pothole on the pavement of a bridge crossing by a truck wheel is influenced by various factors. In this study, the following initial data and conditions have been selected and substantiated:

1. Speed and Movement: The dynamic load is more significant at higher speeds. According to traffic regulations in populated areas of Ukraine, the maximum speed is limited to 50 km/h. Hence, this study considers straight-line, uniform movement with a speed of 13.9 m/s, which corresponds to the maximum permitted speed limit.

2. Truck Characteristics: A truck, such as a tractor with a trailer considered in previous work [18], is characterized by a substantial mass (as shown in Table 1) and consequently possesses significant inertia. Additionally, at the speeds considered in this study, the truck accumulates high kinetic energy in the longitudinal direction of movement. Therefore, it is assumed that there is no change in longitudinal acceleration due to overcoming a pothole on the road surface.

3. Rolling Resistance and Air Environment Resistance: The rolling resistance of the car wheel and the resistance of the air environment are not considered in this study.

Although these factors have minimal effects on the vertical dynamics of the fluctuations of the sprung and unsprung masses of a truck moving over a bridge crossing with a pothole on its surface, their exclusion significantly simplifies and reduces the complexity of mathematical dependencies describing this process. This approach has also been utilized in the works [11, 12, 15-18].

By carefully selecting and justifying these initial data and conditions, the mathematical modeling process in this study aims to provide a clear understanding of the dynamic load experienced by a truck wheel when overcoming a pothole on the pavement of a bridge crossing.

Table 1. The Main Parameters of the Truck [18]

Designation	Parameter name, measurement unit	Front axle	Rear axle
m_s	Equivalent sprung mass reduced to the wheel, kg	2600	2000
m_u	Unsprung weight, kg	250	250
k_s	Reduced stiffness of the elastic suspension device, N/m	365000	566000
c_s^{com}	Reduced coefficient of shock absorber resistance during suspension compression, N·s/m	3500	2750
c_s^{reb}	Reduced coefficient of shock absorber resistance during suspension rebound, N·s/m	14000	11000
r	Free radius of the vehicle wheel 315/70R22.5, m	0.499	0.499
k_t	Radial stiffness of the tire, N/m	103400	103400
c_t	Tire damping coefficient, N·s/m	12000	12000

The most common deformations of the road surface of bridge crossings are potholes. Such types of deformations of the road surface are characterised by accelerated destruction.

Therefore, the paper considers a pothole's geometric parameters with five characteristic destruction stages (Table 2).

Table 2. Geometrical Parameters of the Growing Pothole (from the Destruction Stage №1 to №5)

Stage №	Pothole depth h_q, mm	Pothole length l_q, m
1	60	0,8
2	75	1,2
3	90	1,2
4	105	1,4
5	120	1,6

The geometric parameters of the pothole in Table 2 are based on field observations of Ukrainian bridges [1, 2].

2.3. A Mathematical Model for a Truck Wheel Overcoming a Pothole on a Bridge Pavement

For the convenience of describing these processes, the quarter-car model is used. The equivalent spring-loaded mass of the vehicle ms reduced to the wheel is concentrated at point S (Figure 2, a), and the unsprung mass mu is concentrated in the center of the wheel (at point U).

The interconnection of these masses occurs through the elastic and dissipative suspension device of the car, which, respectively, are modeled by the stiffness of the elastic suspension device ks reduced to the car wheel and the coefficients of the shock absorber resistance cs in the compression stroke   and rebound .

The center of the vehicle wheel (point U) is connected to the bearing surface of the road through an elastic tire. In the wheel's plane of rotation, the tire is characterized by radial stiffness kt and damping coefficient ct.

The bearing surface of the road is characterized by the angle of attack of the pavement defect βq in the center of contact between the tire and the road surface (point Q) and its longitudinal (Qx) and vertical (Qz) coordinates [19].

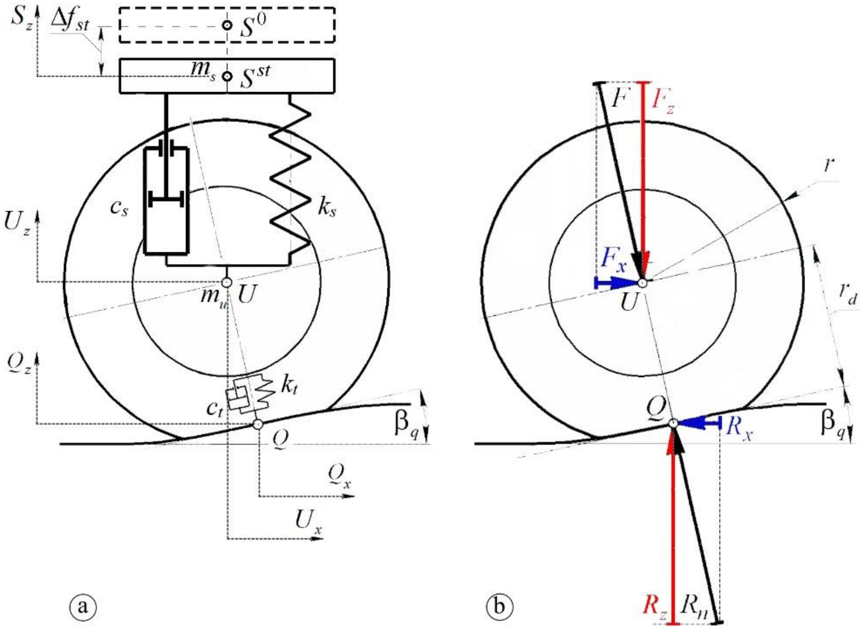

Figure 2. Quarter-Car Model (Part A) and Diagram of the Forces Acting on a Car Wheel while Overcoming a Road Surface Defect (Part B)

Considering the assumptions made, replacing the structural elements of the chassis with the equivalent of the interconnection force (Figure 2, b), a system of equations for the dynamics of vertical vibrations of the vehicle's sprung m_s and unsprung m_u masses was compiled:

$$\begin{cases} m_s \cdot \ddot{S}_z = F_s + F_d - m_s \cdot g \\ m_u \cdot \ddot{U}_z = R_z - F_s - F_d - m_u \cdot g \end{cases} \quad (1)$$

where $\ddot{S}_z$ is the vertical acceleration of the equivalent sprung mass of the vehicle m_s reduced to the wheel, m/s^2;

F_s – the force from the deformation of the elastic suspension device reduced to the plane of rotation of the car wheel, N;

F_d – the force of resistance of the suspension damper device reduced to the plane of rotation of the car wheel, N;

g = 9.81 – acceleration of free fall, m/s^2;

$\ddot{U}_z$ – vertical acceleration of the unsprung mass m_u of the vehicle, m;

R_z – vertical reaction of the bearing surface to the car wheel, N;

The forces from the deformation of the elastic suspension device F_s and the resistance of its damper device F_d, as well as the vertical reaction of the bearing surface to the car wheel, which are part of the system (1), can be represented as:

$$F_s = k_s \cdot \left(f + \Delta f^{st} \right), \qquad (2)$$

$$F_d = \begin{cases} c_s^{com} \cdot \dot{f}, \text{ if } \dot{f} > 0, \\ c_s^{reb} \cdot \dot{f}, \text{ if } \dot{f} < 0 \end{cases} \qquad (3)$$

$$R_z = R_n \cdot \cos\beta_q , \qquad (4)$$

In equations (2)-(4), it is denoted:

f is the current value of the suspension stroke, m;

Δf_{st} – the deformation of the suspension under static load, m;

$\dot{f}$ – the rate of change of the suspension stroke, m/s;

R_n – the normal reaction of the bearing surface to the car wheel, N.

The current value of the suspension stroke f (counting from the position under a static load) and the deformation of the suspension from a static load Δf_{st} (Figure 2, a) are respectively recorded as follows [19]:

$$f = \left(S_z^{st} - U_z^{st} \right) - \left(S_z - U_z \right), \qquad (5)$$

$$\Delta f_{st} = \frac{m_s \cdot g}{k_s}, \qquad (6)$$

Where S_z^{st} is the vertical coordinate of the equivalent sprung mass of the vehicle m_s in the static load position (Figure 2, a), m;

U_z^{st} – the vertical coordinate of the unsprung mass of the vehicle m_u in the position of static loading (before the beginning of overcoming the road surface defect), m.

The normal reaction of the bearing surface included in (4) is equal in modulus and opposite in direction to the action of the potential and dissipative forces of the elastic tire normal to the bearing surface [19], i.e.:

$$R_n = k_t \cdot \left(r - r_d \right) + c_t \cdot \dot{r}_d, \qquad (7)$$

where r is the free radius of the vehicle wheel, m;

r_d – the dynamic radius of the vehicle wheel, m;

$\dot{r}_d$ – the rate of change of the dynamic radius of the vehicle wheel, m/s.

The dynamic radius of the car wheel is determined considering the current vertical coordinate of the wheel center U_z and the contact of the tire with the road surface Q_z, as well as the current angle of attack of the pavement defect β_q [19]:

$$r_d = \begin{cases} \dfrac{U_z - Q_z}{\cos\beta_q}, \text{ if } \dfrac{U_z - Q_z}{\cos\beta_q} < r \\ r \quad - \quad \text{otherwise} \end{cases} \qquad (8)$$

To model a pothole on the road surface, it is proposed to divide its profile into three parts, separated by four key points (a, b, c, d) (Figure 3).

The beginning and the end of the pothole are marked by points a and d, respectively.

The distance between points a and d is equal to the length of the pothole l_q.

Points b and c mark the beginning and end of the most vertically distant section of the pothole bottom, respectively.

The distance between points a and b, and c and d are equal to the depth of the pothole h_q.

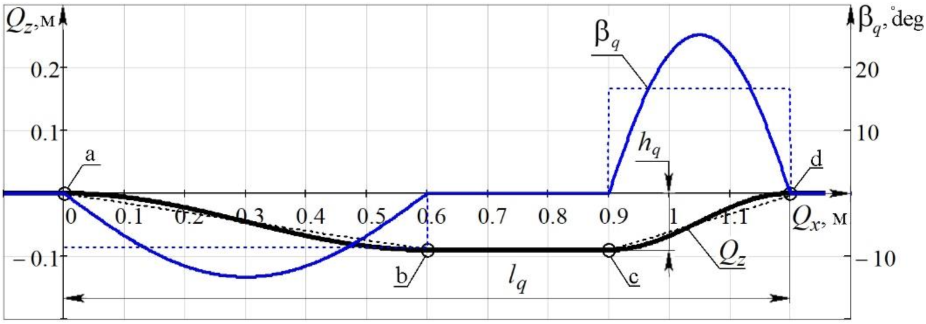

Figure 3. Modelling the Pothole Profile on the Road Surface

The position of each key point of a pothole is determined by its two coordinates. It is proposed to define them as functions of the depth hq and length lq of the pavement defect (see Figure 3).

$$a_x = 0\,;\ b_x = 2\cdot\frac{l_q}{4}\,;\ c_x = 3\cdot\frac{l_q}{4}\,;\ d_x = l_q\,; \quad (9)$$

$$a_z = 0\,;\ b_z = h_q\,;\ c_z = h_q\,;\ d_z = 0\,; \quad (10)$$

By connecting these points with straight lines, we can get a “broken” pothole profile (black dashed curve) (Figure 3), as well as a discrete change in the angle of attack (blue dashed curve). It is known that the sharp edges of potholes are smoothed out by tires, and the roughness of the pavement is absorbed by them. Taking this into account, as well as to avoid discrete changes in the angle of attack (blue dashed curve) (Figure 3) of the pothole, it is proposed to combine the key points a and b, as well as c and d, with harmonic functions. This approach will ensure a gradual change in the pothole attack angle (blue solid curve) (Figure 3).

The current pothole attack angle β_q at the center of contact between the tire and the road surface (point Q) and its vertical coordinate Q_z are determined by considering the current horizontal coordinate of the center of contact between the tire and the road surface Q_x and the position of the key points of the defect (Figure 3).

$$Q_z = \begin{cases} \frac{b_z - a_z}{2} \cdot \left[1 - \cos\left(\frac{Q_x - a_x}{b_x - a_x} \cdot \pi\right)\right] & \text{, if } a_x \le Q_x < b_x \\ b_z & \text{, if } b_x \le Q_x < c_x \\ b_z + \frac{d_z - c_z}{2} \cdot \left[1 - \cos\left(\frac{Q_x - c_x}{d_x - c_x} \cdot \pi\right)\right] & \text{, if } c_x \le Q_x < d_x \\ 0 & - \text{ otherwise} \end{cases}, \quad (11)$$

$$\beta_q = \begin{cases} \arctan\left[\frac{b_z - a_z}{b_x - a_x} \cdot \frac{\pi}{2} \cdot \sin\left(\frac{Q_x - a_x}{b_x - a_x} \cdot \pi\right)\right] & \text{, if } a_x \le Q_x < b_x \\ 0 & \text{, if } b_x \le Q_x < c_x \\ \arctan\left[\frac{d_z - c_z}{d_x - c_x} \cdot \frac{\pi}{2} \cdot \sin\left(\frac{Q_x - c_x}{d_x - c_x} \cdot \pi\right)\right] & \text{, if } c_x \le Q_x < d_x \\ 0 & - \text{ otherwise} \end{cases}, \quad (12)$$

The longitudinal coordinate of the tire contact center with the road surface Q_x is determined considering the longitudinal coordinate of the wheel center U_x, its dynamic radius r_d, and the pothole attack angle β_q (see Figure 2):

$$Q_x = U_x + r_d \cdot \sin\beta_q \qquad (13)$$

Considering the assumptions made in this work, the current longitudinal coordinate of the center of the car wheel U_x is determined considering the speed v_a and time t_a of the truck movement:

$$U_x = v_a \cdot t_a \qquad (14)$$

The set of differential equations (1) and algebraic equations (2)-(14) represents a system for solving which initial conditions are set the parameters of the vehicle movement before the start of overcoming the pothole.

3. Results

The theoretical study involves calculations based on the developed mathematical model and analysis of the dynamic impact of a truck (Table 1) on the pavement of a bridge with an increasing pothole (Table 2). The study focuses on a straightforward, uniform movement with a speed of 13.9 m/s. The calculation step in the modeling is 0.001 s. Based on the calculation results, we obtained the trajectories of motion for the sprung and unsprung masses of the truck (Figure 4), as well as the change in the vertical reactions of the bearing surface to its wheels (Figure 5) during the process of overcoming the pothole.

The analysis of the graphs in Figure 4 and Figure 5 reveals that prior to overcoming the pothole, the vertical coordinates of the vehicle's masses, as well as the vertical reactions of the bearing surface to its wheels, remain constant and correspond to their static values.

However, after the start of overcoming the pothole, a rapid decrease to zero in the vertical reaction of the bearing surface to the wheels of the car is observed (Figure 5), indicating wheel detachment from the road surface. This is accompanied by a downward movement of both the sprung and unsprung masses of the vehicle (Figure 4).

Upon reestablishing contact between the vehicle's wheels and the road surface, there is a rapid increase in the vertical reaction of the bearing surface. For instance, when the front axle wheel of the truck overcomes a pothole with geometric parameters No. 1 (Table 2), the vertical reaction reaches a value of 55 kN; No. 2 – 59 kN; No. 3 – 64 kN; No. 4 – 70 kN; No. 5 – 79 kN (Figure 5, a).

Similarly, when the rear axle wheel of the truck overcomes a pothole with geometric parameters No. 1 (Table 2), the vertical reaction reaches a value of 45 kN; No. 2 – 49 kN; No. 3 – 54 kN; No. 4 – 61 kN; No. 5 – 70 kN (Figure 5, b).

The upward movement of the unsprung mass of the vehicle is observed (Figure 4), both for the front axle (Figure 4, a) and the rear axle (Figure 4, b).

However, the unsprung mass continues to move downward due to inertia (Figure 4). Subsequently, a brief decrease in the vertical reaction to the car wheels is observed, followed by a subsequent increase.

The second local maximum of the vertical reaction, when the front axle wheel of the truck overcomes a pothole with geometric parameters No. 1 (Table 2), reaches a value of 36 kN; No. 2 – 40 kN; No. 3 – 43 kN; No. 4 – 49 kN; No. 5 – 54 kN (Figure 5, a).

Similarly, when the rear axle wheel of the truck overcomes a pothole with geometric parameters No. 1 (Table 2), the second local maximum of the vertical reaction reaches a value of 33 kN; No. 2 – 38 kN; No. 3 – 44 kN; No. 4 – 50 kN; No. 5 – 55 kN (Figure 5, b).

These changes in the vertical reaction are accompanied by a change in the direction of motion of the sprung mass for both the front axle (Figure 4, a) and the rear axle of the truck (Figure 4, b).

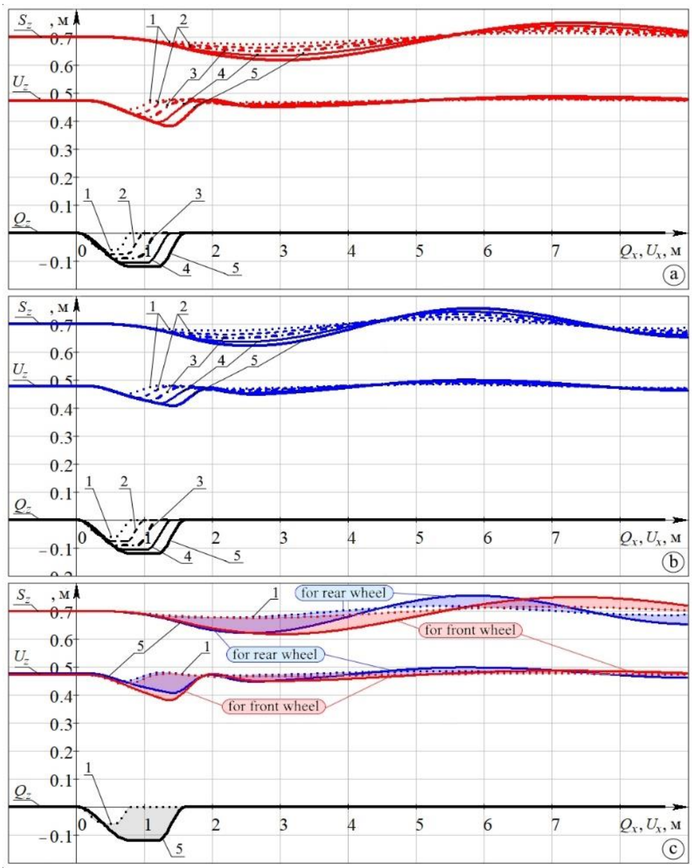

Figure 4. Dependence of the Change in the Vertical Coordinates of the Sprung S_z and Unsprung U_z Masses on the Longitudinal Coordinate of the Wheel Center U_x, as well as the Vertical Coordinate of the Road Surface Q_z on its Horizontal Coordinate Q_x. Part A –for the front axle; Part B – for the rear axle; Part C – for the front and rear axles of the vehicle; 1, 2, 3, 4, 5 – for a pothole with geometric parameters of destruction stages 1 – 5 (Table 2)

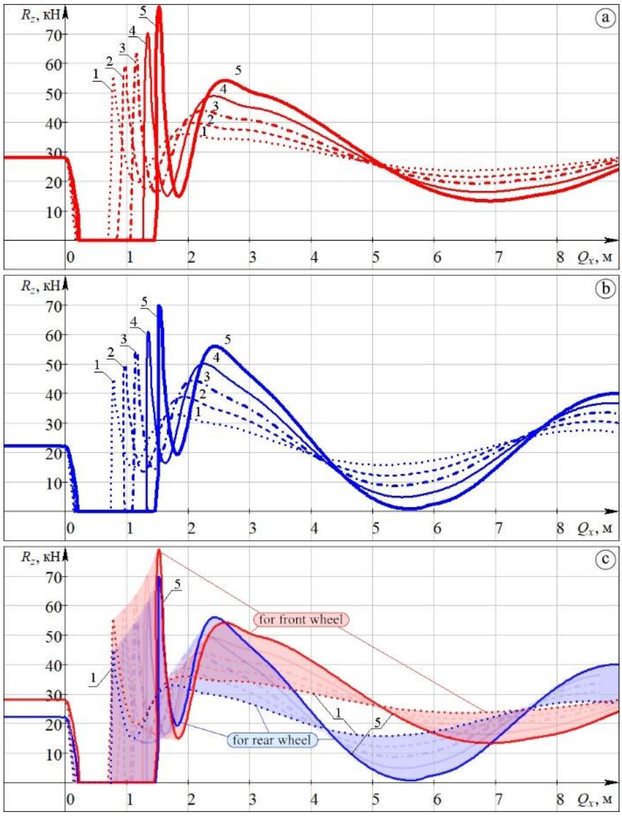

Figure 5. Dependence of the Change in the Vertical Reaction of the Bearing Surface To The Car Wheel R_z on the Horizontal Coordinate of the Road Surface Q_x. Part A – for the front axle; Part B – for the rear axle; Part C – for the front and rear axles of the vehicle; 1, 2, 3, 4, 5 – for a pothole with geometric parameters of destruction stages 1 – 5 (Table 2)

In the area where the front axle wheel of the truck overcomes 7 m of the bridge roadway (measured from the beginning of the pothole, as shown in Figure 4), and the rear axle wheel overcomes 5.5 m, the vertical displacement of the sprung masses reaches its maximum values. Consequently, the vertical reactions between the bridge deck and the truck wheels decrease. For instance, when the front axle wheel of a truck overcomes a pothole with geometric parameters No. 1 (as specified in Table 2), the vertical reaction in this area decreases to 24 kN. Similarly, with pothole parameters No. 2, the reaction decreases to 21 kN, and so forth (as depicted in Figure 5, a). On the other hand, when the rear axle wheel of a truck overcomes a pothole with geometric parameters No. 1, the vertical reaction in this area decreases to 16 kN, and subsequently decreases to 12 kN for No. 2, 9 kN for No. 3, and so on (as shown in Figure 5, b).

The subsequent change in the vertical reactions of the bridge pavement to the truck wheels follows a wavy pattern, but with gradually decreasing amplitudes. To facilitate the analysis of the obtained results, the graphical representations in Figure 4.c and Figure 5.c illustrate the trajectories of movement for the sprung and unsprung masses of the front and rear axles of the truck, along with the changes in the vertical reactions of the roadway to their respective wheels.

The study of the dynamic impact of a truck on the bridge pavement (as shown in Figure 5) confirms the hypothesis that the vertical reaction of the bearing surface to the car wheel (Rz) increases when overcoming a pothole in the pavement. This increase in vertical reaction occurs within a range of 0.5 m to 2 m and can vary depending on the geometric shape of the pothole. Importantly, the length of the span for road bridges of the split static scheme does not influence the change in the vertical bearing reaction from the car wheel.

As per the current regulatory documents in Ukraine [22], the dynamic coefficient for moving temporary loads, required to determine the carrying capacity, is calculated using the following formula:

$$\mu = 1 + 14/(40 + L); \qquad (15)$$

where L – the length of the line of influence.

Accordingly, the dynamic coefficient is assigned in the range from 1.1 to 1.3. The regulated formula 15 accordingly considers only one variable – the length of the line of influence. However, for small and medium-sized bridges up to 100 meters long and with split spans, a significant danger is posed by an increase in load due to a vehicle wheel hitting a defect (pothole) in the pavement.

Based on the studies conducted (Figure 4 & Figure 5), the dynamic coefficient for determining the carrying capacity is proposed to be determined by the formula:

$$\mu = 1 + \tanh\left(\frac{R_{z,f}}{R_{z,i}} - 1\right), \qquad (16)$$

where $R_{z,i}$ – the initial (before overcoming the defect) vertical reaction of the bearing surface to the car wheel R_z, kN.

$R_{z,f}$ – the vertical reaction of the bearing surface to the car wheel after overcoming the defect R_z, kN.

The dynamic coefficient for the wheel of the front axle of a truck (Figure 5) is defined as:

$$\mu = 1 + \tanh\left(\frac{79\text{kN}}{28\text{kN}} - 1\right) = 1{,}95. \qquad (16a)$$

Dynamic coefficient for the rear axle of the car (Figure 5):

$$\mu = 1 + \tanh\left(\frac{70\text{kN}}{22\text{kN}} - 1\right) = 1{,}97. \qquad (16b)$$

This allows us to consider the change in the vertical bearing reaction from the vehicle wheel after overcoming the pothole. Accordingly, the dynamic coefficient will be taken in the range from 1.0 to 2.0, depending on the quality of the bridge pavement and the geometric parameters of the pothole.

This approach to determining the dynamic coefficient further needs to be refined in the process of considering various heavy vehicles that differ in weight, geometric characteristics, suspension stiffness, shock absorber characteristics, as well as elastic and damping properties of tires.

4. Discussion

A mathematical model of truck movement that considers the progression of a pothole has been developed. Based on this model, the dynamic load acting between the car wheel and the road surface of a bridge crossing has been determined. The application of this model will enhance the methodology for calculating the bridge's load capacity, provide clarity on its technical condition, and estimate its service life. The study of the dynamic impact on the bridge's structural components due to pavement deformations and irregularities will facilitate the prediction of the rate of degradation of individual elements of the bridge crossing and the proposal of preventive maintenance measures.

The results of the dynamic impact study include the trajectories of movement for the sprung and unsprung masses of the truck, as well as changes in the vertical reactions of the bridge pavement to the wheels of its front and rear axles overcoming a progressive pothole. As the stage of pothole destruction increases, both the amplitudes of vertical vibrations of the truck's sprung and unsprung masses and the changes in the vertical reactions of the bearing surface to its wheels increase. According to Newton's Third Law, the vertical load exerted by the truck's wheels on the pavement of the bridge crossing will be equal in magnitude but opposite in direction to the vertical reactions. Therefore, under these research conditions, it was observed that when the front axle wheel of a truck overcomes a pothole, its dynamic load can increase by 2.8 times (from 28 kN to 79 kN), and the rear axle wheel can increase by 3.2 times (from 22 kN to 70 kN), which corresponds to a dynamic coefficient for the front axle wheel, where $\mu = 1.95$; for the rear axle wheel $-\mu = 1.97$.

5. Conclusion

An analysis of the factors that determine the dynamic impact of a truck on the pavement of a bridge with a pothole has been conducted. The dynamic load from a truck is dependent on various vehicle parameters, including the mode of movement, mass parameters, geometric parameters, suspension stiffness, shock absorber characteristics, as well as the elastic and damping properties of its tires. Additionally, the geometric parameters of the pothole on the road surface, such as the profile shape, depth, and length, also contribute to the dynamic load.

The initial data and conditions for the mathematical modeling of overcoming a pothole on the pavement of a bridge crossing by a truck wheel have been substantiated. The parameters of the truck used in the mathematical modeling, as well as its mode of movement, have been determined. The study focuses on straight-line, uniform movement with a speed of 13.9 m/s. The geometric parameters of the pothole considered in this study are based on extensive field observations conducted on Ukrainian bridges.

A mathematical model for overcoming a pothole on the pavement of a bridge crossing by a truck wheel has been developed. The quarter-car model is employed to describe the processes involved. The relationship between the equivalent sprung and unsprung masses of the truck, the stiffness of the elastic suspension device reduced to the truck wheel, and the shock absorber resistance coefficients during compression and rebound are considered in the modeling. The car wheel is modeled using the dynamic radius, radial stiffness of the tire, and its damping coefficient. To simulate the pothole on the pavement, the identification of key points is proposed, with each point determined by its two coordinates. These coordinates are defined as functions of the pothole's depth and length, and the key points are connected using harmonic functions to allow for a gradual change in the pothole's angle of attack.

Theoretical studies and analysis of the dynamic impact of a truck on the pavement of a bridge with an increasing pothole have been conducted. As the volume of the pothole increases (considering five stages of destruction), the amplitudes of vertical vibrations in both the truck's sprung and unsprung masses increase, and the vertical reactions of the bearing surface to the wheels change, both increasing and decreasing in magnitude. According to Newton's third law, the vertical load acting from the truck's wheels on the pavement of the bridge crossing will be equal in magnitude but opposite in direction to the vertical reactions. Thus, under the conditions of this study, it was found that when the front axle wheel of a truck overcomes a pothole, its dynamic load can increase by 2.8 times (from 28 kN to 79 kN), and the rear axle wheel can increase by 3.2 times (from 22 kN to 70 kN).

The dynamic coefficient of the truck's impact on the bridge pavement, determined based on its quality condition, has been found to be 1.95 for the front axle and 1.97 for the rear axle. This analysis serves as a foundation for further exploration in determining the load capacity of road bridges under the influence of modern heavy vehicles. Consequently, this research aims to enhance the reliability and durability of bridges, which will be the subject of future investigations conducted by the authors.

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