Nanotechnology Perceptions
ISSN 1660-6795
www.nano-ntp.com

State Estimation of a Second Order Non-Linear System

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State estimation makes control much easier by detecting faults and undesirable changes and allowing remedial actions to be taken priorly. State estimation can be done using a variety of algorithms and filters. One such method of non-linear system's state estimation is by implementation of filters from the Kalman filter family. Even when the exact shape of a dynamic system is unknown, the Kalman filter can nevertheless approximate its state. The filter's ability to support assessments of past, present, and even future situations makes it incredibly potent. Although it is only applicable to linear systems, the conventional Kalman filter is a useful tool for estimating. Most real-world systems are nonlinear, hence Kalman filters are not immediately applicable. Hence, non-linear filters are used more often than linear filters. In this project, state estimation of a non-linear system, a two-tank interacting system is done using two different filters to predict the future behavior of the system. These include the Unscented Kalman Filter (UKF) and the Extended Kalman Filter (EKF). Our research comprises a description of the EKF and UKF, as well as an algorithm with two major steps: prediction and correction. These procedures are used to estimate the heights of the two tanks.
Keywords: Extended Kalman filter (EKF), Unscented Kalman Filter (UKF), non-linear system, Estimation

1. Introduction

Process operating in nonlinear systems require nonlinear filtering techniques to estimate states since linearized filtering techniques, like the extended Kalman filter (EKF), are not accurate enough. Among these techniques are sampling-based approaches like the unscented Kalman filter (UKF). The process of determining a system's true state and parameter values when noisy observations, modeling uncertainties, and undesired perturbations are present is known as estimation [1]. The Kalman filtering investigations demonstrate that the first-order (Hill–Clohessy–Wiltshire) dynamic model is not observable when comparing the filter performance for these estimating scenarios. The benefits of employing higher-order nonlinear dynamic models, which have quicker filter convergence and more observability, are also highlighted [2]. The unscented Kalman filter (UKF) is a type of sigma point Kalman filter that relies on unscented transformation [3]. The UKF consistently performs better than the EKF in terms of accuracy at a comparable level of complexity [4]. The results show that UKF outperforms EKF in terms of estimation accuracy and convergence rate for all battery models [5]. Without developing, implementing, and modifying the two Kalman filters, forecast the UKF's improvement in estimating performance over the EKF for a particular non-linear system and measurement [6].

Contribution of this Work

The following are the key phases of the suggested approach, for further information:

- Highlights the importance of state estimation in enhancing control systems by detecting faults and undesirable changes, enabling proactive remedial actions.
- Identifies the limitations of standard Kalman filters for linear systems and emphasizes the need for nonlinear filters like EKF and UKF to handle real-world nonlinear dynamics.
- The proposed system provides EKF and UKF for state estimation in a practical nonlinear system, specifically estimating the heights of two interacting tanks.
- Provides an overview of the prediction and correction steps in EKF and UKF, offering a structured methodology for state estimation.
- As the result the ability of EKF and UKF to predict the future behavior of a nonlinear system, aiding in better control and decision-making.

The following is describing the way the paper is established: Section 1 illustrates the introduction. Section 2 provides a description of the relevant works. Section 3 describes the proposed techniques; Section 4 presents the experiment's results; and Section 5 offers an overview.

2. Related Works

Simon et.al. [7] have proposed, the optimal method for determining the state of a broad system is predicted using mathematical methodologies. The methods for state estimate that will eventually be used in software are noted. Dan et al. [8] have suggested, The Kalman Filter's modification for nonlinear system state estimation. A thorough algorithm for using the Extended Kalman Filter for state estimation was also provided. Julier et al. [9] have described, one of the most popular techniques for tracking and estimating is the Kalman filter, and an explanation of its implementation challenges was examined. It also explained the conventional strategy to employing the Extended Kalman Filter and its drawbacks. This study introduced a new approach to state estimation using the UKF, another estimator. Murray-Smith et al. [10] have reported, Hydraulic system modeling was introduced, along with techniques for validating and confirming a simulation model of a laboratory-scale system with two connected vessels. A variety of studies pertaining to a two-tank system were acquired. Changela [11] have has introduced for the study purpose two tank interacting level process has been considered. The development of mathematical model for the two-tank system was studied from. Geetha [12] have proposed, a clear understanding of the computation of covariances using the linear propagation in implementation of the Extended Kalman Filter was gained. Lee [13] have discussed, the same linear approximation of the state equation was clearly used to create an ideal prediction equation for the future states. Within the constrained assumptions of measurement noise and external disruptions, the suggested method highlighted its advantages. Julier [14] have recommend, In order to address the practical challenges of applying the Kalman filter to a non-linear system, the Unscented Kalman filter prediction and correction algorithm was presented.

Wan et al. [15] have proposed, an overview on UKF was presented. It discussed the flaws present in EKF and the reasons to prefer UKF over EKF. Wan et al. [16] have reported, the various names of UKF and the most commonly used non-linear filters for state estimation were introduced. The formulae for model forecast and data assimilation steps of Unscented transform was studied. Grewal et al. [17] have introduced, all of the MATLAB software offered the chance to think about the actual arithmetic required to maintain the correctness of findings and to learn how the Kalman filter operates in practice. Kumar et al. [18] have proposed, The EKF and UKF filters were designed and implemented, and thorough simulation experiments were conducted to test their relative performance. It covered how to determine which of the two filters is the most effective.

3. Proposed System

Numerous algorithms can be used for state estimation. Despite the variations in methods, the fundamental processes for state estimation, prediction, and correction are the same. In Figure 1, the proposed block diagram for the state estimator is shown.

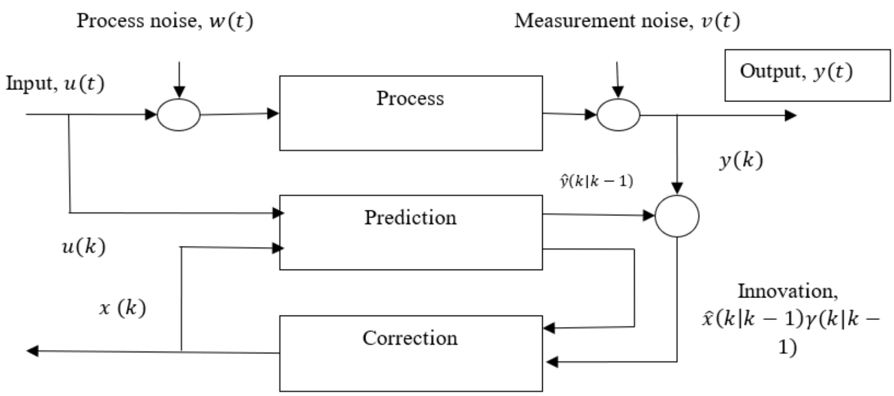

Figure 1. Proposed block diagram of a State estimator

Estimation and updation in real time occur in two steps:

(a) Prediction: Starting at a state $k-1$, response at a new state k is predicted using the system model.

(b) Correction: By using sensors and real time values the response at k is observed. The predicted and the observed states are fused to form the corrected state. The fusion of these random variables produce a final estimate desired unknown state

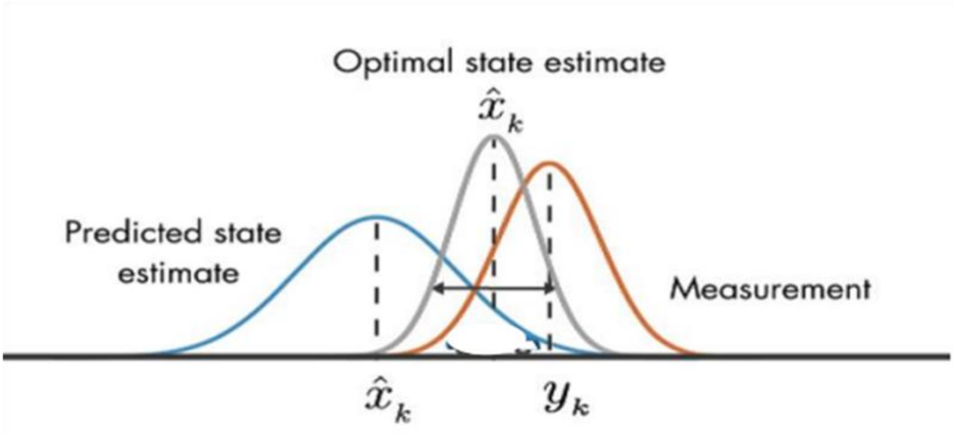

Figure 2. Gaussian representation of state estimation

Figure 2, displays the final optimal state estimate that is obtained by fusing the measurement model with the anticipated state. This chapter has covered the fundamentals of a state and how to estimate it. It is currently unknown how to use Extended and UKF for state estimation in a second order non-linear system. Different outcomes are produced when the same method is applied in distinct steps. A thorough analysis of state estimate with UKF and EKF is,

3.1 Extended Kalman filter

Our estimate of x is used by the EKF as the nominal trajectory of the linearized Kalman filter. An ingenious bootstrapping technique for state estimation is to estimate x using a nominal trajectory and then use the estimated value of x as the nominal trajectory. These modifications to the linearized Kalman filter equations, along with a few mathematical procedures, result in the EKF approach. Figure 3 illustrates the general block diagram of an Extended Kalman Filter.

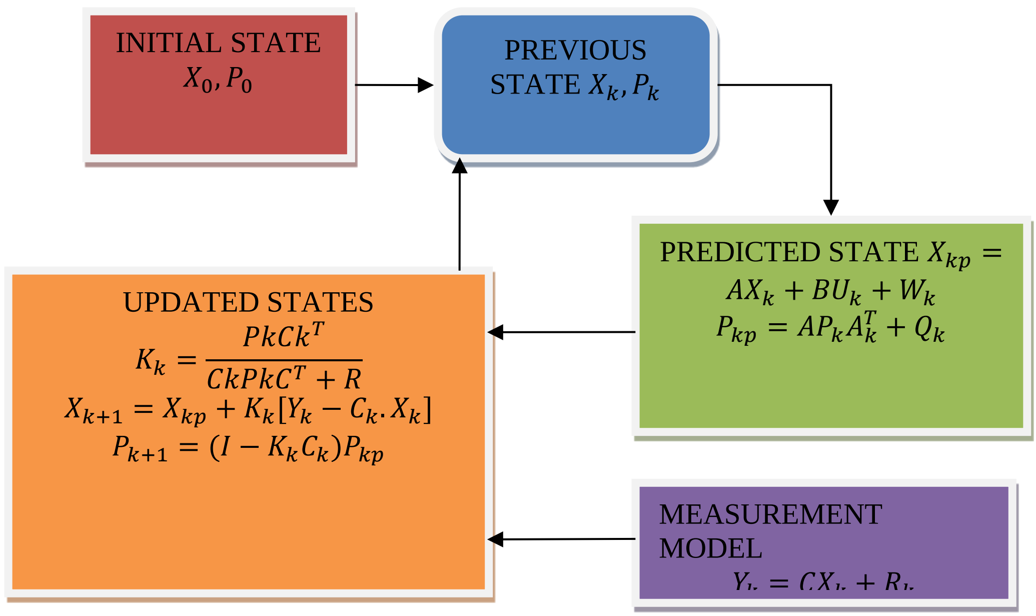

Figure 3. Block diagram of EKF

Where, X - State (height in case of the assumed two tank system), P – Estimated error covariance matrix, Q - Process noise covariance matrix, R - Measurement noise covariance matrix, A - State matrix, C - Output matrix k - Previous state, k_p - Predicted state, $k + 1$ – Updated state.

3.1.1 EKF ALGORITHM

With some mathematical adjustments, they obtain the following EKF method:

(a) The system equations are given as

State equation: $X_{k+1} = f(X_k, U_k) + W_k$

Output equation: $Y_k = h(X_k) + V_k$

(b) Calculate the following derivative matrices at each time step, evaluating them at the current state estimate,

$$C_k = h'(\widehat{x_k})$$

(c) Execute the following Kalman filter equations:

$$K_k = \frac{P_k{C_k}^T}{P_k{C_k}^T + R}$$

3.2 UNSCENTED KALMAN FILTER

The UKF block diagram is displayed in Figure 5. Utilizing the statistical linearization technique, the Unscented Kalman Filter (UKF) is a member of the larger class of filters known as Sigma-Point Kalman Filters or Linear Regression Kalman Filters.

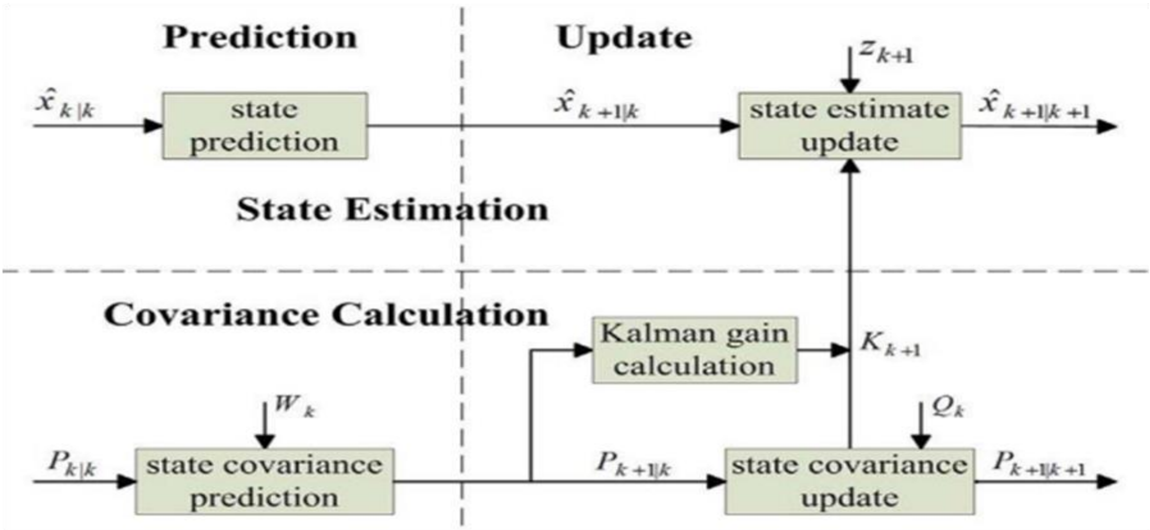

Figure 5. Block Diagram of UKF

This technique can be used to linearize a random variable's nonlinear function between n locations chosen from the random variable's prior distribution. Compared to Taylor series linearization, the method is generally more precise since we account for the dispersion of the random variable. The central difference Kalman filter (CDKF), the divided difference filter, and the square-root counterparts of UKF and CDKF are all members of the same filter family. In EKF, the posterior mean and covariance may be distorted since the state distribution is propagated analytically through the first-order linearization of the nonlinear system. By employing a deterministic sampling strategy, the UKF, a derivative-free substitute for EKF, gets around this issue. Sigma points are a limited set of carefully selected sample points that are used to depict the state distribution.

3.2.1 UKF ALGORITHM

The UKF algorithm is displayed in Figure 6. Deterministically selecting sigma points makes it simpler to approximate a probability distribution using the UKF. By selecting the sigma points, the mean and covariance are precisely x_{k-1} and P_{k-1}. Following the propagation of each sigma point via the nonlinearity, a cloud of changed points is ultimately produced. Then, using their statistics, the new estimated mean and covariances are calculated. Refer to this process as unscented transition. The unscented transformation is one method for determining the statistics of a random variable that undergoes a nonlinear transformation. Examine a nonlinear system that is characterized by additive noise in the observation model and the difference equation.

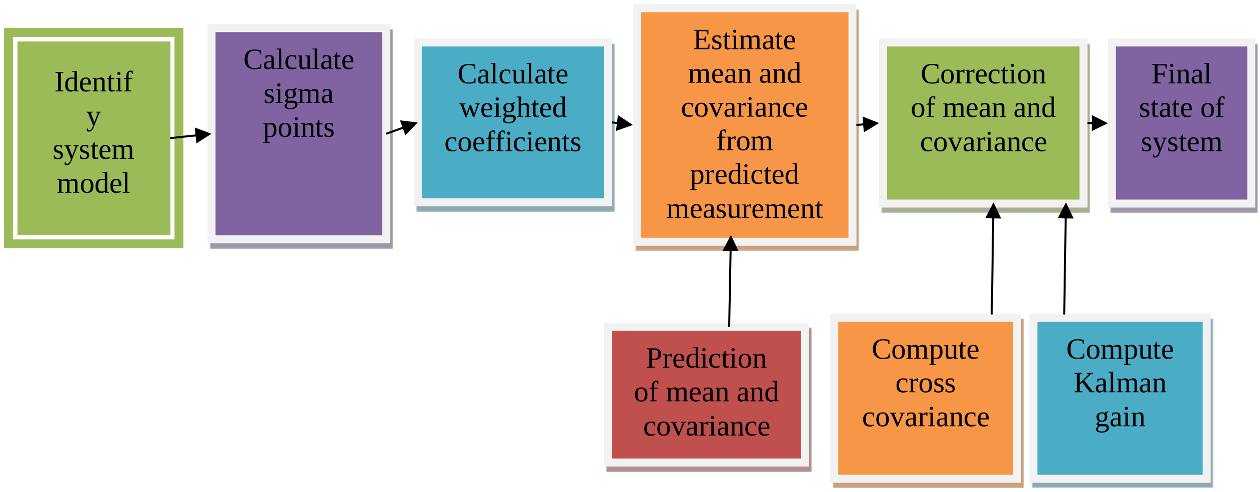

Figure 6. Algorithm of UKF

3.2.2 UKF SIGMA POINTS

Since approximating a probability distribution is simpler than approximating an arbitrary nonlinear function or change, the UT was established. A set of points (sigma points) are selected so that their mean and covariance are x and ∑x, as shown in Figure 7, to demonstrate the methodology. A cloud of transformed points is produced by applying the nonlinear function to each point one at a time. The statistics of the transformed points can then be used to estimate the nonlinearly transformed mean and covariance. There are a number of important distinctions between this approach and particle filters, despite their apparent similarities. The sigma points are first selected deterministically to display certain particular qualities (e.g., having a given mean and covariance) rather than being chosen at random. This allows a fixed, limited number of points to collect high-order information about the distribution.

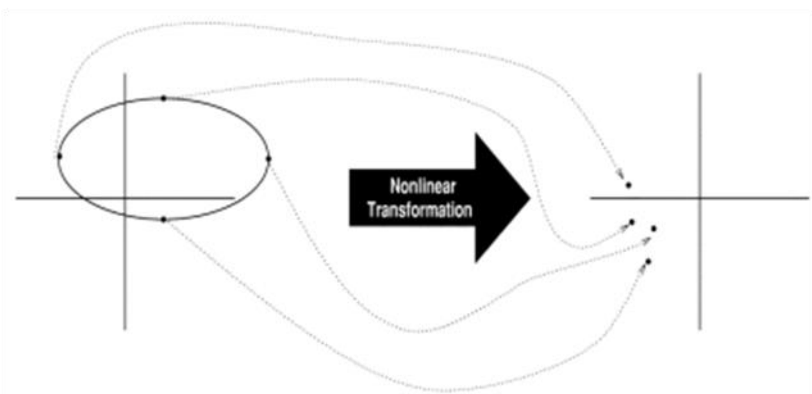

Figure 7. Sigma Points

3.2.3 Set Selection of Sigma Points

Let X_{k-1} be a set of $2n + 1$ sigma points and the weights that go with them, where n is the state space dimension.

1. Compute Cholesky decomposition of Covariance:

$$TT^T = \sum xx \text{ (T} - \text{ Lower Triangular Matrix)} \quad (1)$$

2. Calculate Sigma points: Take into consideration the following sigma point selection, which includes higher order information:

$$X(0)k - 1 = \mu x$$

$$X(i)k - 1 = \left(\sqrt{nP}\right)_i^T \quad \text{for i=1…n}$$

$$X(i+n)k - 1 = -\left(\sqrt{nP}\right)_i^T \quad \text{for i=n+1…2n}$$

$$\text{Kappa}, \kappa = 3 - n$$

3.3 MATHEMATICAL MODELLING

A two-tank interacting system is chosen as the non-linear system whose states are going to be estimated using a nonlinear filter called Extended Kalman Filter.

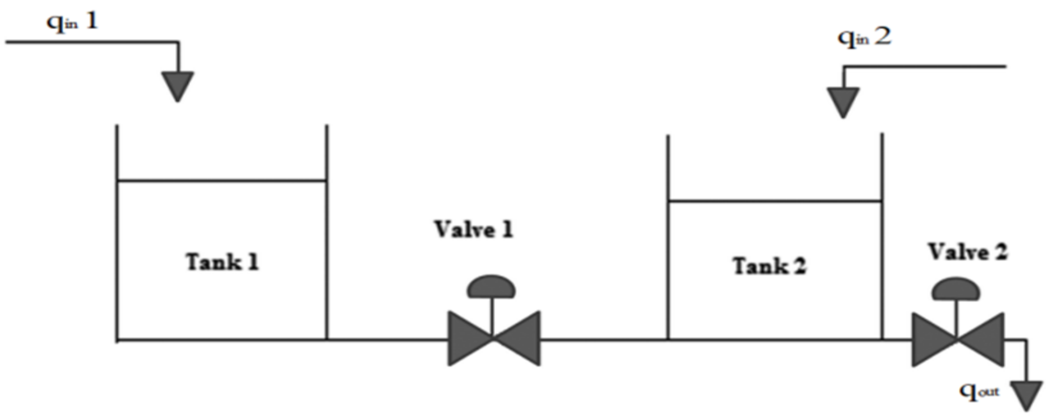

Figure 4. Two tank interacting system – EKF

The system depicted in Figure 4 is made up of two interconnected tanks that are connected to one another by means of circular cross-sectional connecting pipes that have a valve. The physical parameters of the two-tank process are displayed in Table 1.

Table 1. Physical parameters of Two-Tank Process

Parameters	Values
Areas of the tank (A1, A2)	0.0154 m^2
Cross section of the connecting pipes (a)	0.005 m^2
Acceleration due to gravity (g)	9.81 m/sec

Coefficients for joining pipes (s)	0.45
Inflow rate of tank 1, (q_{in1})	0.00315 m^3 /sec
Inflow rate of tank 2, (q_{in2})	0.00231 m^3 /sec

Nonlinearity is introduced into the system by valves 1 and 2. In the dynamic model, the two measurements h1 and h2, or the heights of tanks 1 and 2, respectively, are considered the outputs, and the incoming mass flow q_{in1} and q_{in2} are defined as inputs. Derived from the entering and departing mass flow rates, the dynamic model is characterized by the differential equations (1) and (3).

For tank 1: $A1\frac{dh1}{dt} = q_{in1} - b1\sqrt{h1 - h2}$ (2)

$$f1 = \frac{dh1}{dt} = \frac{qin1}{A1} - \frac{b1\sqrt{h1-h2}}{A1} \quad (3)$$

For tank 2:$A2\frac{dh2}{dt} = qin2 + b1\sqrt{h1 - h2} - b2\sqrt{h2}$ (4)

$$f2 = \frac{dh2}{dt} = \frac{qin2}{A2} + \frac{b1\sqrt{h1-h2}}{A2} - \frac{b2\sqrt{h2}}{A2} \quad (5)$$

Where, the valve coefficients of valves 1 and 2 are $b1 = s_1.a\sqrt{2g}$ and $b1 = s_2.a\sqrt{2g}$.

3.4 MATHEMATICAL MODELLING- UKF

Figure 8, illustrates the selection of a two-tank interacting system for a second order non-linear system.

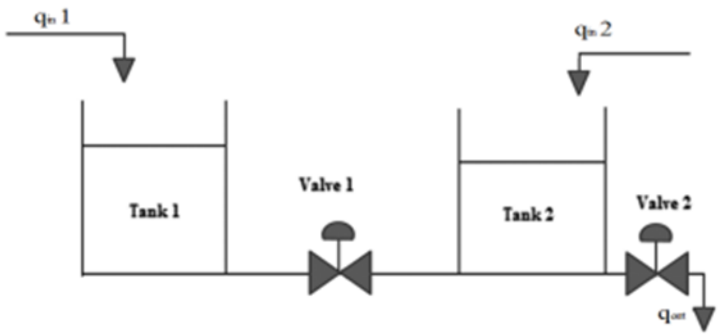

Figure 8. Two tank interacting system – UKF

Table 2, shows that the Parametric values of two tank system. The inflow of tank 1 and tank 2 is qin1, qin2 and the outflow is q1, q0 respectively. The parameters h1 and h2 are the height of the tank 1 and tank 2 which changes with respect to time. The differential equation for the two-tank system is as follows:

Table 2. Parametric values of Two tank system

Parameters	Description	Values
A_1, A_2	Area of tank 1 and tank	2 0.0145 m^2
R_1	Resistance to flow in tank 1	1478.57 s/m^2

R_2	Resistance to flow in tank 2	$642.86\ s/m^2$
q_{in1}, q_{in2}	Inflow of tank 1 and tank 2	$0.000085\ m^3/s$

For tank 1: $\frac{dh_1}{dt} = \frac{R_1 q_{in1} - h_1 h_2}{A_1 R_1}$ (6)

For tank 2: $\frac{dh_2}{dt} = \frac{q_{in2} R_1 R_2 + h_1 R_1 - h_2 + (R_1 + R_2)}{R_1 A_2 R_2}$ (7)

$$\begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} -\frac{1}{A_1 R_1} & \frac{1}{A_1 R_1} \\ \frac{1}{A_1 R_1} & -\frac{(R_1+R_2)}{R_1 A_2 R_2} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{A_1} \\ \frac{1}{A_2} \end{bmatrix} \begin{bmatrix} q_{in1} \\ q_{in2} \end{bmatrix} \quad (8)$$

4 Experimental result and discussion

The two chosen non-linear filters, EKF and UKF were studied and the design of these filters for implementing it to estimate the heights of a two-tank was done. The code written for implementing these two filters is shown in this result section.

4.1 RESPONSE CURVES- EKF

By utilizing all of the values acquired when applying EKF to a two-tank system, MATLAB software is utilized to simulate both the Extended Kalman Filter and the two-tank system and Figures 9, 10, and 11 display the outcomes of the simulation.

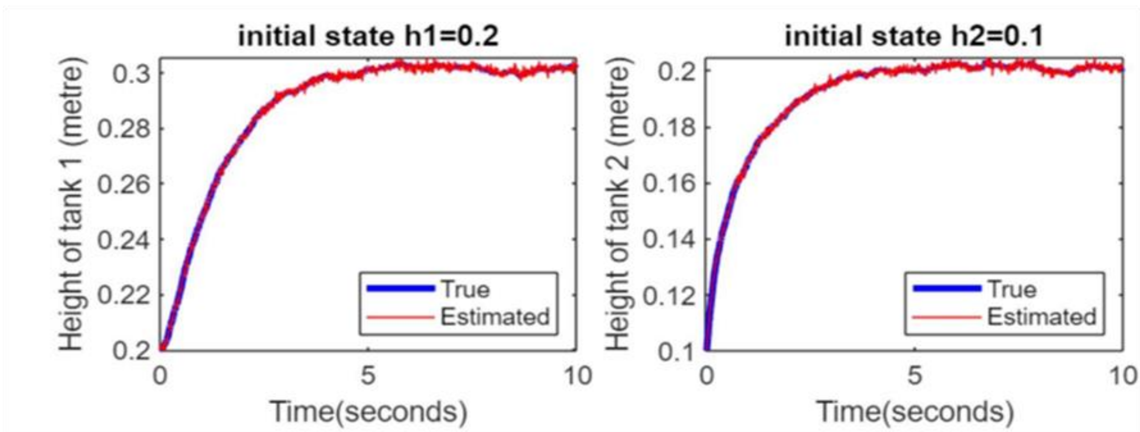

Figure 9. EKF simulation for a two-tank system with control noise= 0.001. Heights h1 and h2 are estimated

Tank 1's water level is first thought to be 0.2 meters, whereas Tank 2's is 0.1 meters. With them as the starting points, the system's future behavior is anticipated. The figures demonstrate the difference between the estimated and true states, which are represented in red and blue, respectively. In Figure 9, it is clearly visible that the estimated state coincides with the actual state and produces a steady output when noise is of a lower order 0.001.

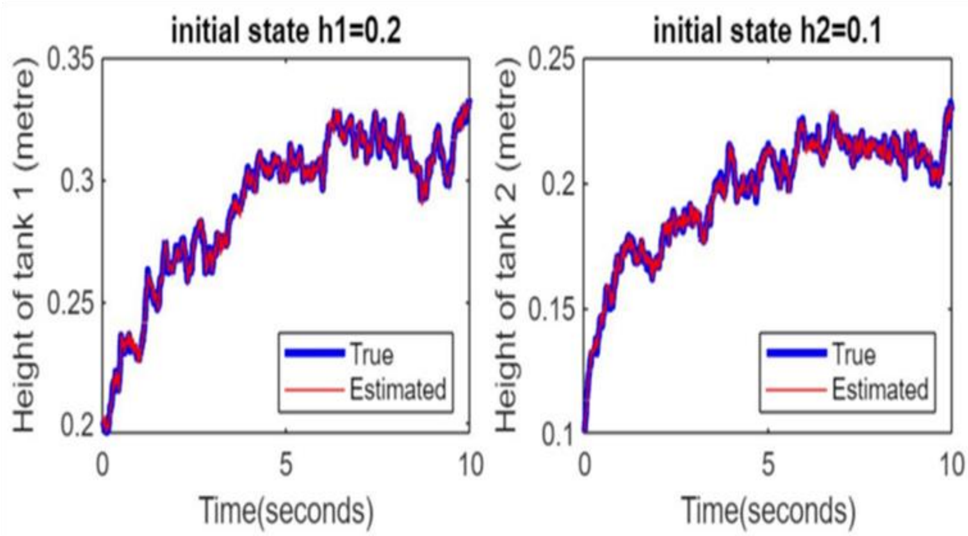

Figure 10. EKF simulation for a two-tank system with control noise= 0.01. Heights h1 and h2 are estimated

When the noise is increased by a factor of 10, as shown in Figure 10, we can understand the irregularities in estimation.

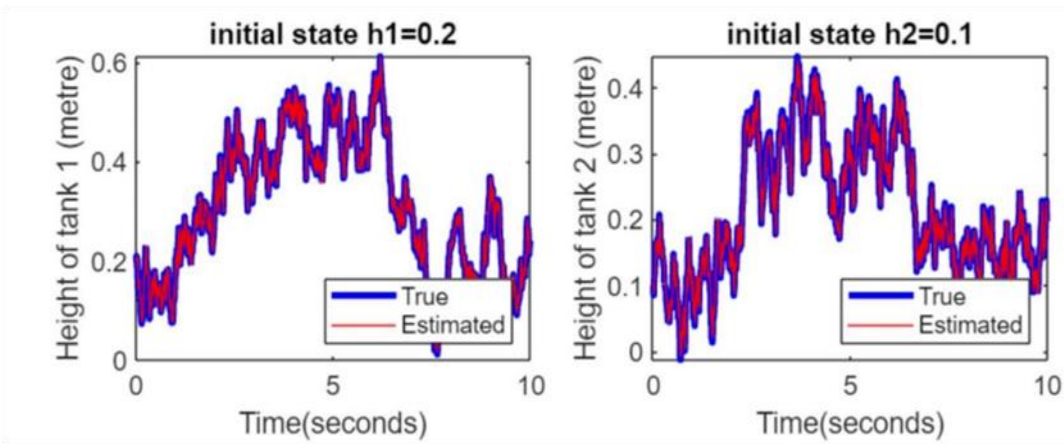

Figure 11. EKF simulation for a two-tank system with control noise= 0.1. Heights h1 and h2 are estimated

The greater the noise, more unstable is the system predicted using EKF. Being aware of EKF's limits is crucial. Figure 9, for instance, was produced using a control noise of 0.001 and Figure 10 with a control noise of 0.01. Observe that, an increase in noise by a factor of 10 produces a distorted response. When, the noise is increased by a factor of 100, EKF generates an essentially worthless response as shown in Figure 11. Hence, it is necessary that, the noise factors are maintained under control in order to obtain a steady estimation. In case of higher noise factors, the EKF estimate might become unstable.

Table 1. Estimated errors for various noise assumptions

Cases	Standard deviation(σ)	Square root of estimated error.
Control noise=0.001 Measurement noise=0.001	0.001	0.015
Control noise=0.01 Measurement noise=0.01	0.01	0.145
Control noise=0.1 Measurement noise=0.01	0.007	0.135

From the table, we can observe that the standard deviation and estimated error is proportional to the noise applied. With increased noise level, the error is also higher. Especially, the measurement noise creates an adverse effect in stable estimation.

4.2 RESPONSE CURVES

The heights in tanks 1 and 2 of the system are estimated using MATLAB R2017a software to demonstrate the UKF for state estimation. This chapter provides detailed documentation of UKF's superior error performance over EKF for state estimation. This section demonstrates the error reduction attained by UKF and focuses on confirming the error performance of UKF and EKF. The Appendix contains the code that was added to the software specifically for UKF. This chapter provides a graphical illustration of the different responses that occur when the system is fed different types of measurement noise. The following section discusses case studies based on measurement noise.

Case 1: In figure 12 shows, Control noise= 0.00122222 Measurement noise= 0.001

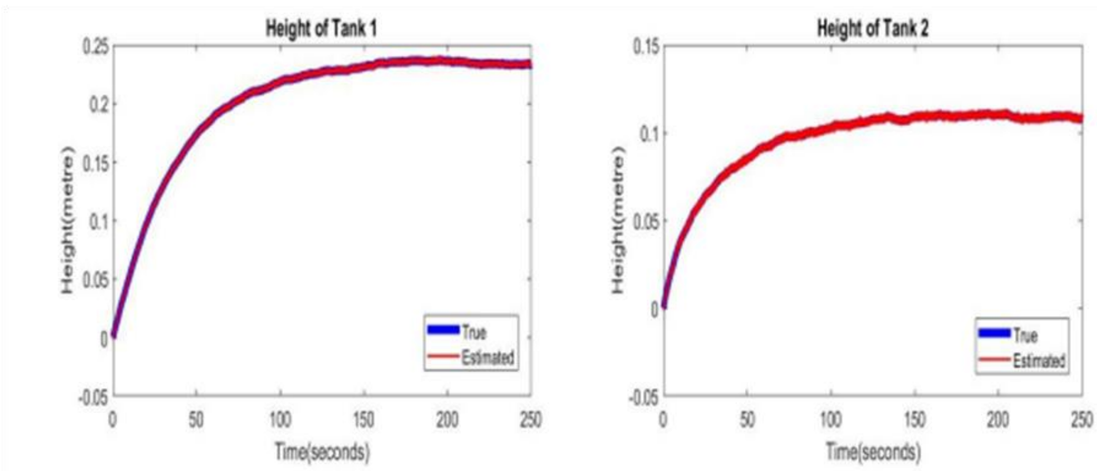

Figure 12. UKF Response when Measurement noise is 0.001

When the measurement noise is varied, the true state and the estimated state changes its response. When measurement noise is high, the estimated state produces large distortion comparing to less noise. Here, the control noise is approximately equal to the measurement noise. Hence, both the states produce deviation from the actual state. When measurement noise is high, it takes a large amount of time to settle as given in figure 9.

Case 2: In figure 13 shows, Control noise= 0.00122222 Measurement noise= 0

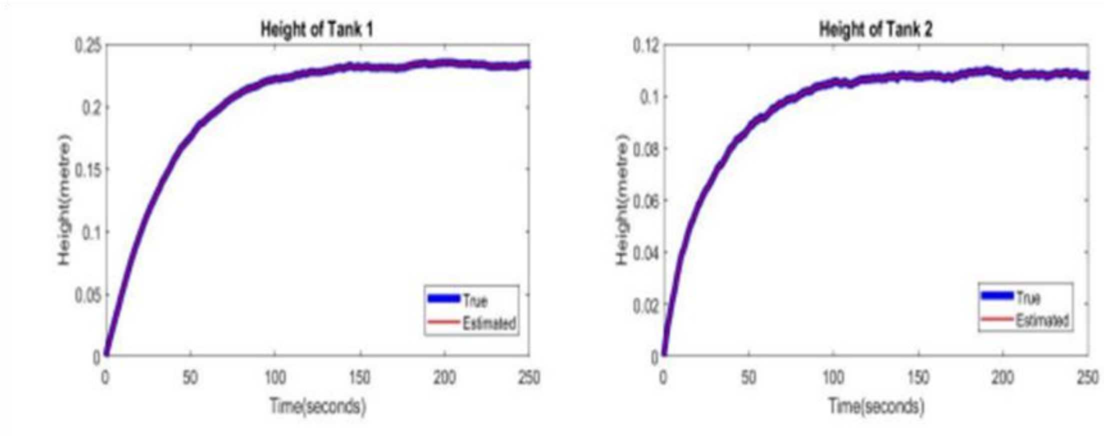

Figure 13. UKF Response when Measurement noise is 0

Since the system is provided with null measurement noise, the estimated state has no distortion. The settling time is quick when compared to other responses. It is noted that the response of system is as provided in figure 10 when control noise is provided without measurement noise.

Case 3: Control noise= 0.00122222 Measurement noise= 0.0005

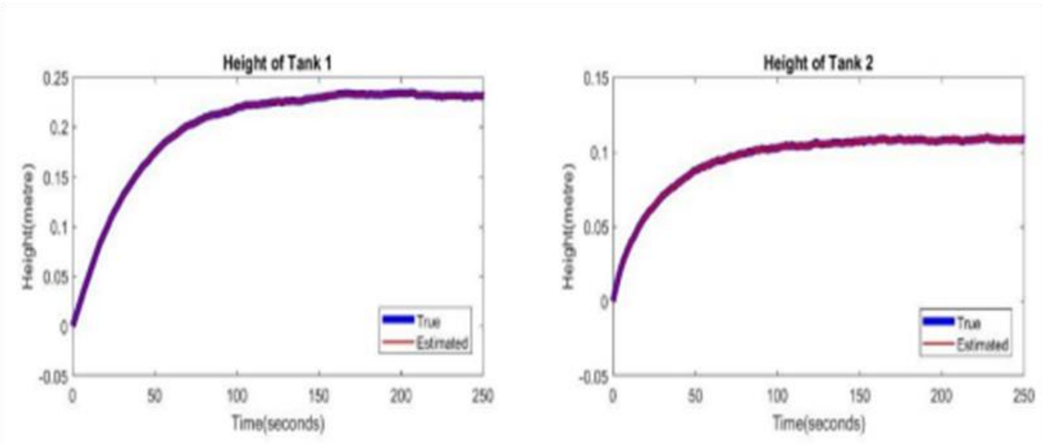

Figure 14. UKF Response when Measurement noise is 0.0005

In Figure 14, a moderate measurement noise of 0.0005 is fed in the system. In this case, the control noise is greater than the measurement noise. Hence the settling time and the distortion is comparatively low as shown in figure 11.

5 Conclusion

The paper State estimation of a second order non-linear system was developed to estimate or predict the states (height) of two-tank interacting system. It gives a clear idea to estimate a system using EKF and UKF. In EKF, the linearization of the system was done using Jacobian and the Kalman gain was calculated for the linearized model. Since, UKF has the ability to work on non-linear systems, the non-linear state space model was derived directly from the systems' differential equation. The sigma points, predicted mean and covariance were calculated to find the updated covariance and Kalman gain. The algorithm of both the filters were implemented in MATLAB software and the desired responses was obtained. In future state estimation of two-tank system by UKF using UT can be done by applying SUT for better estimation.

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