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Isolate D- Eccentric Domination in Graphs

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A subset D of the vertex set $\mathrm{V(G)}$ of a graph G is said to be a dominating set if every vertex not in D is adjacent to at least one vertex in$\mathrm{D}$. A dominating set D is said to be an D-eccentric dominating set if for every$v \in V - D$, there exists at least one D-eccentric vertex$u$ of $v$in D. A D-eccentric dominating set D of G is an isolate D-eccentric dominating set if the induced subgraph $\langle D \rangle$ has atleast one isolated vertex. The minimum of the cardinality of the isolate D-eccentric dominating set of G is called the isolate D-eccentric domination number $\gamma_{oed}^{D}(G)$. In this paper, we obtain some bounds for $\gamma_{oed}^{D}(G)$. Exactvalues of $\gamma_{oed}^{D}(G)$ for some particular classes of graphs are obtained.
Keywords: Domination, Eccentric domination,D-Eccentric domination, Isolate domination, Isolate D-eccentric domination.

1. Introduction

In 1962 O. Ore proposed a new idea dominating set and domination number [5]. In 1972, F. Harary introduced the Graph theory concept[1]. In 1998 T. W. Haynes et al., deliberated various dominating parameters [2]. In 2010, V.R. Kulli introduced the Theory of domination in graphs[4]. In 2010 T. N. Janakiraman et al., illustrated eccentricdomination in graphs [3]. In 2013 L. N. Varma et al., determined D-Distance in graphs [6]. In 2018, Bhanumathi and Niroja developed the isolate eccentric domination in graph[9]. In 2021, Prasanna and M

Azarudeen initiated the concept of D-Eccentric domination in graph[7]. In 2021, Prasanna and M Azarudeen initiated the concept of Detour D-Eccentric domination in graph[8].

In this article, the concept of isolate D-eccentric dominating set and its numbers is found. The isolate D-eccentric domination numbers are procured for various standard graphs. The bounds for isolate D-eccentric domination numbers are found.

2. Preliminaries

Definition 2.1[1]: For any graph G (V, E),V is the vertex set of G or V(G) and E is the edge set of G or E(G). The cardinality |V| of the vertex set V is called order of G and the cardinality |E| of the edge set E is called size of G. Let u, v ∈ V be two vertices of G. The standard or usual distance d (u, v) between u and v is the length of the shortest u – v path in G.

Definition 2.2[7]: If u ,v are any two vertices of a connected graph G, then the D-length of a u – v path s is defined as $l^D(s) = d(u, v) + \deg(u) + \deg(v) + \sum \deg(w)$ where sum runs over all intermediate vertices w of s. The D-distance $d^D(u, v) = \min\{ l^D(s) \}$, where the minimum is taken over all u – v paths in G.

Definition 2.3[7]: For a vertex v, each vertex at a D-distance $e^D(v)$ from v is a D-eccentric vertex of v. D-eccentric set of a vertex v is defined as $E^D(v) = \{ u \in V / d^D(v) = e^D(v) \}$ or any vertex u for which $d^D(u, v) = e^D(v)$ is called D-eccentric vertex of v and a vertex u is said to be D-eccentric vertex of G if it is the D-eccentric vertex of some vertex.

Definition 2.4: The D-eccentricity of a vertexvis defined by$e^D(v) = \max \{ d^D(u,v) / u \in V \}$

Definition 2.5[7]: The D-radius, defined and denoted by $r^D(G) = \min \{ e^D(v) : v \in V \}$.The D-diameter, defined and denoted by $d^D(G) = \max \{ e^D(v) : v \in V \}$.

Definition 2.6[7]: The vertex v in G is a D-central vertex if $r^D(G) = e^D(v)$ and the D-center $C^D(G)$ is the set of all central vertices. The D-central sub graph $< C^D(G) >$ is induced by the center.

Definition 2.7[7]: The D-peripheral of G, $P^D(G) = d^D(G) = e^D(G)$. V is a D-peripheral vertex if $e^D(v) = d^D(G)$. The D-periphery $P^D(G)$ is the set of all peripheral vertices. The D-peripheral sub graph $< P^D(G) >$ is induced by the periphery.

Definition 2.8[3]: A set $D \subseteq V(G)$of vertices in a graph G = (V, E) is called a dominating set of G if every vertex $v \in V - D$ is adjacent to some vertex in D.

Definition 2.9[9] A dominating set $D \subseteq V(G)$ is aisolate dominating set if the induced subgraph $< D >$has at least one isolate vertex.

Definition 2.1.[7] A set $S \subseteq V(G)$ is known as an eccentric point set of G if for every $v \in V - S$ there exist at least one eccentric vertex u in S such that $u \in E(v)$.

Definition 2.11[7] A set $S \subseteq V(G)$ is known as an D-eccentric point set of G if for every $v \in V - S$ there exist at least one D-eccentric vertex u in S such that $u \in E(v)$.

Definition 2.12[3] A set $D \subseteq V(G)$ is an eccentric dominating set if D is a dominating set of G and also for every v in $V - D$ there exist at least one eccentric point of v in D.

Definition 2.13[7] In a dominating set $D \subseteq V$ of a graph G (V, E) if there exists at least one D-eccentric vertex u of v in D for every $v \in V - D$ then it is called a D-eccentric dominating set.

Definition 2.14[1]: The neighbourhoodN(u) of a vertex u is the set of all vertices adjacent to u in G. N [v] = N(v) ∪ {v} is called the closed neighbourhood of v.

Definition 2.15[1]: A subgraph that has the same vertex set as G is called linear factor the degree of all vertices is one.

In this paper, only non trivial simple connected undirected graphs are considered and for all the other undefined terms one can refer [2, 3].

3. Isolate D-Eccentric Vertex Set in Graphs

Definition 3.1: Let $S \subseteq V(G)$ be a set of vertices in a graph $G(V, E)$, Then S is said to be a isolate D-eccentric vertex set of Gif S is a D-eccentric point set and also the induced subgraph $< S >$ has an isolated vertex. A isolate D- eccentric vertex set S of G is called minimal isolate D-eccentric vertex set, if no proper subset S' of S is a isolate D-eccentric vertex set of G. The minimum cardinality of a minimal isolate D-eccentric vertex set of S is called the isolate D-eccentric number and is denoted by $e_o^D(G)$ and simply denoted by e_o^D. The maximum cardinality of a minimal isolate D-eccentric vertex set is called the upper isolate D-eccentric number and is denoted by $E_o^D(G)$ and simply denoted by E_o^D.

Example 3.1

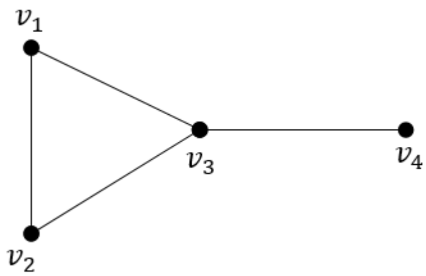

Figure.1

From the graph given in figure1, we observe the following, $e^D(v_1) = e^D(v_2) = e^D(v_4) = 8$, $e^D(v_3) = 6$ and $E^D(v_1) = E^D(v_2) = \{v_4\}, E^D(v_3) = E^D(v_4) = \{v_1, v_2\}$. Also, $r^D(G) = 6, \text{diam}^D(G) = 8$. The D-eccentric vertex sets are $S_1 = \{v_1, v_4\}$and $S_2 = \{v_2, v_4\}$, the sets S_1 and S_2 have isolate vertex. Hence, the sets S_1 and S_2 are minimal isolate D-eccentric vertex set. Therefor, isolate D-eccentric number of Gis $e_o^D(G) = 2$.

Note 3.1: The minimum isolateD-eccentric vertex set is denoted by e_o^D -Set.

Observation 3.1

1. In a Graph G, $e_o^D(G) \leq E_o^D(G)$ and
2. $1 \leq e_o^D(G) \leq n$

3. Every superset of a isolate D-eccentric vertex set is a isolate D-eccentric vertex set .
4. The subset of a isolate D-eccentric vertex set need not be a isolate D-eccentric vertex set.

Observation 3.2

1. $e_o^D(K_n) = 1$
2. $e_o^D(K_{1,n}) = 1$
3. $e_o^D(P_n) = 1, n \leq 3$
4. $e_o^D(P_n) = 2, n \geq 4$

4. Isolated D-Eccentric Dominating Set in Graph

Definition 4.1 A set $D \subseteq V(G)$ is aisolateD-eccentric dominating set if D is an D-eccentric dominating set of G and also the induced subgraph $\langle D \rangle$has an isolated vertex. The isolateD-eccentric dominating set is a minimal isolateD- eccentric dominating set if no proper subset D' of D is a isolateD- eccentric dominating set. The minimal isolateD- eccentric dominating set with minimum cardinality is known as a minimum isolateD-eccentric dominating set. The cardinality of minimum isolateD- eccentric dominating set is known as the isolateD-eccentric domination number and is denoted by $\gamma_{oed}^D(G)$. The cardinality of maximumisolateD-eccentric dominating set is known as the upper isolateD-eccentric domination number and is denoted by $\Gamma_{oed}^D(G)$.

Note 3.1: The minimum isolateD-eccentric dominating set is denoted by γ_{oed}^D -Set.

Example 4.1

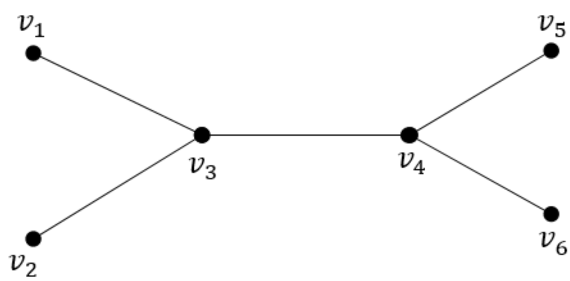

Figure.2

From the graph given in figure.2, $E^D(v_1) = E^D(v_2) = E^D(v_3) = \{v_5, v_6\}$ and $E^D(v_4) = E^D(v_5) = E^D(v_6) = \{v_1, v_2\}$the minimal isolate D-eccentric dominating set is $D = \{v_1, v_2, v_5, v_6\}$. Hence, the isolate D-eccentric domination number of Gis $\gamma_{oed}^D(G) = 4$.

Observation 4.1

1. In a Graph G, $\gamma_{oed}^D(G) \leq \Gamma_{oed}^D(G)$ and

2. $1 \leq \gamma_{oed}^{D}(G) \leq n$

3. Every superset of a isolateD-eccentric dominating set is a isolateD-eccentric dominating set .

4. The subset of aisolateD-eccentric dominating set need not be a isolateD-eccentric dominating set.

5. For any connected graph, $\gamma_{oed}^{D}(G) \leq \gamma(G) + e_{o}^{D}(G)$.

6. The complement of aisolateD-eccentric dominating set need not be a isolateD-eccentric dominating set.

7. Every isolateD-eccentric dominating set is the D-eccentric dominating set and every D-eccentric dominating set is the dominating set. Therefore we have $\gamma(G) \leq \gamma_{ed}^{D}(G) \leq \gamma_{oed}^{D}(G)$.

8. Every isolateD-eccentric dominating set is the isolate dominating set and every isolate dominating set is the dominating set. Therefore, we have $\gamma(G) \leq \gamma_{o}(G) \leq \gamma_{oed}^{D}(G)$.

9. The complement of aisolateD-eccentric dominating set need not be aisolateD-eccentric dominating set.

10. EveryisolateD-eccentric dominating set is a dominating set but the converse is not true.

Observation 4.2

1. $\gamma_{oed}^{D}(K_n) = 1$

2. $\gamma_{oed}^{D}(K_{1,n}) = 2$

3. $\gamma_{oed}^{D}(K_{m,n}) = m \text{ or } n$

Theorem 4.1 $\gamma_{oed}^{D}(P_n) = \begin{cases} \left\lceil \frac{n}{3} \right\rceil & \text{if} \quad n = 3k+1 \\ \left\lceil \frac{n}{3} \right\rceil + 1 & \text{if} \quad n = 3k \text{ or } n = 3k+2 \end{cases}$

Proof:

Case(i) $n = 3m$

$S = \{v_1, v_4, v_7, \dots, v_{3m-2}, v_{3m}\}$is the minimum D-eccentric dominating set in P_n. $\langle S \rangle$ contains isolated vertices. Hence S is also an isolate D-eccentric dominating set.

Therefore, $\gamma_{oed}^{D}(P_n) = \gamma_{ed}^{D}(P_n) = \lceil n/3 \rceil + 1$.

Case(ii) $n = 3m + 1$

$S = \{v_1, v_4, v_7, \dots, v_{3m-2}, v_{3m+1}\}$is the minimum D-eccentric dominating set in P_n. $\langle S \rangle$contains isolated vertices. Hence S is also an isolate D-eccentric dominating set.

Therefore, $\gamma_{oed}^{D}(P_n) = \gamma_{ed}^{D}(P_n) = \lceil n/3 \rceil$.

Case(iii) $n = 3m + 2$

$S = \{v_1, v_2, v_5, v_8, \dots, v_{3m-1}, v_{3m+2}\}$is the minimum D-eccentric dominating set in P_n. $\langle S \rangle$contains isolated vertices. Hence S is also an isolate D-eccentric dominating set.

Therefore, $\gamma_{oed}^{D}(P_n) = \gamma_{ed}^{D}(P_n) = \lceil n/3 \rceil + 1$.

Theorem 4.2 $\gamma_{oed}^{D}(C_n) = \begin{cases} \left\lceil \frac{n}{2} \right\rceil & \text{if} \quad \text{n is even} \\ \left\lceil \frac{n}{3} \right\rceil \text{ or } \left\lceil \frac{n}{3} \right\rceil + 1 & \text{if} \quad \text{n is odd} \end{cases}$

PROOF:

Proof of (A) :

If $p = 4$, any two adjacent vertices of C_4 is an D-eccentric dominating set of C_4 but does not exists an isolate D-eccentric dominating set in C_4.

Let n= $2m$and $m > 2$.

Let the cycle $C_p$be $v_1 v_2 v_3 \dots v_{2m} v_1$. Each vertex of C_n has exactly one D-eccentric vertex (that is C_n is unique eccentric vertex graph).

Hence, $\gamma_{oed}^{D}(C_n) \geq \gamma_{ed}^{D}(C_n) \geq n/2$

Case(i) m-odd.

$S = \{v_1, v_3, \dots, v_m, v_{m+2}, \dots, v_{2m-1}\}$is the minimum D-eccentric dominating set in C_n. $\langle S \rangle$contains isolated vertices. Hence S is also an isolate D-eccentric dominating set.

Therefore, $\gamma_{oed}^{D}(C_n) \leq n/2$.

From (1) and (2), $\gamma_{oed}^{D}(C_n) = n/2$

Case(ii) m-even.

$S = \{v_1, v_3, \dots, v_{m-1}, v_{m+2}, \dots, v_{2m}\}$is the minimum D-eccentric dominating set in C_n. $\langle S \rangle$contains isolated vertices. Hence S is also an isolate D-eccentric dominating set.

Therefore, $\gamma_{oed}^{D}(C_n) \leq n/2$.

From (1) and (3), $\gamma_{oed}^{D}(C_n) = n/2$.

Proof of (B):

When n is odd, each vertex of C_n has exactly two D-eccentric vertices. If $n = 2m + 1, v_i \in V(G)$ has v_{i+m}, v_{i+m+1} as eccentric vertices.

Case(i) $n = 3m$, p-odd ⇒ m-odd.

$S = \{v_1, v_4, v_7, \dots, v_m, v_{m+3}, \dots, v_{2m-1}\}$is the minimum D-eccentric dominating set in C_n. $\langle S \rangle$ contains isolated vertices. Hence S is also an isolate D-eccentric dominating set.

Therefore, $\gamma_{oed}^{D}(C_n) = n/3$.

Case(ii) $= 3m + 1$, n-odd ⇒ m-even. $S = \{v_1, v_4, \dots, v_{m+1}, v_{m+3}, v_{m+6}, \dots, v_{2m-1}\}$is the minimum D-eccentric dominating set in C_n. $\langle S\rangle$contains isolated vertices. Hence S is also an isolate D-eccentric dominating set.

Therefore, $\gamma^D_{oed}\,(C_n) = \lceil n/3\rceil$.

Case(iii) $n = 3m + 2 \Rightarrow 3m$-odd $\Rightarrow m$-odd.

$S = \{v_1, v_4, \ldots, v_{m-1}, v_m, v_{m+3}, \ldots, v_{2m+1}\}$is the minimum D-eccentric dominating set in C_n. $\langle S\rangle$contains isolated vertices. Hence S is also an isolate D-eccentric dominating set.

Therefore, $\gamma^D_{oed}\,(C_n) = \lceil n/3\rceil + 1$.

THEOREM: 4.3 If G is of D-radius seven and D-diameter nine and if G has a pendant vertex h of D-eccentricity three then $\gamma^D_{oed}\,(G) \le \Delta(G)$.

PROOF: If G has a pendant vertex h of D-eccentricity three then its support f is of D-eccentricity two. In this case, $N(f)$ is an isolate D-eccentric dominating set, since h is an isolated vertex in $\langle N(f)\rangle$. Thus, $\gamma^D_{oed}\,(G) \le \deg_G\,(f) \le \Delta(G)$.

THEOREM: 4.4 If G is of D-radius greater than seven, then $\gamma^D_{oed}\,(G) \le n - \Delta(G)$.

PROOF: Let w be a vertex of maximum degree $\Delta(G)$. Then w dominates $N[w]$ and the vertices in $V - N[w]$ dominate themselves. Also, since diam $(G) > 9$, each vertex in $N(w)$ has D-eccentric vertices in $V - N[w]$ only. Therefore, $V - N(w)$ is an D-eccentric dominating set of cardinality $n - \Delta(G)$ and u is an isolated vertex in $\langle V - N(w)\rangle$, so that $\gamma^D_{oed}\,(G) \le p - \Delta(G)$.

OBSERVATION: 4.2

(i) If $G = \overline{K_2} + K_1 + K_1 + \overline{K_2}$, then $\gamma(G) = 2, \gamma^D_{ed}(G) = 4, \gamma_o(G) = 3, \gamma^D_{oed}(G) = 4$.

(ii) If $G = K_p + K_1 + K_1 + K_p, p > 2$ then $\gamma(G) = \gamma^D_{ed}(G) = \gamma_0(G) = \gamma^D_{oed}\,(G) = 2$.

5. Conclusion

Here we have initiated the study of isolate D-eccentric domination in graphs. We have studied isolate Deccentric domination in some families of graphs and also studied some bounds for isolate Deccentric domination number of a graph.

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