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Split 2-Equitable Domination in Fuzzy Graphs

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A sub set $D \subseteq V(G)$ of a fuzzy graph $G(\sigma,\mu)$ is said to be equitable dominating set if each $v \in V - D$, there exists a vertex $u \in D$ such that $uv \in E(G)$ and $|\deg_s(u) - \deg_s(v)| \leq 1$. An equitable dominating set $D$of $V(G)$ is said to be 2-equitable dominating set in a fuzzy graphs $G(\sigma, \mu)$, if every vertex $v \in V - D$there exists a vertex $u \in D$ or v is equitable dominated by at least two vertices in D.A 2-equitable dominating set $D \subseteq V(G)$ of a fuzzy graph $G(\sigma, \mu)$ is said to be split2-equitable dominating set if the induced subgraph $\langle V - D \rangle$ is disconnected. In this study, split2-equitable equitable dominating set, its number in fuzzy graphs are introduced. Bounds and some theorems related to split2-equitable equitable domination numbers are stated and proved.

Keywords: Fuzzy graph, equitable dominating set, equitable domination number,2 - equitable dominating set,connected equitable dominating set and its number, split2 - equitabledominating set, split2 - equitable dominating number.

1. Introduction

Applications of fuzzy graph are include data mining, clustering, image capturing, networking, communications, planning, etc.,L.AZadeh [1] introduced fuzzy sets in 1965.Fuzzy graph theory was initiated by A. Rosenfeld [2] in 1975.Revathiet.al[3] introduced the concept the split equitable domination in number in fuzzy graphs in 2016. Gurubaranet.,all[4] initiated the concept 2- equitable domination in fuzzy graphs in 2018. Complementary nil g-eccentric domination fuzzy graphs concepts introduced by Mohamed Ismayil and Muthupandiyan[5] in 2020. Rao Y., et al[6] stated Equitable domination in vague graphs with application in medical sciences in 2021.S. Muthupandiyan and A. Mohamed Ismayil [7] introduced the concept isolate g-eccentric domination in fuzzy graph in 2023. John JC, Xavier P, Priyanka GB.[8] Divisor 2-equitable domination in fuzzy graphs in 2023.Muthupandiyan S, IsmayilAM[9] introduced the concept ofconnected g-eccentric domination in fuzzy graphs in 2022. RabeehAhamed et.al.,[10] stated the concept complementary nil equitable domination in fuzzy graphs in 2024.

2. Basic Definitions

Definition 2.1[2]: A fuzzy graph $G = (\sigma, \mu)$is characterized with two functions ρ on V and μ on $E \subseteq V \times V$, where $\sigma: V \to [0,1]$ and $\mu: E \to [0,1]$ such that $\mu(x, y) \le \rho(x) \wedge \rho(y) \forall x, y \in V$. We expect that V is finite and non-empty, μ is reflexive and symmetric. We indicate the crisp graph $G^* = (\sigma^*, \mu^*)$ of the fuzzy graph $G(\sigma, \mu)$ where $\sigma^* = \{x \in V: \rho(x) > 0\}and\mu^* = \{(x, y) \in E: \mu(x, y) > 0\}$. The fuzzy graph $G = (\sigma, \mu)$ is called trivial in this case$|\rho *| = 1$.

Definition 2.2[5]: A path Pof lengthn is a sequence of distinct nodes $u_0, u_1, \dots, u_n$ such that $\mu(u_{i-1}, u_i) > 0$, $i = 1,2, \dots, n$ and the degree of membership of a weakest arc is defined as its strength.

Definition 2.3[5]: An edge is said to be strong if its weight is equal to the strength of connectedness of its end nodes. Symbolically, $\mu(u, v) \ge CONN_{G-(u,v)}(u, v)$.

Definition 2.4[5]:The order and size of a fuzzy graph $G(\sigma, \mu)$ are defined by $p = \sum_{u \in V} \sigma(u)$ and $q = \sum_{uv \in E} \mu(u, v)$ respectively.

Definition 2.5[5]: Let $G(\sigma, \mu)$ be a fuzzy graph. The strong degree of a vertex $v \in \sigma^*$ is defined as the sum of membership values of all strong arcs incident at v and it is denoted by $d_s(v)$. Also, it is defined by $d_s(v) = \sum_{u \in N_s(v)} \mu(u, v)$ where $N_s(v)$ denotes the set of all strong neighbors of v.

Definition 2.6[7]: A fuzzy graph $G(\sigma, \mu)$ is connected if $CONNG(u, v) > 0$ where $CoNNG(u, v)$ is strength of connectedness between two vertices u, vin $G(\sigma, \mu)$.

Definition 2.7[7]: In a fuzzy graph $G(\sigma, \mu)$, strength of connectedness between two vertices $u, v \in V(G)$ is maximum strength of all paths between u, v in $V(G)$.

Definition 2.8[7]: A subset D of V is called a dominating set (DS) in G if for every $v \notin D$ there exist $u \in D$ such that udominates v. The minimum scalar cardinality taken over all dominating set is called domination number and is denoted by the symbol γ. The maximum scalar cardinality of a minimal dominating set is called upper domination number and is denoted by the symbol Γ.

Definition 2.9[4]:A sub set $D \subseteq V(G)$ of a fuzzy graphs $G(\sigma, \mu)$ is said to be equitable dominating set (EDS) if each $v \in V - D$, there exists a vertex $u \in D$ such that $uv \in E(G)$ and $|deg_s(u) - deg_s(v)| \le 1$.

Definition 2.1043]:An equitable dominating set $D \subseteq V$ of a fuzzy graph $G = (\sigma, \mu)$ is called 2 – equitable dominating set if for every vertex $v \in V - D$ there exist $v \in D$ or v is equitable dominated by atleast two vertices in D.

Definition 2.11[3]:Let $D \subseteq V(G)$ of a fuzzy graphG is an equitable dominating set. A set $D \subseteq V(G)$ of a fuzzy graph $G(\sigma, \mu)$ is said to be split equitable dominating set, if the induced subgraph $\langle V - D \rangle$ is disconnected.

3. Main Results

The Split2- Equitable Domination in Fuzzy Graphs

In this section discuss about split2- equitable dominating set and its number in fuzzy graphs. Bound and theorem related to split 2- equitable domination number in fuzzy graphs are stated and proved.

Definition 3.1:

A 2-equitable dominating set $D \subseteq V(G)$ of a fuzzy graph $G(\sigma, \mu)$ is said to be split2- equitable dominating set if the induced subgraph $\langle V - D\rangle$ is disconnected. A split2 – equitable dominating set D is said to be minimal if no proper subset of D'isasplit2 – equitable dominating set.The minimum scalar cardinality of a minimal split2 – equitable dominating set of G is called the split2 – equitable dominating number of G and is denoted by $\gamma_{s2eqd}(G)$.The maximum scalar cardinality of a minimal split2 – equitable dominating set of G is called anupper split2 – equitable dominating number of G and is denoted by $\Gamma_{s2eqd}(G)$.

Note 3.1:The minimum split2 - equitable dominating set is denoted by γ_{s2eqd}-set.

Example 3.1:Consider the fuzzy graph $G(\sigma, \mu)$.

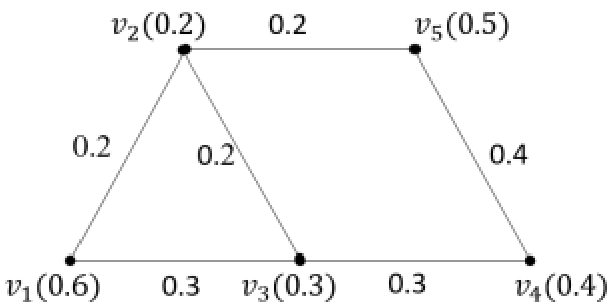

Figure: Split2-Equitable Dominating Set in a Fuzzy Graph

From the fuzzy graph given in example 3.1, the followings are observed.

1. The minimumsplit2- equitable dominating set is $D_1 = \{ v_2, v_3, v_4, \}$, then $\gamma_{s2eqd}(G) = 0.9$.

2. The upper split2- equitable dominating set is,$D_2 = \{ v_2, v_4, v_5 \}$, then $\Gamma_{s2eqd}(G) = 1.0$

Observation 3.1: For any connected fuzzy graphs $G(\sigma, \mu)$

1. $\gamma(G) \leq \gamma_{eqd}(G) \leq \gamma_{2eqd}(G) \leq \gamma_{s2eqd}(G)$

2. $\gamma_{s2eqd}(G) \leq \Gamma_{s2eqd}(G)$.

3. Obviously any minimumsplit2-equitable dominating set is also minimal but the converse is not true.

4. The complement of a split2-equitable dominating set is need not be split.

5. Supper set of a split2-equitable dominating set is also a split2-equitable dominating set.

Proposition 3.1:For any fuzzy graph G with order p, then $\sum_{\substack{v_i,v_j \in G \\ v_i \neq v_j}} \min(\sigma(v_i), \sigma(v_j)) \leq \gamma_{s2eqd}(G) \leq p$.

Proof:
Let D be a splitdominating set of a fuzzy graph G having atleast two vertices has minimum of V which is a sum of minimum value of vertices $v_i, v_j \in D', \gamma_{s2eqd}(G) \leq p$ it is obviously true.

Theorem 3.1:Let G be a fuzzy graph$\gamma_{s2eqd}(G) = p$ iff the fuzzy graph G has adjacent to less than two vertices.

Proof:
Let G be a fuzzy graph, then $\gamma_{s2eqd}(G) = p$ then definition of fuzzy graph has all vertices in dominating set D. which shows that every vertex in G has adjacent to less than two vertices. Conversely, G be a fuzzy graph has adjacent to less than two vertices then every vertex is in regulardominating set. Which is$\gamma_{s2eqd}(G) = p$.

Theorem 3.2:Let G be a connected fuzzy graph has no non - equitable edge and H is spanning subgraph of Gthen $\gamma_{s2eqd}(G) \leq \gamma_{s2eqd}(H)$.

Proof:
Let G be a connected fuzzy graph and H is the spanning subgraph of H. consider D is minimum split2 - equitable dominating set of G, D also asplit 2 - equitable dominate all the vertices in $V(H) - D$ that is D is an split2 - equitable dominating set in H. Hence $\gamma_{s2eqd}(G) \leq \gamma_{s2eqd}(H)$.

Theorem 3.3:For any fuzzy graph G, $\gamma_{2eqd} + \min\sigma(v_i) \leq \gamma_{s2eqd}(G)$, for $v_i \notin D'$.

Proof:
Let D besplit 2 - equitable dominating set with minimum cardinality $\gamma_{s2eqd}$for any vertex $v_i \in D$, $D - \{v_i\}$ is2 - equitable dominating set. Hence $\gamma_{2eqd} + \min\sigma(v_i) \leq \gamma_{s2eqd}(G)$.

Theorem 3.4:Let G be a fuzzy graph without isolated vertices. Then $\gamma(G) \leqslant \gamma_{s2eqd}(G)$

Proof:

Every split2- equitable dominating set is a split dominating set. Thus $\gamma(G) \leqslant \gamma_{s2eqd}(G)$

Theorem 3.5A split2- equitable dominating set exists for any strong fuzzy graph G .

Proof.

Let $G = (\sigma, \mu)$ be a fuzzy graph. Suppose a strong fuzzy graph G has asplit2- equitable dominating set, obviously it contains a 2 equitable dominating set D. Therefore every strong fuzzy graph has a split2 - equitabledominating set and it exists for strong fuzzy graph.

Theorem 3.6For any fuzzy graph G $\gamma_{2eqd}(G) \leqslant \gamma_{s2eqd}(G)$.

Proof.

It is clear that every inverse 2- equitable dominating set is a2-equitable dominating set. we get$\gamma_{2eqd}(G) \leq \gamma_{s2eqd}(G)$.

Theorem 3.7For a fuzzy graph $G = (\sigma, \mu)$ if γ_{s2eqd} is a minimum split2-equitable dominating set then $V - D$ is a dominating set of a fuzzy graph G.

Proof

Let v be any vertex in D, D is a split2-equitableset in G. Since G has no isolated vertex $v \in N_s(u)$.It is clearly every split2-equitable dominating set is a equitable dominating set such that $v \in V - S$.Hence every vertex of D dominates some of the vertices in $V - S$.Therefore,$V - D$ is a dominating set of fuzzy graph G.

Theorem 3.8

Aconnected2-equitable dominating set D of a fuzzy graph$G$ is minimal if and only if for every vertex $v \in D$ one of the following conditions holds

(i) there exists vertex $u \in V - D$ such that $N_s(v) \cap D = \{v\}$

(ii) $N_s(u) \cap D = \emptyset$

(iii) $\langle (V - D) \cup \{v\} \rangle$is disconnected.

Proof

Suppose D is minimal and there exists a vertex $v \in S$ such thatdoes not hold any of the above conditions. Then by conditions (i) and (ii) $D_1 = D - \{v\}$ is a 2-equitable dominating set of G.Also by (iii) $\langle V - D \rangle$ is disconnected. This implies that D_1 is a split 2-equitable dominating set of , a contradiction. Converse is obious.

Theorem 3.9

For any fuzzy graph G without equitable isolated nodes, $\gamma(G) \leq \min\{\gamma(G), \gamma_{2ed}(G), \gamma_{s2eqd}(G)\}$

Proof

Every split2-equitable dominating set is aconnected dominating sets of G and every split dominating set is dominating set, similarly every 2-equitable dominating set is dominating set. Hence $\gamma(G) \leq \min\{\gamma(G), \gamma_{2ed}(G), \gamma_{s2eqd}(G)\}$.

Observation 3.2

1. If G is regular fuzzy graph or $(k, k+1)$ bi regular fuzzy graph for some k then $\gamma_{eq}(G) = \gamma(G)$.

2. If G is regular or bi-regular fuzzy graph with at least one equitable end vertex, then $\gamma(G) = \gamma_{spd}(G) = \gamma_{seqd}(G) = \gamma_{s2eqd}(G)$.

Theorem: 3.10

If D is a split 2-equitable dominating set of a fuzzy graph G then D is a both minimal split 2-equitable dominating set and a maximal split equitable dominating set. Conversely any maximal split2- equitable dominating set Din G is a split 2-equitable dominating set of G.

Proof:
If Dis a split 2-equitable dominating set of G. $D_d = S - \{d\}$is not a split2- equitable dominating set for every $d \in D$ and $D \cup \{x\}$ is not a split 2-equitable dominating set for every $x \notin D$ so that D is a minimal 2- equitable dominating set and a maximal split 2-equitable dominating set.

Conversely, let D be a maximal split 2-equitable dominating set in G.Then for every $x \in V - D$, $D - \{x\}$ is not split 2-equitable dominating set and x is dominated by some element of S.Thus S is a split 2-equitable dominating set of G.

Observation 3.3

1. Let G be a fuzzy graph then $\gamma_{s2eqd}(G) \leq P - \Delta_s\,(G)$
2. For any fuzzy graph P_σ with$|\sigma^*| = n$, $n \geq 5$ vertices $\gamma_{s2eqd}\,(\bar{\bar{p}}_{\bar{\sigma}}) \leq P - 3$.

4. Conclusion

In this article, split2 - equitable dominating set, its number in fuzzy graphs are obtained. Theorems related to split2 -equitable dominating set and number in a fuzzy graphare stated and proved. Bounds and some points related split2-equitable domination number are observed and discussed.

References

[1] Zadeh LA. Fuzzy sets. Information and control, 1965;8(3):338-353. Available at https://doi.org/10.1016/S0019-9958(65)90241-X

[2] Rosenfeld A. Fuzzy graphs. InFuzzy sets and their applications to cognitive and decision Processes, Academic press,1975 ;77-95. Available at https://doi.org/10.1016/B978-0-12-775260-0.50008-6

[3] S. Revathi, C.V.R Harinarayanan and Muthuraj, The Split Equitable Domination number of a Fuzzy graph, 2016;8(2);93-98. ijmrv8n2_03.pdf

[4] C. Gurubaran, A. prasanna and A. Mohamed Ismayil, 2- Equitable domination in Fuzzy Graphs, International Journal of Applied Engineering Research, 2018;13(16), 12531-12535. Available at ijaerv13n16_18.pdf (ripublication.com)

[5] Ismayil AM, Muthupandiyan S. Complementary nil g-eccentric domination in fuzzy graphs. Advances in Mathematics: Scientific Journal. 2020;9(4):1719-1728. Available at https://doi.org/10.37418/amsj.9.4.28

[6] Rao Y, Kosari S, Shao Z, Qiang X, Akhoundi M, Zhang X. Equitable domination in vague graphs with application in medical sciences. Frontiers in Physics. 2021;9:635642. Available at https://doi.org/10.3389/fphy.2021.635642

[7] Muthupandiyan S, Ismayil AM. Isolate g-eccentric domination in fuzzy graph. Ratio Mathematica. 2023;50. Available at http://dx.doi.org/10.23755/rm.v50i0.1548

[8] John JC, Xavier P, Priyanka GB. Divisor 2-Equitable Domination in Fuzzy Graphs. Discrete Mathematics and Mathematical Modelling in the Digital Era: ICDM3DE-2023, Gandhigram, India, March 23–25.:219.

[9] Muthupandiyan S, Ismayil AM. Connected g-Eccentric Domination in Fuzzy Graph. Journal

ofAlgebraic Statistics. 2022;13(2):3377-3382. Available at Connected g-Eccentric Domination in Fuzzy Graph | JOURNAL OF ALGEBRAIC STATISTICS

[10] Ahamed TR, Mohideen SI, Hussain RJ, Complementary Nil Equitable Domination in Fuzzy Graphs. Indian Journal of Science and Technology, 2024:17(38): 3942-3946. Available at https://doi.org/10.17485/IJST/v17i38.2819