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A Mathematical Solution of Fuzzy Transportation Problem by Runge-Kutta Ranking Method for Forecast Analysis

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A Transportation problem is a particular case of the Linear Programming Problem in the subject area of Operation Research. The cost of the payoff outlay values of the Fuzzy Transportation problem are measured as fuzzy numbers, in particular triangular fuzzy numbers. Firstly, the Triangular Fuzzy numbers are renewed into crisp regularvalues using fuzzy ranking techniques.Then, anincredibleoptimum solution of the Fuzzy Transportation Problem is acquired by characteristicVogel's Approximation algorithmprocess.Here, Runge Kutta ranking method is aninnovativeproposed approach which is exhibitedby a numerical example and compared to our proposed technique with three other ranking methods.

Keywords: Fuzzy Transportation-Triangular Fuzzy Number-Runge Kutta Ranking Method-Vogel's Approximation, Algorithm-Optimum Transportation Schedule.

1. Introduction

Transportation problem is used worldwide in solving definite physical humankind problems. The resolution of the problem will make powerful us to establish the quantity of entities to be transported from a particular source to a particular destination so that the cost obtained is minimize theexpenditure. Let a_i be the number of units of a product obtainable at sourcei and b_j be the numberof units of the item for consumptionessential at destination j. Let C_{ij} be the cost of transporting one unit from sourcei to destination j and let x_{ij} be the amount of quantity agreed or shipped from sourcei to destination j. A fuzzy transportation problem is a transportation problem in which the transportation payoff cost, supply and demand quantities are triangular fuzzy numbers.The ranking model of the fuzzy number is aessentialtechniques to fix a number on real line.Alatest method called Runge Kutta ranking method is presented for the ranking of triangular fuzzy numbers. As this proposed ranking method is simple and direct, it is very easy to recognize and to find the optimal solution of fuzzy transportationproblems taking place in the real-life situations. Finally, the Modified distribution method is adopted to validate the optimality conditionsof the Fuzzy transportation solutions.

2. Basic Definitions

2.1 Fuzzy Numbers

A Fuzzy Number is a uniqueoutline of fuzzy sets on R of real numbers.Along with the different types of the fuzzy sets, of extraordinary consequence are fuzzy numbers with the intention of distinct on the set R of real line. The membership functions of these sets in the form $\mu_{\tilde{A}}: R \rightarrow [0,1]$ is viewed as fuzzy numbers. Every fuzzy number can fully and uniquely be represented by its η-cut also is a convex fuzzy set. Also,μ-cut of each fuzzy number are closed intervals [a, b] of real numbers for all real $\mu \in [0, 1]$.

2.2 Triangular Fuzzy Number

A fuzzy number $\tilde{A}$= (p, q, r), p, q, r are in real number R and $p \leq q \leq r$, is a triangular fuzzy number if its membership utility$\mu_{\tilde{A}}$: R → [0, 1]is defined by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x-p)}{(q-p)}, p \leq x \leq q \\ 1 \\ \frac{(x-r)}{(q-r)}, q \leq x \leq r \end{cases}$$

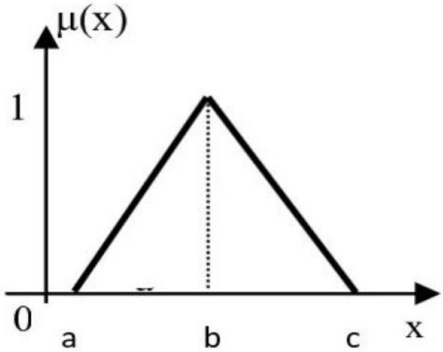
2.3μ- cut of a Triangular Fuzzy Number:

Let A be a fuzzy number and let μ be a real number of the closed interval [0, 1]. Then the μ-cut of A, denoted by A^{μ}, is defined to be the set

$A_{\mu} = \{x \in U / m(x) \geq \mu\} = [p^{\mu}, r^{\mu}] = [p + (q - p)\,\mu, r - (r - q)\,\mu]$

A fuzzy set A on R is said to be convex, if its μ-cut A^{μ} are usual closed real intervals, for all x in [0, 1].A fuzzy number is a normal and convex fuzzy set A on R with a piecewise continuous membership functions.

Figure 1 Triangular Fuzzy Number



2.4 Operations on Triangular Fuzzy Number

Assume that $\tilde{\tilde{A}}$ =(l, m, u) and another B=(p, q, r) are two triangular fuzzy numbers, where l, m, u are in R, l≤ m ≤ u also p, q, r are in R and p ≤ q ≤ r, then

(i) A+B=(l+p, m+q, u+r).

(ii) k + A = (k + l, k + q, k + r}

(iii) AxB =(x, y, z) where F={lp,lr, up,ur}, x =minF, y =mq, z =max F. Suppose if all are non-zero positive real number then A x B = (lp, mq, ur).

(iv) Let $\alpha \in$ R then

$$\alpha\tilde{A} = \begin{cases} (\alpha l, \alpha m, \alpha u), if \alpha \geq 0 \\ (\alpha u, \alpha m, \alpha l), if \alpha < 0 \end{cases}$$

3. Formulation of Fuzzy Transportation Problem

A typical fuzzy transportation problem involving ‘m’ sources and ‘n’ destinations can be stated into mathematically representation is as follows:

a_i - quantity of a product available at source i

b_j - quantity of a product available at destination j

T_{ij} - cost of transporting one unit of a product from source i to destination j

X_{ij} - quantity transported from source i to destination j

Minimum $\tilde{w} = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{T}_{ij} x_{ij}$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i; \quad i = 1, 2, 3, \ldots m, \qquad \sum_{i=1}^{n} x_{ij} = b_j; \quad j = 1, 2. 3, \ldots n$$

and the RIM condition is sum of the total supply and the sum of the total demand are equal.

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

Hence, the objective function and the constraints are all linear. A basic feasible solution for the fuzzy transportation problem must consists of (m + n – 1) occupied allocation cells. Then the basic feasible solution is called non-degenerate feasible solution. The fuzzy transportation table can be thought of as an m × n matrix having 'm.n' square cells.

Table 1 Fuzzy Transportation Problem with Cost Payoff Matrix

Resources	Activity				
	1	2	3	…j…	Supply
1	T11	T12	T13	.. T1j..	Ai
2	T21	T22	T23	.. T2j ...	a2
-					
i	Ti1	Ti2	Ti3	.. Tij..	Ai
-					
Demand	b1	b2	b3	.. bj..	$a_i = b_j$

A set of non-negative values x_{ij} (i = 1, 2, 3, . . . m, j = 1, 2. 3, . . n) that satisfied the constraints is called a feasible solution to the fuzzy transportation problem. The necessary and sufficient condition for the existence of a feasible solution to the fuzzy transportation problem is that

Total Supply = Total Demand

3.1 Modified Distribution Method (MODI)

Various steps involved in solving any transportation problem by Modified Distribution Method may be summarized as follows:

❖ Find an initial basic feasible solution by using Vogel's approximation method.

❖ Check the number of occupied cells, if there are less than (m+n-1), there exists degeneracy and we introduce a very small positive assignment of $\in \rightarrow 0$ in suitable independent positions, so that the number of occupied cells is exactly equal to m+n-1.

❖ For all the occupied cells, determine a set of numbers u_i (i = 1, 2, ... ,m) for each row and v_j(j = 1, 2, ... ,n) for each column by solving the system of equations $u_i + v_j = C_{ij}$. Because the number of unknown u; and v; are m + n, we can assign an arbitrary value (conveniently zero) to one of the unknowns without violating the equations.

❖ Compute the opportunity costs (net evaluations) for all the unoccupied cells by using the relationship $\Delta_{ij} = u_i + v_j - C_{ij}$for i = 1, 2, ... ,m and j = 1, 2, ... ,n.

❖ Check the sign of each opportunity cost. If all the opportunity costs are less than or equal to zero, an optimum solution is reached. Otherwise go to next step.

❖ Select the largest positive opportunity cost computed in the table, the unoccupied (non-basic) cell corresponding to this positive value becomes occupied (basic) in the next iteration.

❖ Determine a closed path for the current entering cell that starts and ends at this unoccupied cell. Right angle turns in this closed path are permitted only at the occupied cells and at the current entering (unoccupied) cell.

❖ Assign alternate plus and minus signs at the corner points of occupied cells on the closed path starting with plus sign at the current entering cell (unoccupied cell).

❖ Determine the maximum number of units that should be shipped to the selected unoccupied cell. This is being done by choosing the least number of units that can be allocated to the occupied cells having minus sign. Add this quantity to all the cells on the closed path marked with (+) sign and subtract the same from the cells marked with (-) sign.

❖ Repeat again and again the procedure until an optimum solution is attained.

4. Fuzzy Ranking Methods

De-fuzzification is the classical route of distinctunique value (crisp value) which signifies the typicalrate of thetriangular fuzzy numbers. Everyranking method represents anextraordinary point of analysison the triangular fuzzy numbers.Now, weestablished the proposed ranking method which is simple and easy in computationand gives better approximation of optimum solution which is explained through the numerical example.

4.1Runge-KuttaRanking Method

The Runge-Kutta ranking method involves fuzzy sets in which a ranking approach $\Re(\tilde{A})$ is calculated for the fuzzy numbers $\tilde{A} = (m, n, \alpha, \beta)_{LR}$ from its μ-level cut is defined by $\tilde{A}_\mu = [m - \alpha L^{-1}(\mu), n + \beta R^{-1}(\mu)]$

$$\Re(\tilde{A}) = \frac{5}{48}[m - \alpha L^{-1}(\mu)] + \frac{27}{56}[\gamma] + \frac{125}{336}[\gamma] + \frac{1}{24}[m + \beta L^{-1}(\mu)]$$

4.2 Centroid Mean Ranking Method

A Centroid mean ranking method is the development for fuzzy sets accomplish of $C(\tilde{A})$ for the fuzzy number $\tilde{A} = (m, n, \alpha, \beta)_{LR}$ is its ημlevel cut is

$$\tilde{A}_\mu = [m - \alpha L^{-1}(\mu), n + \beta R^{-1}(\mu)]$$

$$C(\tilde{A}) = \frac{(m - \alpha L^{-1}(\mu)) + \gamma + (n + \beta R^{-1}(\mu))}{3}$$

4.3Robust's Ranking Method

The Robust's ranking method is defined by $R(\tilde{A}) = \frac{1}{2}\int_0^1 (a_L, a^U)da$, where (a_L, a^U) is the μ-level cut of the fuzzy number $\tilde{A}$. The Robust's ranking index R(a) gives fuzzy number, it satisfies the additive linearity property.

$R(\tilde{A}) = \frac{1}{2}\int_0^1 (a_L, a^U) da$

4.4 PERT Ranking Method

A project evaluation review technique ranking method progressthe result of$P(\tilde{A})$ is defined by triangular fuzzy number $\tilde{A} = (m, n, \alpha, \beta)_{LR}$ from its μ-level cut is

$\tilde{A}_\mu = [m - \alpha L^{-1}(\mu), n + \beta R^{-1}(\mu)]$

$P(\tilde{A}) = \frac{(m - \alpha L^{-1}(\mu)) + 4\gamma + (n + \beta R^{-1}(\mu))}{6}$

5. Numerical Example

There are four ATM centers located in different parts of the Chennai city in India. Anamount of money must be deposited in the fourzonal branch officesat different placesin the Chennai city. The freight shipping cost for the money transfer from the respective branchesto all ATM locations are as follows.

		ATM Centers				Supply
		Mambalam	Tambaram	Adayar	Park	
Branch	Gundy	[62 65 70]	[24 26 30]	[70 73 79]	[20 24 30]	[60 70 80]
	Porur	[70 73 76]	[54 57 60]	[60 62 66]	[30 36 45]	[30 33 39]
	Beach	[22 25 27]	[66 68 70]	[21 27 36]	[76 78 82]	[84 90 99]
	Egmore	[68 72 76]	[33 36 45]	[27 30 36]	[36 38 40]	[42 48 57]
Demand		[51 60 66]	[66 72 84]	[55 59 62]	[44 50 52]	[216 241 275]

The problem is to find the optimal freight shipping cost so that the total cost of transfer the money job become minimum.

Solution

Here, given transportation problem is balanced transportation problem since TP satisfied the RIM condition, i.e., the total supply = the total demand = [216 241 275]

Apply Vogel's approximation method for Runge Kutta ranking payoff cost and find an initial basic feasible solution of the fuzzy transportation problem is

		ATM			
		Mambalam	Tambaram	Adayar	Park
Branch	Gundy		52.6875		16.6875
	Porur				32.9375
	Beach	59.4792		30.2708	
	Egmore		19.1875	28.5625	

The solution is non-degenerate since number of allocations = m + n – 1 = 7 occupied allocation cells. Then the optimum minimum cost is = 6773.046375

Using Vogel's approximation method for PERT ranking payoff cost and find an initial basic feasible solution of the fuzzy transportation problem is

		ATM			
		Mambalam	Tambaram	Adayar	Park
Branch	Gundy		53.5		16.5
	Porur				33.5
	Beach	60.1667		30.3333	
	Egmore		19.5	29	

The solution is non-degenerate since number of allocations = m + n – 1 = 7 occupied allocation cells. Then the optimum minimum cost is = 6967.3888

By Vogel approximation's method for Robust's ranking payoff cost and find an initial basic feasible solution of the fuzzy transportation problem is

		ATM			
		Mambalam	Tambaram	Adayar	Park
Branch	Gundy		53.75		16.25
	Porur				33.75
	Beach	60.25		30.5	
	Egmore		19.75	29	

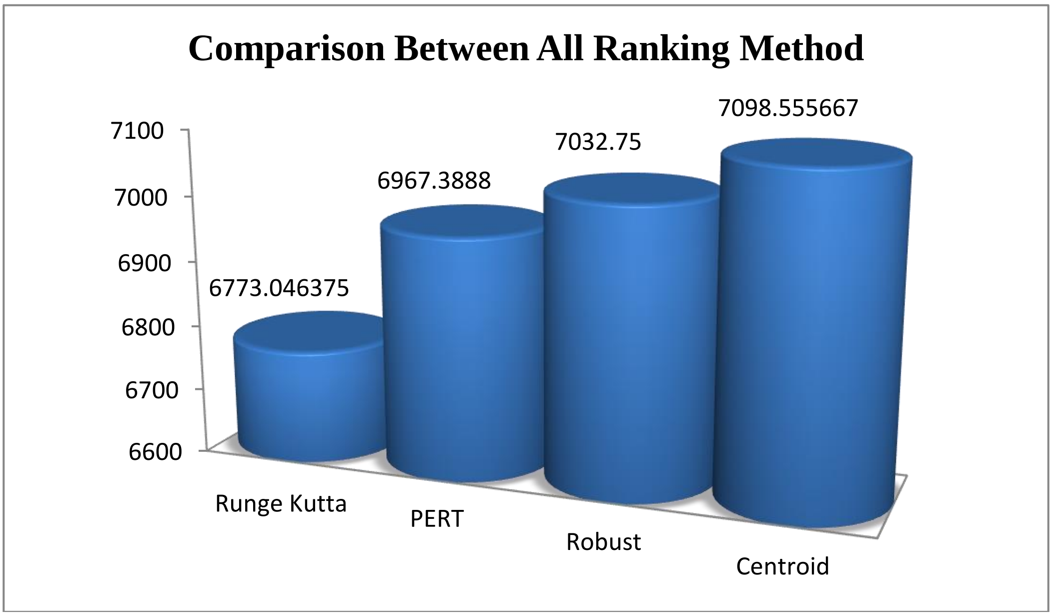
The solution is non-degenerate since number of allocations = m + n – 1 = 7 occupied allocation cells. Then the optimum minimum cost is = 7032.75

With help of Vogel approximation's method for Centroid ranking payoff cost and find an initial basic feasible solution of the fuzzy transportation problem is

		ATM			
		Mambalam	Tambaram	Adayar	Park
Branch	Gundy		54		16
	Porur				34
	Beach	60.3333		30.6667	
	Egmore		20	29	

The solution is non-degenerate since number of allocations = m + n – 1 = 7 occupied allocation cells. Then the optimum minimum cost is = 7098.555667

Figure 2 Solution of Fuzzy Transportation Problem with minimum cost



6. Conclusion

In real-lifesituation, any ranking method is done on one-to-one basis, and it minimizes the total fuzzy transportation cost in all the cases. Here, when comparing all the three ranking methods, PERT, Robust and Centroid against the Runge Kutta method, we notice that the total transportation cost of the problem is very minimum in the Runge Kutta method. Therefore, the proposed Runge Kutta ranking method can clearly be seen as the best method compared to the other three methods used in our case study.We have now established that in ourcomputational practice the consequences of solutions are as per expectations and satisfaction, and it has been proved in this article all the way through the numerical example.

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