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Investigating the Application of Fuzzy Logic and Uncertainty Handling in Case-Based Decision Making

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This work scrutinizes the use of fuzzy logic and uncertainty management in CBDM, paying special focus on the difficulties and approaches associated with knowledge system uncertainty management. Uncertainty naturally pervades human decision making and can, therefore, significantly degrade the effectiveness of decision-support systems. There are factors such as faulty information and assumptions and surprise events behind this uncertainty. There are several stages involved in modeling and reducing impacts of uncertainty using fuzzification as a mathematical tool such as representation of vocabulary, similarity metrics, design of case bases, and transformation of the solution that appear in several phases of CBDM. Fuzzy sets can be applied in combining to solve all those problems like maintaining a case base in dynamic environment, the description of imprecisely case features, and cases retrieval with uncertainties. It also addresses the case when uncertainty can be expressed using formal techniques by possibility theory and linguistic variables with the aid of fuzzy logic so that the performance and effectiveness of case-based reasoning can be enhanced. Major techniques used in it are fuzzy feature vectors, fuzzy similarity functions, fuzzy decision trees, adaptive fuzzy clustering, and fuzzy integrals.Basically, the aim here is to improve decision-making systems through more reliable solutions that deal better with uncertainty, particularly under intricate real-world scenarios. This research advances the development of intelligent systems that use fuzzy logic in order to improve decision support in dynamic and uncertain situations.

Keywords: Fuzzy logic, uncertainty handling, case-based decision making, fuzzification, fuzzy sets, decision support systems.

1. Introduction

A useful mathematical tool for simulating much of what is characterized in a process by subjectivity, uncertainty, and indeterminacy, is fuzzification. Whilst some qualitative criteria and the values of the qualities used for assessment depend on the subjective rating of the decision maker regarding the relative weight of his or her chosen criteria uncertainty also exists in making the decision at all known parameters of the decision(Alakhras et al., 2020). The subjectivity of the decision maker cannot allow real-world problems to be solved. However, uncertainties must be brought into the decision-making process. Every human is constantly being faced with the necessity of having to make decisions, be it personal or work-related.

However simple the issue is, we may consider that ambiguity prevails with every choice that we have made. The problem here lies in how much the decision maker is willing to risk for making the wrong choice and how much he is willing to compromise over the ambiguity (Papageorgiou, 2011). There are two sorts of uncertainty: that arisen due to wrong assumption and that due to force majeure. Whether it is an event that influences the decision and one that is not expected or an event that is expected, but not all of the consequences, or the precise nature of the event, deficient assumptions arise from ignorance, or a lack of information about the nature of upcoming events. A force majeure is an event like bad weather, natural disaster, or any other similar case that has adverse effects, which were neither foreseen nor avoidable.

It is impossible to predict with certainty future events in every sector unless there are specific statistical techniques for us to make assumptions pertaining to the future using such knowledge. Using information culled (resource status, capacity, motivation of the workers, etc.), the assumption made by the decision maker involves assumptions that are generally valid as well as marginally variable with time(Varshavskii et al., 2024). If all factors are considered into account, the following exhibits uncertainty.

Interval mathematics is typically used to address uncertainties associated with imprecision in people-to-people communication and, for instance, refer to high people, low temperatures, and bad sales while the theory of fuzzy set is typically applied to simulate uncertainties that emerge whenever defining terms given to express characterization of a concept fail to specify a single number as expected and probability is typically used to model them (Atmani et al., 2013).

1.1. Uncertainty in Case-Based Reasoning (CBR) and Fuzzy Logic

This decision uncertainty may be due to the incompleteness, imprecision, and ambiguity in the data(Chai et al., 2013). Uncertainty may be caused by many factors in Case-Based Reasoning (CBR) systems. These factors may be due to the vocabulary of attributes used to describe the cases, the measures used for the similarity between cases, the case base itself that might include some old or redundant cases, and the transformation used to solve the problem after retrieval based on retrieved cases (Zarandi et al., 2011). These sources of uncertainty have a great impact on the system's performance and decision accuracy.

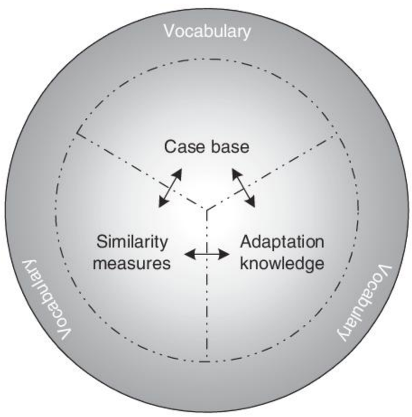

Figure 1: The knowledge containers of CBR system

In an instance where fuzzy logic addresses uncertainty in systems through handling imprecise, incomplete, or vague data (Dubois et al., 2006). It allows attribute representation with fuzzy sets explaining degrees of membership instead of fixed values, enabling greater decision nuance. As applied to the vocabulary in CBR, and similarity measures through solution transformation, systems now handle vague or ambiguous information effectively by improving the results' accuracy and reliability on account of the natural fuzziness associated with real world cases.

1.2. Fuzzy Logic Integration Improves Decision Support Systems

The integration of fuzzy logic into decision support systems (DSS) will enable better management of uncertainty and reproduce human decision-making (Sun and Finnie, 2007). As compared to the binary logic systems, DSS using fuzzy logic enables it to process imperfect and ambiguous data, allowing more flexible and adaptive decision-making. This integration is very useful in complex areas such as business model selection, clinical decision support, and risk assessment where data is usually imprecise. A DSS that uses fuzzy logic can aid executives in deciding on e-business models, considering qualitative and quantitative factors(Sabahi and Akbarzadeh-T, 2014).Fuzzy decision support systems aid dietitians in diet changes for patients with multiple chronic conditions by enhancing treatment planning. Fuzzy set theory aids these systems in accommodating the vagueness of human reasoning, thus being more nuanced and effective in decisions. Fuzzy logic also helps DSS adapt to changing data patterns, keeping relevance and accuracy even in dynamic contexts.

1.3. Objectives of the Study

- To explore the application of fuzzy logic and uncertainty handling in case-based decision-making (CBDM) systems.
- To investigate the integration of fuzzy sets and fuzzification techniques to reduce uncertainty in various stages of CBDM, including case retrieval and solution transformation.
- To enhance decision-making systems by providing reliable solutions that better manage uncertainty in complex, real-world situations.

2. LITERATURE REVIEW

Jahani et al. (2015) argued that to deal with the complexity of modern supply chains, multi-agent systems, fuzzy logic, and case-based reasoning (CBR) should be combined. Common problems in today's supply chains include client dissatisfaction, information overload, and too much uncertainty. While it is true that both qualitative and quantitative considerations are commonly overlooked by existing agent-based systems when it comes to supplier selection and other supply chain processes. Jahani et al. proposed an agent-based paradigm that combines multi-agent systems, fuzzy logic, and case-based reasoning in order to overcome some of these problems. Infrequently were these views united in previous studies. Usefulness of the proposed framework has been demonstrated through an exploratory case study performed within office furniture company. This agent-based architecture would suggest substitute products in cases of stockouts, judge supply offers on customer preference, and facilitate better communication among agents of the supply chain. The F-CBR approach narrows down the whole search space while retrieving cases by arranging information into relevant instances. The fuzzy nature of this approach tackles supplies chain uncertainty, most especially when customers have varying requirements and preferences. It was very short, only the description and retrieval stages of CBR approach, retaining, reusing, and altering phases were not covered and no inventory management and agent bargaining is mentioned(Jahani et al., 2015).

Kasie and Bright (2023)addressed the unexplored area of designing and controlling the cutter process in metal cutting. There have been computer-aided planning, simulation, and optimization techniques used for ages to address the problems encountered in cutter planning and control. They proposed a decision support system that utilized techniques of fuzzy case-based reasoning and fuzzy analytic hierarchy process. This integration provides a more reliable method to solve planning and control issues related to machining cutters, as it determines hybrid similarity measures between new and previous cases. The system is designed to adapt new arrivals of orders through retrieving the best matching of previous cases. For proving the soundness of the DSS, a numerical example was presented, offering fresh views of how fuzzy set theory, CBR, and AHP can be combined for this specific application (Kasie and Bright, 2023).

Uğur and Baykan (2016)investigated the use of fuzzy logic in construction management (CM) for its applicability and feasibility. The authors focused their research on papers in well-known journals listed under Science Citation Index (SCI), since CM spans an array of subjects. The present study found four basic applications of fuzzy logic in the field of CM, including: modelling, optimization, performance evaluation/assessment, and decision-making. The study underlines fuzzy logic's ability to deal with the uncertainty and imprecision that exist in construction projects and, therefore, highlights the potential for betterment in CM's decision-making and optimization procedures (Uğur and Baykan, 2016).

Kamal (2012)provided a framework for use-case-based effort prediction based on adaptive fuzzy logic that is supposed to address imprecision and to be able to consider expert opinions. Software development effort prediction is a challenging task that often involves dealing with imprecision, ambiguity, and incomplete knowledge, especially during the early stages of development. The paper focused on effort prediction using machine learning techniques such as fuzzy logic, neural networks, and genetic algorithms. An empirical evaluation of the proposed framework revealed some promising results and highlighted ways in which different objectives interact to impact prediction accuracy. The work also explored system architecture and its impact on the accuracy of fuzzy-based effort prediction systems. It emerged that the creation of a genetic-fuzzy tool to evolve different designs significantly impacted effort prediction system performance (Kamal, 2012).

Asemi et al. (2014) examined the usage of FMCDM approaches with special emphasis on how versatile these are within a wide variety of disciplines. From reviewing 150 studies using FMCDM approaches for decision-making purposes, the most prevalent usages were identified. Given fuzzy AHP and fuzzy TOPSIS are the most popular methodologies of FMCDM, the most widely examined topics so far are "location management" and "supplier selection." This paper pushes forward knowledge concerning the widespread use and application of FMCDM in the most vital processes of making decisions (Asemi et al., 2014).

Seleem et al. (2020)developed a lean manufacturing roadmap for industrial companies by selecting appropriate lean tools based on firm constraints and predetermined strategic objectives. In addition, the study ranks these tools based on their interrelationships. The first step in the process is to set operational goals using the BSC framework. The theory of constraints (TOC) is used to analyze the manufacturing system and identify its constraints, and then fault tree analysis (FTA) is used to determine the root causes of the constraints identified. Finally, lean efforts are prioritized with the aid of the fuzzy-decision making trial and evaluation laboratory (fuzzy-DEMATEL) method. Practical recommendations made by the study include examining the manufacturing system, monitoring the mastery of specialists' expertise, and directing the efforts by ranking them relative to one another in an effort to accomplish strategic goals without any increased time and cost. The study considers only the case of manufacturing organizations; thus, further study is needed to assess how successful the strategy is for the service organizations (Seleem et al., 2020).

3. RESEARCH METHODOLOGY

This section describes the research approach to deal with uncertainty in Case-Based Reasoning (CBR) systems, particularly using fuzzy set theory (FST). The methodology is laid out from case representation to solution transformation in order to analyze the diverse approaches that have been used to address uncertainty at every step of the CBR process. Depending on what source of uncertainty is drawn from—the knowledge base, the similarity measure, or even the CBR hypothesis itself—research approaches are put into different categories.

3.1. Addressing Uncertainty in the Vocabulary

Introducing uncertainty into the lexicon of the CBR system: fuzzy aspects of case features or when cases have both fuzzy and crisp aspects. The paper proposes an investigation into the utilization of fuzzy feature vectors, representing case features as a composition of fuzzy and crisp values, where the method will incorporate neural networks to efficiently merge all these different feature types at the case retrieval process.

To enhance knowledge acquisition, a fuzzy-based module is also incorporated into the design of CBR. Within this module, fuzzy sets represent uncertain concepts and make the process of creating fuzzy sets more manageable as new concepts come into view. Fuzzy linguistic variables are used as a technique to reduce the dimension in classification tasks, especially in scenarios where an excess number of features may degrade the quality and performance of a system.

3.2. Addressing Uncertainty in the Similarity Measure

With several variables to consider, like indexing, similarity functions, and the summation of individual similarities, this makes the uncertainty associated with the similarity measure less significant. This technique is primarily a fuzzy integral technique that aims to obviate some drawbacks of standard closest neighbor algorithms. As this technique is intended to provide suitable treatment of elements that vary in importance, thereby avoiding the masking of relatively lesser features by more prominent ones, the cancellation effects can be significantly reduced. This strategy also addresses the problems with the additivity assumptions in conventional aggregation techniques(Osiro et al., 2014).

In addition, disagreements on similarity judgments are resolved using fuzzy membership functions to quantify the expert's confidence in similarity between cases. This approach also deals with uncertainty in incomplete cases by allowing incomplete cases to be added to the case base without affecting the materiality of the system's accuracy.

Preference functions in fuzzy form are then adopted to describe how similar conditions are with respect to any feature that an application like plastics color matching may have. Then the simplified representation of similarity resulting from expert evaluation is one that is gotten from the representation of similarity values ensuing from the comparison as fuzzy preference vectors. The other very essential technique that this research used relates to the use of fuzzy membership distributions in the calculation of the value of properties. This sidesteps all the demerits that conventional sharp thresholds have by employing fuzzy predicates to represent preference while in the determination of similarity and assessing dependability.

3.3. Addressing Uncertainty in the Case Base

Heterogeneity in case attributes or the presence of incomplete case information often introduces vagueness to the case base. The uncertainty, in case certain case attributes are not known, introduces various techniques for the alleviation of such problems like ordered weighted averaging-based nearest neighbor rules. The selection and maintenance of relevant cases to the case base use fuzzy techniques with neural networks.

This not only reduces the redundant cases but also organizes cases into clusters that represent solution prototypes by using fuzzy clustering. The adaptive fuzzy clusters use both fuzzy clustering and the statistical process management approach to deal with the uncertainty in case parameters in applications like aircraft engine maintenance. In fact, as parameters do, so does the case base with time.

The study forms granular clusters in the feature space, combining fuzzy and rough sets to enhance case representation for more intricate cases. It uses fuzzy similarity functions and incorporates rough-fuzzy hybridization to facilitate effective case base retrieval and assure more accurate case representations. Finally, to ease the maintenance of the case base, it uses fuzzy decision trees. These trees increase retrieval efficiency, while lowering the case base's complexity without sacrificing important information by complementing a smaller case base and aiding in the learning of adaption rules.

3.4. Addressing Uncertainty in Solution Transformation

Another crucial problem that the research solves to ensure effective reuse of solutions is uncertainty when transforming a solution. For example, fuzzy disjunction is applied in musical composition to combine numerous fuzzy sets into a new fuzzy set representing the change. Then, the fuzzy set is defuzzied to transform it into a specific solution parameter.

Also, mining the case base through adaption knowledge involves using fuzzy association algorithms. With such knowledge principles regarding determining requirements and conditions to make successful solution adaptations according to the prediction accuracy and recall rate, this provides for building an adaptation engine.

4. RESULTS AND DISCUSSION

The modeling of uncertainty in the assumption of Case-Based Reasoning (CBR) is described in this section where, for a collection of possibly related cases found by a closest neighbor algorithm, the main goal is finding typical elements. Typical elements are identified using the principle of typicality. Using the parameters provided by this theory, fuzzy sets, which reflect the uncertainty of the solution, are created(Rajabi et al., 2019). These fuzzy sets then express the result of the CBR system.

This method is an extension of the first attempt to apply the concept of typicality in the retrieval stage of a CBR system, where the aim is to retrieve a set of examples according to the similarity of problems for the purpose of finding similar solutions to a problem. This technique is based on the premise that the solution to a problem may be found in the repository of solutions that have satisfactorily solved related problems. This approach uses typicality for describing the solution, providing for a more flexible and unstable method of determining results than does any other approach that relies on the similarity between issues or solutions.

The uncertainty is diminished by finding the typicality of a group of possibly sound answers. A fuzzy set models the typicality of solutions; the most typical solution would be represented by the center of the set, and the uncertainty that surrounds that solution would be described in terms of the shape of the set. The solution space can be better represented by utilizing fuzzy sets, which not only capture the uncertainty but also the typicality of the solution.

4.1. Finding the Typical Result of a CBR System

From a set of possibly sound solutions that is represented as Sp = {S1, S2,…, Sn}, that collection results when the related cases are found. Next, MTV of that set is calculated. That value which depicts most typically a set's primary tendency is called the MTV of the set. The core value of that solution, representing the solution that is the most likely to solve that issue, is placed centrally in a fuzzy set.

For instance, 30 is the most obvious value that represents the fuzzy set of similar solutions, if the MTV of the set is found to be 30. Then place the center at 30 to create a fuzzy set. The MTD defines the shape of the fuzzy set, which determines the extent to which values in the set deviate from the MTV. The MTD has the range or uncertainty about the typical value much as standard deviation does (El-Sappagh et al., 2015). The fuzzy set that emerges in this process models the most typical answer and the uncertainty about it, and it's why it's called Most typical Fuzzy Set, or MTFS.

4.2. Theory of Typicality

According to the Theory of Typicality, it introduces a framework for determining the Most Typical Value (MTV) of a dataset, representing the characteristic property of data particularly in situations where there exist multiple clusters. In situations where, multiple diverse clusters exist within a given data set, typical statistical metrics such as mean or median may not efficiently capture the usual value for the given dataset. The MTV, on the other hand, uses geometrical fuzzy clustering approaches to determine which value best captures the data given these many clusters.

Algorithms for Geometrical Fuzzy Clustering

Fuzzy clustering loosens the constraints in which the data points are assigned to clusters. Fuzzy clustering, unlike standard clustering, gives a membership value to every data point so that the value determines the proportion of each cluster that a given data point belongs to. These membership values represent the degree of similarity between each data point and the several cluster centers(Sarkheyli-Hägele and Söffker, 2020). They sum up to 1 for each data point.

Often the fuzzy clustering is applied with overlapping clusters or when two data points are not really distinguishable from each other. The geometrical approaches of fuzzy clustering techniques try to divide data space into clusters or areas representing both groups of possible fuzzy membership minimizing internal variance.

1) Fuzzy c-Means Algorithm:

These include the fuzziness parameter, mmm, and the number of clusters, c, and are predetermined for the Fuzzy c-Means algorithm. The algorithm follows these steps:

1. Initialize cluster centers v1, v2, …, vc.
2. For every data point xj, assign the fuzzy memberships μij according to how close it is to the cluster centers.
3. Optimize the objective function to update the centers of clusters:

$$J_m = \sum_{i=1}^{c} \sum_{j=1}^{N} \mu_{ij}^{m} ||x_j - v_i||^2$$

where $\mu^{m}_{ij}$is the membership value of data point xj in cluster i, and mmm is a fuzziness parameter typically set greater than 1.

The algorithm has converged when the membership values do not change much between cycles.

2) Cluster Validity:

Cluster validity measures to which extent the clustering function classifies the data into meaningful groups. A good partition induced by a good clustering algorithm should have:

a) Clear distinction among the clusters.

b) Each cluster's compactness.

c) Following the basic structure of the data.

Table 1 below shows the retrieved sets of solutions for problems PA, PB, and PC. Comparing estimates of solutions obtained for each problem over a variety of sets enables an estimate of the effectiveness of the clustering.

Table 1:Sets A, B, and C of problem-solving results PC, PB, and PA

	PA Estimate	PB Estimate	PC Estimate
1	44747	76557	113163

2	48552	66149	177474
3	48733	77973	44282
4	32102	23551	110574
5	87454	53027	79266
6	55037	77460	109003
7	43121	60366	108144
8	54362	68007	322234
9	45668	56774	113563
10	51898	97839	66725
11	38138	70627	108125
12	51115	85766	102093
13	49471	53105	99256
14	44359	64322	128909
15	40471	31875	150964
16	47250	80030	139318
17	52390	75203	228258
18	39333	87444	140481
19	80211	206627	57072
20	43426	140305	105032
21	45734	72555	67751
22	55481	68178	118193
23	38566	87554	156232
24	48007	55036	85429
25	45735	66381	110396

Many indices such as the Davies-Bouldin Index, Silhouette Score, or Dunn Index, which measures the compactness and separation of the clusters, may be used to assess cluster validity.

3) MTV, Most Typical Value:

After clustering, the core value of a data set is referred to as the Most Typical Value, MTV (Lu et al., 2016). It considers both the distance of each data point from the cluster center and the population, or size, of each cluster.

There are two effects that act as a reference for calculating the MTV:

- The population effect: MTV depends more on bigger clusters regarding data points. That simply means that, for higher clusters, MTV is near the centroid.

- The distance effect: The further away the population is from its centre in all clusters, their contribution will lower it as well.

An equation that considers the contribution of each centroid of a cluster by the size of the cluster and by the distance between the centroids and the MTV is obeyed by the MTV. The obtained MTV values for the clusters in sets A, B, and C are represented in Table 2. For every cluster within the same set, the MTV is calculated independently.

Table 2:Values obtained for clusters' MTV

SETS	MTV
set A cluster 1	45700
set A cluster 2	45700
set A cluster 3	45700
set B cluster 1	75390
set B cluster 2	75390
set B cluster 3	75390
set C cluster 1	121680
set C cluster 2	121680
set C cluster 3	121680

Definition 1: MTV Calculation

Let c clusters Ci=(ki,vi) be derived from the data set X={x1,x2,…,xN}⊂Rp, with vi as the centroid of cluster Ci and ki is the population size. Then, MTV is the solution of the following equation:

$$S = \frac{v_1\gamma_1(|v_1-s|)k^{\lambda}{}_1+v_2\gamma_2(|v_2-s|)k^{\lambda}{}_2+\ldots.+v_c\gamma_c(|v_c-s|)k^{\lambda}{}_c}{\gamma_1(|v_1-s|)k^{\lambda}{}_1+\gamma_2(|v_2-s|)k^{\lambda}{}_2+\ldots.+\gamma_c(|v_c-s|)k^{\lambda}{}_c}$$

Where,

- The distance effect is represented by the monotonically decreasing function γi(u).
- λ>1 is the weight modifier of the population effect

4) Most Typical Deviation (MTD):

The Most Typical Deviation measures the closeness of MTV to each unique data point set. The lower the MTD, the closer the MTV to the data center.

Definition 2: Calculate MTD

The MTD can be calculated as:

$$t = \frac{|v_1-s|^2\gamma_1(|v_1-s|)k_1^{\lambda}+|v_2-s|^2\gamma_2(|v_2-s|)k_2^{\lambda}+\cdots+|v_c-s|^2\gamma_c(|v_c-s|)k_c^{\lambda}}{\gamma_1(|v_1-s|)k_1^{\lambda}+\gamma_2(|v_2-s|)k_2^{\lambda}+\cdots+\gamma_c(|v_c-s|)k_c^{\lambda}}$$

The MTD is approximately distance |vk−s| if one of the clusters is dominant and k is the dominant cluster. Table 3 below gives MTV, MTD values for data sets A, B and C(Avdeenko and Makarova, 2017). With smaller values that indicate the MTV is close to the center, it can be seen that in the MTD values how closely every MTV resembles the center related data set.

Table 3: MTV and MTD values for sets A, B, and C

Set	MTV	MTD
Set A	45700	310.9
Set B	75390	851.2
Set C	121680	761.8

5) Definite Typical Value (DTV)

The DTV or the definite typical value is that which will emerge from the MTV equation with proper tuning (through a parameter, β) and iteration. Now, having looked at population as well as distance effects, it is that final most typical value the dataset settles into.In the Table 4 below are shown the DTVs of the three sets A, B and C. Following the necessary iterations and readjustment with β these represent the final converged solution to the MTVs of the above three sets.

Table 4: DTV for sets A, B and C

Set	DTV
A	45700
B	75390
C	121680

4.3. Finding the Most Typical Fuzzy Set (MTFS)

Most Typical Fuzzy Set: One of the major ideas behind fuzzy logic and clustering is a structure called the Most Typical Fuzzy Set that intends to express the inherent uncertainty present in the data. MTD extends the notion of Most Typical Value by introducing Most Typical Distance. This fuzzy set considers not only the MTV but also the way it interrelates to other aspects of the dataset with the attempt to more exactly describe the vagueness about the typicality of the solution.

- The Most Typical Value or MTV is the center or representative value of a set following the use of clustering techniques. It shows the tendency of central points within the dataset and is an expression of the most frequent result.

- MTD (Most Typical Distance) measures the distance of the MTV from the centers of the clusters. A low value for MTD indicates that MTV is close to the center and very typical of the items of the set, and, conversely, it also has a higher degree of typicality for the MTV (Lavrynenko et al., 2020).

The degree of typicality of the MTV is essentially defined by the MTD. In contrast, fuzzy sets' membership functions cannot be constructed directly employing the MTD as the deviation parameter. Depending upon how close elements are to the MTV, the MTFS is meant to be an expression of a spectrum of possible outcomes. There are three types of outcomes showed in figures 2 to 4.

- Very Close to MTV: Components which are very close to the MTV.

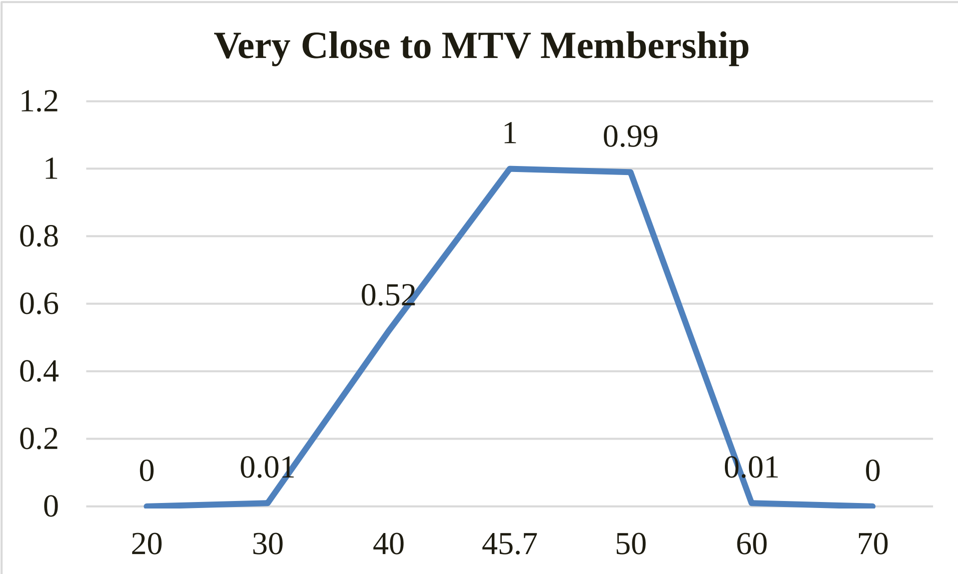

Figure 2:MTFS very close to MTV

- Close to MTV: Those things that are in this category but are not quite as close as the "very close" group.

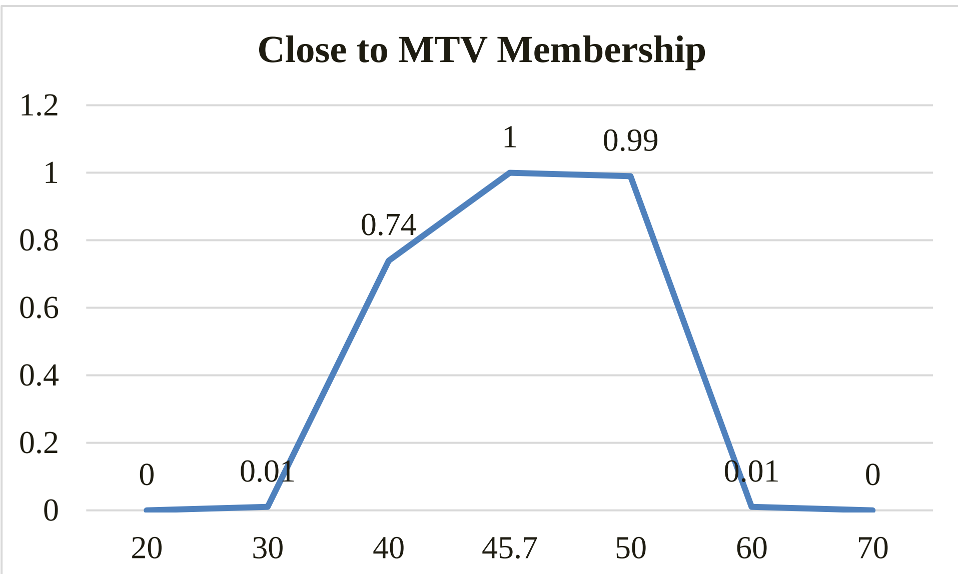

Figure 3:MTFS close to MTV

- Around MTV: Elements which are not included in the "close" category, but still somewhat close to the MTV.

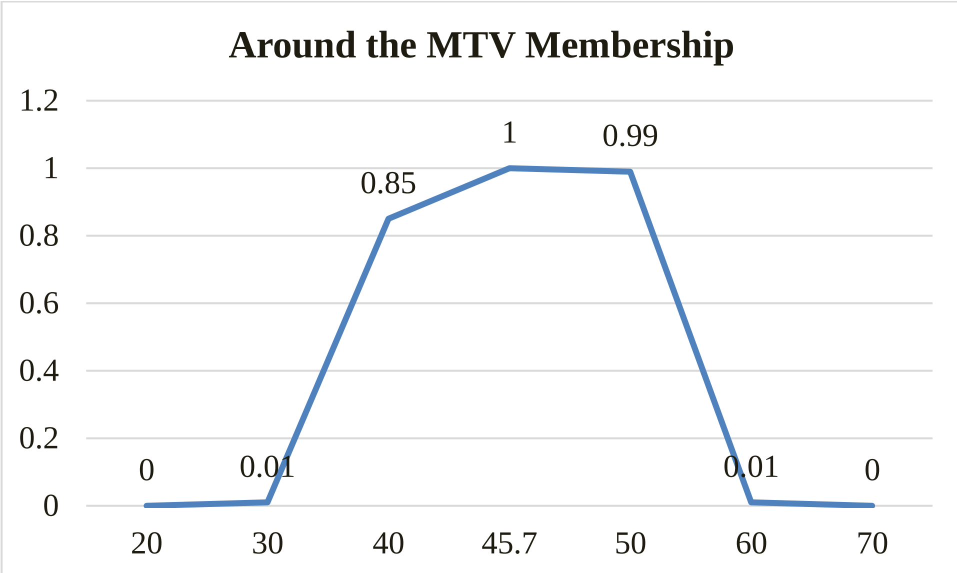

Figure 4: MTFS around MTV

Such descriptions using MTFSs also provide for a more particular categorization in such representations of the data which may well capture the loose concept of proximity to MTV. In the case of MTD, for instance, phrases such as "small, regular and large" are possible words to be understood under proximity.

In the fuzzy set model, MTD is a base variable to which the varying degrees of typicality are given in terms of the language. The MTD is represented by the following three linguistic terms:

○ Small MTD: Indicates that MTV lies well-centered inside the cluster, or in other words, the membership grades of elements around MTV are higher.

○ Regular MTD: It presents a better distribution of the membership classes as it signifies the average distance between the MTV and the cluster center.

○ Large MTD: This gives an impression that there exist several solutions close to the MTV since the MTV is highly distant from the cluster center. Here, pieces further away from the MTV are assigned higher grades of membership.

The MTD ranges between 0 and 5,000; here the smaller values signify larger values of MTV typicality (Choudhury and Begum, 2017). The flexible form of data, thus possible with this range, deals with different levels of uncertainty about the data. Table 5 shows the linguistic variable MTD along with its membership values, which correspond to several different values of MTD. These numbers are converted to "Small MTD," "Regular MTD," and "Large MTD." These numbers provide an imprecise description to the data: The membership values vary between these groups as the MTD increases, which indicates the different levels of typicality.

Table 5:The linguistic variable MTD

MTD Value	Small Membership	Regular Membership	Large Membership
0	1.0	0.0	0.0
290	0.9	0.0	0.0
580	0.7	0.0	0.0
870	0.4	0.0	0.0
1160	0.1	0.0	0.0
1450	0.0	0.0	0.0
1740	0.0	0.1	0.0
2030	0.0	0.4	0.0
2320	0.0	0.7	0.0
2610	0.0	0.9	0.0
2900	0.0	1.0	0.0
3190	0.0	0.9	0.0
3480	0.0	0.7	0.0
3770	0.0	0.4	0.0
4060	0.0	0.1	0.0
4350	0.0	0.0	0.1
4640	0.0	0.0	0.4
4930	0.0	0.0	0.7

According to the MTD and membership grades, certain rules are implemented in order to provide the MTFS (Karatop et al., 2015). Degree of typicality is represented in such rules:

- Extremely small MTD:Very small MTD represents higher degree of typicality by MTV. Elements closer to the MTV will get better memberships from MTFS while those are farther from MTV will acquire lower grade memberships.
- Regular MTD: A regular MTD represents a more balanced typicality by evenly spreading the membership grades over the MTV.
- Big MTD: Since it might have more than one valid solution, elements farther away from the MTV are awarded membership grades, and hence big MTD indicates low typicality.

These guidelines ensure that the MTFS captures appropriately the variability and uncertainty in the data and also help in defining the membership functions(Surma, 2015).

Use the following example to illustrate how the MTFS is constructed by taking the MTV as 45,700 and the corresponding MTD as 310.9 It would be classified as a "small" MTD in the MTD classification. The constructed MTFS would therefore represent the concept of being "very close" to the MTV. Members that are closer to the MTV would have higher membership grades while members that are farther away from the MTV would have lower grades.

5. CONCLUSION

This conclusion of the study, therefore, highlights the importance of fuzzy logic in dealing with uncertainty in CBDM systems. Human decision-making is, in any case, extremely difficult under uncertainty, and the support systems can be very adversely affected. The proposed approach applies fuzzification approaches such as fuzzy sets, fuzzy similarity functions, fuzzy decision trees, and adaptive fuzzy clustering at different steps of CBDM, namely, vocabulary representation, case retrieval, and solution transformation to show how uncertainty may be controlled at those different stages.With fuzzy logic integration, better and more flexible decisions could be made, especially within dynamic and complex contexts in which data is often ambiguous, imprecise, or incomplete. More formally, the formalization of vagueness in fuzzy theory ensures that the answers are much more reliable and consistent than those obtained from the ordinary decision-making process. Therefore, this research contributes to the overall effectiveness and robustness of CBDM systems by furthering the development of intelligent systems that could handle uncertainty in real-life applications. The findings of the study underscore the ability of fuzzy logic to promote decision support particularly when the traditional approach fails to cope with the intricacy of uncertain input.

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