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A Note on Nano D-closed maps in Nano Topological Spaces

Dr. K. Dass[1], R. Uma Mageshwari[2]

[1]*Associate Professor, PG and Research Department of Mathematics, The M.D.T College, Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli, India*
[2]*Research Scholar, PG and Research Department of Mathematics, The M.D.T College, Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli, India*
Email: dassmdt@gmail.com

The purpose of this paper is to introduce the new class of functions in nano topological spaces called nano D-closed maps.,nano D-open maps and nano D*-closed maps .The condition for a map to be a nano D-closed map ,nano D-open map and nano D* -closed map is discussed. Also their relationships with already existing nano closed maps and nano open maps in nano topological spaces are investigated.

Keywords: Nano topology, Nano closed map, Nano open map.

1. Introduction

Continuous function is one of the main concepts of Topology. In [1] Balachandran et.al, introduced and studied the notion of generalized continuous functions. Different types of generalization of continuous functions were studied by various authors in the recent development of Topology. The notion of nano topology was introduced by Lellis Thivagar[7]which was defined in terms of approximation and boundary region of a subset of an universe using an equivalence relation on it and defined Nano closed , Nano interior and Nano closure. He also defined Nano continuous function._

In the paper, nano D-closed maps, D*-closed maps and D-open maps are introduced. Using these new types of maps several characterization and properties are analyzed. Throughout this paper $(U, \tau_R(X))$ is a Nano topological spaces with respect to X where X$\subseteq$ U.R is an equivalence relation on U,U/R denotes the family of equivalence classes of U by R , $(V, \tau_R(Y))$ is a Nano topological spaces with respect to Y,where Y$\subseteq$V.R is an equivalence relation on V,V/R denotes the family of equivalence classes of V by R', (W, $\tau_{R''}(Y)$ is a Nano topological spaces with respect to Z where Z$\subseteq$ W.R is an equivalence relation on W,Z/R`` denotes the family of equivalence classes of W by R``,

Definition: 1:1[7]

Let U be a non-empty finite set of objects called Universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernibility with one another. The pair (U,R) is said to be an Approximation space. Let X $\subseteq$ U

The Lower approximation of X with respect to R is the set of all objects which can be certain classified as X with respect to R and is denoted by $L_R(X)$.

$$L_R(X) = \bigcup_{x \in X} \{R(X):R(X) \subseteq X\}.$$

The Upper approximation of X with respected to R is the set of objects which can be possibly classified as X with respect to R and is denoted by U_R (x).

$$U_R(X) = \bigcup_{x \in X} \{R(X):R(X) \cap X \neq \varphi\}.$$

The boundary region of X with respect to R is the set of all objects which can be classified neither as x nor not X with respect to R and is denoted by B_R (X).

$B_R (X) = U_R (X) – L_R (X)$.

Definition 1.2[7] :

Let U be the universe R be an equivalence relation on U and τ R (X) = {U,φ,L_R (X), U_R (X) , B_R(X) }, where X $\subseteq$U.

Then the property τ R (X), satisfies the following axioms

i.U and φ belongs to τ R (X).

ii.The union of the elements of any sub collection of τ R (X) is in τ R (X).

iii.The intersection of the elements of any finite sub collection of τ R (X) is in τ R (X).

Thus, $\tau_R(X)$ is a topology on U is said to be Nano Topology on U with

Respect to X. (U, $\tau_R(X)$ as the Nano Topological Spaces.

The element of $\tau_R(X)$ are called as Nano-open sets and complement of Nano open sets are called Nanoo closed sets.

Example 1.3:

Let U ={ a,b,c,d} U/R ={{a},{b},{c},{d}}.Let X= {a,c}$\subseteq$U. Then the nano topology is $\tau_R(X)$ ={φ,{a},{c,d},{a,c,d},U}.

Definition 1.4 [6]:

Let (U, $\tau_R(X)$ be a nano topological space and H$\subseteq$U.Then H is said to be

i)Nano semi-open if H$\subseteq$ Ncl(Nint(H)) ii)Nano pre-open if H$\subseteq$ Nint(Ncl(H))

iii)Nano α -open if H$\subseteq$Nint(Ncl(Nint(H))

iv)Nano regular open if H= Nint(Ncl(H))

v)Nano π-open if it is finite union of all nano-regular open sets.

vi)Nano semi pre-open if H$\subseteq$Ncl(Nint(Ncl(H))

Definition 1.5 [7]:

If (U, $\tau_R(X)$) is a nano topological space and if H $\subseteq$U, then i) The nano –interior of H is defined as the union of all nano open subsets contained in H and is denoted by Nint(H) . Nint(H) is the largest nano open subset of H.

ii) The nano closure of the set H is defined as the intersection of all nano closed sets containing H and is denoted by Ncl(H). Ncl(H) is the smallest nano closed set.

Definition 1.6:

A subset H of a nano topological space (U, $\tau_R(X)$) is called a

i)Nano generalized closed (Ng-closed)[2] if Ncl(H)$\subseteq$V ,whenever H$\subseteq$V and V is nano open in (U, $\tau_R(X)$)

ii)Nano generalized pre -closed (Ngp-closed)[3] if Npcl((H))$\subseteq$V, whenever H$\subseteq$V and V is nano open is (U, $\tau_R(X)$ i).

iii)Nano generalized pre regular closed (Ngpr-closed)[8] if Npcl(H)$\subseteq$V ,whenever H$\subseteq$V is nano regular open in (U, $\tau_R(X)$).

iv)Nano generalized semi pre -closed (Ngsp-closed) [9]if Nspcl(H)$\subseteq$V,whenever H$\subseteq$V and V is nano open in (U, $\tau_R(X)$)..

v)Nano $\hat{g}$- closed (N$\hat{g}$ - closed) [5]f Ncl(H)$\subseteq$V whenever H$\subseteq$V and V is semi open in (U, $\tau_R(X)$).

vi) Nano π-generalized pre closed(Nπgp-closed[4] if Npcl(H) $\subseteq$V whenever H$\subseteq$V And V is π-open is (U, $\tau_R(X)$)

<u>Definition 1.6[16]:</u>

Let f: (U, $\tau_R(X)$)– (V, $\tau_R'(Y)$) be a nano topological space. Then the function f: (U,τ R (X))– (V,τ R′ (Y)) is nano D-Continuous(n-D-continuous) on U, if the inverse image of nano open in V is nano D-open in U.

Definition 1.7 :[17]

Let f:(U,τ_R (X))→ (V,$\tau_{R'}$ (Y)) be a map.Then f is said to be

i) nano closed map if the image of nano closed set in $(U,\tau_R(X))$ is nano closed in $(V,\tau_{R'}(Y))$

ii) nano open map if the image of nano open in $(U,\tau_R(X))$ is nano open in $(V,\tau_R(Y))$.

iii)nano pre closed map if the image of nano pre-closed set in $(U,\tau_R(X))$ is nano pre-closed in $(V,\tau_{R'}(Y)$.

iv)nano M-pre closed map if the image of nano M pre-closed set in $(U,\tau_R(X))$ is nano M pre-closed in $(V,\tau_{R'}(Y)$.

v) nano g- closed map if the image of nano g-closed set in $(U,\tau_R(X))$ in nano g-closed in

$(V,\tau_{R'}(Y)$.

vi) nano $\hat{g}$-closed map if the image of nano $\hat{g}$-closed set in $(U,\tau_R(X))$ in nano $\hat{g}$-closed in

$(V,\tau_{R'}(Y)$.

vii)nano $\tilde{g}$-closed map if if the image of nano $\tilde{g}$-closed set in $(U,\tau_R(X))$ in nano $\tilde{g}$-closed in

$(V,\tau_{R'}(Y)$.

viii) nano gp- closed map if the image of nano gp-closed set in $(U,\tau_R(X))$ in nano gp-closed in

$(V,\tau_{R'}(Y))$.

ix) nano gpr- closed map if the image of nano gpr-closed set in $(U,\tau_R(X))$ in nano gpr-closed in

$(V,\tau_{R'}(Y))$.

x) nano gsp-closed map if the image of nano gsp-closed set in $(U,\tau_R(X))$ in nano gsp-closed in

$(V,\tau_{R'}(Y))$.

xi) nano πgpg- closed map if the image of nano g-closed set in $(U,\tau_R(X))$ in nano g-closed in

$(V,\tau_{R'}(Y))$.

Definition 1.8:

A nano topological space $(U,\tau_R(X))$ is said to be

i)nano D-T_S space if every nano D-closed is nano closed in $(U,\tau_R(X))$

ii) nano $T_{1/2-}$ space if every nano g-closed is nano closed in $(U,\tau_R(X))$.

Definition 1.9:

Let $f:(U,\tau_R(X))\rightarrow(V,\tau_{R'}(Y))$ be a map.Then f is said to be nano $\hat{g}$-irresolute if $f^{-1}(F)$ is nano $\hat{g}$-closed in $(U,\tau_R(X))$ for every nano $\hat{g}$-closed F of $(V,\tau_{R'}(Y))$.

2. Nano D-closed map:

In this section, new class of maps called nano D-closed maps are defined and some basic properties of these maps are studied.

Definition 2.1:

A map f: (U, τ R(X)) $\rightarrow$ (V, τ R'(Y)) is said to be nano D-closed if the image every nano closed set of (U, τ R(X)) is nano D-closed in (V, τ R'(Y)).

Example 2.2:

i)Let U={a,b,c}=V; with U/R ={{a},{b,c}} and V/R$'$ ={{a,b}, {c}}

Let X={a,b} $\subseteq$ U; Y={a,b} $\subseteq$ V

Then τ_R(X)={ φ ,{a},{b,c},U} and $\tau_{R'}$(Y)={ φ ,{c},{a,b},V}

Now nC(τ_R(X))={ φ ,{a},{b,c},U} and nC ($\tau_{R'}$(Y))={ φ ,{c},{a,b},V}

Also nD-closed sets={ φ , {a}, {b}, {c}, {a,b},{b,c},{c,a}, U}

Define an identify map f: (U, τ_R(X)) $\rightarrow$ (V, $\tau_{R'}$(Y)) by f(a)=a; f(b)=b; f(c)=c.

Then the identify map f: U $\rightarrow$ V is a nD-closed map.

Theorem 2.3:

Every nano contra closed map and nano pre-closed map f: (U, τ_R(X)) $\rightarrow$ (V, $\tau_{R'}$(Y)) is nano D-closed map.

Proof:

Let f be a nano closed set in (U, τ_R(X)).

Then f(F) is nano open and nano pre-closed in (V, $\tau_{R'}$(Y)).

Since every nano open and nano pre-closed is nano nano D-closed, f(F) is nano D-closed .

Hence f is nano D-closed map.

Remark 2.4:

The converse of the above theorem need not be true as seen from the following example.

Example 2.5:

Let U={a,b,c,d}=V with U/$_R$={{a},{b},{c,d}}.

Let X={a,d} $\subseteq$ U.

Then τ_R(X)={ φ ,{a},{c,d}, {a,c,d},U}.

Let Y={a,b} $\subseteq$ V with V/$_{R'}$={{a},{b,d},{c}}.

Then $\tau_{R'}$(Y)={ φ ,{a}, {b,d},{a,b,d},V}

Then nDC ($\tau_{R'}$(Y))={ φ ,{c}, {a,c}, {b,c}, {c,d}, {a,b,c}, {a,c,d},{b,c,d},U}

Then the function f: (U, τ_R(X)) $\rightarrow$ (V, $\tau_{R'}$(Y)) by f(a)=a; f(b)=c; f(c)=b; f(d)=d is a nano closed map but neither nano contra closed map nor nano pre-closed map.

It is observed that the nano closed set F={a,b} in (U, τ_R(X)) ,f(F)={a,c} is neither nano open nor nano pre-closed in (V, $\tau_{R'}$(Y)).

Theorem 2.6:

Every nano D-closed map f: U $\rightarrow$ V is a ngp-closed (resp ngpr-closed, nπgp-closed) map.

Proof:

Let F be a nano closed set of (U, τ_R(X))

Then f(F) is nano D-closed in (V, $\tau_{R'}$(Y)).

Since every nanoD- closed is ngp-closed set, f(F) is a ngp-closed set in (V, $\tau_{R'}$(Y))..

Since every nano D- closed is ngpr-closed set and nπgp-closed set,f(F) is ngpr-closed and nπgp-closed) in (V, $\tau_{R'}$(Y)).

Hence f is ngp-closed map (resp ngpr-closed mapand nπgp-closed map)

Remark 2.7:

The converse of the above theorem need not be true as seen from the following examples.

Example 2.8:

i)Let U={a,b,c,d}=V.

Let X={a,b} $\subseteq$ U and Y={a,b} $\subseteq$ V.

Then τ_R(X)={ φ ,{c}, {a,b},{a,b,c},U}

nDc($\tau_{R'}$(Y))={ φ ,{c}, {a,c}, {b,c,d},V}

Define f: U $\rightarrow$ V by f(a)=d; f(b)=c; f(c)=a; f(d)=b.

Then a function f is ngp-closed map but not nD-closed map.

It is observed that for the nano closed set F={c,d} in (U, τ_R(X), f(F)={a,b} is not nD-closed in (V, $\tau_{R'}$(Y)).

ii) Let U={a,b,c,d}=V.

Let X={a,b}, Y={a,c} .

Then $\tau_{R'}$(Y)={ φ ,{c}, {a,b}, {a,b,c},U}

Now, ngpr-closed($\tau_{R'}$(Y))={ φ ,{a}….{d},{a,b}…..{c,d}, {a,b,c}….{b,c,d},U}

nDc ($\tau_{R'}$(Y))={ φ ,{d}, {a,b}, {a,d}, {b,d}, {c,d}, {a,c,d}, {b,c,d}, {a,b,d},V}

Define f:U $\rightarrow$ V by f(a)=d; f(b)=c; f(c)=a; f(d)=b.

Then a function f is ngpr-closed map but not nD-closed map.

It is observed for the nano closed set F={b,d} in (U, τ_R(X)),f(F)={b,c} is not nD-closed in (V, $\tau_{R'}$(Y)).

iii) Let U=V={a,b,c,d} with $V/_{R'}$={{a},{b},{c,d}} and $U/_R$={{a},{b,c}, {d}}.

Then τ_R(X)={ φ ,{a},{b,c}, {a,b,c},U}and $\tau_{R'}$(Y)={ φ ,{a},{c,d}, {a,c,d} U}

nDC($\tau_{R'}$(Y))={ φ ,{a,b}, {b,d}, {a,b,c}, {b,c,d}, {a,b,d},U}

and nπgpc ($\tau_{R'}$(Y)={ φ ,{b}, {c}, {d}, {a,b}, {b,d}, {b,c}, {a,b,c},{a,b,d}, {b,c,d},V}

Define f:U$\rightarrow$V by f(a)=c; f(b)=a; f(c)=d; f(d)=b

Then the function f is nπgp-closed but not nD-closed map.

It is observed that for the nano closed set F={a,d} in (U, τ_R(X)), f(F)={b,c} is not nD-closed in (V, $\tau_{R'}$(Y)).

Theorem 2.9:

Every Nano generalized closed map is nano D-closed map.

Proof:

Let F be a nano closed set in (U, $\tau_R(X)$).

Then f(F) is nano generalized closed set in (V $\tau_{R'}$(Y)).

Since every nano closed is nano D-closed,f(F)is nano D-closed in (V $\tau_{R'}$(Y)).

Hence f is nano D-closed map.

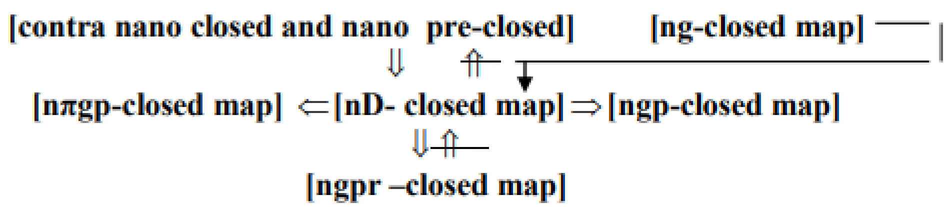
Remark 2.10:

The converse of the above theorem is not true as seen from the following example.

Example 2.11:

As in example 2.2, f is a nD –closed map, but not a nano generalized closed map.

Remark 2.12: We have the following relationship between nano D-closed map and other related nano generalized closed maps A$\Rightarrow$ B (A$\rightarrow$ B) represents A implies but not conversely.



Theorem 2.13:

If f: (U, τ_R(X)) $\rightarrow$ (V, $\tau_{R'}$(Y)) is nano D-closed and g:(V, $\tau_{R'}$(Y)) $\rightarrow$(W, $\tau_{R''}$(Z)) is nano closed and (V, $\tau_{R'}$(Y)) is nD-T_s space, then the composition gof: (U, τ_R(X)) $\rightarrow$(W, $\tau_{R''}$(Z)) is nano D-closed.

Proof:

Let F be a nano closed set in (U, τ_R(X)).

Then f(F) is nano D-closed in (V, $\tau_{R'}$(Y)).

Since (V, $\tau_{R'}$(Y)) is nD-T_s space, f(F) is nano closed in (V, $\tau_{R'}$(Y)).

Since g is nano D- closed map ,g(f(F)) is nano closed in (W, $\tau_{R''}$(Z)).

Since every nano closed is nano D-closed, g(f(F)) is nano D-closed in (W, $\tau_{R''}$(Z)).

Hence (gof)(F)= g(f(F) is nano D-closed in (W, $\tau_{R''}$(Z)).

Therefore, gof: (U, τ_R(X)) $\rightarrow$(W, $\tau_{R''}$(Z)) is nano D-closed.

Theorem 2.14:

If f: (U, τ_R(X)) $\rightarrow$ (V, $\tau_{R'}$(Y)) is a nano $\tilde{g}$ -closed and nano contra closed map and

g:(V, $\tau_{R'}$(Y)) $\rightarrow$(W, $\tau_{R''}$(Z)) is a nano M-pre closed and nano open map, then their composition gof: (U, τ_R(X)) $\rightarrow$(V, $\tau_{R'}$(Y)) is a nano D-closed map.

Proof:

Let F be a nano closed of (U, τ_R(X)).

Then f(F) is nano $\tilde{g}$-closed and nano open in (V, $\tau_{R'}$(Y)).

Since every nano $\tilde{g}$--closed is nano pre-closed and g is nano M-pre closed and nano open, hence (gof)(F) =g(f(F)) is nano pre closed and nano open in (W, $\tau_{R''}$(Z)).

Since every nano closed is nano D-closed, (gof) (F) is nano D-closed in (W, $\tau_{R''}$(Z)).

Therefore , gof is nano D-closed.

Theorem 2.15:

If f: (U, τ_R(X)) $\rightarrow$(V, $\tau_{R'}$(Y)) be a nano closed and g:(V, $\tau_{R'}$(Y)) $\rightarrow$(W, $\tau_{R''}$(Z)) be a nano closed map, then their composition gof: (U, τ_R(X)) $\rightarrow$(W, $\tau_{R''}$(Z)) is nano D-closed map.

Proof:

Let F be a nano closed set of (U, τ_R(X))

Then f(F) is nano closed in (V, $\tau_{R'}$(Y)).

Hence g(f(F))=(gof)(F) is nano closed in (W, $\tau_{R''}$(Z)).

Since every nnao closed is nano D-closed, gof is a nano D-closed map.

Theorem 2.16:

If f: (U, τ $_R$(X)) $\rightarrow$ (V, τ $_{R'}$(Y)) is nano D-closed and g:(V, τ $_{R'}$(Y)) $\rightarrow$(W, τ $_{R''}$(Z)) is M-nano pre closed and nano $\tilde{g}$ -irresolute map, then gof: (U, τ $_R$(X)) $\rightarrow$ (V, τ $_{R'}$(Y)) is nano D-closed map.

Proof:

Let F be a nano closed set of (U, τ $_R$(X)).

Then f(F) is nano D-closed in (V, τ $_{R''}$(Y))

By assumption,since f(F) is nanoD-closed set in (V, τ $_{R''}$(Y)),(gof)(F) = g(f(F)) is nano D-closed in (W, τ $_{R''}$(Z)).

Therefore, gof is a nano D-closed map.

Theorem 2.17:

Let f: (U, τ $_R$(X)) $\rightarrow$(V, τ $_{R'}$(Y)) and g:(V, τ $_{R'}$(Y)) $\rightarrow$(W, τ $_{R''}$(Z)) be two mappings such that their composition gof: (U, τ $_R$(X)) $\rightarrow$(V, τ $_{R'}$(Y)) be a nano D-closed mapping. Then the following statements are true if

i)f is nano continuous and surjective, then g is nano D-closed.

ii)g is nano D-irresolute, injective, then f is nano D-closed .

iii)f is nano $\hat{g}$ -continuous, surjective and (U, τ $_R$(X)) is a T $\hat{g}$ -space then g is nano D-closed.

iv)f is nano g-continuous, surjective and (U, τ $_R$(X)) is a nano $T_{1/2}$-space, then g is nano D-closed.

v) f is nano D-continuous, surjective and (U, τ $_R$(X)) is a nD-T_s space, then g is nano D-closed.

Proof:

i)Let F be a nano closed set of (V, τ $_{R'}$(Y))

Since f is nano continuous, f^{-1}(F) is nano closed in (U, τ $_R$(X))

Since gof is nano D-closed and f is surjective, (gof) (f^{-1}(F))=g(F) is nano D-closed in (W, τ $_{R''}$(Z)).

Therefore, g is nano D-closed map.

ii) Let F be a nano closed set of (U, τ $_R$(X))

Since gof is nano D-closed, (gof) (F) is nano D-closed in (W, τ $_{R''}$(Z)) .

Since g is nano D-irresolute and g is injective g^{-1}(gof)(F)=f(f) is nano D-closed in (V, τ $_{R''}$(Y)).

Thus, f is a nano D-closed map.

iii) Let F be a nano closed set of (V, τ $_{R''}$(Y)).

Since F is nano $\hat{g}$-continuous, f^{-1}(F) is nano $\hat{g}$-closed in (U, τ R(X)).

Since gof is nano D-closed and f is surjective, (gof) f^{-1}(F)=g(F) is nano D-closed in (W, $\tau_{R''}$(Z)).

Thus, g is a nano D-closed map.

iv) Let F be a nano closed set in (V, $\tau_{R''}$(Y)).

Since f is nano g-continuous, f^{-1}(F) is nano g-closed in (U, τ_R(X)).

Since (U, τ_R(X)) is nano $T_{1/2}$ space, f^{-1}(f) is nano closed in (U, τ_R(X)).

Since gof is nano D-closed and f is surjective (gof) (f^{-1}(F))=g(F) is nano D-closed in (W, $\tau_{R''}$(Z)).

Thus, g is a nano D-closed map.

v) Let F be a nano closed set of (V, $\tau_{R''}$(Y)).

Since f is nano D-continuous, f^{-1}(F) is nano D-closed in (U, τ_R(X)).

Since (U, τ_R(X)) is D-T_s space, f^{-1}(F) is nano closed in (U, τ_R(X)).

Since gof is nano D-closed and f is surjective ,(gof)(f^{-1}(F))=f(F) is nano D-closed in

(W, $\tau_{R''}$(Z)).

Thus g is nano D-closed map.

Remark 2.18:

The following examples shows that the composition of two nano D-closed maps need not be nano D-closed.

Example 2.19:

Let U=V=W={a,b,c}with W/$_{R''}$ = U/$_R$={{a}, {b,c}}, V/$_{R'}$ ={{a,b}, {c}}.

Let X={a,b}$\subseteq$U; Y={b,c}$\subseteq$V ; Z={a,b}$\subseteq$W.

Then τ_R(X)={ φ ,{a},{b,c},U}, $\tau_{R'}$(Y)={ φ ,{c},{a,b},V} and $\tau_{R''}$(Z)={ φ ,{a},{b,c},W}

Define f: (U, τ_R(X))$\rightarrow$ (V, $\tau_{R'}$(Y)) define by f(a)=a; f(b)=b; f(c)=c

Also g:(V, $\tau_{R'}$(Y)) $\rightarrow$(W, $\tau_{R''}$(Z)) define by g(a)=b; g(b)=c; g(c)=a

Then both f and g are both nano D-closed map but their composition

gof: (U, τ_R(X)) $\rightarrow$(W, $\tau_{R''}$(Z)) is not a nano D-closed map,since

the nano closed set F={a} in (U, τ_R(X)), (gof)(F)={b} is not nano D-closed in

(W, $\tau_{R''}$(Z)).

Theorem 2.20:

Let (U, τ_R(X)) and (V, $\tau_{R'}$(Y)) be any two nano topological spaces.If f: (U, τ_R(X)) $\rightarrow$ (V, $\tau_{R'}$(Y)) nano D-closed and A is a nano closed subsets of (U, τ_R(X)). Then f/A: (A, τ_{R^*}(S)) $\rightarrow$ (V, $\tau_{R''}$(Y)) is nano D-closed.

Proof:

Let B be a nano closed set in (A, τ_{R^*}(S))

Then B=A $\cap$ F for some nano closed F of (U, τ_R(X)) .

Therefore,B is nano closed in (U, τ_R(X))

Since f is nano D-closed, f(B) is nano D-closed in (V, $\tau_{R''}$(Y)).

But f(B)= (f/A)(B) and therefore f/A is a nano D-closed map.

Theorem 2.21:

A map f: (U, τ_R(X)) $\rightarrow$ (V, $\tau_{R'}$(Y)) is nano D-closed if and only if for each subset A of (V, $\tau_{R'}$(Y)) and for each nano open set G containing f^{-1}(A) there is a nano D-open G' of (V, $\tau_{R'}$(Y)) such that A $\subseteq$ G' and f^{-1}(G') $\subseteq$ G.

Proof:

Suppose f: (U, τ_R(X)) $\rightarrow$ (V, $\tau_{R'}$(Y)) is a nano D-closed map.

Let A $\subseteq$ V and G be an nano open subset of (U, τ_R(X)) such that f^{-1}(A) $\subseteq$ G.

Then G'=[f(G^c)]^c is a nano D-open set containing A such that f^{-1}(A) $\subseteq$ G.

Conversely, let F be a nano closed set of (U, τ_R(X))

Then f^{-1}((f(F))^c) $\subseteq$ F^c and F^c is nano open .

By assumption, there exists a nano D-open set G' of (V, $\tau_{R''}$(Y)) such

that (f(F))^c $\subseteq$ G' and f^{-1}(G') $\subseteq$ F^c and so F $\subseteq$ (f^{-1}(G'))^c.

Hence (G')^c $\subseteq$ f(F) $\subseteq$ f(f-1(G')^c) $\subseteq$ (G')^c which implies f(F)= (G')^c .

Since (G')^c is nano D-closed in (V, $\tau_{R'}$(Y)), f(F) is nano D-closed in (V, $\tau_{R''}$(Y)) and therefore, f is a nano D-closed.

Theorem2.22:

If a mapping f:(U, τ_R(X)) $\rightarrow$ (V, $\tau_{R'}$(Y)) is nano D-closed,then nD-cl(f(A)) $\subseteq$ f (ncl(A)) for every subset A of (U, τ_R(X)).

Proof:

Suppose f is nano D-closed and A $\subseteq$ U

Then f(ncl(A)) is nano D-closed in (V, $\tau_{R'}$(Y))

Since A is nano D-closed,nD-cl(A)=A, nD-cl (f(ncl(A))=f(ncl(A))

Also f(A) $\subseteq$ f(ncl(A)).

Since nD-cl(A)is nano D- closed, we have nD-cl(f(A))$\subseteq$nDcl(f(ncl(A) =f(ncl(A)).

Remark 2.23:

The converse of the above theorem need not be true as seen from the following example

Example 2.24:

Let U={a,b,c}=V with $U/_R$={{a,b}, {c}}and $V/_{R'}$={{a}, {b,c}}

Let X={b,c}and Y={a,b}.

Then τ_R(X)={φ,{c},{a,b},U}.

Now,nDc ($\tau_{R'}$(Y))={φ,{a},{a,b}, {a,c}, {b,c}, V}.

Define a map f: (U, τ_R(X))$\rightarrow$ (V,$\tau_{R'}$(Y)) f(a)=b; f(b)=a; f(c)=c.

For any subset A of U, nDcl(f(A))$\subseteq$Ncl(f(A))

But f is not nD-closed map

It is observed that for the nano closed set F={c} in (U,τ_R(X)), f({c})={c} is not nD-closed in (V,$\tau_{R'}$(Y)).

3. Nano D-open Map:

In this section, new class of maps called nano D-open maps are defined and some basic properties of these maps are studied.

Definition 3.1:

A map f: (U,τ_R(X))$\rightarrow$ (V,$\tau_{R'}$(Y)) is said to be nano D-open if the image f(G) is nano D-open in (V,$\tau_{R'}$(Y)) for every nano open set G is (U,τ_R(X)).

Theorem 3.2:

For any bijection, f: (U,τ_R(X)$\rightarrow$(V,$\tau_{R'}$(Y)), the following statements are equivalent

i)f^{-1}: (V,$\tau_{R'}$(Y))$\rightarrow$(U,τ_R(X)) is nano D-continuous.

ii) f is nD-open map and

iii) f is nD-closed map

Proof:

(i)$\Rightarrow$(ii):

Let G be a nano open set of (U,τ_R(X)).

By assumption, $(f^{-1})^{-1}$(G)=f(G) is nano D-open map in (V,$\tau_{R'}$(Y)).

Hence f is nano D-open map.

(ii)$\Rightarrow$(iii):

Let F be a nano closed set in (U,τ_R(X)) .

Then F^c is nano open in (U,τ_R(X)).

By assumption, $f(F^c)=[f(F^c)]^c$ is nano D-open in (V,$\tau_{R'}$(Y)).

Therefore f(F) is nano D-closed in (V,$\tau_{R'}$(Y)).

Hence f is a nano D-closed map.

(iii)$\Rightarrow$(i):

Let F be a nano closed set in (U,τ_R(X)).

By assumption, f(F) is nano D-closed in (V,$\tau_{R'}$(Y)).

But $f(F)=(f^{-1})^{-1}(F)$ and therefore f^{-1} is nano D-continuous on (V,$\tau_{R'}$(Y)).

Theorem 3.3:

Let f: (U,τ_R(X)$\rightarrow$(V,$\tau_{R'}$(Y)) be a map. If f is an nD-open map, then for each x$\in$X and for each nano neighbourhood of B of x in (U,τ_R(X), there exists a nano D-neighbourhood B_1 of f(x) in (V,$\tau_{R'}$(Y)) such that $B_1 \subseteq f(B)$.

Proof:

Let x$\in$U and B be an arbitrary nano neighbourhood of x.

Then there exists an nano open set G in (U,τ_R(X) such that $x \in G \subseteq B$.

By assumption, f(G) is nano D-open in (V,$\tau_{R'}$(Y)).

Further $f(x) \in f(G) \subseteq f(B)$.

Clearly f(B) is a nano D-neighbourhood of f(x) in (V,$\tau_{R'}$(Y)) such that $B_1 \subseteq f(B)$. .

Theorem 3.4:

A function f: (U,τ_R(X)$\rightarrow$(V,$\tau_{R'}$(Y)) is nano D-open if and only if for any subset B of (V,$\tau_{R'}$(Y)) and for any nano closed set F containing f^{-1}(F), there exists a nano D-closed set F' of (V,$\tau_{R'}$(Y)) containing B such that $f^{-1}(F') \subseteq F$.

Proof:

Similar to Theorem 2.21.

4. Nano D*-closed maps:

In this section, new class of maps called nano D*-closed maps are defined and some basic properties of these maps are studied.

Definition 4.1:

A function f: (U, τ_R(X) $\rightarrow$ (V, $\tau_{R'}$(Y)) is called nano D*-closed if the image of each nano D-closed set of U is nano D-closed in V.

Example 4.2:

Let U=V={a,b,c} with $U/_R$={{a}, {b,c}}& $V/_{R'}$={{a,b}, {c}}

Let X={a,b} $\subseteq$ U & Y={b,c} $\subseteq$ V.

Then τ_R(X)={ φ ,{a},{b,c},U}& $\tau_{R'}$(Y)={ φ ,{c},{a,b},U}

nDC(τ_R(X))={ φ ,{a},{a,b}, {a,c},{b,c},U}&

nDC($\tau_{R'}$(Y)={ φ ,{a},{b},{c},{a,b},{b,c},{a,c}.U}.

Define f: U $\rightarrow$ V by f(a)=a; f(b)=b; f(c)=c is a nD*-closed map.

Theorem 4.3:

A surjection map f:(U, τ_R(X) $\rightarrow$ (V, $\tau_{R'}$(Y)) is nano D*-closed if and only if for each subset B of V and each nano D-open set G of U comtaining f^{-1}(B) there exists a nano D-open G' in V containing B such that f^{-1}(G') $\subseteq$ G.

Proof:

Suppose that f:(U, τ_R(X) $\rightarrow$ (V, $\tau_{R'}$(Y)) is nano D*-closed map.

Let B $\subseteq$ V and G be nano D-open set of U such that f^{-1}(B) $\subseteq$ G.

Since f is nD*-closed, $[f(G^c]^c$=G' is a nano D-open set in V containing B such that f^{-1}(G') $\subseteq$ G.

Conversely, assume that F be any nanoD-closed set of U.

Put B=$(f(F))^c$.

Then we have f^{-1}(B) $\subseteq$ F^c and F^c is nano D-open in U.

By hypothesis, there exists a nano D-open set G' of V such that B $\subseteq$ G' and

f^{-1}(G') $\subseteq F^c$ and so F $\subseteq [f^{-1}(G')]^c = f^{-1}(G'^c)$.

Hence we obtain f(F) = G'^c.

Since G'^c is nano D-closed, f(F) is nD-closed

Hence f is nano D*-closed.

Theorem 4.4:

If f:(U, τ_R(X) $\rightarrow$ (V, $\tau_{R'}$(Y)) is nano $\overset{\Delta}{g}$ -irresolute and nano M- pre closed map, then f is a nano D*-closed map.

Since f is $\overset{\Delta}{g}$ -irresolute and nano M- pre closed map,f(A) is nano D-closed in (V, $\tau_{R'}$(Y)) ,for every nano D- closed set in (U, $\tau_{R'}$(X)) .Therfore f is nano D* -- closed map.

Theorem 4.5:

Every nano D-closed maps is a nano D*-closed map, if (U, τ_R(X) is

nano D- T_s space.

Proof:

Let f: (U, τ_R(X) $\rightarrow$(V, $\tau_{R'}$(Y)) be a nano closed map and F be a nano D-closed set of (U, τ_R(X).

Since (U, τ_R(X) is a nD-T_s space, F is nano closed set in (U, τ_R(X).

Since f is nD-closed, f(F) is a nano D-closed set in (V, $\tau_{R'}$(Y)).

Hence f is a nD*-closed map.

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