Nanotechnology Perceptions
ISSN 1660-6795
www.nano-ntp.com

On (1,2)*- Weakly Delta Generalized Beta Closed Sets and Continuous functions in Bitopological Spaces

C. Nanammal[1] and K. Alli[2]

[1]*PG & Research Department of Mathematics, The M.D.T. Hindu College, Affiliated to Manonmaniam Sundaranar University, India, Email: cnanammal@gmail.com.*
[2]*Associate Professor, PG & Research Department of Mathematics, The M.D.T. Hindu College, Affiliated to Manonmaniam Sundaranar University, India, Email: allimdt@gmail.com*

In this paper, we introduce a new class of sets called (1,2)*-wδgβ-closed sets and explore their fundamental properties. We investigate how these sets relate to other well-known types of τ_1,2-closed sets in bitopology, highlighting both similarities and differences. Additionally we define and study (1,2)*-wδgβ-continuous functions, focusing on their basic characteristics and behavior under various conditions. Furthermore, we provide several characterizations of these functions, establishing their significance within the broader framework of bitopological spaces. Lastly, we discuss potential applications of (1,2)*-wδgβ-closed sets and (1,2)*-wδgβ-continuous functions, demonstrating their utility in both theoretical and applied contexts.

1. Introduction

Kelley [3] initiated the study of bitopological spaces which are equipped with two arbitrary topologies. Sundaram [10] introduced the concept of generalized closed sets in bitopological spaces. The notion has been studied extensively in recent years by many topologists. T. Fukutake [1] introduced the concepts of ω-closed sets, ω-open sets and ω-continuity in bitopological spaces. Sanjay Tahiliani [9] introduced the notion of weakly β-continuous functions in bitopological spaces further and investigate the properties of these functions.

In this paper, we introduce and investigate the notion of (1,2)*-w$\delta g\beta$-closed sets in bitopological spaces. The relationships with other kinds of sets are given. Also, we give some applications of (1,2)*-w$\delta g\beta$-closed sets.

2. Preliminaries

Throughout the paper, $\mathcal{F}_1$, $\mathcal{F}_2$ and $\mathcal{F}_3$ denote bitopological spaces $(\mathcal{F}_1, \tau_1, \tau_2)$, $(\mathcal{F}_2, \sigma_1, \sigma_2)$ and $(\mathcal{F}_3, \rho_1, \rho_2)$ respectively.

Definition 2.1

Let $\mathcal{D}_1$ be a subset of a bitopological space $\mathcal{F}_1$. $\mathcal{D}_1$ is called $\tau_{1,2}$-open [3] if $\mathcal{D}_1 = \mathcal{R}_1 \cup \mathcal{R}_2$, for some $\mathcal{R}_1 \in \tau_1$ and $\mathcal{R}_2 \in \tau_2$. The complement of set is called $\tau_{1,2}$-closed. The family of all $\tau_{1,2}$-open and $\tau_{1,2}$-closed sets of $\mathcal{F}_1$ is denoted by (1,2)*-$\mathcal{O}(\mathcal{F}_1)$ and (1,2)*-$\mathcal{C}(\mathcal{F}_1)$.

Definition 2.2 [3].

Let $\mathcal{U}_1$ be a subset of a bitopological space $\mathcal{F}_1$. Then

(1) $\tau_{1,2}$-$int(\mathcal{F}_1) = \cup \{\mathcal{U}_1 : \mathcal{U}_1 \subseteq \mathcal{F}_1 \ and \ \mathcal{U}_1 is \ \tau_{1,2} - open\}$.

(2) $\tau_{1,2}$-$c\ell(\mathcal{F}_1) = \cap \{\mathcal{U}_1 : \mathcal{F}_1 \subseteq \mathcal{U}_1 \ and \ \mathcal{U}_1 \ is \ \tau_{1,2} - closed\}$.

Remark 2.3

Note that $\tau_{1,2}$-open subsets of $\mathcal{U}_1$ need not necessarily form a topology.

Definition 2.4

Let $\mathcal{U}_1$ be a subset of a bitopological space $\mathcal{F}_1$. Then $\mathcal{U}_1$ is called

(i). (1,2)*-regular open [5] if $\mathcal{U}_1 = \tau_{1,2}$- int ($\tau_{1,2}$- $c\ell(\mathcal{U}_1)$).

(ii). (1,2)*-δ-open [5] if $\mathcal{U}_1 = (1,2)^*$-δ- $int(\mathcal{U}_1)$, where $(1,2)^*$-δ-int(A) is the union of all (1,2)*-regular open set of $\mathcal{F}_1$ contained in $\mathcal{U}_1$.

(iii).(1,2)*-β-open [7] if $\mathcal{U}_1 \subseteq \tau_{1,2}$- $c\ell$ ($\tau_{1,2}$-int ($\tau_{1,2}$-$c\ell(\mathcal{U}_1)$)).

(iv). (1,2)*-α-open [8] if $\mathcal{U}_1 \subseteq \tau_{1,2}$- int($\tau_{1,2}$-$c\ell$($\tau_{1,2}$- $int(\mathcal{U}_1)$)).

Definition 2.5

A subset $\mathcal{U}_1$ of a bitopological space $\mathcal{F}_1$ is called

(i). (1,2)*-δ-generalized closed set [5](briefly (1,2)*-δg closed) if $\tau_{1,2}$-δcl($\mathcal{U}_1$)$\subseteq \mathcal{G}_1$ whenever $\mathcal{U}_1 \subseteq \mathcal{G}_1$ and $\mathcal{G}_1$ is $\tau_{1,2}$-open in $\mathcal{F}_1$.

(ii). a (1,2)*-beta weakly generalized closed [6] (briefly, (1,2)*-βwg-closed) if (1,2)*-$\beta cl(\mathcal{U}_1) \subseteq \mathcal{G}_1$ whenever $\mathcal{U}_1 \subseteq \mathcal{G}_1$ and $\mathcal{G}_1$ is (1,2)*-αg-open in $\mathcal{F}_1$.

(iii). (1,2)* -generalized beta-closed ((1,2)*-gβ-closed) [7] if (1,2)*-βcl($\mathcal{U}_1$) $\subseteq \mathcal{G}_1$ whenever $\mathcal{U}_1 \subseteq \mathcal{G}_1$ and $\mathcal{G}_1$ is $\tau_{1,2}$-open.

(iv). (1,2)* -generalized-closed ((1,2)*-g-closed) [2] if $\tau_{1,2}$-cl($\mathcal{U}_1$) $\subseteq \mathcal{G}_1$ whenever $\mathcal{U}_1 \subseteq \mathcal{G}_1$ and $\mathcal{G}_1$ is $\tau_{1,2}$-open in $\mathcal{F}_1$.

(v). (1,2)*-α-generalized closed (briefly (1,2)*-αg-closed) [8] if (1,2)*-αcl($\mathcal{U}_1$) $\subseteq \mathcal{G}_1$ whenever $\mathcal{U}_1 \subseteq \mathcal{G}_1$ and $\mathcal{G}_1$ is $\tau_{1,2}$-open in $\mathcal{F}_1$.

Remark:

$\tau_{1,2}$-open ⇒ (1, 2)*- δg-open ⇒ (1, 2)*-αg-open ⇒ (1, 2)*-$\delta g\beta$-open.

3. (1,2)*- Weakly Delta Generalized Beta Closed Sets

Definition 3.1

A subset $\mathcal{U}_1$ of a bitopological space $\mathcal{F}_1$ is called a (1,2)*-weakly $\delta g\beta$-closed (briefly, (1,2)*-w$\delta g\beta$-closed) if (1,2)*-$\beta cl(\mathcal{U}_1) \subseteq \mathcal{G}_1$ whenever $\mathcal{U}_1 \subseteq \mathcal{G}_1$ and $\mathcal{G}_1$is (1,2)*-δg-open in $\mathcal{F}_1$.

The complement of a (1,2)*-weakly $\delta g\beta$-closed set is called (1,2)*-weakly $\delta g\beta$-open. We denote the set of all (1,2)*-w$\delta g\beta$-closed sets in by (1, 2)*- w$\delta g\beta$ -$\mathcal{C}(\mathcal{F}_1)$.

Example 3.2

Let $\mathcal{F}_1 = \{a_1, b_1, c_1\}$ with $\tau_1 = \{\phi, \mathcal{F}_1, \{a_1, b_1\}\}$, $\tau_2 = \{\phi, \mathcal{F}_1, \{a_1, c_1\}\}$ and $\tau_{1,2}, \{a_1, c_1\}\}$. Then

(i) $\tau_{1,2}$-closed sets : $\{\phi, \mathcal{F}_1, \{b_1\}, \{c_1\}$.

(ii) (1, 2)*-α-closed sets : $\{\phi, \mathcal{F}_1, \{b_1\}, \{c_1\}$.

(iii) (1, 2)*-β-closed sets : $\{\phi, \mathcal{F}_1, \{a_1\}, \{b_1\}, \{c_1\}, \{b_1, c_1\}$.

(iv) (1, 2)*-δg-closed sets : $\{\phi, \mathcal{F}_1, \{a_1\},, \{b_1, c_1\}$.

(v) (1, 2)*- w$\delta g\beta$-closed sets : $\{\phi, \mathcal{F}_1, \{a_1\}, \{b_1\}, \{c_1\}, \{a_1, b_1\}, \{a_1, c_1\}, \{b_1, c_1\}\}$.

Theorem 3.3

Every (1,2)*-β-closed set is (1,2)*-$w\delta g\beta$-closed set but not conversely.

Theorem 3.4

Every (1,2)*-$g\beta$-closed set is (1,2)*-$w\delta g\beta$-closed set but not conversely.

Proof

Let $\mathcal{U}_1$ be any (1,2)*-$g\beta$-closed set in $\mathcal{F}_1$ and $\mathcal{G}_1$ be any (1,2)*-δg-open set containing $\mathcal{U}_1$. Then $\mathcal{G}_1$ is (1,2)*-β-open set containing $\mathcal{U}_1$. Hence (1, 2)*-βcl($\mathcal{U}_1$) ⊆ $\mathcal{G}_1$. Thus $\mathcal{U}_1$ is (1,2)*-$w\delta$gβ-closed.

Example 3.5

Let $\boldsymbol{\mathcal{F}_1} = \{\boldsymbol{a_1}, \boldsymbol{b_1}, \boldsymbol{c_1}\}$ with $\boldsymbol{\tau_1} = \{\phi, \boldsymbol{\mathcal{F}_1}, \{\boldsymbol{a_1}, \boldsymbol{b_1}\}\}$, $\boldsymbol{\tau_2} = \{\phi, \boldsymbol{\mathcal{F}_1}, \{\boldsymbol{b_1}, \boldsymbol{c_1}\}\}$ and $\boldsymbol{\tau_{1,2}} = \{\phi, \boldsymbol{\mathcal{F}_1}, \{\boldsymbol{a_1}, \boldsymbol{b_1}\}, \{\boldsymbol{b_1}, \boldsymbol{c_1}\}\}$. Where $\{\boldsymbol{b_1}, \boldsymbol{c_1}\}$is (1,2)*-w$\boldsymbol{\delta}g\boldsymbol{\beta}$-closed but not (1, 2)*-g$\boldsymbol{\beta}$-closed.

Theorem 3.6

Every (1,2)*-$\boldsymbol{w\delta g\beta}$-closed set is (1,2)*-$\boldsymbol{\delta g\beta}$-closed set but not conversely.

Proof

Let $\boldsymbol{\mathcal{U}_1}$ be any (1,2)*-w$\boldsymbol{\delta g\beta}$-closed set in $\boldsymbol{\mathcal{F}_1}$ and $\boldsymbol{\mathcal{G}_1}$ be any (1,2)*-$\boldsymbol{\delta}$-open set containing $\boldsymbol{\mathcal{U}_1}$. Then $\boldsymbol{\mathcal{G}_1}$ is (1,2)*-$\boldsymbol{\delta}$g-open set containing $\boldsymbol{\mathcal{U}_1}$. Hence (1, 2)*-$\boldsymbol{\beta}$cl($\boldsymbol{\mathcal{U}_1}$) ⊆ $\boldsymbol{\mathcal{G}_1}$. Thus $\boldsymbol{\mathcal{U}_1}$ is (1,2)*-$\boldsymbol{\delta}$g$\boldsymbol{\beta}$-closed.

Example 3.7

Let $\mathcal{F}_1 = \{a_1, b_1, c_1\}$ with $\tau_1 = \{\phi, \mathcal{F}_1, \{a_1, b_1\}\}$, $\tau_2 = \{\phi, \mathcal{F}_1, \{b_1\}\}$ and $\tau_{1,2} = \{\phi, \mathcal{F}_1, \{b_1\}, \{a_1, b_1\}\}$. Where $\{b_1\}$ is (1,2)*-δgβ-closed but not (1,2)*-wδgβ-closed.

Theorem3.8

Every (1,2)*-βwg-closed set [5] is (1,2)*-$w\delta g\beta$-closed set but not conversely.

Proof

Let $\mathcal{U}_1$ be any (1,2)*-βwg-closed set and $\mathcal{G}_1$ be any (1,2)*-δg-open set containing $\mathcal{U}_1$. Then $\mathcal{G}_1$ is (1,2)*-αg-open set containing $\mathcal{U}_1$. We have (1, 2)*-βcl($\mathcal{U}_1$)⊆ $\mathcal{G}_1$. Thus $\mathcal{U}_1$ is (1,2)*-$w\delta g\beta$-closed.

Example 3.9

Let $\mathcal{F}_1 = \{a_1, b_1, c_1\}$ with $\tau_1 = \{\phi, \mathcal{F}_1, \{a_1, c_1\}\}$, and $\tau_2 = \{\phi, \mathcal{F}_1, \{c_1\}\}$. Then $\tau_{1,2} = \{\phi, \mathcal{F}_1, \{c_1\}, \{a_1, c_1\}\}$. Where $= \{a_1, b_1\}$ is (1,2)*-wδgβ-closed but not (1, 2)*-βwg-closed.

Theorem 3.10

If $\mathcal{U}_1$ is a (1,2)*-$w\delta g\beta$-closed set in $\mathcal{F}_1$. Then (1,2)*-$\beta cl(\mathcal{U}_1)$-$\mathcal{U}_1$ does not contain any non empty (1,2)*-δg-closed set.

Proof

Let $\mathcal{H}_1$ be a (1,2)*-δg-closed subset of (1,2)*-$\beta cl(\mathcal{U}_1)$-$\mathcal{U}_1$. Hence $\mathcal{F}_1 - \mathcal{H}_1$ is an (1,2)*-δg-open set with $\mathcal{U}_1 \subseteq \mathcal{F}_1 - \mathcal{H}_1$ and $\mathcal{U}_1$ is (1,2)*-$w\delta g\beta$-closed, (1,2)*-$\beta cl(\mathcal{U}_1) \subseteq \mathcal{F}_1 - \mathcal{H}_1$. Which implies $\mathcal{H}_1 \subseteq (\mathcal{F}_1$ – (1,2)*-βcl($\mathcal{U}_1$))∩((1,2)*-βcl($\mathcal{U}_1$) -$\mathcal{U}_1$) ⊆ $\mathcal{F}_1$ – (1,2)*-βcl($\mathcal{U}_1$)) ∩ (1,2)*-βcl($\mathcal{U}_1$) $= \boldsymbol{\varphi}$. Therefore $\mathcal{H}_1 = \boldsymbol{\varphi}$.

Corollary 3.11

If $\mathcal{U}_1$ is a (1,2)*-$w\delta g\beta$-closed set in $\mathcal{F}_1$. Then (1,2)*-$\beta cl(\mathcal{U}_1)$-$\mathcal{U}_1$ does not contain any non empty (1,2)*-δ-closed set.

Theorem 3.12

Let $\mathcal{U}_1$ be a (1,2)*-$w\delta g\beta$-closed set in bitoplogical space. Then $\mathcal{U}_1$ is (1,2)*-β-closed iff (1,2)*-βcl($\mathcal{U}_1$)- $\mathcal{U}_1$ is (1,2)*-δg-closed.

Proof

If $\mathcal{U}_1$ is (1,2)*-β-closed, then (1,2)*-βcl($\mathcal{U}_1$) $= \mathcal{U}_1$ and so (1,2)*-βcl($\mathcal{U}_1$) $- \mathcal{U}_1 = \boldsymbol{\varphi}$, that is (1,2)*-$\delta$g closed.

Conversely, let $\mathcal{U}_1$ be a (1,2)*-$w\delta g\beta$-closed set in bitoplogical space and (1,2)*-βcl($\mathcal{U}_1$)- $\mathcal{U}_1$ is (1,2)*-δg closed. By Theorem 3.8, (1,2)*-βcl($\mathcal{U}_1$) $= \boldsymbol{\varphi}$. Therefore $\mathcal{U}_1$ is (1,2)*-β-closed.

Theorem 3.13

If $\mathcal{U}_1$ is (1,2)*-$w\delta g\beta$-closed in $\mathcal{F}_1$ and if $\mathcal{U}_1 \subseteq \mathcal{V}_1 \subseteq$(1,2)*-βcl($\mathcal{U}_1$) then $\mathcal{V}_1$ is also (1,2)*-$w\delta g\beta$-closed in bitopological space.

Proof

Let $\mathcal{G}_1$ be an (1,2)*-δg-open set of bitopological space such that $\mathcal{V}_1 \subseteq \mathcal{G}_1$. Since $\mathcal{U}_1 \subseteq \mathcal{G}_1$ and $\mathcal{U}_1$ is (1,2)*-$w\delta g\beta$-closed, (1,2)*-βcl($\mathcal{U}_1$) $\subseteq \mathcal{G}_1$. Since $\mathcal{V}_1 \subseteq$ (1,2)*-βcl($\mathcal{U}_1$) , we have (1,2)*-βcl($\mathcal{V}_1$) $\subseteq$ (1,2)*-βcl((1,2)*-βcl ($\mathcal{U}_1$)) = (1,2)*-βcl($\mathcal{U}_1$). Thus (1,2)*-βcl ($\mathcal{V}_1$) $\subseteq \mathcal{G}_1$. Hence $\mathcal{V}_1$ is a (1,2)*-$w\delta g\beta$-closed set of bitopological space.

Remark 3.14

The intersection of two (1,2)*-$w\delta g\beta$-closed sets need not be (1,2)*-$w\delta g\beta$-closed.

Example 3.15

Let $\mathcal{F}_1 = \{a_1, b_1, c_1\}$ with $\tau_1 = \{\phi, \mathcal{F}_1, \{a_1, b_1\}\}$, $\tau_2 = \{\phi, \mathcal{F}_1, \{a_1\}, \{a_1, c_1\}\}$and $\tau_{1,2} = \{\phi, \mathcal{F}_1, \{a_1, b_1\}, \{a_1, c_1\}\}$. Here $\mathcal{U}_1 = \{a_1, b_1\}$ and $\mathcal{V}_1 = \{a_1, c_1\}$ are (1, 2)*- $w\delta g\beta$-closed subsets in $\mathcal{F}_1$. Then $\mathcal{U}_1 \cap \mathcal{V}_1 = \{a_1\}$ is not a (1,2)*- $w\delta g\beta$ -closed subset in $\mathcal{F}_1$.

Remark 3.16

In general, the union of two (1,2)*-$w\delta g\beta$-closed subsets of bitopological space is also (1,2)*-$w\delta g\beta$-closed set.

Example 3.17

Let $\mathcal{F}_1 = \{a_1, b_1, c_1\}$ with $\tau_1 = \{\phi, \mathcal{F}_1, \{a_1, b_1\}, \{a_1, c_1\}\}$. and τ_2= {ϕ, $\mathcal{F}_1$, {b}, {b, c}}. We have $\tau_{1,2} = \{\phi, \mathcal{F}_1, \{a_1, b_1\}, \{a_1, c_1\}, \{b_1, c_1\}\}$. Here $\mathcal{U}_1 = \{a_1\}$ and $\mathcal{V}_1 = \{b_1\}$ are (1, 2)*- $w\delta g\beta$-closed subsets in $\mathcal{F}_1$. But $\mathcal{U}_1 \cup \mathcal{V}_1 = \{a_1, b_1\}$ is not (1,2)*-w$\delta g\beta$-closed subset in $\mathcal{F}_1$.

Theorem 3.18

For $f_1 \in \mathcal{F}_1$, then the set $\mathcal{F}_1 - \{f_1\}$ is a (1,2)*-$w\delta g\beta$-closed set (or) (1,2)*-δg-open set.

Proof:

Let $f_1 \in \mathcal{F}_1$. Suppose that $\mathcal{F}_1 - \{f_1\}$ not (1,2)*-δg-open. Then $\mathcal{F}_1$ is the only (1,2)*-δg-open set containing $\mathcal{F}_1 - \{f_1\}$. This implies (1,2)*-βcl($\mathcal{F}_1 - \{f_1\}$)$\subseteq \mathcal{F}_1$. Hence $\mathcal{F}_1 - \{f_1\}$ is (1,2)*-$w\delta g\beta$-closed in $\mathcal{F}_1$.

Theorem 3.19

If $\mathcal{U}_1$ is both (1,2)*-δg-open and (1,2)*- $w\delta g\beta$-closed in $\mathcal{F}_1$ then $\mathcal{U}_1$ is (1,2)*-β-closed.

Proof: Suppose $\mathcal{U}_1$ is (1,2)*-δg-open and (1,2)*-$w\delta g\beta$-closed in $\mathcal{F}_1$. Since $\mathcal{U}_1 \subseteq \mathcal{U}_1$, (1,2)*-βcl ($\mathcal{U}_1$) $\subseteq \mathcal{U}_1$. But always $\mathcal{U}_1 \subseteq$ (1,2)*-βcl($\mathcal{U}_1$). Therefore $\mathcal{U}_1$=(1,2)*-βcl ($\mathcal{U}_1$). Hence $\mathcal{U}_1$ is (1,2)*-β-closed.

Theorem 3.20

For a subset $\mathcal{U}_1$ of $\mathcal{F}_1$ and $f_1 \in \mathcal{F}_1$, (1,2)*-$w\delta g\beta$Cl($\mathcal{U}_1$) contains f_1 iff $\mathcal{M}_1 \cap \mathcal{U}_1 \neq \varphi$ for every (1,2)*-$w\delta g\beta$-open set $\mathcal{M}_1$ containing f_1.

Proof:

Let $\mathcal{U}_1 \subseteq \mathcal{F}_1$ and $f_1 \in \mathcal{F}_1$, where $\mathcal{F}_1$ is a bitopological space. Suppose that there exists a (1,2)*-$w\delta g\beta$-open set $\mathcal{M}_1$ containing f_1 such that $\mathcal{M}_1 \cap \mathcal{U}_1 = \varphi$. Since $\mathcal{U}_1 \subseteq \mathcal{F}_1$, (1,2)*-

$\boldsymbol{w\delta g\beta}$Cl($\boldsymbol{\mathcal{U}_1}$)⊆$\boldsymbol{\mathcal{F}_1 - \mathcal{M}_1}$ and then $\boldsymbol{f_1}$∉(1,2)*-$\boldsymbol{w\delta g\beta}$Cl($\boldsymbol{\mathcal{U}_1}$), which is a contradiction.

Conversely, assume that $\boldsymbol{f_1}$∉(1,2)*-$\boldsymbol{w\delta g\beta}$Cl($\boldsymbol{\mathcal{U}_1}$). Then there exists a (1,2)*-$\boldsymbol{w\delta g\beta}$-closed set $\boldsymbol{\mathcal{L}_1}$ containing $\boldsymbol{\mathcal{U}_1}$ such that $\boldsymbol{f_1}$ ∉ $\boldsymbol{\mathcal{L}_1}$. Since $\boldsymbol{f_1}$∈ $\boldsymbol{\mathcal{F}_1 - \mathcal{L}_1}$ and $\boldsymbol{\mathcal{F}_1 - \mathcal{L}_1}$ is (1,2)*-$\boldsymbol{w\delta g\beta}$-open, ($\boldsymbol{\mathcal{F}_1 - \mathcal{L}_1}$) ∩ $\boldsymbol{\mathcal{U}_1}$= φ which is a contradiction. Hence $\boldsymbol{f_1}$∈ (1,2)*-$\boldsymbol{w\delta g\beta}$Cl($\boldsymbol{\mathcal{U}_1}$) iff $\boldsymbol{\mathcal{M}_1}$∩ $\boldsymbol{\mathcal{U}_1}$ ≠ φ for every (1,2)*-$\boldsymbol{w\delta g\beta}$-open set $\boldsymbol{\mathcal{M}_1}$ containing $\boldsymbol{f_1}$.

Theorem 3.21

Let $\boldsymbol{\mathcal{U}_1}$ and $\boldsymbol{\mathcal{V}_1}$ be subsets of a space $\boldsymbol{\mathcal{F}_1}$. Then

(i) (1,2)*-$\boldsymbol{w\delta g\beta c\ell}$(φ) = φ and (1,2)*-$\boldsymbol{w\delta g\beta c\ell}$($\boldsymbol{\mathcal{F}_1}$) = $\boldsymbol{\mathcal{F}_1}$.

(ii) $\boldsymbol{\mathcal{U}_1}$ ⊆ (1,2)*-$\boldsymbol{w\delta g\beta c\ell}$($\boldsymbol{\mathcal{U}_1}$)

(iii) $\boldsymbol{\mathcal{U}_1}$ ⊆ $\boldsymbol{\mathcal{V}_1}$ ⇒ (1,2)*-$\boldsymbol{w\delta g\beta c\ell}$($\boldsymbol{\mathcal{U}_1}$)⊆ (1,2)*-$\boldsymbol{w\delta g\beta c\ell}$($\boldsymbol{\mathcal{V}_1}$)

(iv) (1,2)*-$\boldsymbol{w\delta g\beta c\ell}$((1,2)*-$\boldsymbol{w\delta g\beta c\ell}$($\boldsymbol{\mathcal{U}_1}$)) = (1,2)*-$\boldsymbol{w\delta g\beta c\ell}$($\boldsymbol{\mathcal{U}_1}$).

(v) (1,2)*-$\boldsymbol{w\delta g\beta c\ell}$($\boldsymbol{\mathcal{U}_1 \cup \mathcal{V}_1}$) = (1,2)*-$\boldsymbol{w\delta g\beta c\ell}$($\boldsymbol{\mathcal{U}_1}$) ∪ (1,2)*-$\boldsymbol{w\delta g\beta c\ell}$($\boldsymbol{\mathcal{V}_1}$).

(vi) (1,2)*-$\boldsymbol{w\delta g\beta c\ell}$($\boldsymbol{\mathcal{U}_1}$ ∩ $\boldsymbol{\mathcal{V}_1}$) ⊆ (1,2)*-$\boldsymbol{w\delta g\beta c\ell}$($\boldsymbol{\mathcal{U}_1}$) ∩ (1,2)*-$\boldsymbol{w\delta g\beta c\ell}$($\boldsymbol{\mathcal{V}_1}$).

Proof follows from definitions.

Theorem 3.22

Let $\boldsymbol{\mathcal{U}_1}$ and $\boldsymbol{\mathcal{V}_1}$ subsets of $\boldsymbol{\mathcal{F}_1}$. If $\boldsymbol{\mathcal{U}_1}$ is (1,2)*-$\boldsymbol{w\delta g\beta}$-closed, then (1,2)*-$\boldsymbol{w\delta g\beta}$Cl($\boldsymbol{\mathcal{U}_1}$ ∩ $\boldsymbol{\mathcal{V}_1}$) ⊆ $\boldsymbol{\mathcal{U}_1}$∩ (1,2)*-$\boldsymbol{w\delta g\beta}$Cl($\boldsymbol{\mathcal{V}_1}$).

Proof: Let $\boldsymbol{\mathcal{U}_1}$ be a (1,2)*-$\boldsymbol{w\delta g\beta}$-closed set, then (1,2)*-$\boldsymbol{w\delta g\beta}$Cl ($\boldsymbol{\mathcal{U}_1}$) = $\boldsymbol{\mathcal{U}_1}$ and so (1,2)*-$\boldsymbol{w\delta g\beta}$Cl ($\boldsymbol{\mathcal{U}_1}$ ∩ $\boldsymbol{\mathcal{V}_1}$) ⊆ (1,2)*-$\boldsymbol{w\delta g\beta}$Cl($\boldsymbol{\mathcal{U}_1}$) ∩ (1,2)*-$\boldsymbol{w\delta g\beta}$gCl($\boldsymbol{\mathcal{V}_1}$) = $\boldsymbol{\mathcal{U}_1}$ ∩ (1,2)*-$\boldsymbol{w\delta g\beta}$Cl($\boldsymbol{\mathcal{V}_1}$), which is the desired result.

Definition 3.23

Let $\boldsymbol{\mathcal{V}_1}$⊆$\boldsymbol{\mathcal{U}_1}$⊆ $\boldsymbol{\mathcal{F}_1}$. Then we say that $\boldsymbol{\mathcal{V}_1}$ is (1,2)*-$\boldsymbol{w\delta g\beta}$-closed relative to $\boldsymbol{\mathcal{U}_1}$ if (1,2)*-$\boldsymbol{\beta cl_{\mathcal{U}_1}}$($\boldsymbol{\mathcal{V}_1}$)⊆$\boldsymbol{\mathcal{G}_1}$ where $\boldsymbol{\mathcal{V}_1}$⊆$\boldsymbol{\mathcal{G}_1}$ and $\boldsymbol{\mathcal{G}_1}$ is (1,2)*-$\boldsymbol{\delta g}$-open in $\boldsymbol{\mathcal{U}_1}$.

Theorem 3.24

Let $\boldsymbol{\mathcal{V}_1}$⊆ $\boldsymbol{\mathcal{U}_1}$ ⊆$\boldsymbol{\mathcal{F}_1}$ and suppose that $\boldsymbol{\mathcal{V}_1}$ is (1,2)*-$\boldsymbol{w\delta g\beta}$-closed in $\boldsymbol{\mathcal{F}_1}$. Then $\boldsymbol{\mathcal{V}_1}$ is (1,2)*-$\boldsymbol{w\delta g\beta}$ -closed relative to $\boldsymbol{\mathcal{U}_1}$.

Proof: Here $\boldsymbol{\mathcal{V}_1}$⊆ $\boldsymbol{\mathcal{U}_1}$ ⊆ $\boldsymbol{\mathcal{F}_1}$ and $\boldsymbol{\mathcal{V}_1}$ is (1,2)*-$\boldsymbol{w\delta g\beta}$-closed in the bitopological space $\boldsymbol{\mathcal{F}_1}$. Let us assume that $\boldsymbol{\mathcal{V}_1}$⊆ $\boldsymbol{\mathcal{U}_1}$ ⊆ $\boldsymbol{\mathcal{M}_1}$, $\boldsymbol{\mathcal{M}_1}$ is (1,2)*-$\boldsymbol{\delta g}$ open in $\boldsymbol{\mathcal{F}_1}$. As $\boldsymbol{\mathcal{V}_1}$ is (1,2)*-$\boldsymbol{w\delta g\beta}$-closed set, $\boldsymbol{\mathcal{V}_1}$⊆$\boldsymbol{\mathcal{M}_1}$, implies (1,2)*-βcl ($\boldsymbol{\mathcal{V}_1}$) ⊆$\boldsymbol{\mathcal{M}_1}$. It implies that (1,2)*-$\boldsymbol{\beta cl_{\mathcal{U}_1}}$($\boldsymbol{\mathcal{V}_1}$) =(1,2)*-βcl($\boldsymbol{\mathcal{V}_1}$)∩ $\boldsymbol{\mathcal{U}_1}$⊆ $\boldsymbol{\mathcal{M}_1}$ ∩ $\boldsymbol{\mathcal{U}_1}$. Hence $\boldsymbol{\mathcal{V}_1}$ is (1,2)*-$\boldsymbol{w\delta g\beta}$-closed relative to $\boldsymbol{\mathcal{U}_1}$.

Remark 3.25

From the above discussion, we obtain the following:

$\boldsymbol{\tau_{1,2}}$-closed ⇒ (1,2)*-β-closed ⇒ (1,2)*-gβ-closed ⇒ (1,2)*-$\boldsymbol{w\delta g\beta}$-closed ⇒ (1,2)*-$\boldsymbol{\delta g\beta}$-closed.

Definition 3.26

A subset $\mathcal{U}_1$ of a bitopological space $\mathcal{F}_1$ is called (1,2)*-$\boldsymbol{w\delta g\beta}$-open set if $\mathcal{F}_1 - \mathcal{U}_1$ is (1,2)*-$\boldsymbol{w\delta g\beta}$- closed in $\mathcal{F}_1$. We denote the family of all (1,2)*-$\boldsymbol{w\delta g\beta}$-open sets in $\mathcal{F}_1$ by (1,2)*-$\boldsymbol{w\delta g\beta}\mathcal{O}(\mathcal{F}_1)$.

Theorem 3.27

If (1,2)*-β$\boldsymbol{int}(\mathcal{U}_1) \subseteq \mathcal{V}_1 \subseteq \mathcal{U}_1$ and if $\mathcal{U}_1$ is (1,2)*-$\boldsymbol{w\delta g\beta}$-open in the bitopological space $\mathcal{F}_1$ then B is (1,2)*-$\boldsymbol{w\delta g\beta}$-open in $\mathcal{F}_1$.

Proof: Suppose that (1,2)*-β$\boldsymbol{int}$ $(\mathcal{U}_1) \subseteq \mathcal{V}_1 \subseteq \mathcal{U}_1$ and $\mathcal{U}_1$ is (1,2)*- $\boldsymbol{w\delta g\beta}$-open in $\mathcal{F}_1$. Then $\mathcal{F}_1 - \mathcal{U}_1 \subseteq \mathcal{F}_1 - \mathcal{V}_1 \subseteq$ (1,2)*-β$\boldsymbol{c\ell}(\mathcal{F}_1 - \mathcal{U}_1)$. Since $\mathcal{F}_1 - \mathcal{U}_1$is (1,2)*-$\boldsymbol{w\delta g\beta}$-closed in $\mathcal{F}_1$, we have $\mathcal{F}_1 - \mathcal{V}_1$ is (1,2)*-$\boldsymbol{w\delta g\beta}$-closed in $\mathcal{F}_1$. Hence $\mathcal{V}_1$ is (1,2)*-$\boldsymbol{w\delta g\beta}$-open in $\mathcal{F}_1$.

Remark 3.28

The intersection of two (1,2)*-$\boldsymbol{w\delta g\beta}$-open sets need not be a (1,2)*-$\boldsymbol{w\delta g\beta}$-open.

Example 3.29

Let $\mathcal{F}_1 = \{\boldsymbol{a_1}, \boldsymbol{b_1}, \boldsymbol{c_1}\}$ with $\boldsymbol{\tau_1} = \{\phi, \{\boldsymbol{a_1}\}, \{\boldsymbol{a_1}, \boldsymbol{b_1}\}, \{\boldsymbol{a_1}, \boldsymbol{c_1}\}\}$ and $\boldsymbol{\tau_2} = \{\phi, \{\boldsymbol{b_1}\}, \{\boldsymbol{b_1}, \boldsymbol{c_1}\}\}$. We have $\boldsymbol{\tau_{1,2}} = \{\phi, \{\boldsymbol{a_1}\}, \{\boldsymbol{b_1}\}, \{\boldsymbol{a_1}, \boldsymbol{b_1}\}, \{\boldsymbol{a_1}, \boldsymbol{c_1}\}, \{\boldsymbol{b_1}, \boldsymbol{c_1}\}\}$. Here $\mathcal{U}_1 = \{\boldsymbol{a_1}, \boldsymbol{c_1}\}$ and $\mathcal{V}_1 = \{\boldsymbol{b_1}, \boldsymbol{c_1}\}$ are (1,2)*-$\boldsymbol{w\delta g\beta}$-closed subsets in $\mathcal{F}_1$. But $\mathcal{U}_1 \cap \mathcal{V}_1 = \{\boldsymbol{c_1}\}$ is not a (1,2)*-w$\boldsymbol{\delta g\beta}$-closed subset in $\mathcal{F}_1$.

Theorem 3.30

A subset $\mathcal{U}_1$ is (1,2)*-$\boldsymbol{w\delta g\beta}$-open iff $\mathcal{L}_1 \subseteq$ (1,2)*-β$\boldsymbol{int}$(A) whenever $\mathcal{L}_1$ is (1,2)*-$\boldsymbol{\delta g}$-closed and $\mathcal{L}_1 \subseteq \mathcal{U}_1$.

Proof

Let $\mathcal{U}_1$ be an $\boldsymbol{\tau_{1,2}}$-open set and suppose $\mathcal{L}_1 \subseteq \mathcal{U}_1$ where F is (1,2)*-$\boldsymbol{\delta g}$-closed. By definition, $\mathcal{F}_1 - \mathcal{U}_1$ is contained in (1,2)*-$\boldsymbol{w\delta g\beta}$-closed. Also, $\mathcal{F}_1 - \mathcal{U}_1$ is contained in the (1,2)*-$\boldsymbol{\delta g}$-open set $\mathcal{F}_1 - \mathcal{L}_1$. This implies (1,2)*-β$\boldsymbol{c\ell}(\mathcal{F}_1 - \mathcal{U}_1) \subseteq \mathcal{F}_1 - \mathcal{L}_1$. Now (1,2)*-β$\boldsymbol{c\ell}(\mathcal{F}_1 - \mathcal{L}_1)$ = $\mathcal{F}_1$-(1,2)*-β$\boldsymbol{int}(\mathcal{U}_1)$. Hence $\mathcal{F}_1$-(1,2)*-β$\boldsymbol{int}(\mathcal{U}_1)$ $\subseteq$ $\mathcal{F}_1 - \mathcal{L}_1$, that is $\mathcal{L}_1 \subseteq$ (1,2)*-β$\boldsymbol{int}(\mathcal{U}_1)$.

Conversely, If $\mathcal{L}_1$ is (1,2)*-$\boldsymbol{\delta g}$-closed set with $\mathcal{L}_1 \subseteq$ (1,2)*-β$\boldsymbol{int}(\mathcal{U}_1)$ where $\mathcal{L}_1 \subseteq \mathcal{U}_1$, it follows that $\mathcal{F}_1 - \mathcal{U}_1 \subseteq \mathcal{F}_1 - \mathcal{L}_1$ and (1,2)*-β$\boldsymbol{int}(\mathcal{U}_1) \subseteq \mathcal{F}_1 - \mathcal{L}_1$. That is (1,2)*-β$\boldsymbol{c\ell}(\mathcal{F}_1 - \mathcal{U}_1) \subseteq \mathcal{F}_1 - \mathcal{L}_1$. Hence $\mathcal{F}_1 - \mathcal{U}_1$ is (1,2)*-$\boldsymbol{w\delta g\beta}$-closed and $\mathcal{U}_1$ becomes (1,2)*-$\boldsymbol{w\delta g\beta}$-open.

Proposition 3.31

For subsets $\mathcal{U}_1$ and $\mathcal{V}_1$ of $\mathcal{F}_1$, the following assertions are valid.

(i) $\mathcal{U}_1$ is (1,2)*-$\boldsymbol{w\delta g\beta}$-open if and only if $\mathcal{U}_1$ is (1,2)*-$\boldsymbol{int}(\mathcal{U}_1)$.

(ii)) (1,2)*-$\boldsymbol{w\delta g\beta}$ $\boldsymbol{int}$((1,2)*-$\boldsymbol{w\delta g\beta int}(\mathcal{U}_1)$) = (1,2)*-$\boldsymbol{w\delta g\beta}$ $\boldsymbol{int}(\mathcal{U}_1)$.

(iii) $\mathcal{F}_1 -$ (1,2)*-$\boldsymbol{w\delta g\beta}$ $\boldsymbol{int}(\mathcal{U}_1)$ = (1,2)*-$\boldsymbol{w\delta g\beta}g\boldsymbol{c\ell}(\mathcal{F}_1 - \mathcal{U}_1)$.

(iv) $\mathcal{F}_1 -$(1,2)*-$w\delta g\beta c\ell(\mathcal{U}_1)$ = (1,2)*-$w\delta g\beta\ int(\mathcal{F}_1 - \mathcal{U}_1)$.

(v) $\mathcal{U}_1 \subseteq \mathcal{V}_1 \Rightarrow$ (1,2)*-$w\delta g\beta\ int(\mathcal{U}_1) \subseteq$ (1,2)*-$w\delta g\beta int(\mathcal{V}_1)$.

(vi) (1,2)*-$w\delta g\beta\ int(\mathcal{U}_1) \cup$ (1,2)*-$w\delta g\beta\ int(\mathcal{V}_1) \subseteq$ (1,2)*-$w\delta g\beta\ int(\mathcal{U}_1 \cup \mathcal{V}_1)$.

(vii) (1,2)*-$w\delta g\beta\ int(\mathcal{U}_1 \cap \mathcal{V}_1)$ = (1,2)*-$w\delta g\beta\ int(\mathcal{U}_1) \cap$ (1,2)*-$w\delta g\beta int(\mathcal{V}_1)$.

4. (1,2)*-Weakly delta generalized beta continuous functions

Definition 4.1

Let $\mathcal{F}_1$ and $\mathcal{F}_2$ be two bitopological spaces. A function $\wp: \mathcal{F}_1 \to \mathcal{F}_2$ is called (1,2)*-weakly $\delta g\beta$-continuous (briefly, (1,2)*-$w\delta g\beta$-continuous) if $f^{-1}(\mathcal{G}_1)$ is a (1,2)*-$w\delta g\beta$-open set in $\mathcal{F}_1$ for each $\sigma_{1,2}$-open set $\mathcal{G}_1$ of $\mathcal{F}_2$.

Example 4.2

Let $\mathcal{F}_1 = \mathcal{F}_2 = \{a_1, b_1, c_1\}$, with $\tau_1 = \{\emptyset, \mathcal{F}_1, \{b_1\}\}$, $\tau_2 = \{\emptyset, \mathcal{F}_1, \{b_1, c_1\}\}$, $\sigma_1 = \{\emptyset, \mathcal{F}_2, \{a_1, b_1\}\}$, $\sigma_2 = \{\emptyset, \mathcal{F}_2, \{b_1, c_1\}\}$. Then $\tau_{1,2}$-open sets: $\{\emptyset, \mathcal{F}_1, \{b_1\}, \{b_1, c_1\}\}$,

$\tau_{1,2}$-closed sets : $\{\emptyset, \mathcal{F}_1, \{a_1\}, \{a_1, c_1\}\}$, $\sigma_{1,2}$- open sets : $\{\emptyset, \mathcal{F}_1, \{a_1, b_1\}, \{b_1, c_1\}\}$,

$\sigma_{1,2}$- closed sets : $\{\emptyset, X, \{a_1\}, \{c_1\}\}$. Let $\wp: \mathcal{F}_1 \to \mathcal{F}_2$ be an identity function. Where (1,2)*-$w\delta g\beta O(\mathcal{F}_1) = \{\emptyset, \mathcal{F}_1, \{b_1\}, \{c_1\}, \{a_1, b_1\}, \{b_1, c_1\}\}$. Then $\wp$ is (1,2)*-w$\delta g\beta$-continuous.

Theorem 4.3

Every (1,2)*-$g\beta$-continuous function is (1,2)*-w$\delta g\beta$-continuous.

Proof

By Theorem 3.3, the result is true.

The converse is false as seen in the following example.

Example 4.4

Let $\mathcal{F}_1 = \mathcal{F}_2 = \{a_1, b_1, c_1\}$, $\tau_1 = \{\emptyset, \mathcal{F}_1, \{a_1, b_1\}\}$, $\tau_2 = \{\emptyset, \mathcal{F}_1, \{b_1, c_1\}\}$, $\sigma_1 = \{\emptyset, \mathcal{F}_2, \{b_1\}\}$, $\sigma_2 = \{\emptyset, \mathcal{F}_2, \{b_1, c_1\}\}$. Then $\tau_{1,2}$-open sets: $\{\emptyset, \mathcal{F}_1, \{a_1, b_1\}, \{b_1, c_1\}\}$, $\tau_{1,2}$-closed sets : $\{\emptyset, \mathcal{F}_1, \{a_1\}, \{b_1\}\}$,

$\sigma_{1,2}$- open sets : $\{\emptyset, \mathcal{F}_2, \{b_1\}, \{b_1, c_1\}\}$, $\sigma_{1,2}$- closed sets : $\{\emptyset, \mathcal{F}_2, \{a_1\}, \{a_1, c_1\}\}$. Let $\wp: \mathcal{F}_1 \to \mathcal{F}_2$ be the function defined by $\wp(a_1) = a_1$, $\wp(b_1) = c_1$ and $\wp(c_1) = b_1$. Then f is (1,2)*-w$\delta g\beta$-continuous but not (1,2)*-$g\beta$-continuous.

Theorem 4.5

Every (1,2)*-w$\delta g\beta$-continuous function is (1,2)*-$\delta g\beta$-continuous.

Proof

By Theorem 3.5, the proof is straight forward.

The converse need not be true as seen in the following example.

Example 4.6

Let $\mathcal{F}_1 = \mathcal{F}_2 = \{a_1, b_1, c_1\}$, $\tau_1 = \{\emptyset, \mathcal{F}_1, \{b_1\}\}$, $\tau_2 = \{\emptyset, \mathcal{F}_1, \{b_1, c_1\}\}$, $\sigma_1 = \{\emptyset, \mathcal{F}_2, \{a_1, b_1\}\}$, $\sigma_2 = \{\emptyset, \mathcal{F}_2, \{b_1, c_1\}\}$,. Then $\tau_{1,2}$- open sets : $\{\emptyset, \mathcal{F}_1, \{b\}, \{b, c\}\}$, $\tau_{1,2}$-closed sets : $\{\emptyset, \mathcal{F}_1, \{a_1\}, \{a_1, c_1\}\}$, $\sigma_{1,2}$-open sets: $\{\emptyset, \mathcal{F}_2, \{a_1, b_1\}, \{b_1, c_1\}\}$, $\sigma_{1,2}$-closed sets : $\{\emptyset, \mathcal{F}_2, \{a_1\}, \{b_1\}\}$. Let $\wp: \mathcal{F}_1 \to \mathcal{F}_2$ be the function defined by $\wp(a_1) = a_1$, $\wp(b_1) = c_1$ and $\wp(c_1) = b_1$. Then f is (1,2)*- $\delta g\beta$-continuous but not (1,2)*-w$\delta g\beta$-continuous.

Theorem 4.7

A function $\wp: \mathcal{F}_1 \to \mathcal{F}_2$ is (1,2)*-$w\delta g\beta$-continuous iff $\wp^{-1}(\mathcal{G}_1)$ is a (1,2)*-$w\delta g\beta$-closed set in $\mathcal{F}_1$ for every $\sigma_{1,2}$-closed set $\mathcal{G}_1$ of $\mathcal{F}_2$.

Proof

Let $\mathcal{G}_1$ be any $\sigma_{1,2}$-closed set of $\mathcal{F}_2$, by hypothesis $\wp^{-1}(\mathcal{G}_1{}^c) = \mathcal{F}_1 - \wp^{-1}(\mathcal{G}_1)$ is a (1,2)*-$w\delta g\beta$-closed set in $\mathcal{F}_1$ for every $\sigma_{1,2}$-closed set $\mathcal{G}_1$ of $\mathcal{F}_2$.

The converse can be proved in the similar manner.

Definition 4.8

A bitopological space $\mathcal{F}_1$ is said to be locally (1,2)*-$\delta g\beta$-indiscrete if every (1,2)*-$\delta g\beta$-open set of $\mathcal{F}_1$ is $\tau_{1,2}$-closed in $\mathcal{F}_1$.

A function $\wp: \mathcal{F}_1 \to \mathcal{F}_2$ is called contra (1,2)*-$\delta g\beta$-continuous if the preimage of every $\sigma_{1,2}$-open subset of $\mathcal{F}_2$ is (1,2)*-$\delta g\beta$-closed in $\mathcal{F}_1$.

Theorem 4.9

A function $\wp: \mathcal{F}_1 \to \mathcal{F}_2$ is contra (1,2)*-$\delta g\beta$-continuous iff the inverse image of every $\sigma_{1,2}$-closed set of $\mathcal{F}_2$ is (1,2)*-$\delta g\beta$-open in $\mathcal{F}_1$.

Proof

Let $\mathcal{G}_1$ be any $\sigma_{1,2}$-closed set of $\mathcal{F}_2$. Then $\mathcal{F}_2 - \mathcal{G}_1$ is $\sigma_{1,2}$-open. Since $\wp: \mathcal{F}_1 \to \mathcal{F}_2$ is contra (1,2)*-$\delta g\beta$-continuous, it follows that $\wp^{-1}(\mathcal{F}_2 - \mathcal{G}_1) = \mathcal{F}_1 - \wp^{-1}(\mathcal{G}_1)$ is (1,2)*-$\delta g\beta$-closed. Hence $\wp^{-1}(\mathcal{G}_1)$ is (1,2)*-$\delta g\beta$-open in $\mathcal{F}_1$.

Similarly to prove the converse.

Theorem 4.10

Let $\wp: \mathcal{F}_1 \to \mathcal{F}_2$ be a function. If $\wp$ is contra (1,2)*-$\delta g\beta$-continuous and $\mathcal{F}_1$ is locally (1,2)*-$\delta g\beta$-indiscrete, then $\wp$ is (1,2)*-continuous.

Proof. Let $\mathcal{M}_1$ be a $\sigma_{1,2}$-closed in $\mathcal{F}_2$. Since $\wp$ is contra (1,2)*-$\delta g\beta$-continuous, $\wp^{-1}(\mathcal{M}_1)$ is (1,2)*-$\delta g\beta$-open in $\mathcal{F}_1$. Since $\mathcal{F}_1$ is locally (1,2)*-$\delta g\beta$-indiscrete, $\wp^{-1}(\mathcal{M}_1)$ is $\tau_{1,2}$-closed in $\mathcal{F}_1$. Hence $\wp$ is (1,2)*-continuous.

Theorem 4.11.

Let $\wp: \mathcal{F}_1 \to \mathcal{F}_2$ be a function. If $\wp$ is contra (1,2)*-$\delta g\beta$-continuous and $\mathcal{F}_1$ is locally (1,2)*-$\delta g\beta$-indiscrete, then $\wp$ is (1,2)*-w$\delta g\beta$-continuous.

Proof. Let $\wp: \mathcal{F}_1 \to \mathcal{F}_2$ be contra (1,2)- $\delta g\beta$-continuous and $\mathcal{F}_1$ is locally (1,2)*-$\delta g\beta$-indiscrete. By Theorem 4.7, $\wp$ is (1,2)*-continuous, then $\wp$ is (1,2)*-w$\delta g\beta$-continuous.

Definition 4.12.

A bitopological space $\mathcal{F}_1$ is called (1,2)*-$w\delta g\beta$-compact (resp. (1,2)*-$\delta g\beta$-compact) if every cover of $\mathcal{F}_1$ by (1,2)*- $w\delta g\beta$-open (resp. (1,2)*-$\delta g\beta$-open) sets has finite subcover.

Theorem 4.13.

Let $\wp: \mathcal{F}_1 \to \mathcal{F}_2$ be surjective (1,2)*-w$\delta g\beta$-continuous function. If $\mathcal{F}_1$ is (1,2)*-w$\delta g\beta$ -compact, then $\mathcal{F}_2$ is (1,2)*-compact.

Proof. Let $\{\mathcal{U}_i : i \in I\}$ be an $\sigma_{1,2}$-open cover of $\mathcal{F}_2$. Then $\{\wp^{-1}(\mathcal{U}_i) : i \in I\}$ is a (1,2)*-w$\delta g\beta$-open cover in $\mathcal{F}_1$. Since $\mathcal{F}_1$ is (1,2)*-w$\delta g\beta$-compact, it has a finite subcover, say $\{\wp^{-1}(\mathcal{U}_1)$, $\wp^{-1}(\mathcal{U}_2)$),...., $\wp^{-1}(\mathcal{U}_n)\}$. Since $\wp$ is surjective $\mathcal{U}_1, \mathcal{U}_2, \ldots, \mathcal{U}_n$ is a finite subcover of $\mathcal{F}_2$ and hence $\mathcal{F}_2$ is (1,2)*-compact.

Theorem 4.14

For a bitopological space $\mathcal{F}_1$, the following are equivalent:

(i) $\mathcal{F}_1$ is (1,2)*-$w\delta g\beta$-connected.

(ii) The empty set $\emptyset$ and $\mathcal{F}_1$ are only subsets which are both (1,2)*-$w\delta g\beta$-open and (1,2)*-$w\delta g\beta$-closed.

(iii) Each (1,2)*-$w\delta g\beta$-continuous function from $\mathcal{F}_1$ into a discrete space $\mathcal{F}_2$ which has atleast two points is a constant function.

Proof

(i)⇒(ii). Let $\mathcal{S}_1 \subseteq \mathcal{F}_1$ be any proper subset, which is both (1,2)*-$w\delta g\beta$-open and (1,2)*-$w\delta g\beta$-closed. Its complement $\mathcal{F}_1 - \mathcal{S}_1$ is also (1,2)*-$w\delta g\beta$-open and (1,2)*-$w\delta g\beta$-closed. Then ($\mathcal{F}_1 = \mathcal{S}_1 \cup (\mathcal{F}_1 - \mathcal{S}_1)$ is a disjoint union of two non-empty (1,2)*-$w\delta g\beta$-open sets which is a contradiction that $\mathcal{F}_1$ is (1,2)*-$w\delta g\beta$-connected. Hence, $\mathcal{S}_1 = \emptyset$ or $\mathcal{F}_1$.

(ii)⇒(i). Let $\mathcal{F}_1 = \mathcal{U}_1 \cup \mathcal{V}_1$ here $\mathcal{U}_1 \cap \mathcal{V}_1 = \emptyset, \mathcal{U}_1 \neq \emptyset$, $\mathcal{V}_1 \neq \emptyset$ and $\mathcal{U}_1, \mathcal{V}_1$ are (1,2)*-$w\delta g\beta$-open. Since $\mathcal{U}_1 = \mathcal{F}_1 - \mathcal{V}_1$, $\mathcal{U}_1$ is (1,2)*-$w\delta g\beta$-closed. According to the assumption $\mathcal{U}_1 = \emptyset$, which is a contradiction.

(ii)⇒(iii). Let $\wp: \mathcal{F}_1 \to \mathcal{F}_2$ be a (1,2)*-$w\delta g\beta$-continuous function where $\mathcal{F}_2$ is a discrete bitopological space with at least two points. Then $\wp^{-1}(\{y\})$ is (1,2)*-$w\delta g\beta$-closed and (1,2)*-$w\delta g\beta$-open for each $y \in \mathcal{F}_2$ and $\mathcal{F}_1 = \cup \{\wp^{-1}(\{y\}): y \in \mathcal{F}_2\}$. According to the assumption, $\wp^{-1}(\{y\}) = \emptyset$ or $\wp^{-1}(\{y\}) = \mathcal{F}_1$. If $\wp^{-1}(\{y\}) = \emptyset$ for all $y \in \mathcal{F}_2$, f will not be a function. Also there is no exist more than one $y \in \mathcal{F}_2$ such that $\wp^{-1}(\{y\}) = \mathcal{F}_1$. Hence, there exists only one $y \in \mathcal{F}_2$ such that $\wp^{-1}(\{y\}) = \mathcal{F}_1$ and $\wp^{-1}(\{y_1\}) = \emptyset$ where $y \neq y_1$. This shows that $\wp$ is a constant function.

(iii)⇒(ii). Let $\mathcal{S}_1 \neq \emptyset$ be both (1,2)*-$w\delta g\beta$-open and (1,2)*-$w\delta g\beta$-closed in $\mathcal{F}_1$. Let $\wp: \mathcal{F}_1 \to \mathcal{F}_2$ be a (1,2)*-$w\delta g\beta$-continuous function defined by $\wp(\mathcal{S}_1) = \{a_1\}$ and $\wp(\mathcal{F}_1 - \mathcal{S}_1) = \{b_1\}$

where $a_1 \neq b_1$. Since $\wp$ is a constant function, we get $\mathcal{S}_1 = \mathcal{F}_1$.

5. Conclusion:

This paper introduced (1,2)*-$w\delta g\beta$-closed sets and examined their fundamental properties and relationships with other (1,2)*-closed sets in bitopology. We also defined and studied (1,2)*-$w\delta g\beta$-continuous functions, highlighting their significance through multiple characterizations.

References

1. T. Fukutake, P. Sundaram and M. Sheik John, Bull. Fukuoka Univ. Educ., Part III 51, 2002, 1–9.
2. Hamant Kumar, (1, 2)∗-generalized -closed Sets in Bitopological Spaces, IJMR, 8(9), 2022, 319-326.
3. J.C. Kelley, Bitopological Spaces, Proc. London math. Soc., 13, 1963, 71-89.
4. M.Lellis Thivagar, O.Ravi and M.E.Abd El-Monsef, Remarks on bitopological (1, 2)-quotient mappings, J. Egypt Math. Soc., 16(1), 2008, 17-25.
5. B. Meera Devi, P. Subbulakshmi, (1,2)*-δgp Closed Sets in Bitopological Spaces, Paripex-Indian Journal Of Research, 6(7), 2017, 27-32.
6. Navalagi and Kantappa, M. Bhavikatti, Beta Weakly Generalized Closed sets in Topology, Govindappa, Journal of Computer and Mathematical Sciences, 9(5), 2018, 435-446.
7. M. Ramaboopathi and K. M. Dharmalingam, On(1,2)*-ˇg-closed sets in bitopological spaces, Malaya Journal of Matematik, 7(3), 2019, 463-467.
8. O.Ravi and M.Lellis Thivagar, On Stronger forms of (1, 2)*- quotient mappings in bitopological spaces, Internat. J. Math. Game by and Algebra, 1(6), 2004, 481-492.
9. Sanjay Tahiliani, On Weakly β-Continuous Functions in Bitopological Spaces, Faculty of Sciences and Mathematics, University of Niš, Serbia, 1, 2008, 77–86.
10. P. Sundaram. Studies on generalizations of continuous maps in topological spaces, Ph. D. Thesis, Bharathiar University, Coimbatore, 1991.