

# Computational study of Nanofluid flow over a Stretching Sheet in a Darcy-Forchheimer Porous Medium with Radiative Heat Transfer

Bhavya N. P.<sup>1</sup>, M. S. Gayathri<sup>2</sup>, P. A. Dinesh<sup>3</sup>

<sup>1</sup>*Department of Mathematics, BGS College of Engineering and Technology, India; bhavya201983@gmail.com*

<sup>2</sup>*Department of Mathematics, B.M.S College of Engineering, India; msgayathri.maths@bmsce.ac.in*

<sup>3</sup>*Department of Mathematics, Ramaiah Institute of Technology, India; dineshdpa@msrit.edu*

The Darcy-Forchheimer model been preferred for its accuracy in simulating fluid flow in porous media, and found much useful for applications such as fluid separation, heat exchange, fluid transport, filtration, and purification. This exercise discusses heat transfer in nanofluid flow over a stretching sheet within a porous medium, incorporating the Darcy-Forchheimer effects into the velocity equation. The ordinary differential equations are obtained from governing partial differential equations employing appropriate similarity transformations and solved numerically using both the shooting technique and the spectral collocation technique. The model accounts for the effects of various parameters, including the Forchheimer number, permeability number, Prandtl number, Eckert number, Lewis number, Brownian motion, and thermophoresis. The various parameters influence on the velocity, thermal, and concentration boundary layers is analysed and presented graphically. The results revealed that thickness of velocity boundary layer rises with the Forchheimer parameter but decreases with higher permeability. The thickness of thermal boundary layer increases with higher Forchheimer and permeability parameters as well as with increased Brownian motion, and thermophoresis, while it decreases with higher Eckert and Lewis numbers. Meanwhile, thickness of concentration boundary layer decreases as the Lewis and Prandtl numbers increase.

**Keywords:** Darcy-Forchheimer flow, porous medium, nanofluid, stretching sheet, thermal radiation.

## 1. Introduction

The colloidal suspensions of nano-particles in a fluid called as nanofluids, gained much more research strength because of their remarkable thermal properties, such as enhanced conductivity and heat transfer capabilities. The ample characteristics make nanofluids highly preferable for many engineering applications, particularly in systems that require efficient heat

management. When nanofluids flow through porous media, the interaction between the porous material and fluid plays a main role in determining the overall system performance.

The boundary layer flow, a phenomenon that occurs in the thin region near solid surfaces where viscous forces dominate, significantly influences nanofluid transport in porous media. This boundary layer is crucial in heat transfer practices and frictional resistance hence plays a major role in flow and thermal performance. In the case of nanofluids, these effects seem more complex by the enhanced fluid properties, such as increased thermal conductivity and in some cases, augmented viscosity because of the presence of nanoparticles.

In fluid dynamics, the Darcy-Forchheimer flow model provides a more comprehensive understanding of fluid behaviour in porous medium by accounting for both viscous and inertial forces. While Darcy's law applies to low velocity, laminar flows, the Forchheimer extension incorporates the effects of higher velocity flow making relevant for systems observing considerable inertia forces. This model becomes very important in the study of nanofluids, as the higher velocity and thermal conductivity of nanofluids can lead to significant inertial effects which would be otherwise ignored in the traditional Darcy approach.

Amir Abbas et al. [1] have exercised a CFD case study of a non-Newtonian fluid flow over inclined stretching sheet in a permeable media using the Darcy–Forchheimer equation. It has noticed that with an increasing magnitude of the diffusion-thermo parameter, there was a rise in the fluid temperature and velocity but a reverse trend in mass concentration. Amir Abbas et al. [2] had a numerical approach with the help of Darcy–Forchheimer equation for non – Newtonian fluid flow and heat dissipation characteristics in permeable media subjected to magneto hydrodynamic heat dissipation. The results of the exercise have compared with available results and a good agreement is observed.

Nihaal et al. [3] have studied the behaviour of bio-convection on heat source, sink and activation energy with radially stretching disc. It has observed that with a rise in Forchheimer number porosity and drag coefficient rise, resulting in more resistive forces thus lower the fluid velocity. Sumera Dero et al. [4] have studied the Darcy-Forchheimer porous medium influence on a nanofluid flow onto a shrinking surface. A heat transfer rate diminishing is noticed with a rise in the parameter of the copper solid volume fraction. Anwar Saeed et al. [5] have studied on hybrid nanofluid flow through a spinning Darcy–Forchheimer porous space with thermal radiation. The nanoparticles of titanium dioxide and aluminium oxide have been suspended in water as base fluid. It is reported that heat is transmitted more quickly in hybrid nanofluid than in traditional nanofluid.

Oyinkepreye D. Orodu et al. [6] have experimented on non-Darcy and Darcy flow in porous media. Anwar Saeed et al. [7] inspected heat transfer and electro-magnetic effects influence on MHD flow of nanofluids over a Darcy-Forchheimer model in a symmetric flow with variable viscosity.

Umer Farooq et al. [8] studied the impact of non-similar modelling on Darcy-Forchheimer-Brinkman model for forced convection of Casson nano-fluid in non-Darcy porous media. The effect of Casson parameter, Schmidt number, thermophoresis parameter, Prandtl number, Brownian motion parameter on the dimensionless velocity, temperature and concentration distributions are comprehensively studied. Belhaj et al. [9] have exercised the numerical

analysis of Forchheimer equation to describe non-Darcy and Darcy flow. It is noticed that a rise in the Darcy number causes a decrease in velocity field. Sadaf Masood et al. [10] did the experiments on nanofluid flow with a Forchheimer porous medium. It is noted that a rise in the Darcy number results decrease in the velocity field.

Himanshu Upreti et al. [11] reported the numerical solution of nanofluid flow through stretching surface in a radiative porous media using Darcy–Forchheimer. It has observed that an increase in Forchheimer number and material parameter values, heat transfer decreases. Farwa Haider et al. [12] have studied the flow of hybrid nanofluid through Darcy Forchheimer porous space with variable characteristics. It is observed that rate of heat transfer of Titanium dioxide nanofluid is higher when compared with hybrid nanofluid.

Ahmed Alshehri and Jahir Shah [13] studied the computational analysis of viscous dissipation and Darcy-Forchheimer porous medium on radioactive hybrid nanofluid. The effects of heat flow and radiative flux are examined in this work. A mathematical modelling was used to create and renovate a system of PDEs for fluid flow based on a few standard ones. Farooq et al. [14] have studied heat transfer of nanofluid flow through non-Darcy Forchheimer medium, concluded that there was a diminish in temperature field with rise in melting parameter and velocity augments for larger melting parameter. Larger Brownian diffusion parameter caused dominant temperature and concentration fields.

Alzahrani et al. [15] investigated the impact of Darcy-Forchheimer effects and nanoparticles on fluid flow and heat dissipation characteristics of  $H_2O$  as a working fluid, exploring its potential to enhance the efficiency of solar collectors using an inclined plate. Vishnu Ganesh et al. [16] have observed the Darcy–Forchheimer flow for hydro-magnetic nanofluid over a stretching sheet in a thermally stratified porous media with viscous and Ohmic transfer effects. A unique solution is noticed for stretching sheet and dual solutions obtained for shrinking sheet which depend on the suction parameter and local inertia coefficient.

Tasawar Hayat et al. [17] have studied about minimization of entropy in case of Darcy-Forchheimer nanofluid flow onto curved stretching sheet and concluded that temperature and concentration have direct relation with thermal and Biot numbers. Ghulam Rasool et al. [18] reported the Darcy-Forchheimer relation in Casson type magneto hydrodynamic nanofluid flow over non-linear stretching surface and concluded that momentum boundary layer reduces for stronger inertial impact and the resistance offered by the porous media to the fluid flow.

Syed Naqvi et al. [19] reported the numerical solution for a nanofluid with Darcy-Forchheimer flow with a rotating disk and concluded that concentration and temperature are dominant for higher thermophoresis parameter. Muhammad Ijaz Khan et al. [20] have done the numerical analysis for Darcy-Forchheimer flow in presence of homogeneous-heterogeneous reactions. It is concluded that larger local inertia coefficient and inverse Darcy number yield reduction in velocity.

Ravuri Mohana Ramana et al. [21] have done numerical investigation of 3-D rotating hybrid nanofluid Forchheimer flow with radiation absorption over a stretching sheet and reported that Nusselt number exhibits an upward trend when Forchheimer number, porosity parameter, radiation absorption, magnetic parameter and aligned magnetic field experience an increase whereas the Sherwood number exhibits a contrary behaviour with these identical parameters.

Adil Sadiq and Hayat [22] have studied the Darcy–Forchheimer flow of magneto Maxwell liquid bounded by convectively heated sheet and revealed that the temperature field has an inverse relationship with the thermal relaxation parameter and Prandtl number.

Gireesha et al. [23] have done a study on numerical solution for hydro-magnetic boundary layer flow and heat transfer past a stretching surface in non-Darcy porous medium with fluid-particle suspension and concluded that suspending fine dust particles in clean fluid reduces thickness of thermal boundary layer. Therefore, the dusty fluids are preferable in engineering and scientific applications, involving cooling processes. Zhengwen Zeng and Red Grigg [24] reviewed on criterion for porous media non-Darcy flow and concluded that Forchheimer number and Reynolds number been used in the past to notice commence of non-Darcy flow. The Forchheimer number has a direct relation to non-Darcy effect.

Osama et al. [25] study the electromagnetic non-Darcy Forchheimer flow, heat transfer over a nonlinearly stretching sheet of non-Newtonian Casson fluid in presence of a non-uniform heat source. Swati Mukhopadhyay et al. [26] discussed the steady forced convection flow and heat transfer past a porous plate placed in a fluid-saturated porous medium using the Darcy-Forchheimer model.

This paper explores the application of the Darcy-Forchheimer model to describe nanofluids flow in porous media focusing on the effects of boundary layer flow and its implications for heat and mass transfer. A key application such as groundwater remediation is discussed with an emphasis on how nanofluid properties and flow characteristics can improve system performance. This work aims to provide valuable insights into advantages of nanofluid based systems for a range of engineering and environmental application.

## 2. MATHEMATICAL MODELLING

A two-dimensional Darcy-Forchheimer flow of an incompressible nanofluid over a stretching sheet in a porous medium has been considered. The fluid linear velocity at the surface is given by  $u_w(x) = ax$  where  $a$  is a positive real constant and  $x$  is the position variable along the stretching surface. The temperature and volume fraction of nanoparticle at the surface are represented by  $T_w$  and  $C_w$  respectively while  $T_\infty$  and  $C_\infty$  represents the temperature and nanoparticle concentration in the ambient fluid. Figure 1 illustrates a schematic representation of the flow problem. The fundamental governing equations of the current problem can be expressed in terms of following equations.

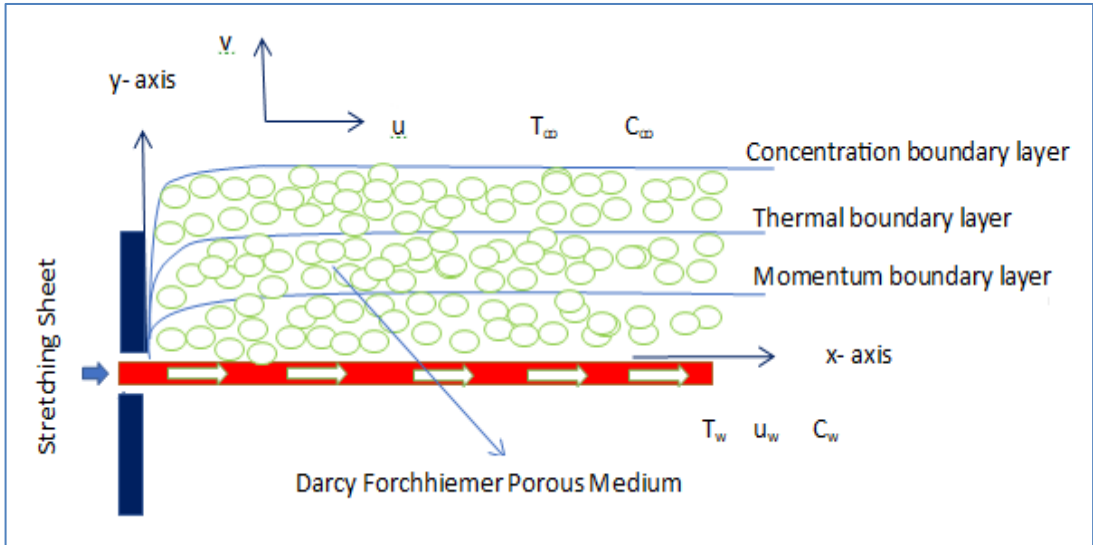


Figure 1 Physical model of flow problem.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

(1)

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho_f} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \frac{\bar{\mu}}{k \rho_f} u - \frac{C_b u^2}{\sqrt{k}} \quad (2)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho_f} \frac{\partial p}{\partial y} + \nu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \frac{\bar{\mu}}{k \rho_f} v - \frac{C_b v^2}{\sqrt{k}} \quad (3)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \tau \left\{ D_B \left( \frac{\partial C}{\partial x} \frac{\partial T}{\partial x} + \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} \right) + \left( \frac{D_T}{T_\infty} \right) \left[ \left( \frac{\partial T}{\partial x} \right)^2 + \left( \frac{\partial T}{\partial y} \right)^2 \right] \right\} - \frac{\bar{\mu} u^2}{(\rho c)_f k}$$

(4)

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \left( \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) + \left( \frac{D_T}{T_\infty} \right) \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)$$

(5)

where  $u$  and  $v$  are velocity components along  $x$  and  $y$  directions respectively,  $p$  : fluid pressure,  $\rho_p$ : particle density,  $\rho_f$ : density of the fluid,  $\nu$ : kinematic viscosity,  $\alpha$ : thermal diffusivity,  $D_T$ : co-efficient of thermophoretic diffusion,  $D_B$ : co-efficient of Brownian motion diffusion,  $\tau = \frac{(\rho c)_p}{(\rho c)_f}$  : ratio between nanoparticles effective heat capacity and fluid heat capacity,  $\rho$  : density,  $k$ : permeability of the porous medium,  $c$ : volumetric expansion coefficient,  $\bar{\mu}$  : effective dynamic viscosity of nanofluid,  $C_b$ : drag co-efficient,  $T$  : Fluid Temperature,  $C$  : Nanoparticle Concentration.

The Boundary conditions for the considered problem are given in equation (7) ;

At  $y = 0$ ;  $v = 0, u = u_w(x) = ax, T = T_w, C = C_w$

At  $y \rightarrow \infty$ ;  $u = 0, v = 0, T = T_\infty, C = C_\infty$

(6)

We have introduced the similarity variable to transform the physical quantities into dimensionless forms and presented in equations (8)–(10).

$$\eta = \sqrt{\frac{a}{v}} y, \Psi = \sqrt{av} x f(\eta)$$

$$\theta(\eta) = \frac{T - T_{\infty}}{T_w - T_{\infty}}, \phi(\eta) = \frac{C - C_{\infty}}{C_w - C_{\infty}}$$

$$u = \frac{\partial \Psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \Psi}{\partial x}$$

(7)

This will result in a set of dimensionless equations

$$f''' + ff'' - (1 + D)f'^2 - k_p f' = 0$$

(8)

$$\frac{1}{Pr} \theta'' + f\theta' + Nb\phi'\theta' + Nt\theta'^2 - k_p E f'^2 = 0 \quad (9)$$

$$\phi'' + Le f\phi' + \frac{Nt}{Nb} \theta'' = 0$$

(10)

and the dimensionless boundary conditions are

$$f(0) = 0, f'(0) = 1, \theta(0) = 1, \phi(0) = 1$$

$$f'(\infty) = 0, \theta(\infty) = 0, \phi(\infty) = 0$$

(11)

Let  $Pr = \frac{\nu}{\alpha}$ : Prandtl number,  $Le = \frac{\nu}{D_B}$ : Lewis number,  $Nb = \frac{(\rho c)_p D_B (C_w - C_{\infty})}{(\rho c)_f \nu}$ : Brownian motion parameter,  $Nt = \frac{(\rho c)_p D_T (T_w - T_{\infty})}{(\rho c)_f T_{\infty} \nu}$ : thermophoresis parameter,  $k_p = \frac{\bar{\mu}}{k \rho_f a}$ : permeability parameter,  $D = \frac{C_{bx}}{\sqrt{k}}$ : Forchheimer number and  $E = \frac{a^2 x^2}{c_f (T_w - T_{\infty})}$ : Eckert number. The reduced boundary problem is given by equations (8) –(10) and applicable boundary conditions are given in equation (11).

The analytical solution of equation (8) with boundary conditions (11) is given by equation (12).

$$f(\eta) = \frac{1}{s} (1 - e^{-s\eta}) \quad (12)$$

$$\text{where } s = \sqrt{1 + D + k_p}$$

The equations (8) – (10) subject conditions outlined in equation (11) were solved using fourth-order Runge – Kutta Fehlberg method and the Newton-Raphson method. The accuracy of these techniques was validated by employing Gauss-Lobatto points within the quasi – linear spectral collocation approach.

### 3. RESULTS AND DISCUSSION

The Equations (8) – (10) with conditions (11) have been solved with the aid of numerical methods. The study investigates how the nanofluid is influenced and modified by various parameters such as permeability parameter, Forchheimer number, Eckert number, Lewis number, Prandtl number, thermophoresis and Brownian motion parameter. The effects of parameters mentioned above on temperature, volume fraction of nanoparticle and stream wise velocity is thoroughly analyzed. Initially, the paper highlights the effect of primary parameter under investigation: the Forchheimer number.

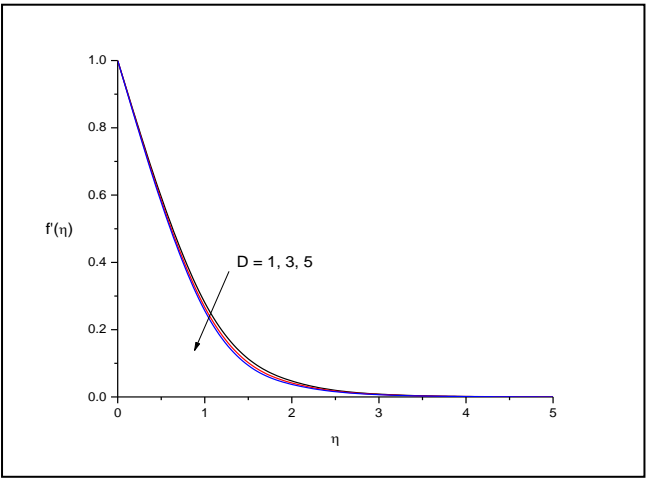


Figure 2 Influence of Forchheimer number (D) on  $f'(\eta)$

Figure 2 illustrates the velocity boundary layer structure for various values of Forchheimer number. The observed changes in the fluid flow pattern are attributed to the decline in flow as the Forchheimer number increases. This slowdown occurs because the resistive force, acting perpendicular to the fluid flow, reduces the fluid's motion.

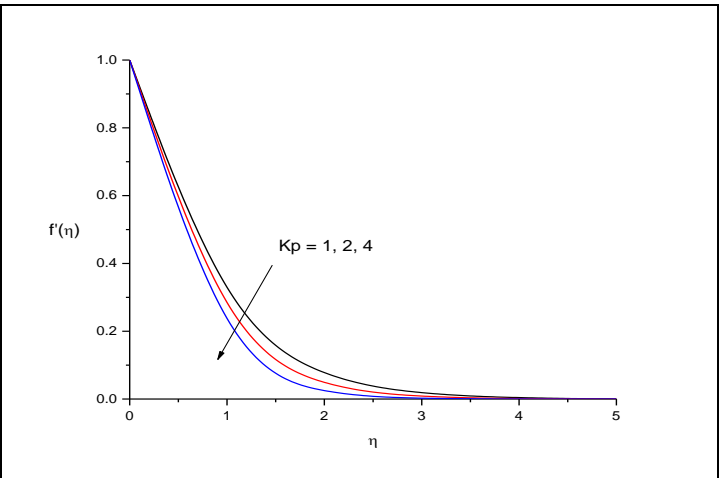


Figure 3 Influence of permeability parameter ( $k_p$ ) on  $f'(\eta)$

Figure 3 depicts the variation in velocity with increasing permeability parameter values. The porous medium permeability is related to significant drag and frictional forces. The rise in permeability number causes a rise friction hence reduction in the fluid’s motion.

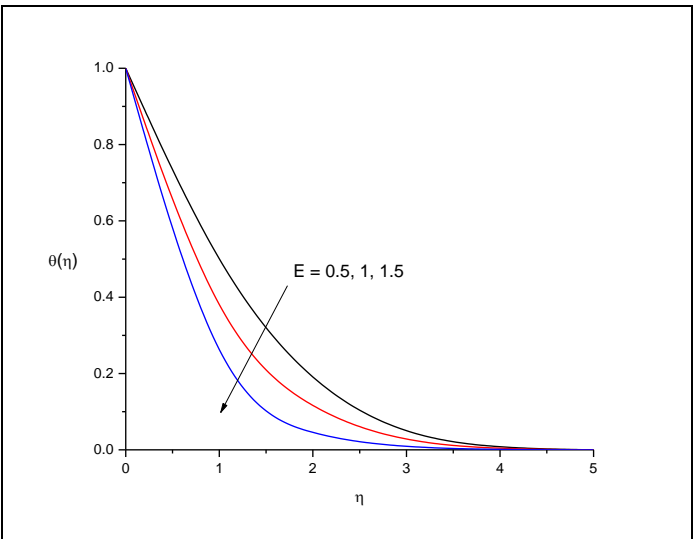


Figure 4 Influence of Eckert number (E) on  $\Theta(\eta)$

Figure 4 shows the decreasing trend of the temperature field as the Eckert number becomes more dominant. As the Eckert number increases, the fluid’s kinetic energy surpasses its thermal energy, leading to greater energy dissipation and less time for heat to accumulate in fluid. This results into a reduction in temperature for porous media flows.

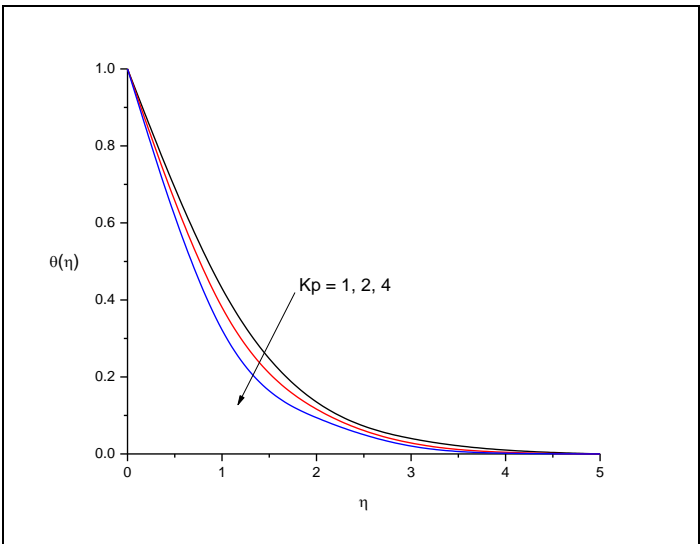


Figure 5 Influence of permeability parameter ( $k_p$ ) on  $\Theta(\eta)$

Figure 5 demonstrates the permeability parameter effect on temperature profile. In Darcy-Forchheimer flow, as permeability parameter increases, the velocity of fluid also rises, *Nanotechnology Perceptions* Vol. 20 No. S10 (2024)



reducing the time available for heat transfer and retention. This leads to less heat absorption by the fluid. Additionally, the larger flow velocity increases heat advection, causing heat to be removed more quickly and resulting in a lower temperature.

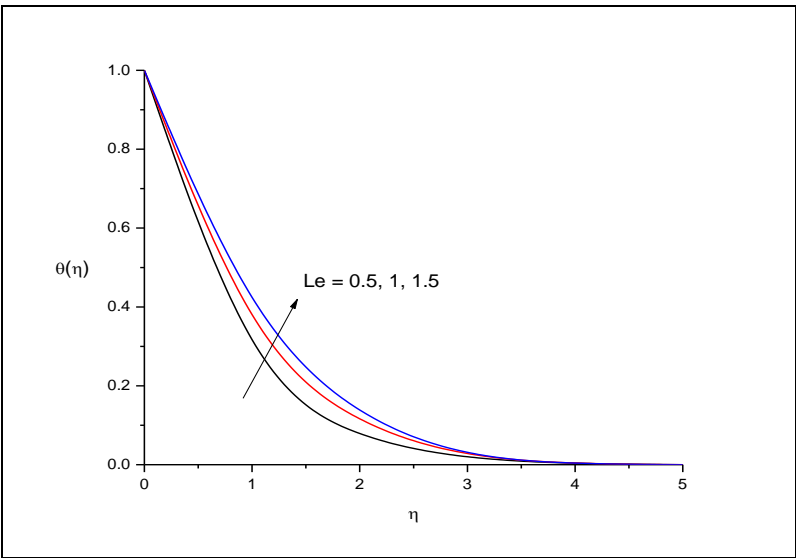


Figure 6 Influence of Lewis number (Le) on  $\theta(\eta)$

Figure 6 clearly shows that as the Lewis number increases, the temperature also rises due to two main factors. First, a rise in kinematic viscosity leads to greater viscous dissipation, which enhances friction between fluid layers and generates more heat. Second, a higher Lewis number indicates slower heat and mass diffusion, as it corresponds to weaker Brownian motion diffusivity. The fluid temperature rises due to slower thermal diffusion and increased viscous heating.

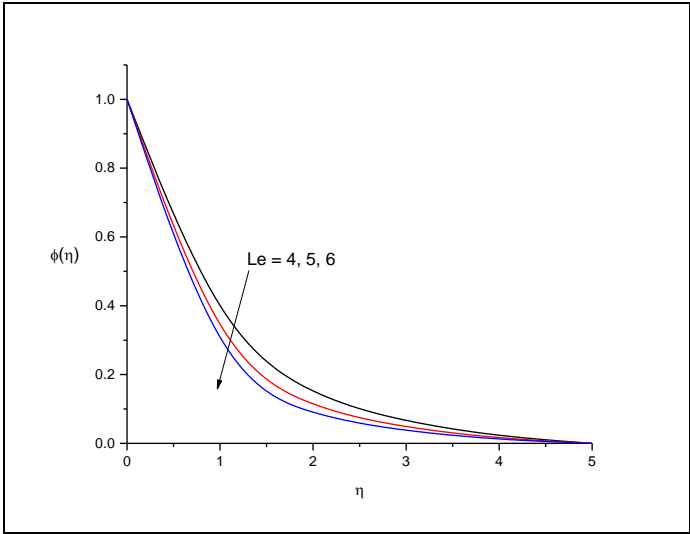


Figure 7 Influence of Lewis number (Le) on  $\phi(\eta)$

Figure 7 highlights Lewis number effect on the concentration distribution. As expected, the concentration field diminishes with rising dominant values of the Lewis number. This is due to a reduction in mass diffusivity, which leads to a lower concentration profile. A higher Lewis number results in a slower mass transfer rate and the thickness of the solutal boundary layer becomes more prominent.

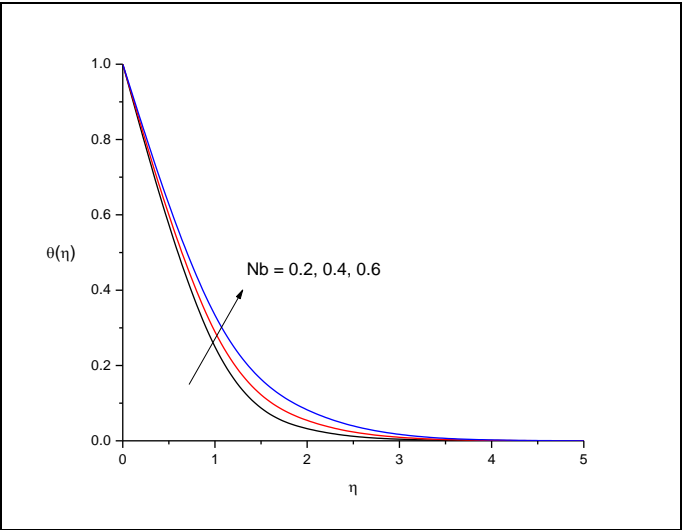


Figure 8 Influence of Brownian motion parameter (Nb) on  $\Theta(\eta)$

Figure 8 indicates Brownian motion parameter effect on the temperature field. A rise in the Brownian motion parameter leads to a higher fluid temperature, as more collisions between fluid particles generate additional heat. However, this also contributes to creation of a thick thermal boundary layer.

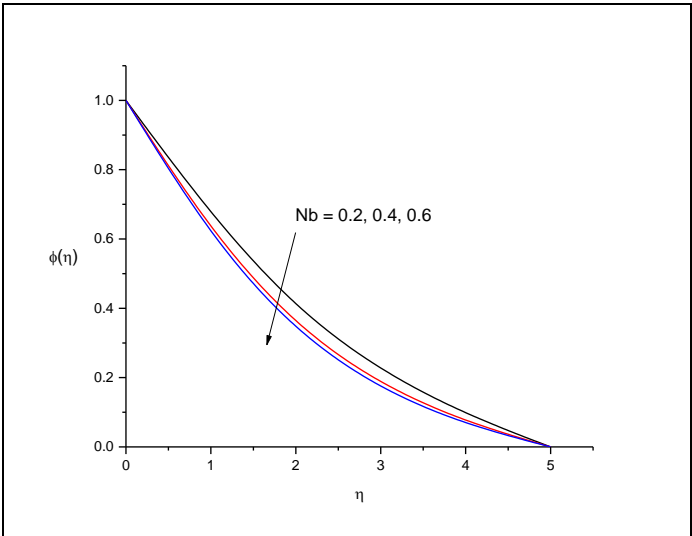


Figure 9 Influence of Brownian motion parameter (Nb) on  $\phi(\eta)$

Figure 9 discloses the impact of the Brownian motion parameter on the concentration profile. As the Brownian motion parameter increase, the fluid concentration decreases. In fact, higher values of Brownian motion parameter caused more frequent collisions between fluid particles, which reduces the mass transfer rate and consequently, lowers the concentration field. Additionally, as the Brownian motion parameter rises, the boundary layer corresponding to the concentration field becomes thinner.

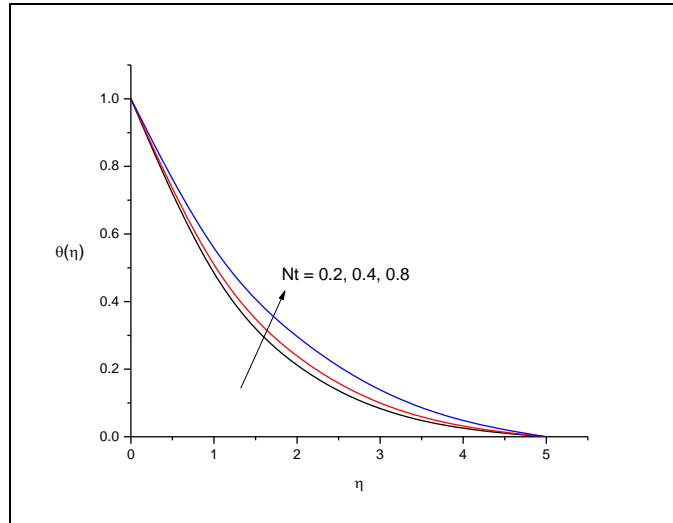


Figure 10 Influence of thermophoresis parameter ( $Nt$ ) on  $\Theta(\eta)$

Figure 10 discloses the variation in the temperature field associated with the thermophoresis parameter. As the thermophoretic force between the fluid particles intensifies, more heat is transferred to the fluid. Consequently, both the distribution of temperature and the thickness of corresponding boundary layer increase.

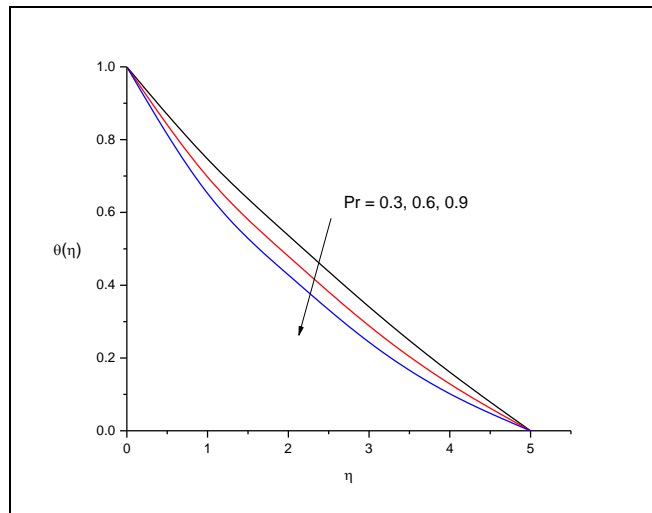


Figure 11 Influence of Prandtl number ( $Pr$ ) on  $\Theta(\eta)$

Figure 11 shows the impact of Prandtl number variation on temperature distribution. It is evident that an increase in Prandtl number leads to a reduction in both the temperature field and the thickness of thermal boundary layer. As the Prandtl number rises, thermal diffusivity decreases, resulting in a slower heat transfer rate from the sheet to fluid and a subsequent decrease in the fluid temperature.

#### 4. CONCLUSIONS

The Darcy-Forchheimer relation has been employed to quantitatively analyze the steady, incompressible nanofluid flow over a linearly stretched sheet considering the effects of momentum, temperature and concentration. To obtain the numerical solutions, the Spectral Collocation Method and Shooting method are utilized. A comprehensive discussion is provided regarding the effects of various fluid properties such as the permeability parameter, Forchheimer number, Eckert number, Lewis number, Prandtl number, Thermophoresis and Brownian motion parameter on the flow profiles. Finally, a graphical representation of the results is presented. The following key points are highlighted:

- The velocity field decreases sharply as the Forchheimer number increases.
- It is noticed that the permeability parameter diminishes the flow momentum and the associated boundary layer.
- Thermophoresis, Brownian motion parameter and Lewis number all lead to an increase in temperature while Prandtl number, Eckert number and permeability parameter contribute to a decrease in temperature.
- As the Lewis number and Brownian motion parameter increases the concentration distribution follows a similar declining trend.

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