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Impacts of Thermophoresis and Brownian Motion on MHD Nanofluid Flow over an Inclined Non-Linear Stretching Surface with Chemical Reaction

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This research paper investigates the combined effects of thermophoresis and Brownian motion on the flow characteristics of non-Darcy magnetohydrodynamic nanofluids over an inclined, non-linear stretching surface in the presence of a chemical reaction. Utilizing mathematical modeling, we derive governing equations that capture the dynamics of the nanofluid under varying thermal and concentration gradients. The research employs similarity variables to transform the partial differential equations into a set of dimensionless ordinary differential equations. The study employs Keller-box technique to solve these equations, revealing significant insights into how thermophoretic forces and Brownian motion influence the flow behavior and heat transfer rates of nanofluids. The results illustrate how these factors affect velocity profiles, temperature distributions, and concentration levels within the fluid, highlighting their critical roles in enhancing or impeding flow characteristics. The findings demonstrate that a hike in the thermophoresis factor causes an enhancement in both the concentration and temperature profiles. Furthermore, increasing Brownian parameter boosts the nanofluid temperature and concurrently lowering its concentration.

Keywords: Magnetohydrodynamics, Thermophoresis, Brownian Motion, Nanofluid.

1. Introduction

The study of nanofluids has garnered significant attention in recent years due to their enhanced thermal properties and wide-ranging applications in engineering and technology. Thermophoresis and Brownian motion are critical mechanisms that influence the behavior of nanoparticles suspended in a fluid, particularly under magnetic fields as described by magnetohydrodynamics (MHD). The interaction between these phenomena can significantly alter flow characteristics, heat transfer rates, and mass transfer processes in non-Darcy flows, which deviate from traditional Darcy's law. Understanding these interactions is essential for optimizing the performance of nanofluids in various applications, including cooling systems, heat exchangers, and advanced manufacturing processes. Researchers are increasingly focused on developing predictive models that can accurately simulate the behavior of nanofluids, taking into account factors such as particle size, shape, and concentration, which play crucial roles in determining their overall performance. Advancements in computational techniques and experimental methodologies are paving the way for deeper insights into these complex systems, enabling researchers to fine-tune nanofluid properties for enhanced efficiency in thermal management applications.

Nanofluids have demonstrated significantly enhanced thermophysical properties, most notably a superior thermal conductivity compared to their base fluids, even when the nanoparticles are present at very low volume fractions, typically ranging from less than 0.1% to around 4%. Several key mechanisms contribute to this remarkable enhancement in thermal conductivity. The unique thermophysical properties of nanofluids have spurred extensive research and development into their applications across a remarkably diverse range of industrial, technological, and scientific sectors. These applications include the design of more efficient heat exchangers for various thermal systems, advanced cooling systems for electronic devices, improved refrigeration systems, various applications in non-conventional energy resources, tribological applications for reducing friction and wear in machinery, metallurgical processes for enhancing material properties during casting and refining , diverse chemical engineering applications involving heat and mass transfer , and an ever-expanding array of medical applications, such as targeted drug delivery systems, advanced medical imaging techniques, and innovative cancer therapies utilizing magnetic nanoparticles for hyperthermia. [1] analysed the non-linear convection dynamics within a thin nanofluid film flowing over an inclined, stretching surface. Employing the Homotopy Analysis Method, the researchers specifically examined how thermophoresis, Brownian motion, and magnetic factors influence the resulting velocity, temperature, and concentration distributions. [2] studied nanofluid flow over stretched surfaces, including nonlinear convection and aggregation effects, but excluded inclined non-linear surfaces and focused on linear/exponential heat sources. [3] studied the flow of Casson nanofluid over a nonlinearly slanted, extending sheet, including the influences of chemical reaction and heat generation/absorption. [4] numerically analysed nanofluid flow over general stretched/ shrinking surfaces, considering partial slips and temperature jumps affecting heat/mass transfer, but excluded inclined non-linear stretching surfaces.

MHD is the study of the intricate interplay between magnetic fields and electrically conducting fluids. This interdisciplinary field elegantly merges the fundamental principles of fluid dynamics with the well-established laws of electromagnetism to analyse the behaviour of conductive fluids, such as plasmas, liquid metals, strong electrolyte solutions, salt water, and nanofluids containing conductive nanoparticles, when subjected to magnetic fields. The application of MHD in microfluidic devices offers elegant ways to manipulate and control fluid flow at small scales without the need for mechanical components. MHD is also being actively explored and utilized in advanced cooling systems for various electronic and industrial applications, where the ability to control fluid flow with magnetic fields can lead to more efficient heat removal. [5] examined the hydromagnetic flow of a nanofluid over a non-linear stretching sheet, focusing on magnetic fields, velocity slip, and thermal radiation within a porous medium, but did not consider inclined surfaces. [6] explored magnetohydrodynamic nanofluid flow and heat transfer over a nonlinear stretching/shrinking sheet with convective boundary conditions. The research considered buoyancy effects and found two solutions for specific shrinking conditions. [7] focused on MHD Carreau nanofluid flow over a radially stretching surface, analysing nonlinear thermal radiation, heat generation, and chemical reactions using Buongiorno's model for thermophoresis and Brownian motion. [8] found that the Nusselt number decreases as the Biot number increases in the nonlinear thermal radiation flow of a micropolar nanofluid past a nonlinear vertically stretching surface.

The functionality of numerous natural and human-made systems relies heavily on porous materials, which have a substantial influence on how fluids move. Considering non-Darcy flow effects is crucial for accurate modelling and analysis in a wide range of applications involving fluid flow through porous media, particularly when high flow rates are involved or when the porous structure is complex. [9] analysed how the geometry and inclination of a wavy surface affect temperature and concentration in 2D mixed convection of a non-Darcy nanofluid. [10] explored dual solutions for 3D nanofluid flow over a permeable, non-linearly shrinking surface, focusing on velocity slips and heat transfer while excluding non-Darcy flow and inclined surfaces. [11] studied nonlinear nanofluid flow over a porous stretching sheet with magnetic forces and Arrhenius kinetics.

Boundary layer flow over stretching surfaces is a fundamental concept in fluid dynamics that describes the viscous flow of a fluid induced by the continuous stretching of a solid surface. Beyond the initial focus on linear stretching, significant research efforts have been directed towards investigating fluid flow induced by non-linear stretching surfaces. In non-linear stretching, the velocity of the stretching surface is not directly proportional to the distance from the origin but rather follows a non-linear relationship. [12] found that the heat transfer rate in the three-dimensional nonlinear convective flow of viscoelastic nanofluid, caused by stretching an impermeable surface, increases with Prandtl number and radiation parameter. A mathematical model was developed by [13], for stagnation point flow of a nano-biofilm containing spherical nanoparticles and bio-convecting gyrotactic micro-organisms towards a stretching/shrinking sheet. [14] studied 3D nanofluid flow and heat/mass transfer on a nonlinearly stretching sheet using a novel operational-matrix method, revealing velocity and thermal insights. [15] found that the sensitivity of surface friction coefficient and local Nusselt number to the heat transfer Biot number was greater than to nanoparticle volume fraction and magnetic field parameters in their investigation of MHD nanofluid flow with non-uniform heat flux across an elongating surface.

Thermophoresis is a phenomenon observed in nanofluids where nanoparticles, when subjected to a temperature gradient, tend to migrate from regions of higher temperature towards regions of lower temperature. Brownian motion, on the other hand, refers to the continuous, random, and unpredictable movement of nanoparticles that are suspended within a fluid. [16] explored how Brownian motion and thermophoresis affect the flow of an electrically conducting Prandtl-Eyring nanofluid over a stretched surface. [17] used the Keller box method to analyze how Brownian motion and thermophoretic force affect the MHD flow of a nanofluid over a non-linear stretching surface. [18] found that increasing the stretching parameter reduces fluid velocity and thickens the thermal boundary layer in a non-Newtonian Williamson fluid on a stretching/shrinking surface, considering thermophoresis and Brownian motion. [19] examined the similarity solution of hydromagnetic nanofluid flow across a slendering stretching sheet, including thermophoresis and Brownian effects.

The incorporation of chemical reactions within the boundary layer of nanofluid flow over stretching surfaces has garnered significant attention in the research community. These studies often consider various types of chemical reactions, including first-order chemical reactions, where the reaction rate is directly proportional to the concentration of the reacting species, as well as more complex homogeneous and heterogeneous reactions that occur within the fluid or at the stretching surface. [20] examined steady MHD incompressible hybrid nanofluid flow and mass transfer over a porous stretching surface with quadratic velocity, including mass transpiration and chemical reaction. MHD mixed convective flow of higher-order reacting fluids with activation energy over a stretching surface with chemical reactions were analyzed in [21]. [22] studied hybrid nanofluid flow over a slender stretching surface, considering chemical reactions, Hall current, and variable magnetic fields, with a focus on how nanoparticle volume fraction and sheet wall thickness affect axial velocity. [23] studied heat transfer in nanofluid flow over a stretching surface, considering chemical reactions and heat generation.

The Keller box method is a numerical technique widely applied to analyze nanofluid flow over stretching surfaces, particularly in complex geometries and under various physical conditions. This method effectively transforms partial differential equations (PDEs) into a system of algebraic equations, facilitating the study of fluid dynamics and heat transfer phenomena. [24] utilized the Keller box method to solve the flow of micropolar fluids over exponentially curved surfaces, revealing significant effects of parameters like magnetic field and curvature on fluid velocity and heat transfer rates. In the context of Casson nanofluids, [25] employed the Keller box method to analyze flow over inclined surfaces, incorporating Soret and Dufour effects.

A comprehensive analysis of the existing literature reveals several key research gaps that motivate the present study. While significant research has been conducted on nanofluid flow, MHD effects, non-Darcy flow in porous media, flow over stretching surfaces, and the influence of thermophoresis, Brownian motion, and chemical reactions individually or in certain combinations, there is a noticeable lack of comprehensive studies that simultaneously consider the combined impacts of all these phenomena. The proposed research aims to address this gap by investigating the synergistic and counteracting effects of these parameters on the fluid flow and transport phenomena. The findings of such a study would contribute significantly to a more fundamental understanding of complex nanofluid behaviour under conditions relevant to a wide range of advanced engineering applications, potentially leading to more efficient designs and improved performance in various technological processes.

2. Mathematical Formulation

This study investigates the interplay of thermal slip, velocity slip, and MHD on the structural characteristics of a boundary layer in a steady, incompressible 2D nanofluid flow through a homogeneous and isotropic porous medium. The analysis considers low Reynolds number laminar flow over an inclined stretched surface with non-linear velocity ($u_w = ax^n$) and constant temperature (T_w) and nanoparticle concentration (C_w). The fluid far from the surface maintains a constant temperature (T_∞) and concentration (C_∞). The model incorporates spatially varying magnetic field effects, velocity slip and thermal slip. Figure 1 illustrates the problem's physical setup. The mathematical formulation and boundary conditions, derived from prior work ([17], [26], [27]), involve following four equations governing the nanofluid flow.

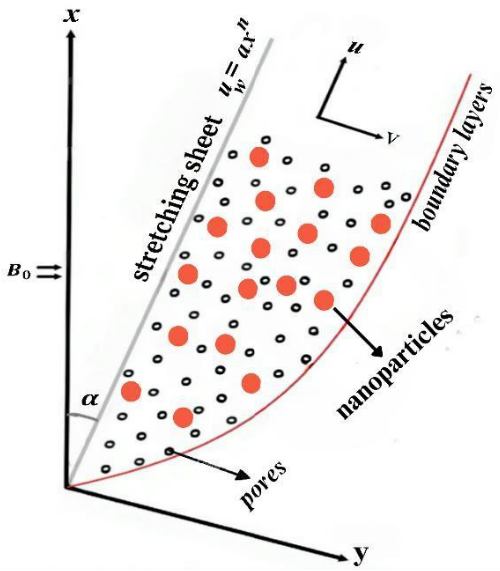

Figure 1 Graphical illustration of the present research problem

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2} + [g\beta_T(T - T_\infty) + g\beta_C(C - C_\infty)]Cos(\alpha) - \frac{\sigma(B(x))^2}{\rho_{nf}}u - \frac{\mu_{nf}}{\rho_{nf}}\frac{1}{K_d}u \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\mu_{nf}}{(\rho C_p)_{nf}}\left(\frac{\partial u}{\partial y}\right)^2 + \alpha_{nf}\frac{\partial^2 T}{\partial y^2} + \tau\left[D_m\frac{\partial T}{\partial y}\frac{\partial C}{\partial y} + \frac{D_r}{D_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right] \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_m\frac{\partial^2 C}{\partial y^2} + \frac{D_r}{T_\infty}\frac{\partial^2 T}{\partial y^2} - Kc(C - C_\infty) \tag{4}$$

The analysis considers a spatially varying magnetic field, expressed as $B(x) = B_0 x^{\left(\frac{n-1}{2}\right)}$. The thermal diffusivity of the nanofluid is $\alpha_{nf} = \frac{\kappa_{nf}}{(\rho C_p)_{nf}}$, the effective viscosity of the nanofluid is determined by $\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}$, while the effective heat capacity ratio is defined as $\tau = \frac{(\rho C_p)_s}{(\rho C_p)_f}$. The concentration and thermal expansion coefficients are denoted by β_C and β_T, respectively, and α represents the angle of inclination.

$$u = u_w + N v_f \frac{\partial u}{\partial y} = ax^n + N_0 x^{-\left(\frac{n+1}{2}\right)} v_f \frac{\partial u}{\partial y}, C = C_w$$

$$v = 0 \text{ and } T = T_w + D\frac{\partial T}{\partial y} = T_\infty + ax^{2n} + D_0 x^{-\left(\frac{n+1}{2}\right)} \; at\; y = 0$$

$$and\; v \to 0, u \to 0, T \to T_\infty, C \to C_\infty \; if\; y \to \infty \tag{5}$$

Equation (5) details the boundary conditions for velocity and thermal slips near the stretching sheet. The velocity slip (N) is $N_0 x^{-(\frac{n+1}{2})}$, and the thermal slip (D) is $D_0 x^{-(\frac{n+1}{2})}$, where N_0 and D_0 are their initial values and n is the stretching factor. The sheet temperature (T_w) is $T_\infty + ax^{2n}$, and the stretching velocity (u_w) is ax^n, with 'a' being the stretching rate.

Similarity transformations are defined by:

$$v = -x^{-(\frac{n+1}{2})}[\frac{a(n+1)v_f}{2}]^{\frac{1}{2}}\,[f(\eta) + \eta\frac{n-1}{n+1}f'(\eta)]\,,\theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty}$$

$$\eta = yx^{-(\frac{n+1}{2})}(\frac{a(n+1)}{2v_f})^{\frac{1}{2}}, u = ax^n f'(\eta)\ and\ \xi(\eta) = \frac{C-C_\infty}{C_w-C_\infty} \tag{6}$$

The stream function $\psi(x,y)$, adheres to equation (1), represented as:

$$u = \frac{\partial\psi}{\partial y}\ \&\ v = -\frac{\partial\psi}{\partial x} \tag{7}$$

The following non-dimensional ODEs are obtained by transforming equations $(2-4)$ via equations $(6-7)$:

$$f'''(\eta) + f''(\eta)f(\eta) - [Gr_x\theta(\eta) + Gr_x\xi(\eta)]Cos(\alpha) - \frac{2n}{n+1}(f'(\eta))^2 - \left(\frac{1}{M} + K\right)f'(\eta) = 0 \tag{8}$$

$$\theta''(\eta) + Pr\left[[\xi'(\eta)N_b + f(\eta)]\theta'(\eta) - \frac{4n}{n+1}\theta(\eta)f'(\eta) + (\theta'(\eta))^2N_t + (f''(\eta))^2Ec\right] = 0 \tag{9}$$

$$\xi''(\eta) + \frac{N_t}{N_b}\theta''(\eta) - \xi'(\eta)f(\eta)Le - Kr\xi(\eta) = 0 \tag{10}$$

Applying equation (6), the boundary restrictions in equation (5) are transformed to:

$$f'(\eta) = 1 + \lambda f''(0),\ \theta(\eta) = 1 + \beta\theta'(0)\,, \xi(\eta) = 1, f(\eta) = 0\ \ for\ \ \eta = 0$$

$$and\ \xi(\eta) \to 0, f'(\eta) \to 0, \theta(\eta) \to 0\ when\ \eta \to \infty \tag{11}$$

Parameters are set to:

$$N_b = \frac{(\rho c)_s D_m(C_w-C_\infty)}{v_f(\rho c)_f}, Kr = \frac{Kc}{D_m},\ Gc_x = \frac{g\beta_C(C_w-C_\infty)}{a^2x}, N_t = \frac{D_r(\rho c)_s(T_w-T_\infty)}{v_fT_\infty(\rho c)_f},$$

$$\beta = D_0\,(\frac{a(n+1)}{2v_f})^{\frac{1}{2}}, K = \frac{2v_f}{a(n+1)K_d}, Pr = \frac{v_f}{\alpha_f}, \lambda = N_0\,(\frac{av_f(n+1)}{2})^{\frac{1}{2}},$$

$$M = \frac{a(n+1)\rho_f}{2\sigma B_0^2}, Ec = \frac{x^{2n}a^2}{(T_w-T_\infty)(c_p)_f}, Gr_x = \frac{g\beta_T(T_w-T_\infty)}{a^2x}, Le = \frac{\alpha_f}{D_m} \tag{12}$$

The key physical measures in this analysis are the mass transfer rate ($-\xi'(0)$), skin friction coefficient ($f''(0)$), and heat transmission rate ($-\theta'(0)$). These are mathematically defined as:

$$-\xi'(0) = Sh(\frac{2}{n+1}Re_x)^{1/2}, f''(0) = C_f\,(\frac{2}{n+1}Re_x)^{1/2}$$

$$and\ \ -\theta'(0) = Nu\frac{\kappa_f}{\kappa_{nf}}\,(\frac{n+1}{2}Re_x)^{-1/2} \tag{13}$$

Sherwood number $Sh = \frac{-xq_m}{D_m(C_\infty-C_w)}$ quantifies convective mass transfer. Local Reynolds number $Re_x = \frac{xu_w}{f}l$ characterizes the flow regime. Skin friction $C_f = \frac{\tau_w}{\rho_f u_w^2}$ represents surface shear stress. Nusselt number $Nu = \frac{-xq_w}{\kappa_f(T_\infty-T_w)}$ signifies convective heat transfer.

3. Results and Discussion

This study numerically investigates the flow of a magnetohydrodynamic nanofluid over an inclined, non-linearly stretching sheet within a porous medium, considering velocity and thermal slip. It analyses how parameters like Chemical reaction factor (Kr), Brownian motion factor (N_b), velocity slip (λ), magnetic factor (M), stretching factor (n), thermophoresis factor (N_t), permeability factor (K), inclination angle (α) and thermal slip (β) affect the fluid's concentration, velocity, and temperature profiles, as well as boundary layer characteristics. This research aims to provide insights into complex fluid dynamics relevant to industrial applications. The Keller box method was used for computations, and results are presented graphically. Default parameter values were: $\alpha = 60^0$, $M = 0.4$, $Le = 5$, $Pr = 6.8$, $N_t = K = \lambda = Ec = N_b = Kr = 0.1$, $n = 2$ and $\beta = 0.25$, unless stated otherwise.

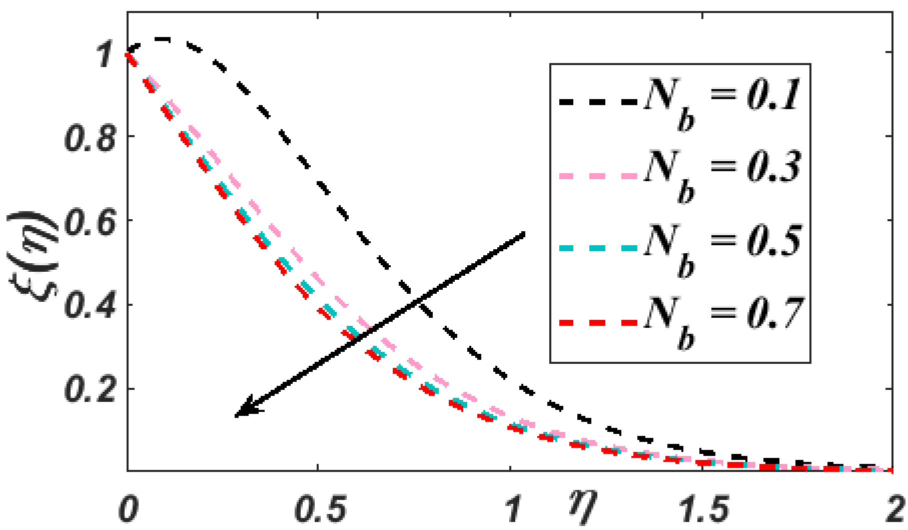

Figure 2 Impact of Brownian motion parameter on Concentration

The Brownian motion factor N_b, which reflects the erratic mobility of nanoparticles across the base fluid, considerably impact on the temperature and concentration trends. Brownian motion causes nanoparticles to migrate from high to low concentration regions. Figures 2 and 3 visually demonstrate these effects, indicating that increasing N_b boosts the nanofluid temperature and concurrently lowering its concentration. This gain in temperature arises from the increased diffusion of nanoparticles within the fluid.

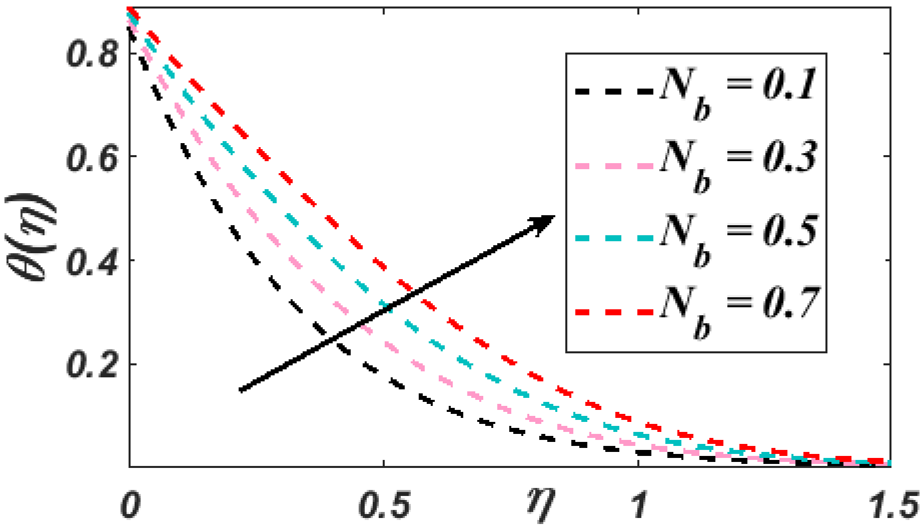

Figure 3 Impact of Brownian motion parameter on Temperature

The thermophoresis factor N_t, which accounts for the diffusion of nanoparticles caused by temperature swings, has a substantial effect on the patterns observed in temperature and concentration. A rise in the thermophoresis factor N_t enhances both the temperature and concentration profiles, clearly illustrated in Figures 4 and 5. This phenomenon is attributed to the thermophoretic force, which drives hotter nanoparticles towards the cooler regions near the surface.

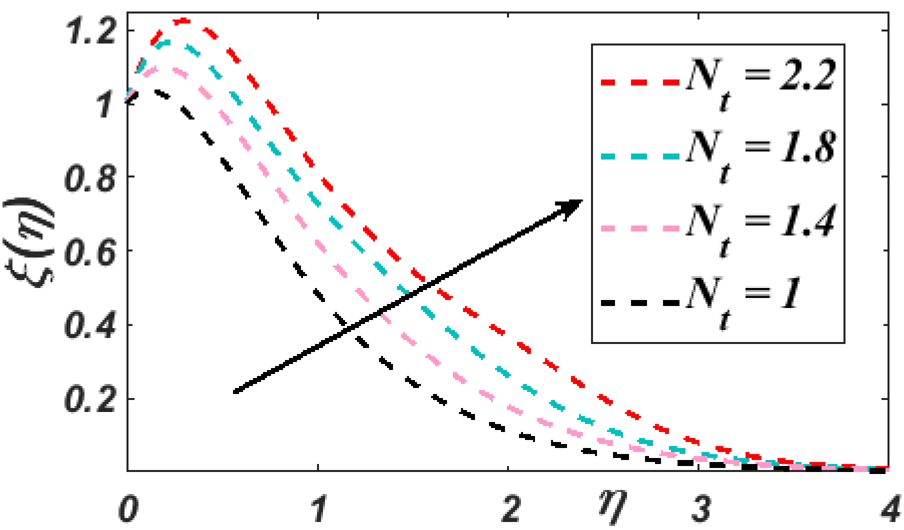

Figure 4 Impact of Thermophoresis parameter on Concentration

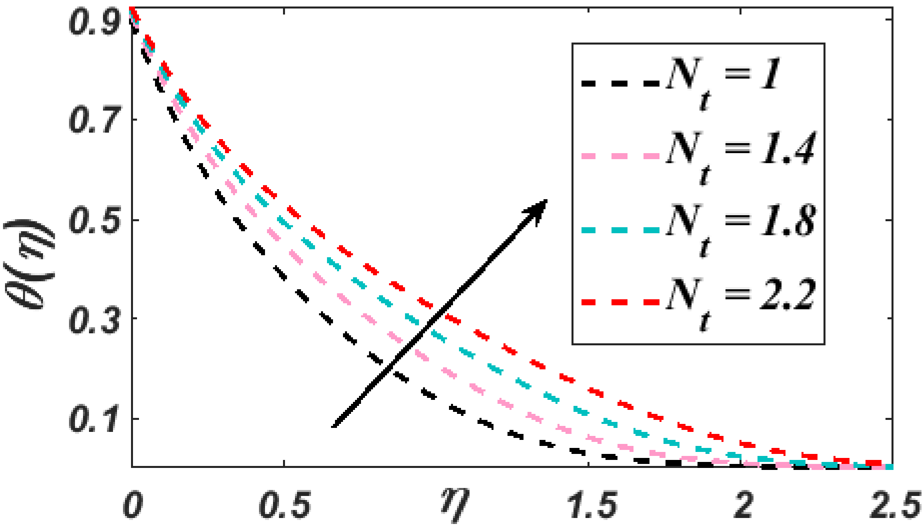

Figure 5 Impact of Thermophoresis parameter on Temperature

The numerical results detailed in Table 1 summarizes the computational results for $-\xi'(0)$ and $-\theta'(0)$ across numerous values of N_b, N_t and Kr. These computations were carried out with consistent settings for parameters $= 60^0$, $Ec = K = \beta = 0.1$, $\lambda = 0.2$, $Le = 5$, $Pr = 6.8$ and $M = n = 2$.

Table 1 Computational results for $-\xi'(0)$ and $-\theta'(0)$ across numerous values of N_b, N_t and Kr, when $\alpha = 60^0$, $Ec = K = \beta = 0.1$, $\lambda = 0.2$, $Le = 5$, $Pr = 6.8$ and $M = n = 2$.

N_b	N_t	$-\xi'(0)$		$-\theta'(0)$	
		$Kr = 0.1$	$Kr = 0.5$	$Kr = 0.1$	$Kr = 0.5$
0.1	0.1	1.141579	2.217297	1.597325	1.517753
0.2	0.1	1.988006	3.561319	1.523922	1.448107
0.3	0.1	3.448181	5.143527	1.453892	1.381456
0.1	0.2	0.326165	0.633551	1.544638	1.467691
0.2	0.2	0.568231	1.103679	1.493689	1.419283
0.3	0.2	0.989144	1.921231	1.444542	1.372581
0.1	0.3	0.321348	0.624156	1.473656	1.410245
0.2	0.3	0.559611	1.086936	1.425049	1.354059
0.3	0.3	0.974535	1.892846	1.378045	1.299404

4. Conclusions

The present study explores the impact of a nanofluid's magnetohydrodynamic boundary layer flow over a non-linearly inclined stretching surface embedded within a porous medium. This investigation considers the effects of both velocity and thermal slip at the boundary. To obtain numerical solutions, the Keller box method was employed. The detailed findings of this research highlight the intricate relationships between the considered variables and their crucial role in determining the flow behaviour. Furthermore, a thorough analysis of these variables reveals their individual and combined influence on the temperature, concentration, and velocity profiles. Key results emerging from this research are:

1. Increasing Brownian parameter boosts the nanofluid temperature and concurrently lowering its concentration.
2. A hike in the thermophoresis factor causes an enhancement in both the concentration and temperature profiles.
3. The influence of the thermophoresis and Brownian motion parameters on dimensionless heat and mass transfer rates was studied and analysed, considering various values for the chemical reaction parameter.

ABBREVIATIONS	
B_0	Uniform magnetic field strength
$B(x)$	Variable magnetic field strength
Kc	Rate of chemical reaction
a	Stretching rate
C	Nanofluid concentration
n	Non-linear stretching factor
Kr	Non-dimensional Chemical reaction factor
C_f	Skin-friction
N_b	Brownian motion parameter
C_p	Nanoparticle specific heat
C_∞	Concentration of the free stream
C_w	Concentration of the stretching sheet
D_r	Thermophoresis diffusion Coefficient
N_t	Diffusion coefficient of thermophoresis
D_m	Brownian diffusion Coefficient
$f(\eta)$	Dimensionless stream function
Ec	Eckert number
$-f''(0)$	Skin friction coefficient
D_0	Initial thermal slip
g	Acceleration due to gravity
$f'(\eta)$	Non dimensional velocity
M	Magnetic parameter
Gr_x	Local Grashof number
N_0	Initial velocity slip

Gc_x	Local modified Grashof number
Pr	Prandtl number
K_d	Dimensional permeability parameter
K	Non-dimensional permeability parameter
Nu	Nusselt number
Sh	Sherwood number
u	Velocity component parallel to the x direction
T	Dimensional temperature of the nanofluid
Re	Reynolds number
T_∞	Free stream temperature
x, y	Cartesian coordinate axis
T_w	Surface temperature
v	Velocity component parallel to the y direction
D	Thermal slip factor
u_w	Stretching sheet velocity
Greek Letters	
η	Non-dimensional similarity variable
κ_s	Nanoparticles thermal conductivity
β_T	Thermal expansion coefficient
β_C	Concentration expansion coefficient
α	Inclination parameter
$\xi(\eta)$	Non-dimensional concentration
κ_{nf}	Nanofluid thermal conductivity
κ_f	Base fluid thermal conductivity
$\theta(\eta)$	Non-dimensional temperature
σ	Electric conductivity
ϕ	Nanoparticles solid volume fraction
α_{nf}	Nano fluid thermal diffusivity
τ_w	Wall shear stress
ν_f	Base fluid kinematic viscosity
μ_{nf}	Nano fluid viscosity
μ_f	Base fluid viscosity
ρ_{nf}	Nano fluid density
ρ_f	Base fluid density
$(\rho C_p)_{nf}$	Nano fluid heat capacitance
ρ_s	Nanoparticles density
$(\rho C_p)_f$	Base fluid heat capacitance
$-\theta'(0)$	Local Nusselt number
$(\rho C_p)_s$	Nanoparticles heat capacitance
ψ	Stream function
$-\xi'(0)$	Local Sherwood number

Conflict of Interest: The authors declare no conflicts of interest.

Data Availability: Data will be made available on reasonable request.

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