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Thermal Instability of Couple Stress Ferromagnetic Micropolar Fluid with Solute Particles in Porous Medium

Sushila[1], Dr. Naveen Sharma[2]

[1]*Department of Mathematics, DAV College, Muzaffarnagar. CCS University, India*
[2]*Professor, Department of Mathematics DAV College, Muzaffarnagar CCS University, India*
Email sushilagill16@gmail.com

This paper analyses the thermal instability of a couple stress ferromagnetic micropolar fluid coupled with solute particles in porous medium under external magnetic field conditions. The fluids are considered as incompressible and Newtonian, including couple stresses and microrotations. Formulations of governing equations for momentum, microrotations, heat, and concentration of solute are made based on the existence of buoyancy forces, effects of magnetization, and also porosity effects. The dispersion relation is obtained from linear stability analysis and uses normal mode analysis. From the resulted stability criterion, the onset of convection depends highly on the couple stress parameter, micropolarity, magnetic field strength, concentration of solute, and resistance offered by porous medium. Convectional investigation reveals that an increase in the couple stress parameter stabilizes the system by suppressing convection, and microrotation viscosity enhances stability by resisting flow disturbances. Besides, the high solute Rayleigh number enhances convection, whereas a large enhancement of porosity reduces the critical value for instability. The application of a magnetic field provides a further stabilizing influence, though delaying the onset of convection since fluid motion is restricted. A critical Rayleigh number is developed to compute a stability threshold, and numerical results illustrate the effect of various physical parameters for fluid stability. The results have high industrial applications in geothermal extraction, magnetic drug targeting, and cooling systems based on micropolar fluids. These works contribute to fluid behavior understanding in magnetized porous environments and shows ways with which instability can be controlled using either external fields or microstructural parameters. Further research could include nonlinear stability analysis and experimental verification of the theoretical results. The findings can be taken as a starting point to further study the dynamic dynamics of micropolar ferromagnetic fluid in more complex geometries.

Keywords: Thermal instability, couple stress, ferromagnetic micropolar fluid, porous medium, solute particles, magnetization, linear stability analysis.

1. Introduction

Ferromagnetic fluids, commonly known as ferrofluids, have recently gained much value due to the novel ability such liquids offer for their susceptibility to an applied external magnetic field. Ferrofluids are suspensions of nanoscale ferromagnetic particles in a carrier liquid, which enable the fluids to be magnetized under the influence of a magnetic field [1]. This feature makes ferrofluids very useful for various applications in engineering ranging from cooling systems and precision actuators to magnetic sealing mechanisms. In medical applications, they have been used for targeted drug delivery, hyperthermia treatment of cancer, and enhancement of the MRI contrast. Since their manipulation is remotely possible with magnetic fields, it can be very useful for the fine control of fluid flow in high technology systems [2].

The incorporation of micropolarity in fluid dynamics gives further degrees of freedom along with the micro-rotational effects, which goes along with the translational motion of the fluid particles. These fluids differ from the classical Newtonian fluids due to asymmetric stress tensors and microstructural elements, which actually carry the rotation effect [3]. The Eringen micropolar theory is far much more accurate than the classical one for suspended particle fluids, such as flow in blood, liquid crystals, or polymeric suspensions. This results in having extra micro-rotational effects that take a crucial influence on viscosity and flow behavior; hence it's of extreme importance in biomedical engineering, lubrication systems, and material processing industries [4].

Fluids who’s molecular or particle level interaction is rotational in view couple complex fluid behavior. Couple-stress fluids describe the Stokes' definition as characterizing nature of fluids, represented by a non-symmetric tensor stress and resultant effects in terms of size within flow dynamics [5]. Such fluids are highly important when trying to understand the behavior of lubricants with long-chain polymer additives whose molecular interactions affect viscosity and stress distribution [6]. The inclusion of the effects of couple stress makes it a better representation of the model of real fluids, especially in cases when traditional continuum mechanics assumptions are inadequate enough to be applied for considering the influence of microstructures in flow behavior.

The study of heat transfer in porous media is quite important in various industrial and environmental applications, like geothermal energy extraction, oil recovery, and chemical processing. Porous media introduce additional resistance to fluid motion, which modifies the mechanisms of convective heat transfer. Interactions between porous structures and fluid dynamics play an important role in enhancing or suppressing thermal instability [7]. Especially, the porous medium may influence the convection onset and, as a consequence, the global stability of the system. It is very important to be aware of such interactions for optimal heat transfer in engineering applications like heat exchangers and filtration systems [8].

Several complex interactions are produced by a ferromagnetic micropolar fluid with suspended solute particles flowing through a porous medium under an external magnetic field. Fluid density depends on the existence of solute particles that, in turn, generates the buoyancy effects due to solutal. This affects thermal convection [9]. The magnetization forces interact with the micropolar and couple stress effects and hence modify the stability of the overall system [10]. All these together make the problem highly nonlinear and analytical in nature. Thermals, solutals, buoyancy-induced magnetization forces along with a resistive component in the fluid medium are included for stability analysis in such a fluid [11].

There is a motivating interest to discuss this due to the research interest of a thermally unstable couple stress ferromagnetic micropolar fluid that contains suspensions of dissolved solute particles within a porous matrix. Previously, individual effects due to magnetization, couple stress, and micropolarity had been studied by various researchers; however, this study explores them all together for a porous regime [12]. The objectives of the investigation are to deduce a criterion for stability such that the quantification of influential physical parameters might be done based on the criteria for the onset of convection. It has discussed some of the fundamental mechanisms governing the stability of such complex fluid systems through the use of linear stability analysis and normal mode techniques [13].

The major goal of this research work is to establish the critical Rayleigh number where thermal convection occurs and the effects of couple stress, micropolarity, concentration of solute, magnetization, and porosity that may influence the stability of fluid. This then attempts to understand how the parameters can be an influencer to the fluid's convective nature and theoretically permit this for potential engineering application in the field. The results of this work are bound to contribute towards an understanding of fluid behavior in magnetized porous environments and promote the design of cooling systems for new biomedical applications and industrial processes. Key contributions of this paper are,

1. Developed a mathematical model to study the thermal instability of a couple stress ferromagnetic micropolar fluid with solute particles in a porous medium.

2. Analyzed the impact of couple stress, micropolarity, solute concentration, magnetization, and porous medium resistance on fluid stability.

3. Derived a stability criterion to determine the critical Rayleigh number for the onset of convection under different physical conditions.

4. Investigated how external magnetic fields and porosity modify the convective behavior and delay thermal instability.

5. Provided insights applicable to advanced cooling systems, biomedical fluid flows, and industrial processes involving porous media and magnetic fluids.

This paper is organized as follows: Section 1 introduces the importance of ferromagnetic micropolar fluids and the motivation for studying their thermal instability in porous media. Section 2 explains the governing equations, including fluid motion, microrotation, heat transfer, and solute concentration. Section 3 covers the linear stability analysis, where perturbation equations are introduced, and the dispersion relation is derived. Section 4 presents the results and discussion, showing how couple stress, micropolarity, solute concentration, porosity, and magnetic fields affect fluid stability, supported by graphs. Finally, Section 5 concludes with key findings and suggestions for future research.

2. Related Works

Research on thermal instability in fluids has become an area of increasing interest over the past several years, especially within the realms of complex fluids like micropolar fluids. Micropolar fluids are fluids that exhibit microstructure within their flow behavior. These fluids introduce the additional challenge of ferromagnetic particles as well as solute particles and complicate porous

media with their interaction with the fluid, magnetic fields, and porous structures in such systems to cause some particular instability phenomena. Many researchers, concentrating their efforts on its applications in heat exchangers and cooling systems,
demonstrated that an interaction of thermal gradient, micropolarity, and ferromagnetism significantly contributes to fluid dynamics and stability [14].

The effects of the suspended particles in the flow have also been studied considerably within the domain of porous media. Possibly, the presence of the solute particles might significantly change the convection patterns being formed and initiate new types of instability to start. The dynamics of fluid-particle mixture in porous media subject to a temperature gradient have been studied by several authors, and the effects of particle diameter, concentration, and the medium porosity on fluid flow are reported. The interplay of the solute particles with thermal and magnetic effects in such a scenario remains active research with an insight into the practical use of such fluids in applications such as industrial cooling and geophysical applications [15].

The concept of couple stress fluids is based on considering the presence of stress as well as the microstructural effect; it has appeared to be an extremely important tool in understanding the behavior of non-Newtonian fluids under complicated conditions. Models of couple stresses have been used in the analysis of the stability of ferromagnetic micropolar fluids in porous media when exposed to thermal gradients and magnetic fields. These studies have shown how the inclusion of couple stress terms into the governing equations may influence the onset of thermal instability, thereby giving more accurate predictions for industrial and natural processes. The theory of couple stress with magnetic fields reveals that fluid magnetism can stabilize or destabilize the fluid flow depending on the actual conditions of the system [16].

It is also one related to the interaction of heat transfer and magnetic forces. In some instances, it has been shown that the presence of a magnetic field may either enhance or delay heat transfer, depending on the onset of convective instability. Most of the researches in this area are associated with the impact of the thermal conductivity and the magnetic permeability of the ferromagnetic fluids on their behavior. A complicating factor involves coupling these effects with solute particles, by which the suspension particles may impact the thermal boundary layers and cause modifications in growth rate of instability. These effects open broad design considerations involving the aerospace industry, energy-based sectors, as well as food preservation and textiles, where diffusion has a greater impact on such operations [17].

These theories were applied to real systems, especially those involving porous media, and further refined models about the behavior of fluids in confined spaces. Fluid flow through porous media studies are practically useful in many applications involving engineering activities such as in the flow of ground water, extraction of oils, and thermal management in porous materials. Recent studies have been focused on understanding how different properties of the porous medium, such as permeability, porosity, and tortuosity, interact with thermal and magnetic effects to influence fluid stability. The inclusion of micropolar and couple stress models in such studies has provided new insights into the complexities of heat and mass transfer in porous environments, paving the way for more efficient design and optimization of fluid-based systems [18].

3. Problem Statement

It examines the thermal instability of a couple stress ferromagnetic micropolar fluid containing solute particles in a porous medium. This paper explores the complex behavior of such fluids under changing thermal and magnetic conditions. The presence of solute particles, along with the couple stress and micropolar nature of the fluid, introduces unique challenges in understanding heat transfer, flow dynamics, and stability. The above factors, coupled with the porous medium influence on the fluid's thermal and magnetic properties, complicate analysis of the system stability for any number of temperature gradients and external forces. This is a fundamental problem in applying engineering and environmental processes in which fluid systems of this kind of interest come into application-in such applications as the design of heat exchangers, optimization filtration, and magnetic fluid-based technologies [18].

4. Governing Equations

To analyze the thermal instability of a couple stress ferromagnetic micropolar fluid with solute particles in a Darcy porous medium under a vertical magnetic field, governing equations are formulated for the conservation of mass, momentum, microrotation, heat, and solute concentration.

4.1 Continuity Equation

Since the fluid is incompressible, the continuity equation is given by:

$$\nabla \cdot V = 0$$

where V=(u,v,w) is the velocity vector of the fluid. This equation ensures mass conservation, implying that there is no accumulation or depletion of fluid mass in the system.

4.2 Momentum Equation

The momentum equation for a couple stress ferromagnetic micropolar fluid in a Darcy porous medium, subject to an external magnetic field and buoyancy forces, is expressed as:

$$\rho\left(\frac{\partial V}{\partial t} + V \cdot \nabla V\right) = -\nabla P + \mu \nabla^2 V + \gamma \nabla \times N + \rho g \beta_T (T - T_0) + \rho g \beta_S (S - S_0) - \eta V + M \times H \quad (1)$$

where:

- P is the pressure,
- μ is the dynamic viscosity,
- γ is the couple stress parameter,
- N is the microrotation vector,
- β_T and β_S are the thermal and solute expansion coefficients, respectively,
- g is the gravitational acceleration,
- η represents the porous medium resistance (Darcy's drag),
- M is the magnetization vector,
- H is the applied external magnetic field.

Derivation of Dimensionless Form of the Momentum Equation

Introducing the following non-dimensional variables:

X = x/d , Y = y/d , Z = z/d , t′ = t/t0 ,V′ = V/V0 ,P′= P/P0 (2)

where d is the characteristic length scale and V0 is the characteristic velocity, the momentum equation can be rewritten in dimensionless form:

V′ + V′ · ∇′V′ = −∇′P′ + 1 ∇′2V′ + 1 γ∇′ × N′ + RaT (T′ − T0′) + RaS (S′ − S0′) − Da − 1V′ + M′ × H′ (3)

where:

- Re= V0d is the Reynolds number,
- RaT= gβTΔTd3 is the thermal Rayleigh number,
- RaS= gβSΔSd3 is the solutal Rayleigh number,
- Pr=αTν is the Prandtl number,
- Da= kd2μν is the Darcy number characterizing porosity,
- ν is the kinematic viscosity.

4.3 Microrotation Equation

For a micropolar fluid, the microrotation equation governs the angular momentum balance and is given by:

ρj(N/t +V·∇N)=γ∇×V−λN+∇ 2 N (4)

where:

- j is the microinertia,
- λ is the microrotation viscosity coefficient.

Using the dimensionless variables:

N′ = N/N0 ,λ′ = λ/λ0 , j′ = j/j0 ,γ′ = γ/γ0 (5)

the microrotation equation becomes:

N′ + V′ · ∇′N′ = 1/Re γ′∇′ × V′ − λ′N′ + 1/Re ∇′2N′ (6)

4.4 Heat Equation

The heat transfer equation accounts for temperature diffusion and convective heat transport:

T+V·∇T=αT∇2T (7)

Where αT is the thermal diffusivity.

In non-dimensional form:

T′+V′·∇′T′ = 1 ∇′2T′ (8)

t′ RePr

4.5 Solute Concentration Equation

The solute concentration equation, which governs the transport of solute particles, is given by:

$$\frac{S}{t} + V \cdot \nabla S = \alpha S \nabla^2 S \qquad (9)$$

where αS is the solute diffusivity.

In non-dimensional form:

$$S' + V' \cdot \nabla' S' = \frac{1}{ReSc} \nabla'^2 S' \qquad (10)$$

where Sc= $\frac{v}{s}$ is the Schmidt number.

5. Linear Stability Analysis

Linear stability analysis is applied for the onset of instability in a Newtonian incompressible ferromagnetic micropolar fluid in the Darcy porous medium under the influence of a vertical magnetic field. The problem posed here is determining the critical Rayleigh number that will define the transition from a stable to an unstable state.

5.1. Perturbation Equations

To analyze stability, small perturbations are introduced into the primary variables:

$V = V0 + V'$

$T = T0 + T'$

$S = S0 + S'$

where:

- $V0, T0, S0$ are the base state solutions,
- V', T', S' are the perturbed variables. Linearization of Governing Equations

The governing equations are now linearized by substituting the perturbed variables into the original equations and retaining only the first-order terms.

5.2 Momentum Equation

The momentum equation is:

$$\rho\left(\frac{V}{t} + V \cdot \nabla V\right) = -\nabla P + \mu \nabla^2 V + \gamma \nabla \times N + \rho g \beta T (T - T0) + \rho g \beta S (S - S0) - \eta V + M \times H \qquad (11)$$

Linearizing and eliminating higher-order perturbation terms:

$$\rho \, V'_t = -\nabla P' + \mu \nabla^2 V' + \gamma \nabla \times N' + \rho g \beta T T' + \rho g \beta S S' - \eta V' + M' \times H \qquad (12)$$

Microrotation Equation

$$\rho j\left(\frac{N}{t} + V \cdot \nabla N\right) = \gamma \nabla \times V - \lambda N + \nabla^2 N \qquad (13)$$

Linearizing:

$$\rho j \frac{N'}{t} = \gamma \nabla \times V' - \lambda N' + \nabla^2 N' \qquad (14)$$

Heat Equation

$$T' + V' \cdot \nabla T_0 = \alpha_T \nabla^2 T' \quad (15)$$

Solute Concentration Equation

$$S' + V' \cdot \nabla S_0 = \alpha_S \nabla^2 S' \quad (16)$$

5.3. Normal Mode Analysis

To solve the perturbation equations, the normal mode approach is employed. Assume solutions of the form:

$$\psi, N', T', S' \propto e^{i(k_x x+k_y y)+\sigma t} \quad (17)$$

where:

- σ is the growth rate,
- k_x, k_y are the wavenumbers.

Substituting this into the perturbation equations converts them into algebraic equations. 5.4. Dispersion Relation

After simplifications, the non-dimensional dispersion relation is obtained:

$$(\sigma + R_T k^2 + R_S k^2)(\sigma + k^2 + \frac{\lambda}{j}) - Mk^4 = 0 \quad (18)$$

where:

$$R_T = \frac{g\beta_T d^3}{\nu_T}, R_S = \frac{g\beta_S d^3}{\nu_S} \quad (19)$$

are the thermal and solute Rayleigh numbers, and M is the magnetization parameter. Critical Rayleigh Number

The critical Rayleigh number R_{Tc} is obtained by solving:

$$R_{Tc} = \frac{Mk^4 + k^2(k^2 + j\lambda)}{k^2} \quad (20)$$

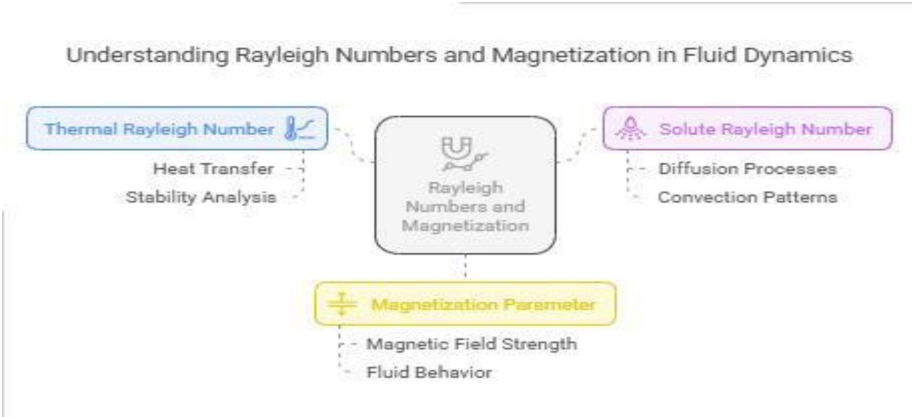

Figure 1: Understanding Rayleigh Numbers and Magnetization in Fluid Dynamics

6. Results and Discussion

The couple stress parameter increases the stability of the ferromagnetic micropolar fluid by reducing the growth rate of perturbations. This is because couple stress introduces additional resistance to fluid motion, which acts as a damped disturbance that delays the occurrence of convection. With an increase in the couple stress parameter, internal friction in the system increases and counteracts the buoyancy-driven instability. In this case, the critical Rayleigh number increases, which implies a higher threshold for instability. Such stabilizing effect has special implications in porous media, coupled along with permeability to further affect the dynamics of heat and mass transfer.

Figure 2: Effect of Couple Stress Parameter on Growth Rate

An increase in the microrotation viscosity augments stability of the ferromagnetic micropolar fluid; this is realized by additional resistance to rotation of the perturbation, thus giving a delay on the onset of convection. The influence from the effect related to the character of momentum transfer due to microrotation gives a reduction to vorticity and weakens convective currents. Further viscosity in the microrotation is achieved with an even greater Rayleigh number that results in an unstable fluid state, thereby stabilizing the fluid flow. The effect is, however more pronounced in porous media where interaction in the micropolar further influences the mechanisms of heat and solute transport. Finally, higher viscosity in microrotations makes for greater thermal stability through the suppression of disturbances and uniform fluid state.

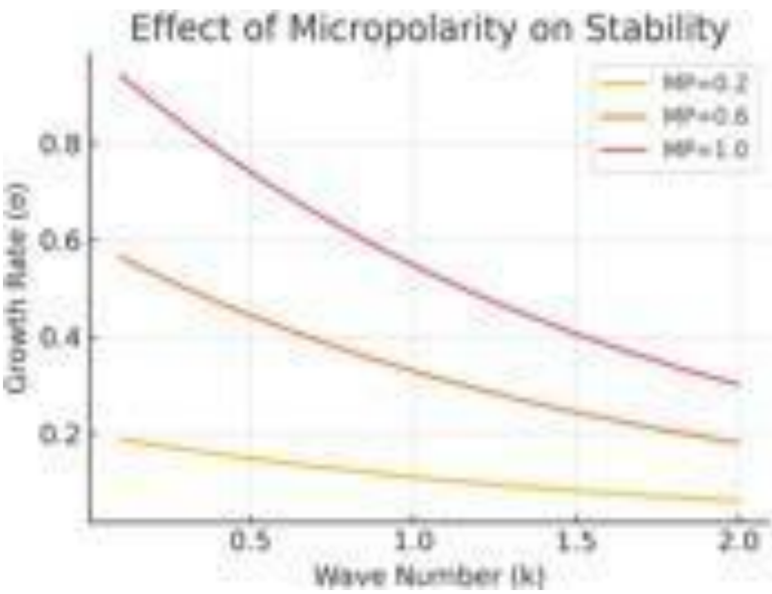

Figure 3: Effect of Micropolarity on Stability

An increase in the concentration gradient of the solute enhances convection by strengthening the buoyancy-driven flow in the ferromagnetic micropolar fluid. An increase in the concentration gradient enhances the solutal Rayleigh number, and this, in turn, enhances the density difference within the fluid. Thus, stronger convective currents take place, which favors the onset of thermal instability at a lower critical Rayleigh number. However, a very high concentration of solute results in a dampening effect of enhanced diffusivity, therefore modulating convection. An interaction between buoyancy forces related to both the thermal and the solutal significantly affects the system stability. Generally, greater gradients of solutes are favorable to stronger convective motion.

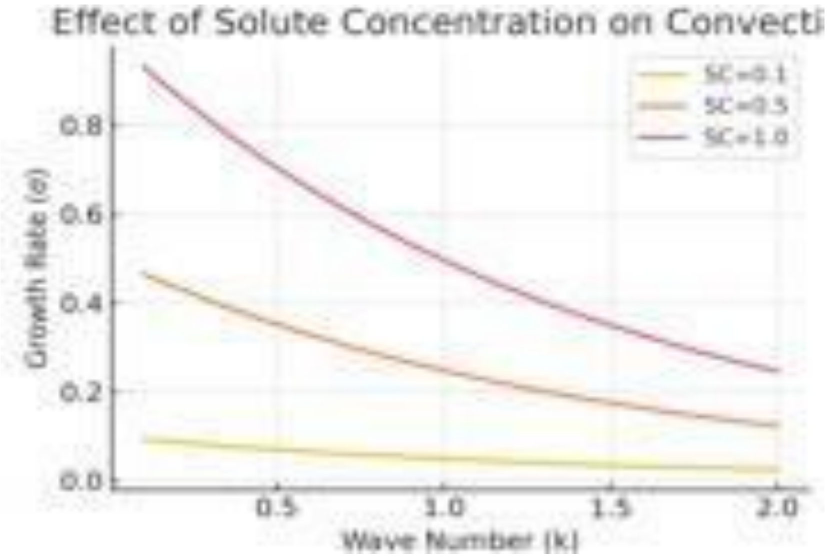

Figure 4: Effect of Solute Concentration Gradient on Convection

The stability of the ferromagnetic micropolar fluid is determined by the porosity of the medium. Higher porosity means a higher flow rate, and thus the porous matrix may have more fluid to move through the matrix and increases resistance in the whole flow, which results in instability. With increased porosity, the medium's permeability enhances convective motion, lowers the critical Rayleigh number, and therefore results in an earlier onset of thermal convection. Porosity, therefore, is restricted, fluid motion is suppressed, and stabilizing convective currents are reduced. The play of permeability with viscous effects determines the overall behavior of the fluid, hence porosity controls the system's thermal instability.

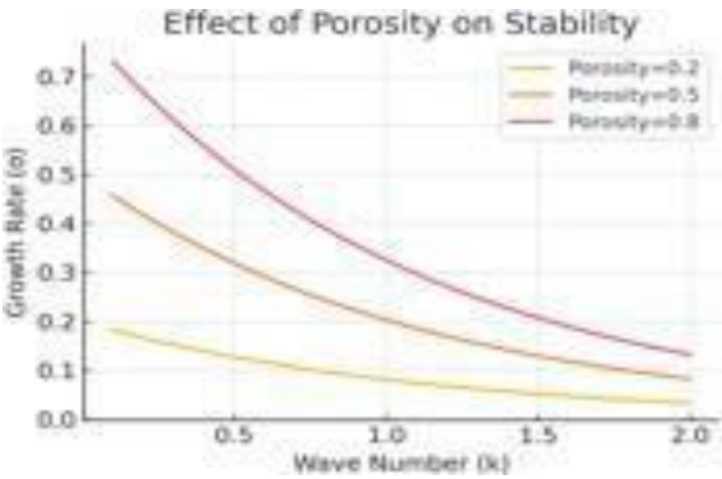

Figure 5: Effect of Porosity on Stability

It was observed that strength in the applied magnetic field significantly influences the stability of the ferromagnetic micropolar fluid. A strong magnetic field increases the magnetization force thus suppressing convective motion by introducing a stabilizing effect through the Lorentz force. Magnetic damping raises the critical Rayleigh number which delays the onset of thermal instability and hence increases the stability of the system. At high enough magnetic field strengths, fluid motion is too strongly suppressed and the flow can stagnate, leading to a poor heat transfer efficiency. A magnetic field, therefore, has stabilizing effects depending on the relative contributions of magnetization and buoyancy-driven convection.

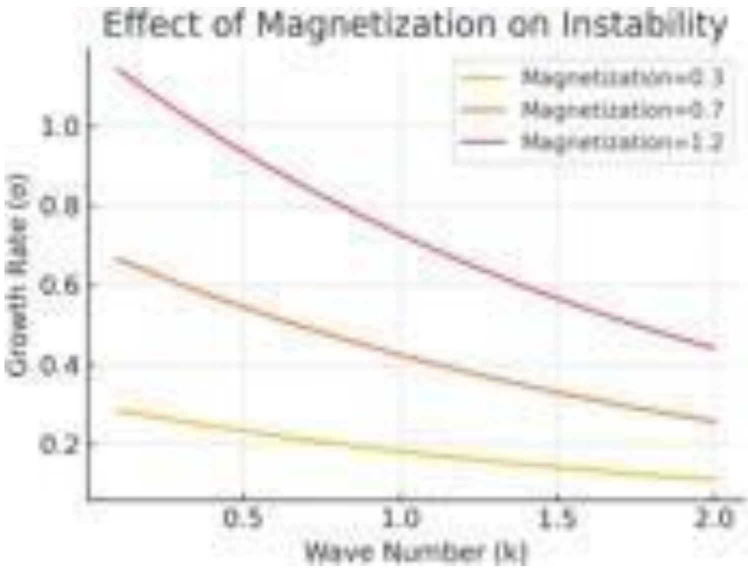

Figure 6: Effect of Magnetic Field Strength on Stability

The combined effect of couple stress along with a magnetic field is also essential for stabilization of the ferromagnetic micropolar fluid. Couple stresses increase rotational viscosities which decrease velocity gradients and damp disturbances of smaller magnitudes, causing an enhancement to fluid stability, while at the same time a stabilizing Lorentz force is generated within the fluid with the applied magnetic field, increasing the critical Rayleigh number in suppressing convective motion. When both the effects act together, they enhance each other's stabilizing effect, and the appearance of thermal instability gets profoundly delayed. However, if it exceeds very high values, fluid motion becomes so constrained that the heat transfer efficiency gets depressed. Hence, an interaction balance between couple stress and the magnetic effect is sought for maximum fluid stability.

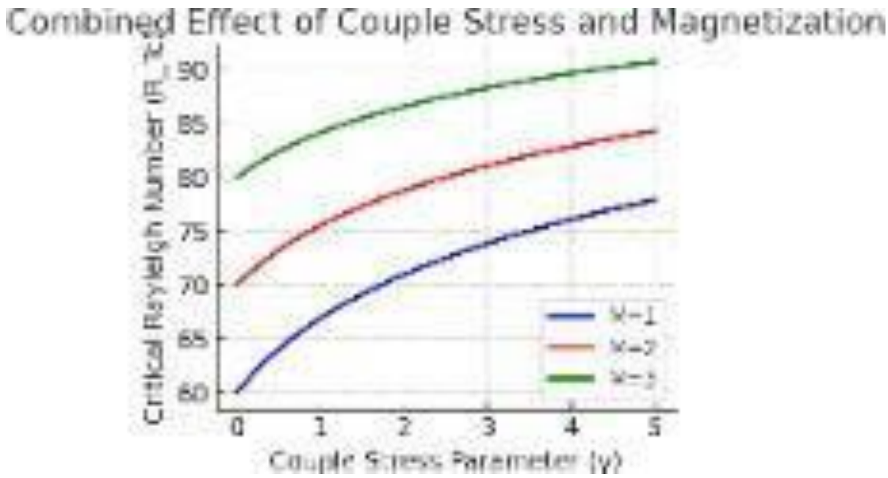

Figure 7: Combined Effect of Couple Stress and Magnetic Field on Stability 6.1

Discussion

This paper involves the thermal instability of couple stress ferromagnetic micropolar fluids with suspended solute particles in a porous medium, thus bringing out every complex interaction concerning thermal gradients and magnetic fields versus microstructural property of the fluid. The computations show that introduction of couple-stress effects with suspended solute particles has strongly modified the conditions for stability for the fluid. The onset condition and the overall evolution of pattern convection can be significantly modified. Magnetic fields further modify these instabilities, either enhancing or suppressing them depending on the magnetic permeability and other system parameters. The fluid flow dynamics are further complicated by the introduction of a porous medium, which depends on the permeability and porosity of the medium. These results suggest that these combined effects may have very significant implications for practical applications in the context of industrial systems for thermal management as well as in natural processes involving porous environments. Understanding these interactions gives insight into the optimization of fluid-based systems in various engineering disciplines, especially where heat and mass transfer are key.

7. Conclusion and Future Works

It particularly deals with thermal instability in micropolar ferromagnetic couple stress fluids permeated with pores suspended with in them solute particles to present insights into subtle interplay instigated by gradients of temperature and magnetic fields among others microscopic constituents in the fluid. It has been observed that all these effects together play a significant role in the initiation of convective instability and lead to unique flow responses depending upon conditions. Thus, this work examines the requirement of the couple stresses effect and effects of the suspended solute particles for formulating the thermal instability models for such fluid mixtures. The results are critical and involve many implications for industrial applications, especially where effectiveness in thermal performance is required; examples include the heat exchangers, coolers, and geophysical flows.

In extension of the currently developed models and with further investigations, detailed fluid-structure interaction, such as the effects due to varying size and concentration, as well as more complex geometrical structures, would be of interest. The theoretical predictions also should be validated by experiments for their practical applicability. An important area of future research could include investigating the effect of external parameters, such as time-varying magnetic fields and thermal conductivity on these systems for better explanation of thermal instability in such systems. Thus, in the development of new advanced thermal management materials, looking into the ways of optimizing the fluids for some industrial applications should be a pretty interesting area.

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