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Non-Homogeneous Orthotropic Rectangular Plate With Parabolic Temperature And Thickness Varying Circularly In One Dimension

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The main study of this paper is to find the effect in parabolic temperature behavior of orthotropic rectangular plate with thickness varies circular in one dimension. Rayleigh Ritz technique is used to solve the fundamental frequencies for the first two modes. For non-homogeneity of the plate material density vary linear in one dimension. Various parameters such as thermal gradient, taper constant, non-homogeneity and aspect ratio with clamped boundary condition have been taken. Numerical result is obtained with the help of the graphs and tables by using the MATLAB software. A comparison is also given to justify our paper study.

Keywords: Parabolic temperature, circular thickness, clamped boundary condition, aspect ratio, taper constant, rectangular plate, thermal gradient, linear density and non-homogeneity.

1. Introduction

Plates are important for the structure of engineering design. Thus the study of vibration has huge impact on the variety of applications in engineering field. These plates are also useful in civil, marine structure, aeronautical and jet engines. Due to the increase in modern technology the effect of high temperature on plates is changing day by day. To deal with that kind of change the structure should have more intensity and controlled heat fluxes. Orthotropic rectangular plates are widely used in modern structures application. Thermal effect, taper constant and non-homogeneity play an important role in vibration problems. Also there are others factor which effect the vibration, such as shear deformation, variable thickness and the boundary condition. So to avoid the vibration we have to use the material which is better in strength, efficiency and good in flexibility. This type of new material is used in modern technology like space craft, earthquake resistance, telephone, jet engine, nuclear and power plants. Such materials are result of the applications which is used after the reduction of weight and size. This material is low in expanse and has a great strength which is effective in many applications.

A lot of work has done on orthotropic rectangular plate with various thicknesses. But a few work is done, non-homogeneity with thermal effect on orthotropic rectangular plates of parabolically varying thickness.

Holzweissig [1] studied Vibration of Plates.(Nasa Sp-160). Mizusawa, Kajita and Naruoka [2] have studied the analysis of skew plate problems with various constraints. Gupta, Johri, and Vats [3] have studied the thermal gradient effect on vibrations of a non-homogeneous orthotropic rectangular plate having bi-direction linearly thickness variations. Sharma and Sharma [4] have discussed the mechanical vibration of orthotropic rectangular plate with two-dimensional linearly varying thickness and thermal effect. Gupta and Khanna [5] have discussed the vibration of visco-elastic rectangular plate with linearly thickness variations in both directions. Singh and Jain [6] have discussed the free asymmetric transverse vibration of polar orthotropic annular sector plate with thickness varying parabolically in radial direction. Khanna and Sharma [7] explained the thermally induced vibration of non-homogeneous visco-elastic plate of variable thickness. Gupta and Johri [8] have discussed the thermal effect on vibration of non-homogeneous orthotropic rectangular plate having bi-directional parabolically varying thickness. Khanna and Kaur [9] have discussed effect of non-homogeneity on free vibration of visco-elastic rectangular plate with varying structural parameters. Kumar Sharma and Sharma [10] have discussed the free vibration analysis of visco-elastic orthotropic rectangular plate with bi-parabolic thermal effect and bi-linear thickness variation. Wu and Lu [11] have discussed the free vibration analysis of rectangular plates with internal columns and uniform elastic edge supports by pb-2 Ritz method. Khanna and Kaur [12] have discussed the thermally induced vibrations of non-homogeneous tapered rectangular plate. Sharma and Sharma [13] have discussed the effect of bi-parabolic thermal and thickness variation on vibration of visco-elastic orthotropic rectangular plate. Khanna and Kaur [14] have discussed the effect of thermal gradient on vibration of non-uniform visco-elastic rectangular plate. Wang and Zu [15] have discussed the vibration behaviors of functionally graded rectangular plates with porosities and moving in thermal environment. Sharma, Sharma, Raghav and Kumar [16] explained the vibrational study of square plate with thermal effect and circular variation in density. Khanna and Sharma [17] have discussed natural vibration of visco-elastic plate of varying thickness with thermal effect. Sharma and Sharma [18] have discussed the mathematical modeling of vibration on parallelogram plate with non homogeneity effect. Tomar and Gupta [19] have discussed the orthotropic rectangular plate whose thickness vary to a thermal gradient. Lal [20] studied the non-uniform orthotropic rectangular plate with two dimension thickness variation. Khanna and Sharma [21] have discussed the effect of thermal gradient on vibration of visco-elastic plate with thickness variation. Gupta, Kumar and Gupta [22] explained the vibration of visco-elastic orthotropic parallelogram plate with parabolically thickness variation. Çeribaşı and Altay [23] have discussed the free vibration of super elliptical plates with constant and variable thickness by Ritz method. Gupta and Kumar [24] have discussed the thermal effect on vibration of non-homogenous visco-elastic rectangular plate of linearly varying thickness. Dokainish and Kumar [25] explained the vibrations of orthotropic parallelogramic plates with variable thickness.

On this paper we studied orthotropic rectangular plate behavior with temperature effect parabolically and circular thickness in x-direction which also include the linear density. We optimize mode values with temperature which varies parabolically and thickness circular in one dimension, so we can avoid the unwanted vibration in our structural design.

The vibration of orthotropic rectangular plates with specific thickness and temperatures was examined by the researchers mentioned above. However, some work is done with linear temperature in both x and y dimensions, respectively and circular thickness in x-direction. Our current work's primary goal is to investigate the linearly varying temperature influence on the x-direction vibration of an orthotropic rectangular plate with a circular thickness that has clamped boundary conditions on all ends.
Ritz method is used to make use of a deflection function with frequency parameter is determined for the first two vibrational modes. Circular thickness is varying in one dimension; density varies linear with parabolic temperature. Thermal gradient is denoted by **(A)** and taper parameter with **(B)**. Also non homogeneity is denoted by $\mathbf{(m_1)}$. The table and graphs demonstrate the effect of non-homogeneity, aspect ratio, temperature gradient, and taper constant.

2. Analysis of differential equation

An orthotropic rectangular plate with variable thickness has a governing differential equation of transverse motion that is define as

$$D_x \frac{\partial^4 W}{\partial x^4} + D_y \frac{\partial^4 W}{\partial y^4} + 2H \frac{\partial^2 W}{\partial x^2}\frac{\partial^2 W}{\partial y^2} + 2\frac{\partial H}{\partial x}\frac{\partial W}{\partial x}\frac{\partial^2 W}{\partial y^2} + 2\frac{\partial H}{\partial y}\frac{\partial W}{\partial y}\frac{\partial^2 W}{\partial x^2} + 2\frac{\partial D_x}{\partial x}\frac{\partial^3 W}{\partial x^3} + 2\frac{\partial D_y}{\partial y}\frac{\partial^3 W}{\partial y^3} + \frac{\partial^2 D_x}{\partial x^2}\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 D_y}{\partial y^2}\frac{\partial^2 W}{\partial y^2} + \frac{\partial^2 D_1}{\partial x^2}\frac{\partial^2 W}{\partial y^2} + \frac{\partial^2 D_1}{\partial y^2}\frac{\partial^2 W}{\partial x^2} + 4\frac{\partial^2 D_{xy}}{\partial x \partial y}\frac{\partial^2 W}{\partial x \partial y} + \rho h \frac{\partial^2 W}{\partial t^2} = 0. \quad (1)$$

And $H = D_1 + 2D_{xy}$. Let a be the length and b be the width side of the plate, thickness i and density ρ, of a non- homogeneous orthotropic rectangular plate with natural frequency δ^2.From governing differential equation of motion the strain energy and kinetic energy for orthotropic rectangular plate vibration are define as in [1]:

$$S_t = \frac{1}{2}\int_0^a \int_0^b [D_x \left(\frac{\partial^2 W}{\partial x^2}\right)^2 + D_y \left(\frac{\partial^2 W}{\partial y^2}\right)^2 + 2D_1 \left(\frac{\partial^2 W}{\partial x^2}\right)^2 \left(\frac{\partial^2 W}{\partial y^2}\right)^2 + 4\,D_{xy} \left(\frac{\partial^2 W}{\partial x \partial y}\right)^2]dxdy. \quad (2)$$

And $$K_t = \frac{1}{2}\,\delta^2 \rho \int_0^a \int_0^b i\,W^2\,dxdy. \quad (3)$$

W is the deflection function.

$$D_x = \frac{E_x i^3}{12(1-v_x v_y)}, D_y = \frac{E_y i^3}{12(1-v_x v_y)}, D_{xy} = \frac{G_{xy} i^3}{12(1-v_x v_y)}. \quad (4)$$

D_x and D_y are define as the flexural rigidities in x–direction and y-direction respectively. D_{xy} is known as a torsional rigidity. $D_{1=} v_x D_y (= v_x D_x)$, D is the Rhelogical operator, E_x and E_y is used for the elastic moduli in the x-direction and y-directions, respectively, v_x and v_y applies for the Poisson ratio, and G_{xy} signifies the shear modulus.

2.1 Frequency Equation with Rayleigh Ritz Method
With the help of Rayleigh Ritz Method, the maximum strain energy and the maximum kinetic energy is solved and are equal which satisfied the following equation

$$D = \delta(S_t - K_t) = 0. \tag{5}$$

$$D = \frac{1}{2}\int_0^a \int_0^b [D_x \left(\frac{\partial^2 W}{\partial x^2}\right)^2 + D_y \left(\frac{\partial^2 W}{\partial y^2}\right)^2 + 2D_1 \left(\frac{\partial^2 W}{\partial x^2}\right)^2 \left(\frac{\partial^2 W}{\partial y^2}\right)^2 + 4\, D_{xy} \left(\frac{\partial^2 W}{\partial x\, \partial y}\right)^2]dxdy$$

$$-\frac{1}{2}\,\delta^2 \rho \int_0^a \int_0^b i\, W^2\, dxdy = 0. \tag{6}$$

Plate with two-dimensional parabolic temperature distribution is describe as

$$\tau = \tau_0 \left(1 - \frac{x^2}{a^2}\right)\left(1 - \frac{y^2}{b^2}\right). \tag{7}$$

τ_0 is the excess of the temperature which mostly used in many field such as marine, engineering and aerospace. This temperature distribution can be represented at the end of the plate materials in such form as

$$E_x = E_1\ [1 - \alpha\tau],$$

$$E_y = E_2\,[1 - \alpha\tau],$$

$$G_{xy} = G_0[1 - \alpha\tau]. \tag{8}$$

E_1 and E_2 here are the Young's modulus values in the x and y axis, respectively. Where α is the slope of the modulus whose elasticity change with τ. Change in modulus can be presented by using equation (7), in equation (8) we get

$$E_x(x) = E_1\ \left[1 - A\left(1 - \frac{x^2}{a^2}\right)\left(1 - \frac{y^2}{b^2}\right)\right],$$

$$E_y(x) = E_2\ \left[1 - A\left(1 - \frac{x^2}{a^2}\right)\left(1 - \frac{y^2}{b^2}\right)\right],$$

$$G_{xy}(x) = G_0\left[1 - A\left(1 - \frac{x^2}{a^2}\right)\left(1 - \frac{y^2}{b^2}\right)\right]. \tag{9}$$

The temperature gradient parameter is denoted by $A = \alpha\,\tau_0$ $(0 \le A < 1)$, Using equation (9) in equation (4), we get

$$D_x = \frac{E_1 i^3}{12(1-v_x v_y)}\left[1 - A\left(1-\frac{x^2}{a^2}\right)\left(1-\frac{y^2}{b^2}\right)\right],$$

$$D_y = \frac{E_2 i^3}{12(1-v_x v_y)}\left[1 - A\left(1-\frac{x^2}{a^2}\right)\left(1-\frac{y^2}{b^2}\right)\right], \qquad D_{xy} = \frac{G_0 i^3}{12}\left[1 - A\left(1-\frac{x^2}{a^2}\right)\left(1-\frac{y^2}{b^2}\right)\right].$$
(10)

let us assume that the plate's thickness I, varies circular in one dimension, i.e.,

$$I = I_0\left[1 + B\left(1 - \sqrt{1-\frac{x^2}{a^2}}\right)\right]. \tag{11}$$

And B denote the taper constant. For non-homogeneity of the plate material the density vary a one-dimensional linear fluctuation, describe as

$$\rho = \rho_0\left(1 + m_1\frac{x}{a}\right). \tag{12}$$

And the non- homogeneity constants is denoted by m_1 , where $0 \le m_1 < 1$. Let us assume the non-dimensional variable's such as

$X_1 = \frac{x}{a}, \qquad Y_1 = \frac{y}{b},$

By using equation (10), (11), (12) and non dimensional variable in (6), we get

$$D = \frac{P}{2}\int_0^a\int_0^b\left[1 - A\left(1-\frac{x^2}{a^2}\right)\left(1-\frac{y^2}{b^2}\right)\right]\left[1 + B\left(1-\sqrt{1-\frac{x^2}{a^2}}\right)\right]^3\left[\left(\frac{\partial^2 W}{\partial x^2}\right)^2 + \frac{E_2}{E_1}\left(\frac{\partial^2 W}{\partial y^2}\right)^2 + 2v_x\frac{E_2}{E_1}\frac{\partial^2 W}{\partial x^2}\frac{\partial^2 W}{\partial y^2} + 4\frac{G_0}{E_1}(1-v_x v_y)\left(\frac{\partial^2 W}{\partial x\,\partial y}\right)^2\right]dxdy - d^2\int_0^a\int_0^b\left(1+m_1\frac{x}{a}\right)\left[1 + B\left(1-\sqrt{1-\frac{x^2}{a^2}}\right)\right]W^2 dxdy = 0. \tag{13}$$

Now the limits of X_1 and Y_1 are 0 to 1 and 0 to $\frac{b}{a}$, respectively. It replaces the values of S_t & K_t in equation (13) with the help of equation (5), we define the equation as

$$(S_t^* - d^2 K_t^*) = 0. \tag{14}$$

This gives us value for S_t^* and K_t^* define as,

$$S_t^* = P/2\int_0^1 \int_0^{\frac{b}{a}} \left[1 - A(1-X_1{}^2)\left(1 - \frac{Y_1{}^2 a^2}{b^2}\right)\right]\left[1 + B\left(1 - \sqrt{1-X_1{}^2}\right)\right]^3 \left[\left(\frac{\partial^2 W}{\partial x^2}\right)^2 + \frac{E_2}{E_1}\left(\frac{\partial^2 W}{\partial y^2}\right)^2 + 2v_x \frac{E_2}{E_1}\frac{\partial^2 W}{\partial x^2}\frac{\partial^2 W}{\partial y^2} + 4\frac{G_0}{E_1}(1 - v_x v_y)\left(\frac{\partial^2 W}{\partial x \partial y}\right)^2\right] dX_1\, dY_1 .$$
(15)

And

$$K_t^* = \int_0^1 \int_0^{b/a} (1 + m_1 X_1)\left[1 + B\left(1 - \sqrt{1-X_1{}^2}\right)\right] W^2 dX_1\, dY_1 . \tag{16}$$

Where, $P = \frac{1}{2}\ \frac{E_1\ i_0{}^3}{12(1-v_x v_y)}$ and $d^2 = \frac{12\ \delta^2 a^4 \rho(1-v_x v_y)}{E_1 i_0{}^2}$.

By using the clamped boundary condition, which define as

$$W(x,y) = \left[\left(\left(\frac{x}{a}\right)\left(\frac{y}{b}\right)\left(1 - \frac{x}{a}\right)\left(1 - \frac{y}{b}\right)\right)^2\right]\left[C_1 + C_2\left(\frac{x}{a}\right)\left(\frac{y}{b}\right)\left(1 - \frac{x}{a}\right)\left(1 - \frac{y}{b}\right)\right]. \tag{17}$$

Two unknown C_1 & C_2 in equation (14) are the results from the clamped boundary condition used in equation (17). Now to evaluate the values of these two constant C_1 & C_2, we can use the following equation describe as:

$$\frac{\partial}{\partial A_n}[S_t^* - d^2 K_t^*] = 0\ , \quad \text{n=1, 2.} \tag{18}$$

Equation (18) can be simplified to provide the following form

$$D_{k1}\, A_1 + D_{k2}\, A_2 = 0. \tag{19}$$

The parametric constants and the frequency parameter are involved in D_{k1} & D_{k2} where k=1, 2. When the coefficients of equation (19) are determined to be non-zero, they must disappear. In this manner, the frequency equation was

$$\begin{vmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{vmatrix} = 0. \tag{20}$$

When equation (20) is solved, a quadratic equation in d^2 is obtained, yielding two roots. When $C_1 = 1$ is chosen to be substituted in equation (17), $C_2 = -\frac{d_{11}}{d_{12}}$ is obtained, and W becomes

$$W(x,y) = \left[\left(\left(\frac{x}{a}\right)\left(\frac{y}{b}\right)\left(1-\frac{x}{a}\right)\left(1-\frac{y}{b}\right)\right)^2\right]\left[1+\left(-\frac{c_{11}}{c_{12}}\right)\left(\frac{x}{a}\right)\left(\frac{y}{b}\right)\left(1-\frac{x}{a}\right)\left(1-\frac{y}{b}\right)\right].$$

Additionally, it can be expressed as

$$W(x,y) = \left[\left(X_1 Y_1 \frac{a}{b}(1-X_1)\left(1-\frac{a}{b}Y_1\right)\right)^2\right]\left[1+\left(-\frac{c_{11}}{c_{12}}\right)X_1 Y_1 \frac{a}{b}(1-X_1)\left(1-\frac{a}{b}Y_1\right)\right].$$

3. Result and discussion

Table 1. Thermal gradient (A) vs Frequency (d) of clamped rectangular plate for Aspect Ratio 1.5

A	B=m_1=0.2,v=0.345		B=m_1=0.4, v=0.345		B= m_1=0.8,v=0.345	
	d_1	d_2	d_1	d_2	d_1	d_2
0.0	46.2992	185.7404	46.4354	186.1372	47.2641	189.6615
0.2	44.2252	177.5717	44.4416	178.3987	45.3859	182.6628
0.4	42.0393	169.0110	42.3411	170.3122	43.4070	175.3898
0.6	39.7195	159.9964	40.1129	161.8266	41.3074	167.8083
0.8	37.2350	150.4477	37.7272	152.8772	39.0593	159.8765

Figure 1. For fixed aspect ratio a/b = 1.5, Thermal gradient (A) against Frequency (d)

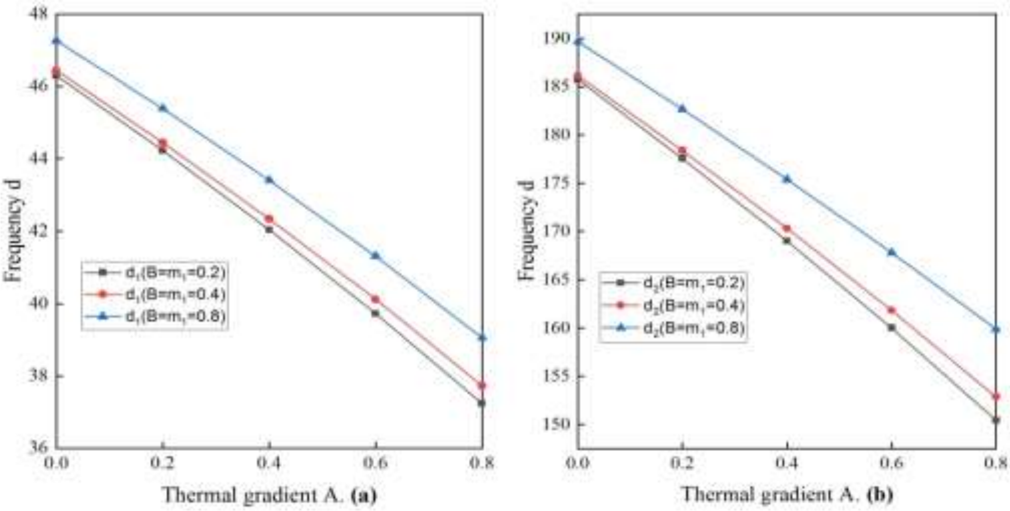


Table 2. Non-homogeneity (m_1) vs Frequency (d) of clamped rectangular plate for Aspect Ratio 1.5

m_1	A=B=0.2, v=0.345		A=B=0.4, v=0.345		A=B=0.8, v=0.345	
	d_1	d_2	d_1	d_2	d_1	d_2
0.0	46.3946	186.3390	46.4198	186.9257	47.5128	190.3548
0.2	44.2252	177.5717	44.2400	178.0402	45.2310	181.1335
0.4	42.3343	169.9359	42.3411	170.3122	43.2832	173.1346

0.6	40.6669	163.2074	40.6675	163.5105	41.5149	166.1099
0.8	38.5222	154.1267	38.8625	155.3219	39.0593	159.8765
1.0	37.74901	151.8460	37.8410	152.0417	38.5234	154.2961

Figure 2. For fixed aspect ratio a/b = 1.5, Non homogeneity constant (m_1) against Frequency (d)

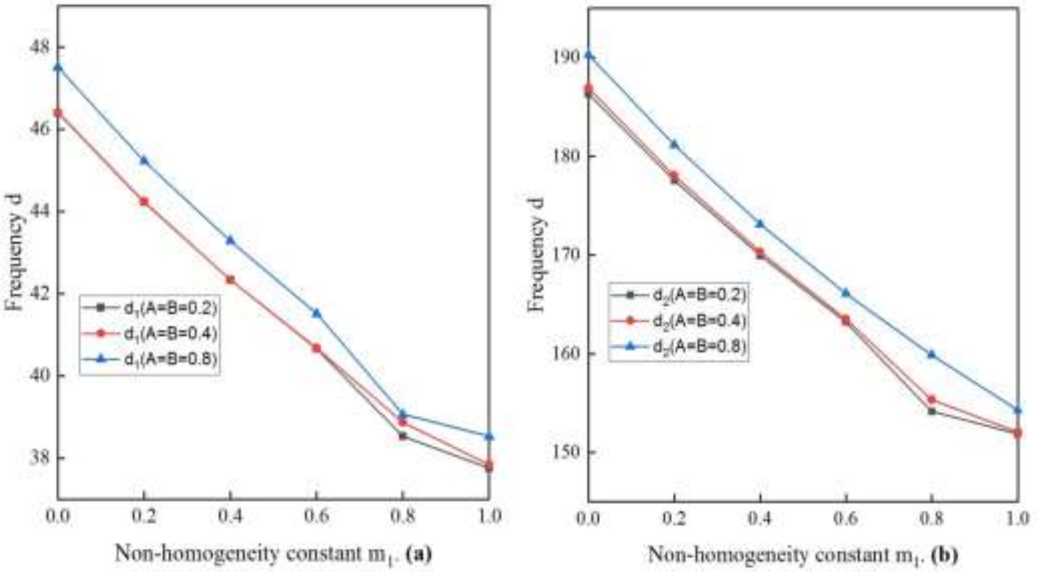


Table 3. Taper constant (B) vs Frequency (d) of clamped rectangular plate for Aspect Ratio 1.5

B	**A=m_1=0.2, v=0.345**		**A=m_1=0.4, v=0.345**		**A=m_1=0.8, v=0.345**	
	d_1	**d_2**	**d_1**	**d_2**	**d_1**	**d_2**
0.0	42.1973	169.6000	38.3096	154.0868	31.2067	125.9123
0.2	44.2252	177.5717	40.2419	161.7431	32.9897	133.2026
0.4	46.4349	186.4930	42.3411	170.3122	34.9094	141.3421
0.6	48.7989	196.3039	44.5801	179.7310	36.9399	150.2589
0.8	51.2915	206.9362	46.9341	189.9291	39.0593	159.8765
1.0	53.8902	218.3182	49.3823	200.8339	41.2507	170.1199

Figure 3. For fixed aspect ratio a/b = 1.5, Tapering constant (B) against Frequency (d)

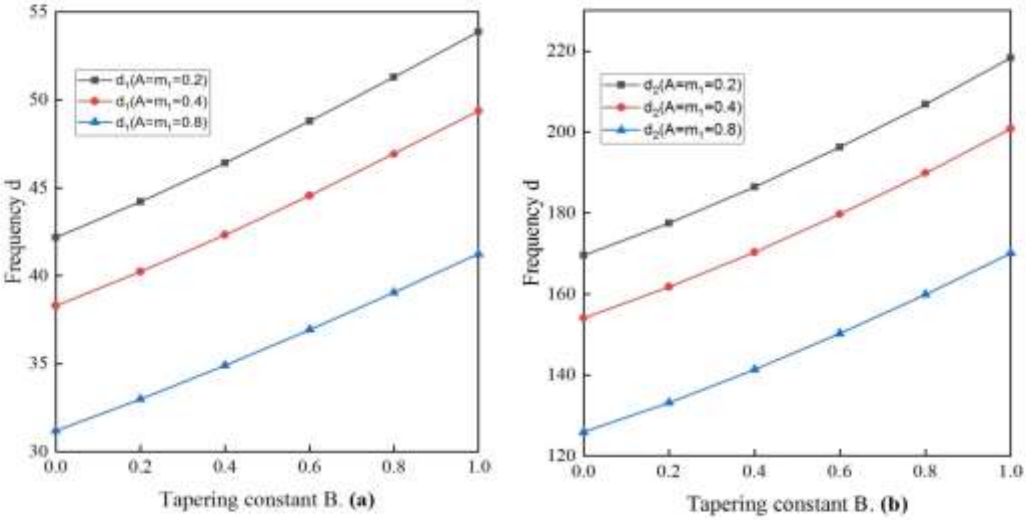


Table 1, we take the value of thermal gradient (A) from (0.0 to 0.8), non homogeneity (m_1) and taper constant (B), from ($B=m_1=0.2$, $B=m_1=0.4$ and $B=m_1=0.8$). With the increase of the value thermal gradient (A) (0.0 to 0.8), the value of non homogeneity m_1 and taper constant B decrease at constant value of ($B=m_1=0.2$, $B=m_1=0.4$ and $B=m_1=0.8$).

Table 2, we take the value of non-homogeneity (m_1) from (0.0 to 1.0), thermal gradient (A) and taper constant (B), from (A=B=0.2, A=B=0.4 and A=B=0.8). Increasing the value of non homogeneity from (0.0 to 1.0), the value of thermal gradient (A) and taper constant (B) decrease at constant value of (A=B=0.2, A=B=0.4 and A=B=0.8).

In Table 3, when we take the value of taper constant(B)from (0.0 to 1.0), thermal gradient (A) and non-homogeneity (m_1)values which lies from ($A=m_1=0.2$, $A=m_1=0.4$ and $A=m_1=0.8$), with the increase of taper constant (B) from (0.0 to 1.0), the constant value of thermal gradient (A) and non-homogeneity (m_1) also increase instead of decreasing for values ($A=m_1=0.2$, $A=m_1=0.4$ and $A=m_1=0.8$).

From Table1, Table2 and Table3 the main results for aspect ratio a/b=1.5, are:

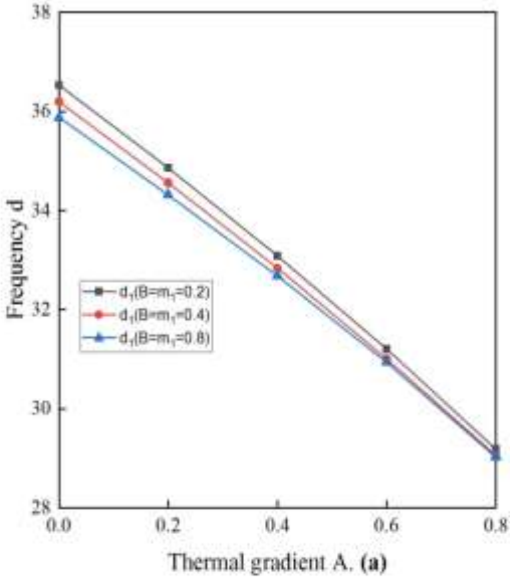
- First and second natural frequencies and corresponding mode shapes $d_1$and d_2 values in Table1, ($B=m_1=0.2$, $B=m_1=0.4$ and $B=m_1=0.8$), the values show a horizontal rise with thermal gradient (A) values ranging from (0.0 to 0.8).
- First and second natural frequencies and corresponding mode shapes $d_1$and d_2 values in Table1, ($B=m_1=0.2$, $B=m_1=0.4$ and $B=m_1=0.8$), the values show a vertical decrease with thermal gradient (A) values ranging from (0.0 to 0.8).
- First and second natural frequencies and corresponding mode shapes $d_1$and d_2 values in Table2, (A=B=0.2, A=B=0.4 and A=B=0.8), decreasing vertically and increasing horizontally for different values of non homogeneity (m_1) from (0.0 to 1.0).

- First and second natural frequencies and corresponding mode shapes $d_1$and d_2 values in Table3, ($A=m_1=0.2$, $A=m_1=0.4$ and $A=m_1=0.8$), the values show a horizontal decrease with taper constant (B) values ranging from (0.0 to 1.0).
- First and second natural frequencies and corresponding mode shapes $d_1$and d_2 values in Table3, ($A=m_1=0.2$, $A=m_1=0.4$ and $A=m_1=0.8$), the values show a vertical increase with taper constant (B) values ranging from (0.0 to 1.0).

Table 4. Thermal gradient (A) vs Frequency (d) of clamped rectangular plate for Aspect Ratio 2.5

A	B=m₁=0.2,v=0.345		B=m_1=0.4, v=0.345		B= m₁=0.8,v=0.345	
	d_1	d_2	d_1	d_2	d_1	d_2
0.0	36.5347	147.6456	36.1889	145.7340	35.8758	143.4432
0.2	34.8545	140.9501	34.5563	139.2555	34.3174	137.3272
0.4	33.0831	133.9217	32.8360	132.4623	32.6768	130.9278
0.6	31.2027	126.5056	31.0112	125.3036	30.9383	124.2018
0.8	29.1885	118.6300	29.0580	117.7138	29.0299	117.0942

Figure 4. For fixed aspect ratio a/b = 2.5, Thermal gradient (A) against Frequency (d)



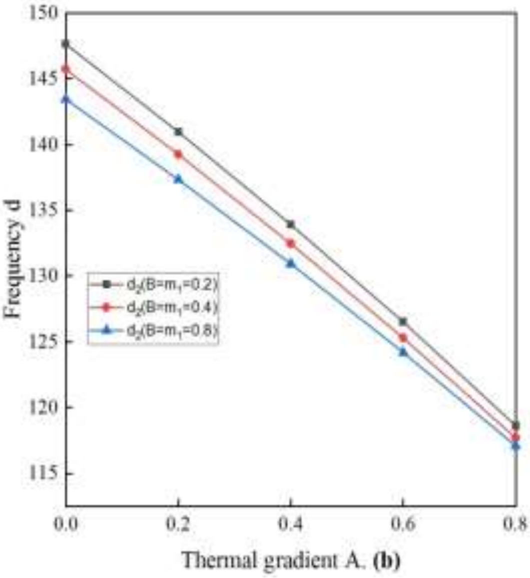


Table 5. Non-homogeneity (m_1) vs Frequency (d) of clamped rectangular plate for Aspect Ratio 2.5

m_1	A=B=0.2, v=0.345		A=B=0.4, v=0.345		A=B=0.8, v=0.345	
	d_1	d_2	d_1	d_2	d_1	d_2
0.0	36.5641	147.9092	35.9990	145.3821	34.4934	139.4027
0.2	34.8545	140.8501	34.3086	138.4721	32.8618	132.6538
0.4	33.3643	134.8891	32.8360	132.4623	31.4420	126.7992
0.6	32.0503	129.5483	31.5382	127.1727	30.1918	121.6572

0.8	30.8803	124.7955	30.3831	122.4701	29.0299	117.0942
1.0	29.8297	120.5301	29.3464	118.2535	28.0826	113.0090

Figure 5. For fixed aspect ratio a/b = 2.5, Non homogeneity constant (m_1) against Frequency (d)

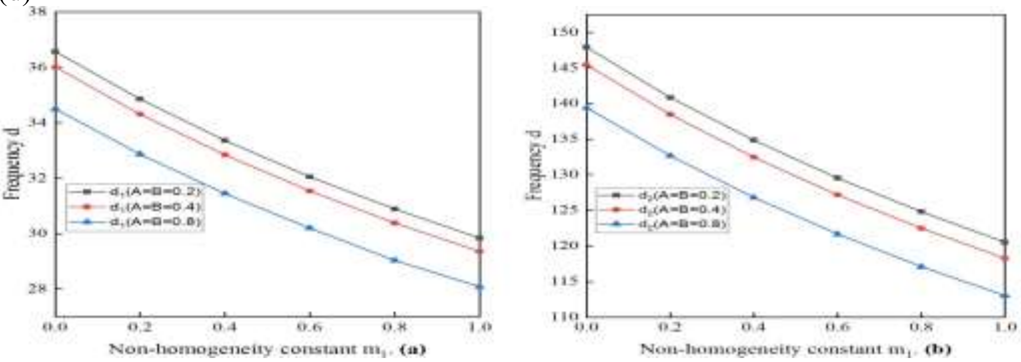
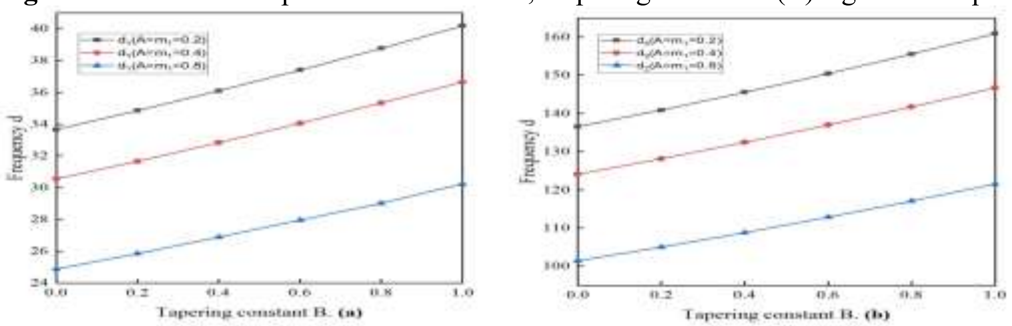


Table 6. Taper constant (B) vs Frequency (d) of clamped rectangular plate for Aspect Ratio 2.5

B	**A=m₁=0.2, v=0.345**		**A=m₁=0.4, v=0.345**		**A=m₁=0.8, v=0.345**	
	d_1	**d_2**	**d_1**	**d_2**	**d_1**	**d_2**
0.0	33.6672	136.5942	30.5614	124.1193	24.8858	101.4692
0.2	34.8545	140.8501	31.6688	128.1628	25.8609	105.0323
0.4	36.1061	145.5732	32.8360	132.4623	26.8878	108.8330
0.6	37.4174	150.4505	34.0588	137.0047	27.9623	112.8583
0.8	38.7835	155.5677	35.3324	141.7762	29.0299	117.0942
1.0	40.1997	160.9105	36.6521	146.7624	30.2364	121.5264

Figure 6. For fixed aspect ratio a/b = 2.5, Tapering constant (B) against Frequency (d)



In Table4 ,we take the value of thermal gradient (A)from(0.0 to 0.8), non homogeneity(m_1)and taper constant(B), from (B=m_1 =0.2, B=m_1=0.4 and B=m_1=0.8). With the increase of the value thermal gradient (A) (0.0 to 0.8), the value of non homogeneity (m_1) and taper constant B decrease at constant value of (B=m_1 =0.2, B=m_1=0.4 and B=m_1=0.8).

In Table5, we take the value of non homogeneity (m_1) from (0.0 to 1.0), thermal gradient (A) and taper constant (B), from (A=B=0.2, A=B=0.4 and A=B=0.8). Increasing the value of non homogeneity from (0.0 to 1.0), the value of thermal gradient (A) and taper constant(B) decrease at constant value of (A=B=0.2, A=B=0.4 and A=B=0.8).

But in Table6, when we take the value of taper constant (B)from (0.0 to 1.0), thermal gradient (A) and non homogeneity(m_1)values which lies from(A=m_1=0.2, A=m_1=0.4 and A=m_1=0.8), with the increase of taper constant(B)from (0.0 to 1.0), the constant value of thermal gradient (A) and non homogeneity (m_1) also increase instead of decreasing.

From Table4, Table5 and Table6 the main results for aspect ratio a/b=2.5, are:

- First and second natural frequencies and corresponding mode shapes $d_1$and d_2 values in Table4, (B=m_1=0.2, B=m_1=0.4 and B=m_1=0.8), decreasing both side horizontally and vertically for different values of thermal gradient (A) from (0.0 to 0.8).
- First and second natural frequencies and corresponding mode shapes $d_1$and d_2 values in Table5, (A=B=0.2, A=B=0.4 and A=B=0.8) decreasing both side horizontally and vertically for different values of non homogeneity (m_1) from (0.0 to 1.0).
- First and second natural frequencies and corresponding mode shapes $d_1$and d_2 values in Table6, (A=m_1=0.2, A=m_1=0.4 and A=m_1=0.8), the values show a horizontal decrease with taper constant (B) values ranging from (0.0 to 1.0).
- First and second natural frequencies and corresponding mode shapes $d_1$and d_2 values in Table6, (A=m_1=0.2, A=m_1=0.4 and A=m_1=0.8), the values show a vertical increase with taper constant (B) values ranging from (0.0 to 1.0).

4. RESULT COMPARISON

- A comparison of data which are presented in Table1, Table2 and Table3 as well as in Table4, Table5 and Table6. Also it is presented in graphical form Figure1 to Figure6. It shows the variation in frequency modes d_1 and d_2.
- In Table1, 2 and 3 we have taken the aspect ratio 1.5, while in Table4, 5 and 6 we take 2.5.
- In Table1 the thermal gradient (A), frequency mode values d_1 and d_2 has highest point value at 0.0 for (B=m_1=0.8) and lowest point value at 0.8 for (B=m_1=0.2) with aspect ratio 1.5. For same thermal gradient (A) in Table4 mode values d_1 and d_2 has the highest values for (B=m_1=0.2) at 0.0 and lowest point values for (B=m_1=0.8) at 0.8 with aspect ratio 2.5, but mode values in Table1 are greater than of Table4.
- In Table2 non homogeneity m_1 has the highest mode values d_1 and d_2 for (A=B=0.8) at 0.0, and lowest mode values d_1 and d_2 for (A=B=0.2) at 1.0 with aspect ratio 1.5. In Table5 for same non homogeneity m_1 highest and lowest point for mode values d_1 and d_2 lies for (A=B=0.2) at 0.0 and for (A=B=0.8) at 1.0 respectively with aspect ratio 2.5. But mode values of d_1 and d_2 in Table2 are greater than Table5.
- In Table3 and Table6 taper constant (B) has the same highest and lowest mode values d_1 and d_2 for (A=m_1=0.2) at 1.0 and (A=m_1=0.8) at 0.0. But mode values of d_1 and d_2 in Table3 with aspect ratio 1.5 are greater than of Table6 include aspect ratio 2.5.

- In all graphs there is no sudden increase or decrease for frequency mode values d_1 and d_2. The frequency mode values of d_1 and d_2 has the same order for increasing or decreasing.

5. Numerical evaluations

We use the material 'Duralumin' which is a mixture of aluminium (95%), copper (4%), magnesium (0.5%) and manganese (0.5%). Different values of temperature gradient (A_1), taper constant (B_1), non- homogeneity (m_1) and aspect ratio (a/b) calculated for the first two modes of vibration ($d_1$and d_2) by using the equation (18), which is quadratic in d^2. The results of orthotropic rectangular plate material are shown by using the tables (1 to 6) and graph figures from (1 to 6).

Following are the parameters for the clamped orthotropic rectangular plate:

ρ_0 = 2.80× 10^3,

G_0 = 2.632× 10^10, v = 0.345, for the aspect ratio of (a/b) =1.5, the dimensions are a=3 and b=2; for (a/b) = 2.5, a=5 and b=2.

I_0 = 0.01 meter.

$\frac{E_2}{E_1}$ = 0.32

$\frac{E}{E_1}$ = 0.04, $\frac{G_0}{E_1}$ = 0.09 and E = 7.08× 10^10.

6. Conclusions

In this present paper we find the results of orthotropic rectangular plates with parabolic temperature whose thickness vary circular in one dimension and density linearly. The thermal gradient (A) is shown in Table1 and Table4. In Table1 the gradient increase horizontally but decrease vertically for aspect ratio (a/b) 1.5, with different values of B=m_l=0.2, B=m_l=0.4 and B=m_l=0.8. But same gradient decrease both side horizontally and vertically for different values of B=m_l=0.2, B=m_l=0.4 and B=m_l=0.8 at aspect ratio (a/b) 2.5. However, Table2 and Table5 show same increase and decrease for frequency mode for non homogeneity (m_l)for different values A=B=0.2, A=B=0.4 and A=B=0.8 with aspect ratio (a/b) 1.5 and 2.5. Also Table3 and Table6 shows the same behaviour for frequency mode for taper constant (B) horizontally and vertically along with values A=m_l=0.2, A=m_l=0.4, A=m_l=0.8 at aspect ratio (a/b) 1.5 and 2.5. Due to the circular variation implementation, an increase and decrease in the variation in frequency mode d_1 and d_2 values in the temperature gradient (A), non-homogeneity (m_1) and taper constant (B) are less among the frequency. Frequency mode d_1 and d_2 shows no high difference values between two modes, weather increase or decrease at different values of thermal gradient (A), non homogeneity (m_l) and taper constant (B). The frequency mode values d_1 and d_2 for thermal gradient (A), non homogeneity (m_l) and taper constant (B) at aspect ratio (a/b) 1.5 are higher than the same mode values of aspect ratio (a/b) 2.5. Present study shows us the vibration behaviour due to the thermal gradient, non homogeneity and density. This study helps us to create such a mechanism which increases the potential energy, intensive strength and efficiency of the material. Accurate and effective mathematical structure is the demand of the current scenario in the world. All the researchers, authors can get the basic concept of the material that has a balanced effect of temperature and density. These types of balanced materials can help to develop such a mathematical design which can use in different branches of the engineering. This present study can help the researchers, scientists and the participators to get the authentic and practical approach information towards the mode vibration, which can be used to make an economy and technology friendly structure for the benefit of the science and the human being.

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