

Study Of Isotropic Non-Homogeneous Rectangular Plate With Non-Uniform Circularly Varying Thickness And Parabolically Varying Temperature Effect

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Rectangular visco-elastic plates are commonly used in many industries, aviation, and mechanical structures. Accurately determining the behaviour and strength properties of plates is necessary for the right design of plate structures and the effective use of material. The frequency of free vibrations of a rectangular visco-elastic plate with varying thickness is examined in relation to two-dimensional thermal effects. In this article, the thermal effect varies parabolically in two directions while the thickness varies circularly in the x-direction. The fundamental frequencies are evaluated using the Rayleigh Ritz method. MATLAB is used to calculate first two modes of frequencies over various values of temperature gradient, non-homogeneity, and taper parameter values.

Key Words:- Vibration, parabolically, circularly, non-homogeneity, isotropic, aspect ratio, taper parameter, thermal gradient, flexural rigidity, strain energy, kinetic energy.

Introduction

Engineers and researchers must always be aware of the system's vibration characteristics when building machines, structures, and other mechanical designs. It is impossible to overlook the significance of researching the vibrations of non-homogeneous tapered plates, which are frequently utilized in the building of bridges, ships, airplanes, and other structures. The material is stronger and lighter due to non-homogeneity and plate tapering. By creating appropriate and precise machine and building designs, the primary goal of vibration research is to prevent unnecessary and excessive vibration. The majority of engineering structures in the current technological period, including nuclear reactors, rockets, and missiles, work with adequate temperature fields under various boundary conditions. Hence, it becomes essential to investigate how temperature changes affect the structures vibrational characteristics. Vibration research primary goal is to minimize unnecessary and excessive vibration by accurately and appropriately constructing mechanical structures.

Studied the effect of bi-parabolic temperature variation on the vibration of an orthotropic rectangular plate, highlighting the importance of thermal sensitivity in structural analysis [1]. A simple model analyses the thermal effect on the vibration of a non-homogeneous orthotropic visco-elastic rectangular plate with parabolically varying thickness and clamped edges, using the Rayleigh-Ritz technique to derive an approximate frequency equation[2]. This paper presents a unified nonlinear analytical solution for bending, buckling, and vibration of temperature-dependent functionally graded rectangular plates under thermal load, comparing three mathematical models for effective material properties [3]. The study analyses the vibration frequencies of a rectangular plate with a linearly varying thickness and a circularly varying Poisson's ratio[4]. The paper presents a mathematical model to analyse the temperature-thickness coupling in a non-homogeneous isotropic viscoelastic rectangular plate with bi-parabolic temperature variation, linear thickness variation, and exponential Poisson's ratio variation[5]. The paper analyses the buckling loads of rectangular composite plates under non-uniform in-plane loading using higher-order shear deformation theory, solving for stress distribution and buckling equations to obtain critical loads and mode shapes[6]. The paper investigates the buckling behaviour of symmetrically laminated rectangular plates under parabolic in-plane compressive loading using the Rayleigh-Ritz method, incorporating Chebyshev polynomials to derive buckling loads for various boundary conditions and validating results against DQM and FEM[7]. The paper analyses the vibrational frequencies of a non-homogeneous viscoelastic parallelogram plate with circular thickness variation and bi-parabolic temperature distribution using the Rayleigh-Ritz method and MAPLE software[8]. The paper presents a nonlinear vibration analysis of functionally graded rectangular micro-plates with variable thickness and a central partial crack using Classical Plate Theory and modified couple stress theory, demonstrating how thickness variation can mitigate crack effects on vibration characteristics[9]. The effect of two-parameter foundation on the transverse vibrations and critical buckling loads of nonhomogeneous rectangular plates under linearly varying in-plane forces has been analysed using the Kirchhoff plate theory and solved numerically via the Levy approach and Differential Quadrature Method[10]. The natural transverse vibration of a nonhomogeneous skew plate with variable thickness and temperature field is analysed using the Rayleigh-Ritz technique under CCCC and CSCS edge conditions[11]. Free transverse vibrations of isotropic rectangular plates with arbitrarily varying non-homogeneity are analysed using the Generalized Differential Quadrature Method based on Kirchhoff plate theory[12]. The Rayleigh-Ritz method is used to analyse the natural vibration time period of an isotropic viscoelastic square plate with circular thickness variation and Poisson's ratio under clamped and simply supported conditions[13]. This paper investigates the natural vibration of a non-uniform, non-homogeneous square plate with clamped boundaries, considering circular thickness variation, bi-linear temperature distribution, and linear density variation using the Rayleigh-Ritz method[14]. This paper analyzes the natural vibration of a non-uniform, non-homogeneous square plate with clamped boundaries, considering thickness variation, bi-linear temperature distribution, and density variation using the Rayleigh-Ritz method[15]. The vibration of a tapered isotropic rectangular plate under thermal conditions is analyzed using the Rayleigh-Ritz method for various boundary conditions[16]. The effect of linear thickness variation on the vibration of a viscoelastic rectangular plate with clamped edges is analyzed using the Rayleigh-Ritz

method[17]. The Rayleigh Ritz method is used to solve the differential equation involving the linear density variation and the circular Poisson's ratio fluctuation on the vibration time period of the rectangular plate[18]. A tapered, non-homogeneous rectangular plate with a variety of boundary conditions is used to examine plate properties[19]. The effect of bilinear temperature variation on the vibration of a non-homogeneous viscoelastic rectangular plate with non-uniform thickness is analyzed using the Rayleigh–Ritz method[20].

In addition to using aspect ratios of 1.5 and 2.5, the deflection function is utilized to identify the modes of frequency for various values of temperature gradient, taper constant, and non-homogeneity. With SSSS boundary conditions and a two-dimensional temperature field distribution, the Rayleigh-Ritz technique is used to study the natural vibration of an isotropic non-homogeneous rectangular plate with a one-dimensional circularly variable thickness and density parameter.

A rectangular plate composed of duralumin material with a circular variation in thickness in one direction was used to achieve the solution of the first two modes of vibration in the current experiment. Both tabular and graph formats are used to display the numerical values of the first two modes of the frequency at different structural parameter values for the SSSS boundary condition, assuming that the plate is simply supported on all four edges.

Analysis Of Equation Of Motion

The differential equation of motion of visco-elastic isotropic plate may be written as [16]

$$(1) \quad \frac{\partial^2 \tau_x}{\partial x^2} + 2 \frac{\partial^2 \tau_{xy}}{\partial x \partial y} + \frac{\partial^2 \tau_y}{\partial y^2} = \rho h \frac{\partial^2 \xi}{\partial t^2}$$

where x and y represent the plate geometry's coordinates, τ_x and τ_y represent the bending moments, τ_{xy} represents the twisting moment per unit plate length, ρ represents the mass per unit volume, h represents the plate's thickness, and ξ represents the displacement at time t .

The expressions for τ_x , τ_y and τ_{xy} are given by [17]

$$(2) \quad \begin{aligned} \tau_x &= -\tilde{D}D_1 \left[\frac{\partial^2 \xi}{\partial x^2} + \nu \frac{\partial^2 \xi}{\partial y^2} \right], \\ \tau_y &= -\tilde{D}D_1 \left[\frac{\partial^2 \xi}{\partial y^2} + \nu \frac{\partial^2 \xi}{\partial x^2} \right] \text{ and} \\ \tau_{xy} &= -\tilde{D}D_1 (1 - \nu) \frac{\partial^2 \xi}{\partial x \partial y} \end{aligned}$$

where $\tilde{D}$ is the representation for visco-elastic operator.

In this case, D_1 represents the material's flexural rigidity and is written as[18]

$$D_1 = \frac{Eh^3}{12(1-\nu^2)} \quad (3)$$

When τ_x , τ_y and τ_{xy} are substituted in equation (1), one obtains

$$\begin{aligned} \bar{D} \left[D_1 \left(\frac{\partial^4 \xi}{\partial x^4} + 2 \frac{\partial^4 \xi}{\partial x^2 \partial y^2} + \frac{\partial^4 \xi}{\partial y^4} \right) + 2 \frac{\partial D_1}{\partial x} \left(\frac{\partial^3 \xi}{\partial x^3} + 2 \frac{\partial^3 \xi}{\partial x \partial y^2} \right) + 2 \frac{\partial D_1}{\partial y} \left(\frac{\partial^3 \xi}{\partial y^3} + 2 \frac{\partial^3 \xi}{\partial y \partial x^2} \right) \right. \\ \left. + \frac{\partial^2 D_1}{\partial x^2} \left(\frac{\partial^2 \xi}{\partial x^2} + \nu \frac{\partial^2 \xi}{\partial y^2} \right) + \frac{\partial^2 D_1}{\partial y^2} \left(\frac{\partial^2 \xi}{\partial y^2} + \nu \frac{\partial^2 \xi}{\partial x^2} \right) + 2(1-\nu) \frac{\partial^2 D_1}{\partial x \partial y} \frac{\partial^2 \xi}{\partial x \partial y} \right] + \rho h \frac{\partial^2 \xi}{\partial t^2} = 0 \end{aligned} \quad (4)$$

Deflection ξ can be considered the product of two functions using the variable separation method[19]

$$\xi(x, y, t) = \phi(x, y) \cdot T(t) \quad (5)$$

where $T(t)$ is a time function for the vibration of a rectangular plate and $\phi(x, y)$ is the deflection function in x and y . Equation (5) can be substituted into equation (4), to get

$$\begin{aligned} \left[D_1 \left(\frac{\partial^4 \phi}{\partial x^4} + 2 \frac{\partial^4 \phi}{\partial x^2 \partial y^2} + \frac{\partial^4 \phi}{\partial y^4} \right) + 2 \frac{\partial D_1}{\partial x} \left(\frac{\partial^3 \phi}{\partial x^3} + 2 \frac{\partial^3 \phi}{\partial x \partial y^2} \right) + 2 \frac{\partial D_1}{\partial y} \left(\frac{\partial^3 \phi}{\partial y^3} + 2 \frac{\partial^3 \phi}{\partial y \partial x^2} \right) \right. \\ \left. + \frac{\partial^2 D_1}{\partial x^2} \left(\frac{\partial^2 \phi}{\partial x^2} + \nu \frac{\partial^2 \phi}{\partial y^2} \right) + \frac{\partial^2 D_1}{\partial y^2} \left(\frac{\partial^2 \phi}{\partial y^2} + \nu \frac{\partial^2 \phi}{\partial x^2} \right) + 2(1-\nu) \frac{\partial^2 D_1}{\partial x \partial y} \frac{\partial^2 \phi}{\partial x \partial y} \right] / \rho h \phi = - \left(\frac{\partial^2 T / \partial t^2}{\bar{D} T} \right) \end{aligned} \quad (6)$$

When both sides of equation (6) are equal to a constant p^2 , the result is

$$\begin{aligned} \left[D_1 \left(\frac{\partial^4 \phi}{\partial x^4} + 2 \frac{\partial^4 \phi}{\partial x^2 \partial y^2} + \frac{\partial^4 \phi}{\partial y^4} \right) + 2 \frac{\partial D_1}{\partial x} \left(\frac{\partial^3 \phi}{\partial x^3} + 2 \frac{\partial^3 \phi}{\partial x \partial y^2} \right) + 2 \frac{\partial D_1}{\partial y} \left(\frac{\partial^3 \phi}{\partial y^3} + 2 \frac{\partial^3 \phi}{\partial y \partial x^2} \right) \right. \\ \left. + \frac{\partial^2 D_1}{\partial x^2} \left(\frac{\partial^2 \phi}{\partial x^2} + \nu \frac{\partial^2 \phi}{\partial y^2} \right) + \frac{\partial^2 D_1}{\partial y^2} \left(\frac{\partial^2 \phi}{\partial y^2} + \nu \frac{\partial^2 \phi}{\partial x^2} \right) + 2(1-\nu) \frac{\partial^2 D_1}{\partial x \partial y} \frac{\partial^2 \phi}{\partial x \partial y} \right] - \rho p^2 h \phi = 0 \end{aligned} \quad (7)$$

and

$$\frac{\partial^2 T}{\partial t^2} + p^2 \bar{D} T = 0 \quad (8)$$

Assumption Required

one dimensional circular variation in thickness as [18]

$$h = h_0 \left[1 + \beta \left(1 - \sqrt{1 - \frac{x^2}{a^2}} \right) \right] \quad (9)$$

where β , ($0 \leq \beta \leq 1$) is known as tapering parameter and thickness of plate becomes constant at $x = 0$ and for non-homogeneity (ρ) consideration, assumed one dimensional circular variation in Poisson's ratio as

$$\rho = \rho_0 \left[1 + m_2 \frac{x}{a} \right], \quad (10)$$

$$v = v_0 \left[1 - m_1 \left(1 - \sqrt{1 - \frac{x^2}{a^2}} \right) \right] \quad (11)$$

where $m_2, (0 \leq m_2 \leq 1)$ and $m_1 (0 \leq m_1 < 1)$ are known as non-homogeneity constant corresponding to density and Poisson's ratio.

The plate is subjected to steady two-dimensional parabolically varying temperature distributions as [11]

$$\bar{\tau} = \bar{\tau}_0 \left(1 - \frac{x^2}{a^2} \right) \left(1 - \frac{y^2}{a^2} \right) \quad (12)$$

Therefore the temperature dependent modulus of elasticity is taken as [16]

$$E(\tau) = E_0 (1 - \gamma \bar{\tau}) \quad (13)$$

$$E(\tau) = E_0 \left[1 - \gamma \bar{\tau}_0 \left(1 - \frac{x^2}{a^2} \right) \left(1 - \frac{y^2}{a^2} \right) \right] \quad (14)$$

$$E(\tau) = E_0 \left[1 - \alpha \left(1 - \frac{x^2}{a^2} \right) \left(1 - \frac{y^2}{a^2} \right) \right]$$

Where $\alpha = \gamma \bar{\tau}_0, (0 \leq \alpha < 1)$

Boundary Condition

For SSSS, the boundary conditions are [19]

$$\phi = \frac{\partial^2 \phi}{\partial x^2} = 0 \text{ at } x = 0, a \quad \text{and} \quad \phi = \frac{\partial^2 \phi}{\partial y^2} = 0 \text{ at } y = 0, b$$

The deflection function (i.e. maximum displacement) which satisfy boundary condition given in as:

$$\phi(x, y) = \left(\frac{x}{a} \right) \left(\frac{y}{b} \right) \left(1 - \frac{x}{a} \right) \left(1 - \frac{y}{b} \right) \left[A_1 + A_2 \left(\frac{x}{a} \right) \left(\frac{y}{b} \right) \left(1 - \frac{x}{a} \right) \left(1 - \frac{y}{b} \right) \right] \quad (15)$$

where A_1 and A_2 are arbitrary constants.

Rayleigh Ritz Method In Rectangular Plate

We are using Rayleigh Ritz technique (i.e., maximum strain energy S_E must equal to maximum kinetic energy K_E) in order to obtain frequency equation for both modes of vibrations. As a result, we need to have:

$$\delta(S_E - K_E) = 0$$

(16)

The K_E and S_E formula are provided by [20]

$$K_E = \frac{1}{2} P^2 \int_0^a \int_0^b \rho h \phi^2 dx dy$$

(17)

$$S_E = \frac{1}{2} \int_0^a \int_0^b D_1 \times \left[\left(\frac{\partial^2 \phi}{\partial x^2} \right)^2 + \left(\frac{\partial^2 \phi}{\partial y^2} \right)^2 + 2\nu \frac{\partial^2 \phi}{\partial x^2} \frac{\partial^2 \phi}{\partial y^2} + 2(1 - \nu) \left(\frac{\partial^2 \phi}{\partial x \partial y} \right)^2 \right] dx dy \quad (18)$$

The non-dimensional variables are outlined to make the computation simple and convenient:

$$X = \frac{x}{a}, \quad Y = \frac{y}{b}, \quad \bar{h} = \frac{h}{a} \quad \text{and} \quad \bar{\phi} = \frac{\phi}{a}$$

(19)

Solution Of Frequency Equation

on using the above assumptions along with (19); equation (17) and (18) becomes

$$K_E = \frac{1}{2} p^2 \int_0^a \int_0^b \rho_0 \left[1 + m_2 \frac{x}{a} \right] h_0 \left[1 + \beta \left(1 - \sqrt{1 - \frac{x^2}{a^2}} \right) \right] \phi^2 dy dx$$

$$= \frac{1}{2} \rho_0 p^2 h_0 \int_0^a \int_0^b \left[1 + m_2 \frac{x}{a} \right] \left[1 + \beta \left(1 - \sqrt{1 - \frac{x^2}{a^2}} \right) \right] \phi^2 dy dx$$

$$\text{Substituting } X = \frac{x}{a} \quad \bigg| \quad Y = \frac{y}{a}$$

$$\begin{aligned} x \rightarrow 0 &\Rightarrow X \rightarrow 0 & y \rightarrow 0 &\Rightarrow Y \rightarrow 0 \\ x \rightarrow a &\Rightarrow X \rightarrow 1 & y \rightarrow b &\Rightarrow Y \rightarrow \frac{b}{a} \end{aligned}$$

$$K_E = \frac{1}{2} \rho_0 p^2 h_0 \int_0^1 \int_0^{b/a} [1 + m_2 X] [1 + \beta (1 - \sqrt{1 - X^2})] \left(\frac{\phi}{a} \right)^2 a^2 dX dY$$

$$= \frac{1}{2} \rho_0 p^2 h_0 \int_0^1 \int_0^{b/a} [1 + m_2 X] [1 + \beta (1 - \sqrt{1 - X^2})] \bar{\phi}^2 a^4 dX dY$$

$$\begin{aligned}
 &= \frac{1}{2} \rho_0 P^2 \frac{h_0}{a} a \times a^4 \int_0^1 \int_0^{b/a} [1 + m_2 X] [1 + \beta (1 - \sqrt{1 - X^2})] \bar{\phi}^2 dX dY \\
 &= \frac{1}{2} p_0 p^2 \bar{h}_0 a^5 \int_0^1 \int_0^{b/a} [1 + m_2 X] [1 + \beta (1 - \sqrt{1 - X^2})] \bar{\phi}^2 dX dY
 \end{aligned}
 \tag{20}$$

And

$$\begin{aligned}
 S_E &= \frac{1}{2} \int_0^a \int_0^b D_1 \left[\left(\frac{\partial^2 \phi}{\partial x^2} \right)^2 + \left(\frac{\partial^2 \phi}{\partial y^2} \right)^2 + 2\nu \frac{\partial^2 \phi}{\partial x^2} \frac{\partial^2 \phi}{\partial y^2} + 2(1 - \nu) \left(\frac{\partial^2 \phi}{\partial x \partial y} \right)^2 \right] dx dy \\
 &= \frac{1}{2} \int_0^a \int_0^b \frac{E h^3}{12(1 - \nu^2)} \left[\left(\frac{\partial^2 \phi}{\partial x^2} \right)^2 + \left(\frac{\partial^2 \phi}{\partial y^2} \right)^2 + 2\nu \frac{\partial^2 \phi}{\partial x^2} \frac{\partial^2 \phi}{\partial y^2} + 2(1 - \nu) \left(\frac{\partial^2 \phi}{\partial x \partial y} \right)^2 \right] dx dy \\
 &= \frac{1}{24(1 - \nu^2)} \int_0^a \int_0^b E_0 \left[1 - \alpha \left(1 - \frac{x^2}{a^2} \right) \left(1 - \frac{y^2}{a^2} \right) \right] h_0^3 \left[1 + \right. \\
 &\quad \left. \beta \left(1 - \sqrt{1 - \frac{x^2}{a^2}} \right) \right]^3 \left[\left(\frac{\partial^2 \phi}{\partial x^2} \right)^2 + \left(\frac{\partial^2 \phi}{\partial y^2} \right)^2 + \right. \\
 &\quad \left. + 2\nu \frac{\partial^2 \phi}{\partial x^2} \frac{\partial^2 \phi}{\partial y^2} + 2(1 - \nu) \left(\frac{\partial^2 \phi}{\partial x \partial y} \right)^2 \right] dx dy \\
 &= \frac{E_0 h_0^3}{24(1 - \nu^2)} \int_0^a \int_0^b \left[1 - \alpha \left(1 - \frac{x^2}{a^2} \right) \left(1 - \frac{y^2}{a^2} \right) \right] \left[1 + \beta \left(1 - \sqrt{1 - \frac{x^2}{a^2}} \right) \right]^3 \left[\left(\frac{\partial^2 \phi}{\partial x^2} \right)^2 + \left(\frac{\partial^2 \phi}{\partial y^2} \right)^2 + \right. \\
 &\quad \left. + 2\nu \frac{\partial^2 \phi}{\partial x^2} \frac{\partial^2 \phi}{\partial y^2} + 2(1 - \nu) \left(\frac{\partial^2 \phi}{\partial x \partial y} \right)^2 \right] dx dy \\
 &= \frac{E \bar{h}_0^3 a^3}{24(1 - \nu^2)} \int_0^1 \int_0^{b/a} [1 - \alpha(1 - X^2)(1 - Y^2(a/b))] [1 + \beta(1 - \sqrt{1 - X^2})]^3 \left[\left(\frac{\partial^2 \phi}{\partial x^2} \right)^2 + \left(\frac{\partial^2 \phi}{\partial y^2} \right)^2 + \right. \\
 &\quad \left. + 2\nu \frac{\partial^2 \phi}{\partial x^2} \frac{\partial^2 \phi}{\partial y^2} + 2(1 - \nu) \left(\frac{\partial^2 \phi}{\partial x \partial y} \right)^2 \right] a^2 dX dY \\
 &= \frac{E \bar{h}_0^3 a^3}{24(1 - \nu^2)} \int_0^1 \int_0^{b/a} [1 - \alpha(1 - X^2)(1 - Y^2(a/b))] [1 + \beta(1 - \sqrt{1 - X^2})]^3 \left[\left(\frac{\partial^2 \bar{\phi}}{\partial x^2} \right)^2 + \left(\frac{\partial^2 \bar{\phi}}{\partial y^2} \right)^2 + \right. \\
 &\quad \left. + 2\nu \frac{\partial^2 \bar{\phi}}{\partial x^2} \frac{\partial^2 \bar{\phi}}{\partial y^2} + 2(1 - \nu) \left(\frac{\partial^2 \bar{\phi}}{\partial x \partial y} \right)^2 \right] dX dY
 \end{aligned}
 \tag{21}$$

Using equation (19) and (20) in equation (16) represents the necessary frequency parameter.

$$\delta(S_E^* - \lambda^2 K_E^*) = 0 \quad (22)$$

$$S_E^* = \int_0^1 \int_0^{b/a} [1 - \alpha(1 - X^2)(1 - Y^2(a/b))] \left[1 + \beta(1 - \sqrt{1 - X^2})^3 \left(\frac{\partial^2 \bar{\phi}}{\partial x^2} \right)^2 + \left(\frac{\partial^2 \bar{\phi}}{\partial y^2} \right)^2 + 2\nu \frac{\partial^2 \bar{\phi}}{\partial x^2} \frac{\partial^2 \bar{\phi}}{\partial y^2} + 2(1 - \nu) \left(\frac{\partial^2 \bar{\phi}}{\partial x \partial y} \right)^2 \right] dXdY \quad (23)$$

$$K_E^* = \int_0^1 \int_0^{b/a} [1 + m_2 X] [1 + \beta(1 - \sqrt{1 - X^2})] \bar{\phi}^2 dXdY \quad (24)$$

Here expression of the required frequency parameter is

$$\lambda^2 = \frac{12\rho_0 p^2 a^2 (1 - \nu^2)}{E_0 \bar{h}_0^2} \quad (25)$$

Equation (22) contains two unknown constants, A_1 and A_2 which result from the substitution of deflection function $\phi(x, y)$.

The following formula could be used to determine these two unknowns:

$$\frac{\partial}{\partial A_n} [S_E^* - \lambda^2 K_E^*] = 0 \quad (26)$$

After simplifying equation (26) we get system of homogeneous eq. as

$$C_{11}A_1 + C_{12}A_2 = 0$$

and

$$C_{21}A_1 + C_{22}A_2 = 0 \quad (27)$$

The determinant of the coefficient matrix obtained from equation (27) must be zero in order to produce a non-zero solution (frequency equation).

$$\begin{vmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{vmatrix} = 0 \quad (28)$$

After simplifying above equation we get a quadratic equation in λ . With λ representing frequency modes derived from equation (28), the time period of frequency modes is computed as $k = \frac{2\pi}{\lambda}$.

Result And Discussion

Duralumin, an aluminium alloy, is a visco-elastic material that produces the intended results. The calculations for Duralumin make use of the following parameters:

$$E_0 = 7.08 \times 10^{10} \text{ N/M}^2,$$

$$G = 2.632 \times 10^{10} \text{ N/M}^2,$$

$$\eta = 14.612 \times 10^5 \text{ N s/M}^2,$$

$$\rho_0 = 2.8 \times 10^3 \text{ kg/M}^3,$$

$$\nu = 0.345 \text{ and } h_0 = 0.01 \text{ M}$$

- I) For aspect ratios of 1.5 and 2.5, calculations were carried out for the first two frequency modes for various values of the thermal gradient (α), non-homogeneity(m_2), and taper parameter (β).
- II) The first two modes of the frequency parameter in Tables (1) and (4) increases continuously for both aspect ratios of 1.5 and 2.5 for every fixed value of the thermal gradient (α) while the taper parameter (β) increases from 0.2 to 0.6 and non-homogeneity (m_2) stays constant at $\nu = 0.345$. In every cases, the first two modes of the frequency parameter decrease as the thermal gradient (α) values rise from 0.0 to 0.8.
- III) For any fixed value of non-homogeneity (m_2) in Tables (2) and (5), the first two modes of the frequency parameter increase steadily for both aspect ratios of 1.5 and 2.5 as the values of the thermal gradient (α) and taper parameter (β) rise from 0.2 to 0.8 with $\nu = 0$. In every scenario, the first two modes of the frequency parameter drop as the non-homogeneity (m_2) values rise from 0.0 to 1.0.
- IV) In Table (3) and (6), for each fixed value of taper parameter(β), the first two mode of frequency parameter decrease continuously for both aspect ratio 1.5 and 2.5 as value of the thermal gradient (α) and non-homogeneity(m_2) increase from 0.2 to 0.6 with $\nu = 0.345$. As the values of the taper parameter(β) increases from 0.0 to 1.0, the first two mode of frequency parameter increases for all cases.

Table1. Frequency of simple supported rectangular plate vs Thermal gradient(α) for Aspect Ratio 1.5

α	$\beta = 0.2, m_2 = 0, \nu=0.345$		$\beta = 0.4, m_2 = 0, \nu=0.345$		$\beta = 0.6, m_2 = 0, \nu=0.345$	
	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2

0	57.6608	430.5953	60.2813	448.4819	63.0982	467.9346
0.2	54.9658	413.0808	57.5806	431.0280	60.3886	450.5353
0.4	52.1281	394.7905	54.7424	412.8375	57.5460	432.4374
0.6	49.1219	375.6113	51.7426	393.8084	54.5479	413.5491
0.8	45.9121	355.3994	48.5489	373.8130	51.3648	393.7572

Graphical representation of the table-1 :

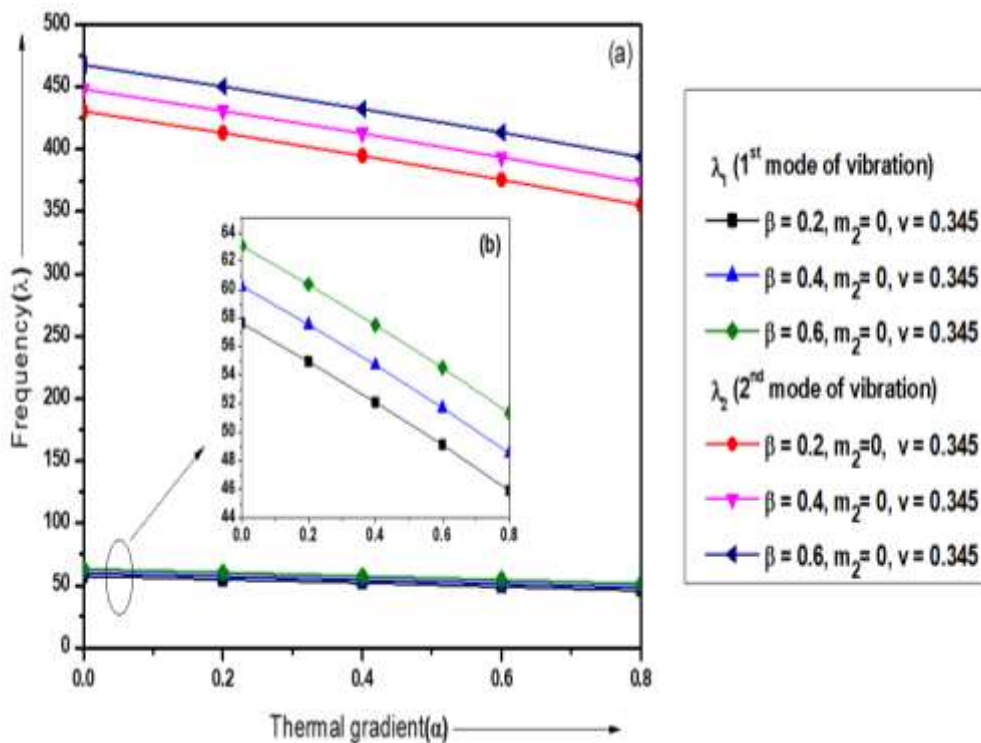


Figure-1 : Thermal gradient vs Frequency

Table2. Frequency of simple supported rectangular plate vs Non-Homogeneity(m_2) for Aspect Ratio 1.5

m_2	$\alpha = \beta = 0.2, \nu = 0$		$\alpha = \beta = 0.6, \nu = 0$		$\alpha = \beta = 0.8, \nu = 0$	
	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
0.0	51.9895	388.0317	52.7129	389.3578	53.2293	391.3813
0.2	49.5520	369.6947	50.2089	370.4594	50.6862	372.1593
0.4	47.4281	353.7337	48.0308	354.0710	48.4758	355.5184
0.6	45.5558	339.6762	46.1137	339.6819	46.5315	340.9281
0.8	43.8890	327.1717	44.4092	326.9160	44.8038	327.9989
1	42.3928	315.9538	42.8807	315.4893	43.2553	316.4377

Graphical representation of the table-2 :

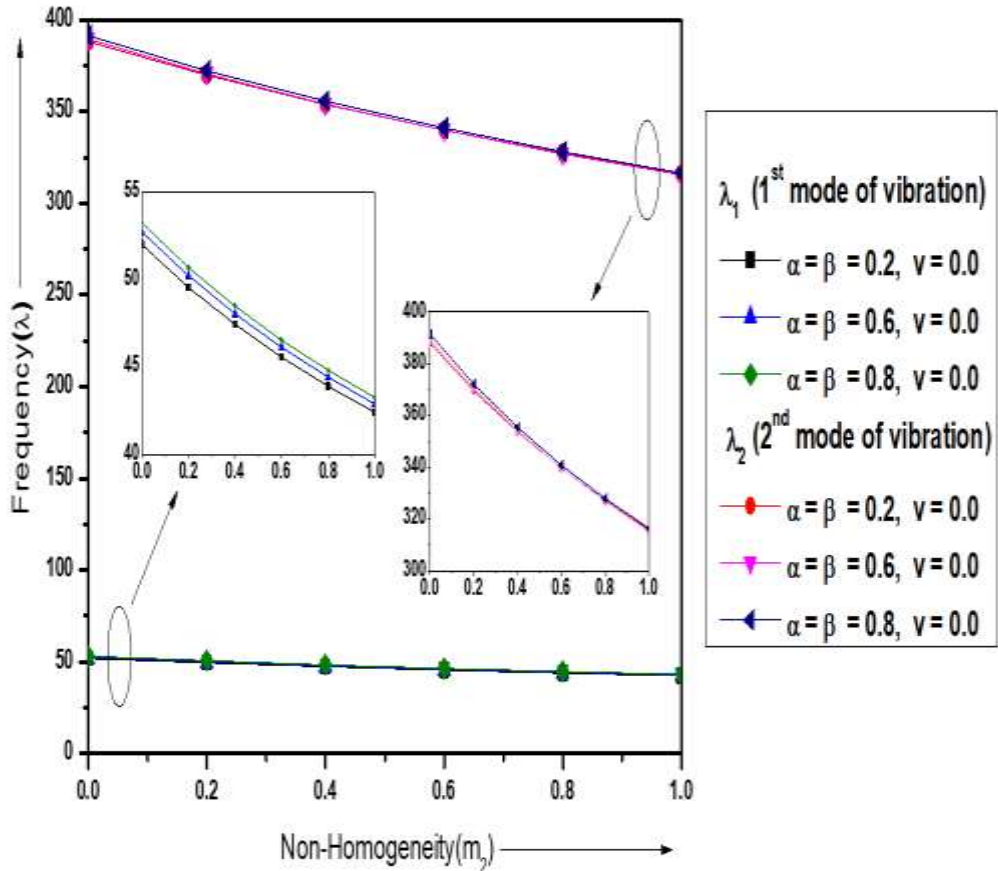


Figure-2 : Non-Homogeneity vs Frequency

Table 3. Frequency of simple supported rectangular plate vs Taper constant(β) for Aspect Ratio 1.5

β	$\alpha = m_2 = 0.2, \nu=0.345$		$\alpha = m_2 = 0.4, \nu=0.345$		$\alpha = m_2 = 0.8, \nu=0.345$	
	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
0	50.1170	378.3390	45.3905	345.4425	36.7436	286.2095
0.2	52.3888	393.5601	47.5547	359.8946	38.7589	299.6551
0.4	54.8624	410.3737	49.9086	375.8674	40.9417	314.4937
0.6	57.5196	428.6701	52.4343	393.2489	43.2739	330.6070
0.8	60.3422	448.3324	55.1139	411.9211	45.7383	347.8744
1	63.3130	469.2423	57.9309	431.7667	48.3197	366.1790

Graphical representation of the table-3 :

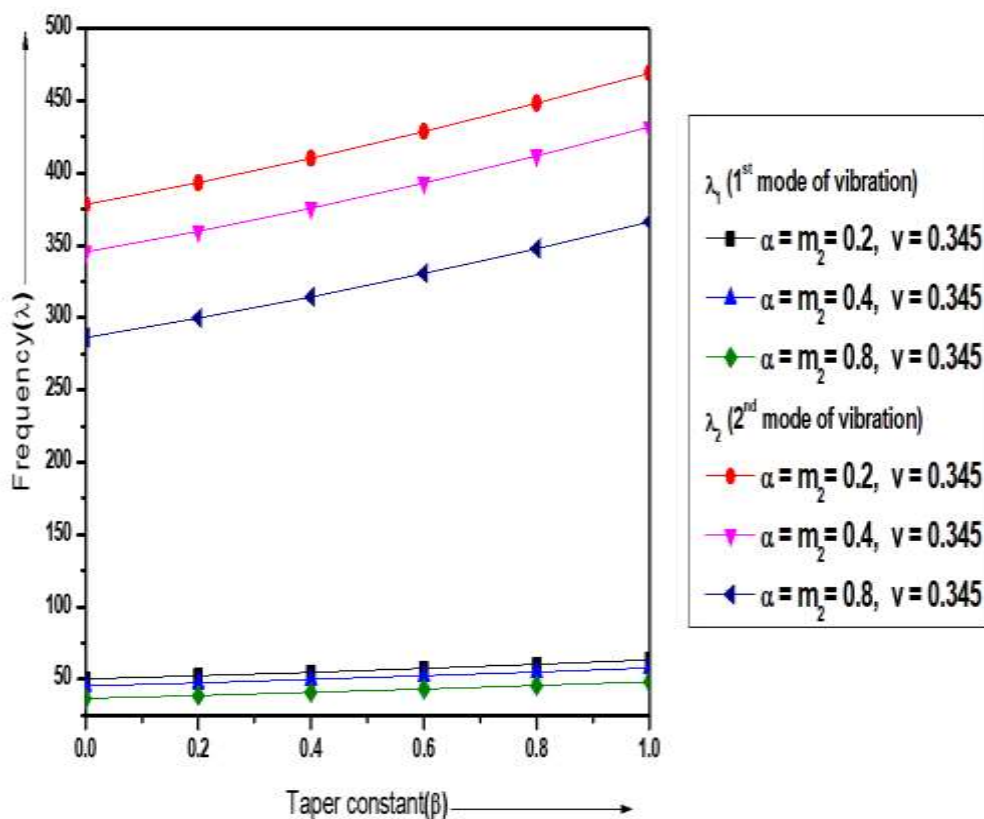


Figure-3 : Taper constant vs Frequency

Table 4. Frequency of simple supported rectangular plate vs Thermal gradient(α) for Aspect Ratio 2.5

α	$\beta = 0.2, m_2 = 0, \nu = 0.345$		$\beta = 0.4, m_2 = 0, \nu = 0.345$		$\beta = 0.6, m_2 = 0, \nu = 0.345$	
	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
0	46.1175	385.4861	47.9804	397.5019	49.9649	410.1599
0.2	43.8635	369.4820	45.7102	381.3286	47.6766	393.8089
0.4	41.4850	352.7527	43.3186	364.4385	45.2697	376.7493
0.6	38.9587	335.1899	40.7834	346.7271	42.7231	358.8802
0.8	36.2524	316.6550	38.0745	328.0615	40.0087	340.0742

Graphical representation of the table-4 :

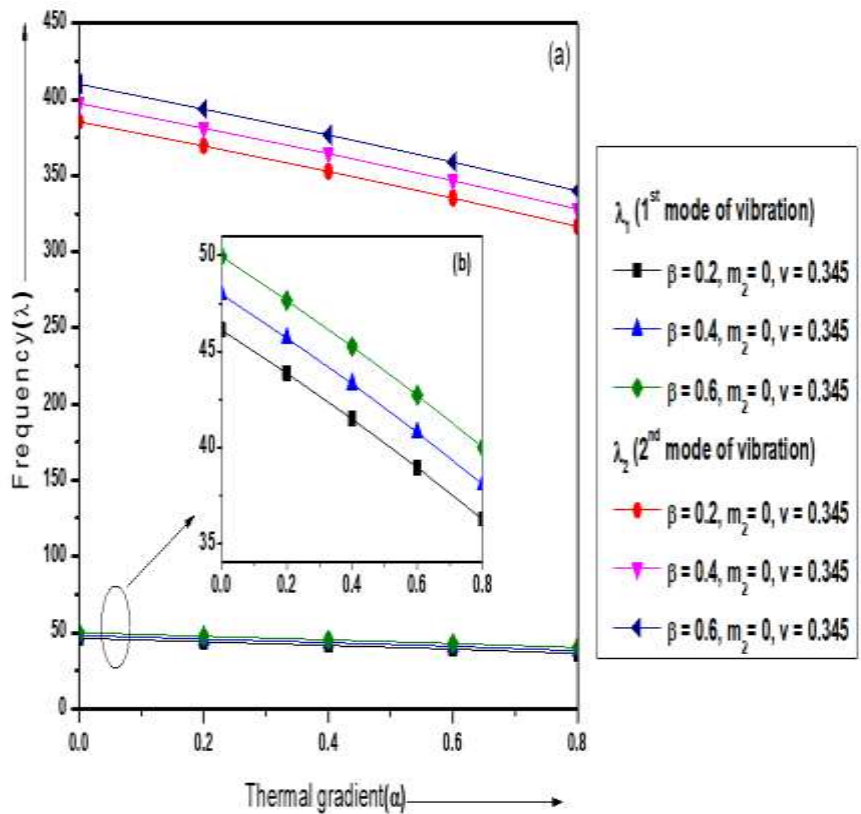


Figure-4 : Thermal gradient vs Frequency

Table 5. Frequency of simple supported rectangular plate vs Non-Homogeneity(m_2) for Aspect Ratio 2.5

m_2	$\alpha = \beta = 0.2, \nu = 0$		$\alpha = \beta = 0.6, \nu = 0$		$\alpha = \beta = 0.8, \nu = 0$	
	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
0.0	41.3505	346.9228	40.8019	337.3437	40.5111	331.7484
0.2	39.4118	330.5286	38.8635	320.9718	38.5752	315.4584
0.4	37.7225	316.2587	37.1774	306.7741	36.8926	301.3554
0.6	36.2333	303.6906	35.6934	294.3083	35.4127	288.9900
0.8	34.9076	292.5109	34.3739	283.2486	34.0976	278.0322
1	33.7176	282.4815	33.1907	273.3490	32.9189	268.2336

Graphical representation of the table-5 :

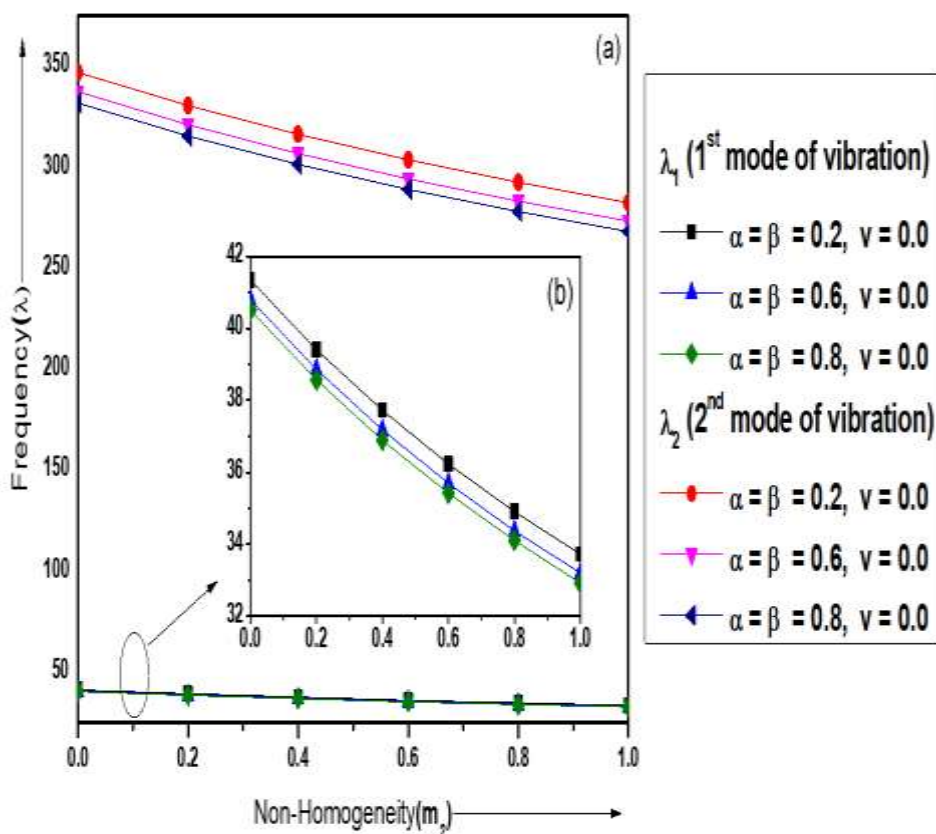


Figure-5 : Non-Homogeneity vs Frequency

Table 6. Frequency of simple supported rectangular plate vs Taper constant(β) for Aspect Ratio 2.5

β	$\alpha = m_2 = 0.2, \nu = 0.345$		$\alpha = m_2 = 0.4, \nu = 0.345$		$\alpha = m_2 = 0.8, \nu = 0.345$	
	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
0.0	40.1848	341.6386	36.3133	311.9610	29.2029	258.5319
0.2	41.8070	352.0218	37.8453	321.5730	30.6041	266.9886
0.4	43.5523	363.0564	39.4934	331.8040	32.1082	276.0051
0.6	45.4114	374.6983	41.2481	342.6104	33.7059	285.5391
0.8	47.3746	386.9037	43.1001	353.9494	35.3877	295.5494
1	49.4325	399.6306	45.0401	365.7798	37.1448	305.9970

Graphical representation of the table-6 :

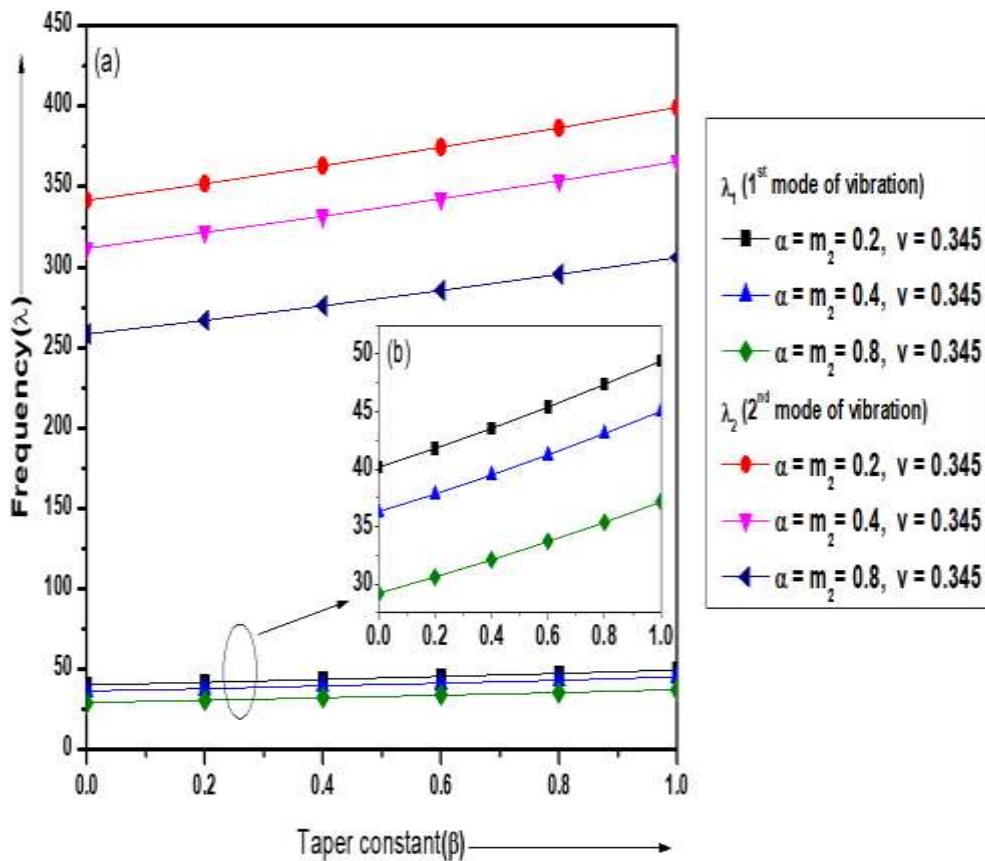


Figure-6 : Taper constant vs Frequency

Conclusion

This study investigated the frequencies of isotropic rectangular plates with circular thickness and variations in density and linear temperature. The above result indicates that when the tapering constant (β) increases, the frequency λ_1 and λ_2 decreases at varying values of the temperature gradient (α) and non-homogeneity (m_2). However, an increase in temperature gradient (α) and non-homogeneity (m_2) leads to an increase in frequency. The implementation of circular variation causes the frequency modes λ_1 and λ_2 to vary very slowly, whether they are growing or decreasing. There is no significant increase or decrease in the frequencies.

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