Intuitionistic Fuzzy Semihypergraphs

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Abstract
A Semihypergraph is a connected hypergraph in which each hyperedge must have atleast three vertices and each pair of hyperedges has atleast one vertex in common. In this article, the intuitionistic fuzzy semihypergraphs, semi μ- strong, semi ν- strong, strong IFSHGs and effective IFSHGs are introduced and some kinds of IFSHG such as simple, support simple, elementary, sectionally elementary IFSHGs are discussed.
Keywords: Intuitionistic fuzzy semihypergraphs (IFSHGs), Semi $\mu-$strong, semi $\nu-$ strong, strong IFSHGs, Effective IFSHGs, simple, support simple, elementary and sectionally elementary.
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1. INTRODUCTION

In 1736, the idea of graph theory was first proposed by Euler. Graph theory concepts have applications in many fields of computer science, including picture capture, networking, data mining and so on. Mathematical models are created to deal with many types of systems that involve components of uncertainty. Zadeh [12] defined the notion of fuzzy relation as a technique of communicating uncertainty and ambiguity in 1965. After that, the theory of fuzzy sets has evolved into a thriving field of study in a variety of areas. When compared to fuzzy models, intuitionistic fuzzy models provide higher precision, adaptability, and compatibility to the system.

The hypergraph theory was introduced by Berge [3] in 1976 and it has been regarded as a valuable tool for analyzing the structure of a system. In 1983, as an extension of fuzzy sets, Atanassov [1] created the ideas of intuitionistic fuzzy graphs. The notion of hypergraphs has been extended in the fuzzy theory and the ideas of fuzzy hypergraphs was provided by Kaufmann.

In 2000, Mordeson, N. J. and Nair, S. P.[4] introduced the concepts of fuzzy graph and fuzzy hypergraph. In 2006[7], intuitionistic fuzzy graph and in 2009,[8] Intuitionistic fuzzy hypergraph (IFHG) was introduced.

Semigraph is a generalization of the concept of graph which was introduced by E.Sampathkumar[11] in 2000, which has applications in road network, telecommunications etc., In a semigraph the edges are assumed to have many parts, and vertices are classified according to their positions. Various works have been carried out on Semigraph concepts. In 2020, Fuzzy Semigraph has been introduced by K.Radha and P.Renganathan[10].

Intuitionistic fuzzy directed hypergraph was introduced in 2013[9] whereas in [5], certain types intuitionistic fuzzy directed hypergraph and their properties were defined. In [6] the concepts of intuitionistic fuzzy $k-$partite hypergraphs and some its types are introduced.

In this paper, the concept of Intuitionistic Fuzzy Semihypergraph(IFSHG) and some types of IFSHGs have been introduced. This may lead to wide scope for research. In Section 2, the definitions of intuitionistic fuzzy sets, cartesian product of IFS, intuitionistic fuzzy hypergraphs and semigraphs are provided. In section 3, the notion of semihypergraphs, fuzzy semihypergraph and intuitionistic fuzzy semihypergraphs are introduced. The concept of strong IFSHG, effective IFSHG are also provided with examples. Section 4 the ideas of simple, support simple, elementary, sectionally elementary IFSHGs, $(\mu,\nu)-$tempered IFSHGs are introduced. Section 5 gives the conclusion for this paper.

2. PRILIMINARIES

Definition 2.1. [2] Let a set E be fixed. An *intuitionistic fuzzy set (IFS) V* in *E* is an object of the form $V=\{\langle v_i,\mu_i(v_i),\nu_i(v_i)\rangle / v_i\in E\}$, where the function $\mu_i:E\to[0,1]$ determine the degree of membership and $\nu_i:E\to[0,1]$ denote the degree of non-membership of the element $v_i\in E$, respectively and for all $v_i\in E, 0\le\mu_i(v_i)+\nu_i(v_i)\le 1$.

Definition 2.2. [7] The five cartesian products of two IFSs V_1,V_2 of V over E is defined as

$V_1\times_1 V_2=\{\langle(v_1,v_2),\mu_1.\mu_2,\nu_1.\nu_2\rangle \mid v_1\in V_1, v_2\in V_2\}$,

$V_1\times_2 V_2=\{\langle(v_1,v_2),\mu_1+\mu_2-\nu_1.\nu_2\rangle \mid v_1\in V_1, v_2\in V_2\}$,

$V_1\times_3 V_2=\{\langle(v_1,v_2),\mu_1.\mu_2,\nu_1+\nu_2-\nu_1.\nu_2\rangle \mid v_1\in V_1, v_2\in V_2\}$,

$V_1\times_4 V_2=\{\langle(v_1,v_2),min(\mu_1,\mu_2),max(\nu_1,\nu_2)\rangle \mid v_1\in V_1, v_2\in V_2\}$,

$V_1\times_5 V_2=\{\langle(v_1,v_2),max(\mu_1,\mu_2),min(\nu_1,\nu_2)\rangle \mid v_1\in V_1, v_2\in V_2\}$

It must be noted that $v_i\times_t v_j$ is an IFS, where $t=1,2,3,4,5$.

Definition 2.3. [8] An *intuitionistic fuzzy hypergraph* (IFHG) H is an ordered pair $H=\langle V,E\rangle$ where, (i) $V=\{v_1,v_2,...,v_n\}$ is a finite set of vertices,

(ii) $E=\{E_1,E_2...,E_m\}$, a family of intuitionistic fuzzy subsets of V,

(iii) $E_j=\{(v_i,\langle\mu_j,\gamma_j\rangle(v_i)):\langle\mu_j,\gamma_j\rangle(v_i)\ge 0\}$ and $0\le\mu_j+\gamma_j\le 1$, for all j. Here $E_j\subseteq V\times V$ where $\mu_{ij}:V\times V\to[0,1]$ are such that $\mu_{ij}\le min(\mu_i,\mu_j)$; $\gamma_{ij}\le max(\gamma_i,\gamma_j)$ and $0\le\mu_{ij}+\gamma_{ij}\le 1$, where μ_{ij} & γ_{ij} are the membership and non-membership values of the edge (v_i,v_j),

(iv) $E_j\ne\varnothing, j=1,2,\cdots,m$ and $\bigcup_j supp(E_j)=V,\ j=1,2,\cdots,m.$

Definition 2.4. [11] A *semigraph* S is a pair (V,X) where V is a nonempty set whose elements are called vertices of S and X is a set of ordered $n-$tuples $n\ge 2$ of distinct vertices called edges of S satisfying the following conditions:

- any two edges have atmost one vertex in common
- two edges $E_1=(u_1,u_2,..u_m)$ and $E_1=(v_1,v_2,..,v_n)$
- are said to be equal if and only if $(a)\, m=n$ and

(b) either $u_i=v_i$ or $u_i=v_n-v_{i+1}$ *for* $1\le i\le n.$

Notations:

$H_s = (V, E_{h_j}, <)$ - Semihypergraph

$F_{sh} = \left\langle V, C_v, E_{h_j} \right\rangle$ - Fuzzy Semihypergraphs (FSHG)

$I_{sh} = \left\langle V, C_v, E_{h_j} \right\rangle$ - Intuitionistic Fuzzy Semihypergraphs (IFSHG)

E_{h_j} - Hyperedges;

C_v - Consecutive vertices of hyperedges

$\langle \mu_i, \nu_i \rangle$ - membership and non-membership values of vertices $\langle \mu_{ij}^c, \nu_{ij}^c \rangle (v_i, v_j)$ or $\langle \mu_{ij}^c, \nu_{ij}^c \rangle$ - membership and non-membership values of consecutive vertices $\langle \mu, \nu \rangle (E_{h_j})$ -membership and non-membership values of hyperedges.

X - Set of all Intuitionistic Fuzzy hyperedges E_{h_j}

I'_{sh} - IF-subSHG

Note: Throughout this paper, it is assumed that the fourth cartesian product, $V_1 \times_4 V_2 = \{\langle (v_1, v_2), min(\mu_1, \mu_2), max(\nu_1, \nu_2) \rangle \mid v_1 \in V_1, v_2 \in V_2\}$, is used to find the membership and non-membership values of consecutive vertices $\langle \mu_{ij}^c, \nu_{ij}^c \rangle (v_i, v_j)$ and hyperedges $\langle \mu, \nu \rangle (E_{h_j})$.

3. Main Definitions

Definition 3.1. A *Semihypergraph* is a connected hypergraph $H_s = (V, E_{h_j}, <)$ where $V = \{v_i / i = 1, 2, \cdots, n\}$ be a non-empty, vertex order preserving finite set and $E_{h_j} = \{E_{h_1}, E_{h_2}, \cdots, E_{h_p}\}$ such that $E_{h_j} \in V$, where, $j = \{1, 2, \cdots, p\}$ with minimum of three vertices satisfying the conditions: (i) $E_{h_j} \neq \emptyset$ and $\cup E_{h_j} = V, 1 \le j \le p.$

(ii) Each pair of hyperedges have minimum of one vertex in common.

(iii) The order of vertices $(v_1, v_2, ... v_n)$ in hyperedge is equal to the reverse order of vertices $(u_1, u_2, ... u_m)$ iff $(a) n = m$ and (b) either $u_i = v_i$ or $u_i = v_{n-i+1}$ for $1 \le i \le n.$

Definition 3.2. Consider a semihypergraph $H_s = (V, E_h, <)$. A *fuzzy semihypergraph (FSHG)* $F_{sh} = \left\langle V, C_v, E_{h_j} \right\rangle$ is a semihypergraph where,

(i) $V = \{v_i / i = 1, 2, ..., n\}$ such that $\mu_i : V \to [0,1]$, $\forall v_i \in V$, where V is a finite, non-empty, vertex order preserving vertex set,

(ii) For every consecutive vertex (C_v) of hyperedges, $\mu_{ij}^c : V \times V \to [0,1]$ such that

$$\mu_{ij}^c(v_i, v_j) \le min(\mu_i(v_i, v_j)), \ 0 \le \mu_{ij}^c(v_i, v_j) \le 1,$$

(iii) For every $E_{h_j} \subseteq V \times V$, $\mu : V \times V \to [0,1]$ such that $\mu(E_{h_j}) = min(\mu(v_i, v_j)) \le \min(\mu_i(v_1, v_n))$, $\forall E_{h_j}, j = 1, 2, ...n$ where $0 \le \mu(E_{h_j}) \le 1.$

$\left\langle \mu_i, \mu_{ij}^c, \mu \right\rangle$ denotes the membership values of vertices, consecutive vertices and hyperedges respectively.

Definition 3.3. Consider a semihypergraph $H_s = (V, E_h, <)$. An *IFSHG* $I_{sh} = \left\langle V, C_v, E_{h_j} \right\rangle$ is a connected semihypergraph where,

(i) $V=\{v_i / i=1,2,...,n\}$ such that $\mu_i : V \to [0,1]$ and $\nu_i : V \to [0,1]$ for each element $v_i \in V$, where V be a finite, non-empty and vertex order preserving finite set,

(ii) For every consecutive vertices (C_v) of hyperedges, $\mu_{ij}^c : V \times V \to [0,1]$ and $\nu_{ij}^c : V \times V \to [0,1]$ are such that $\mu_{ij}^c(v_i, v_j) \le min(\mu_i(v_i, v_j))$ & $\nu_{ij}^c(v_i, v_j) \le max(\nu_i(v_i, v_j))$ where $0 \le \mu_{ij}^c(v_i, v_j) + \nu_{ij}^c(v_i, v_j) \le 1$,

(iii) For every $E_{h_j} \subseteq V \times V$, $\mu : V \times V \to [0,1]$ and $\nu : V \times V \to [0,1]$ such that

$\mu(E_{h_j}) = min(\mu_{ij}^c(v_i, v_j)) \le min(\mu_i(v_1, v_n))$ & $\nu(E_{h_j}) = max(\nu_{ij}^c(v_i, v_j)) \le max(\nu_i(v_1, v_n))$,

for all E_{h_j}, $j=1,2,3....n$ where $0 \le \mu(E_{h_j}) + \nu(E_{h_j}) \le 1$.

$\langle \mu_i, \nu_i \rangle$ denotes the membership and non-membership values of vertices,

$\langle \mu_{ij}^c, \nu_{ij}^c \rangle$ denote the membership and non-membership values of consecutive vertices,

$\langle \mu, \nu \rangle$ denotes the membership and non-membership values of hyperedges respectively.

Example 1. Consider the IFSHG. Here v_1, v_4 and v_8 are the end vertices, v_2, v_3, v_6 and v_7 are middle vertices, v_5 is middle end vertices.

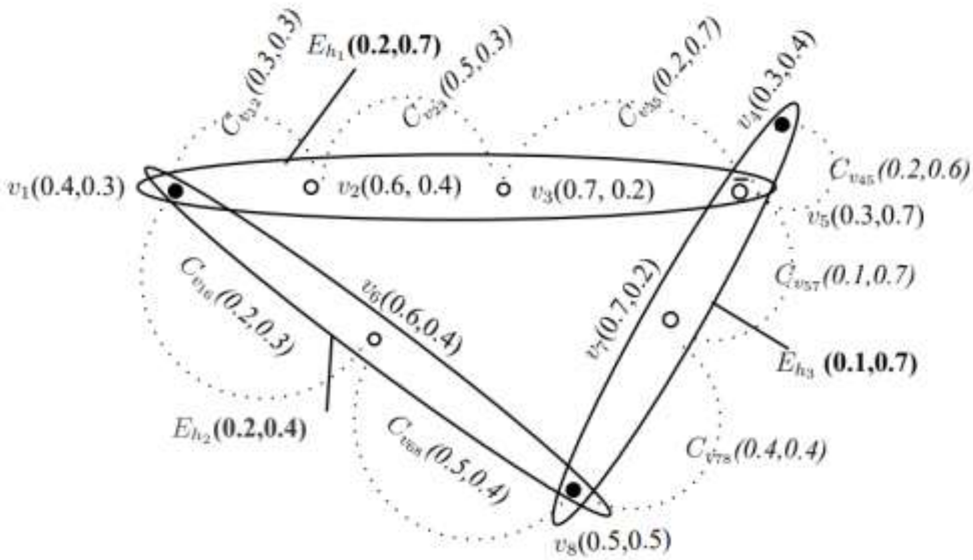


Figure 1: Intuitionistic Fuzzy Semihypergraphs

Definition 3.4. An IFSHG I_{sh} is called as *Semi* μ *- Strong* if

$$\mu(E_{h_j}) = min(\mu_{ij}^c(v_i, v_j)) = min(\mu_i(v_1, v_n)), \quad \nu(E_{h_j}) \le max(\nu_i(v_1, v_n)), \quad \forall E_{h_j}$$

Definition 3.5. An IFSHG I_{sh} is said to be *Semi* ν *- Strong* if

$$\nu(E_{h_j}) = max(\nu_{ij}^c(v_i, v_j)) = max(\nu_i(v_1, v_n)), \quad \mu(E_{h_j}) \le min(\mu_i(v_1, v_n)), \quad \forall E_{h_j}$$

Definition 3.6. The hyperedge $E_{h_j} = \langle \mu, \nu \rangle (E_{h_j})$ is called as *strong hyperedge* in IFSHG, if it satisfies the following conditions:

$\mu(E_{h_j}) = min(\mu_{ij}^c(v_i, v_j)) = min(\mu_i(v_1, v_n))$ & $\nu(E_{h_j}) = max(\nu_{ij}^c(v_i, v_j)) = max(\nu_i(v_1, v_n))$

Definition 3.7. An IFSHG I_{sh} is said to be *strong* IFSHG if all the hyperedges of I_{sh} are strong hyperedges.An example of strong hypergraph is given below. Here $E_{h_1}(v_1,v_2,v_3,v_4), E_{h_2}(v_4,v_5,v_8), E_{h_3}(v_1,v_2,v_7,v_8,v_9)$ and $E_{h_4}(v_3,v_6,v_7,v_8)$ are hyperedges.

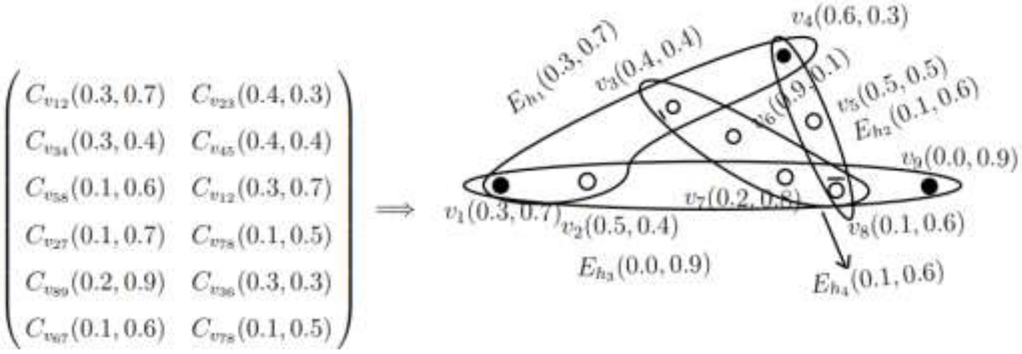


Figure 2: An example of strong IFSHG

Definition 3.8. An IFSHG I_{sh} is said to be *Complete μ Strong* if

$\mu(E_{h_j}) = min(\mu_{ij}^c(v_i,v_j)) = min(\mu_i(v_1,v_n))$ and $\nu(E_{h_j}) = max(\nu_{ij}^c) < max(\nu_i(v_1,v_n)), \ \forall E_{h_j}$.

Definition 3.9. An IFSHG I_{sh} is said to be *Complete ν Strong* if

$$\mu(E_{h_j}) = min(\mu_{ij}^c < min\left(\mu_i\left(v_1,v_n\right)\right) \text{ and } \nu(E_{h_j}) = max\left(\nu_{ij}^c\left(v_i,v_j\right)\right) = max\left(\nu_i\left(v_1,v_n\right)\right), \ \forall E_{h_j}.$$

Definition 3.10. The hyperedge E_{h_j} in IFSHG satisfying the following conditions:

$$i) \ \ \mu_{ij}^c(v_i,v_j) = min(\mu_i(v_i,v_j)) \text{ and } \nu_{ij}^c(v_i,v_j) = max(\nu_i(v_i,v_j)), \ \ \forall i \neq j,$$

$$ii) \ \ \mu(E_{h_j}) = min(\mu_i(v_1,v_n)) \text{ and } \nu(E_{h_j}) = max(\nu_i(v_1,v_n)), \ \ \forall (v_i,v_j) \in C_v \subseteq E_{h_j},$$

is said to be an *Effective hyperedge* in I_{sh}.

Definition 3.11. If all the hyperedges of I_{sh} are effective hyperedges, then an IFSHG is known as an effective IFSHG. Figure 3 is an example of effective IFSHG with $E_{h_1}(v_1,v_2,v_3), E_{h_2}(v_1,v_4,v_5,v_6)$ and $E_{h_3}(v_3,v_6,v_7)$ as hyperedges.

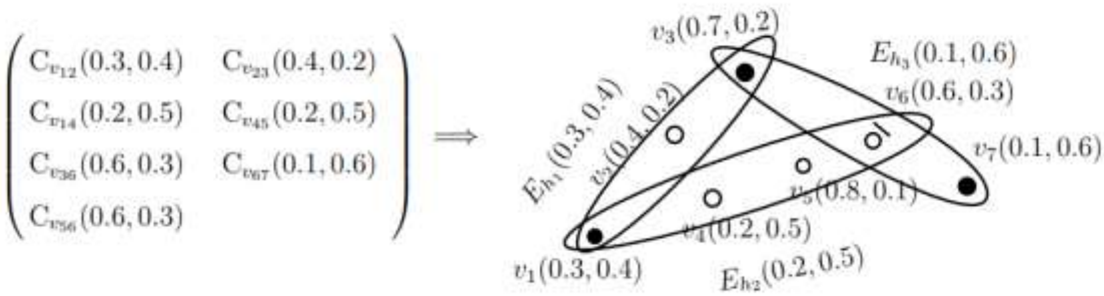


Figure 3: Effective IFSHG

Definition 3.12. An IFSHG I'_{sh} is said to be IF-subSHG of I_{sh} if

(i) $E_{h_j}(I'_{sh}) \subseteq E_{h_j}(I_{sh}), \ \forall h_j,$

(ii) $\mu'_i(v_i) \leq \mu_i(v_i) \,\&\, \nu'_i(v_i) \leq \nu_i(v_i), \ \ \forall v_i \in V',$

(iii) $(\mu_{ij}^c)'(v_i,v_j) \leq \mu_{ij}^c(v_i,v_j) \ \ \&\, (\nu_{ij}^c)'(v_i,v_j) \leq \nu_{ij}^c(v_i,v_j), \ \ \forall (v_i,v_j) \in V \times V$

$(iv)\quad \mu'(E_{h_j}) \le \mu(E_{h_j}) \,\&\, \nu'(E_{h_j}) \le \nu(E_{h_j}),\quad \forall E_{h_j} \in \mathrm{I}'_{sh}.$

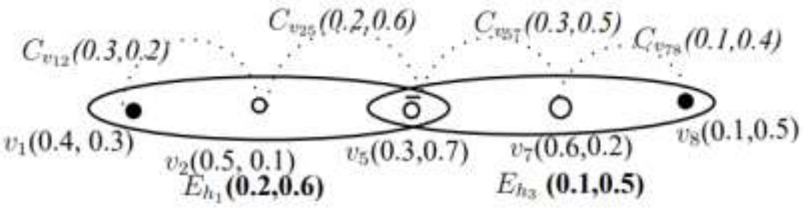


Figure 4: IF Sub-Semihypergraph

Definition 3.13. In IFSHG $\mathrm{I}_{sh}, V = \{v_1, v_2, \ldots, v_n\}$ and $E_{h_j} = \{E_{h_1}, E_{h_2}, ..., E_{h_n}\}$, its incident matrix is a matrix, denoted as $I = [a_{ij}]_{n\times m}$ and is define as

$$[a_{ij}] = \begin{cases} \langle \mu_{ij}^c, \nu_{ij}^c \rangle, & \forall (v_i, v_j) \in E_{h_j}, \\ \langle 0,1 \rangle, & otherwise \end{cases}$$

where n represents number of pair of consecutive vertices and m represents number of hyperedges and the element $\left[a_{ij}\right]$ represents the membership and non-membership of consecutive vertex for the corresponding hyperedges.

4. Types of IFSHGs

The types of IFSHGs such as simple, support simple, elementary, sectionally elementary and (μ,ν) - tempered have been introduced.

Definition 4.1. The support of a hyperedge E_{h_j}, in IFSHG denoted by $supp(E_{h_j})$, is defined as $supp(E_{h_j}) = \{v_i : \mu(E_{h_j}) > 0,\ \nu(E_{h_j}) > 0\}$.

Definition 4.2. Let I_{sh} be an IFSHG. The *height* of I_{sh} is defined by,

$$h(I_{sh}) = \left\{ max\Big(min\Big(\mu\Big(E_{h_j}\Big)\Big)\Big), max\Big(max\Big(\nu\Big(E_{h_j}\Big)\Big)\Big)\right\},\ \forall E_{h_j} \in I_{sh}$$

Definition 4.3. An IFSHG $\mathrm{I}_{sh} = \left\langle V, C_v, E_{h_j} \right\rangle$ is *simple* if E_{h_j} has no repeated intuitionistic fuzzy hyperedges and whenever $E_{h_x} = \langle \mu,\nu \rangle (E_{h_x}),\ E_{h_y} = \langle \mu,\nu \rangle (E_{h_y}) \in E_{h_j}$ and $E_{h_x} \subseteq E_{h_y}$ then $E_{h_x} = E_{h_y}$ for all x and y.

Example 2. Consider an IFSHG, I_{sh} such that $V = \{v_1, v_2, v_3, v_4, v_5, v_6\},\ E_{h_j} = \{E_{h_1}, E_{h_2}, E_{h_3}\}$.
The corresponding IFSHG, incidence matrix $\langle \mu_{ij}^c, \nu_{ij}^c \rangle$ and thevalues of each E_{h_j} is given in Figure 5, $a = C_{v_{12}}(0.5, 0.4),\ b = C_{v_{14}}(0.6, 0.3),\ c = C_{v_{23}}(0.5, 0.4),\ d = C_{v_{24}}(0.7, 0.2),$
$e = C_{v_{35}}(0.5, 0.4),\ f = C_{v_{45}}(0.6, 0.3),\ g = C_{v_{46}}(0.6, 0.3)$

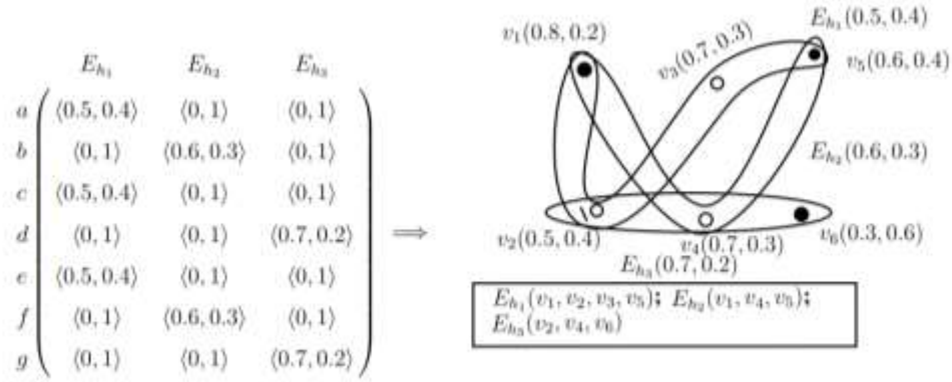


Figure 5: An example of simple IFSHG

Definition 4.4. An IFSHG I_{sh} is *support simple* if whenever $E_{h_x}, E_{h_y} \in E_{h_j}$, $E_{h_x} \subseteq E_{h_y}$ and $supp(E_{h_x}) = supp(E_{h_y})$ then $E_{h_x} = E_{h_y}$ for all h_x and h_y.

The hyperedges E_{h_x} and E_{h_y} are called as *supporting hyperedges*.

Example 3. Consider an IFSHG I_{sh} with vertex set $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$, $E_{h_j} = \{E_{h_1}, E_{h_2}, E_{h_3}\}$. $\langle \mu_{ij}^c, \nu_{ij}^c \rangle$ the corresponding IFSHG, it's values of each E_{h_j} and incidence matrix are given below: $a = C_{v_{12}}(0.5, 0.3)$, $b = C_{v_{13}}(0.5, 0.3)$, $c = C_{v_{25}}(0.2, 0.6)$, $d = C_{v_{23}}(0.4, 0.2)$, $e = C_{v_{34}}(0.3, 0.4)$, $f = C_{v_{35}}(0.3, 0.7)$, $g = C_{v_{46}}(0.3, 0.7)$.

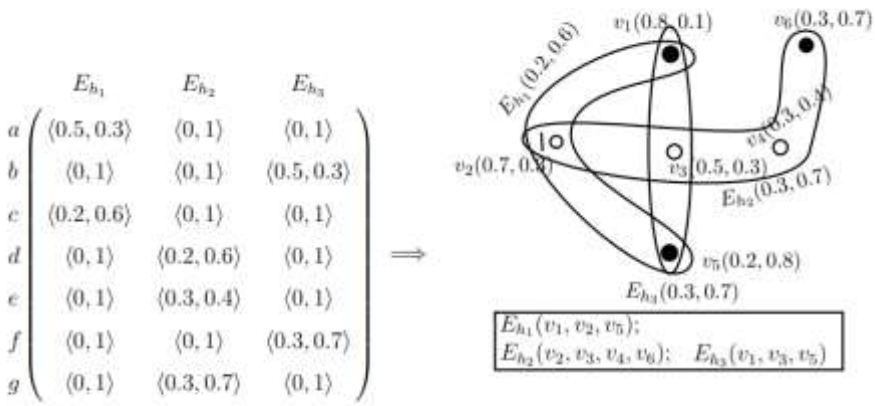


Figure 6: Support Simple IFSHG

Here E_{h_2} and E_{h_3} are supporting hyperedges.

Definition 4.5. Consider an IFSHG $I_{sh} = \left\langle V, C_v, E_{h_j} \right\rangle$, let $(E_{h_x}, E_{h_y}) \in E_{h_j}$ and $(\alpha, \beta) \in (0,1]$. Then, (x_i, y_i) $-$ *level* is defined as $E_h^{(x_i, y_i)} = \{v_i \in V / max(\mu^{\alpha}(E_{h_j})) \geq \alpha,\ min(\nu^{\beta}(E_{h_i})) \leq \beta\}$

Definition 4.6. Let $I_{sh} = \left\langle V, C_v, E_{h_j} \right\rangle$ be an IFSHG and $I_{sh}(x_i, y_i) = (V^{x_i, y_i}, E_h^{x_i, y_i})$ be the $\langle x_i, y_i \rangle$ - level IFSHG of I_{sh}. The sequence of real numbers $\{x_1, x_2, ..., x_n; y_1, y_2, ..., y_n\}$, such that $0 \leq x_i \leq h_{\mu}(I_{sh})$ and $0 \leq y_i \leq h_{\nu}(I_{sh})$, that satisfies the following properties:

(i) If $x_1 < \alpha \leq 1$ and $0 \leq \beta < y_1$ then $E_h^{\alpha, \beta} = \varnothing$,

(ii) If $x_{i+1} \le \alpha \le x_i$; $y_i \le \beta \le y_{i+1}$ then $E_h^{\alpha,\beta} = E_h^{x_i,y_i}$ and

(iii) $E^{x_{i+1},y_{i+1}}$ ð E^{x_i,y_i} is called the *fundamental sequence* $F(\mathrm{I}_{sh})$ of I_{sh}.

Definition 4.7. The *core set* of I_{sh}, is defined as $C(\mathrm{I}_{sh}) = \{\mathrm{I}_{sh}^{x_1,y_1}, \mathrm{I}_{sh}^{x_2,y_2}, ..., \mathrm{I}_{sh}^{x_n,y_n}\}$.
$I(\mathrm{I}_{sh}) = \mathrm{I}_{sh}^{x_k,y_k} \subset .... \subset \mathrm{I}_{sh}^{x_2,y_2} \subset \mathrm{I}_{sh}^{x_1,y_1}$ is called the I_{sh} *induced fundamental sequence.*

Definition 4.8. An IFSHG is said to be *elementary* if $\mu(E_{h_j}) : V \to [0,1]$ and $\nu(E_{h_j}) : V \to [0,1]$ are single valued on support of I_{sh} or constant functions or has range $(0,a]$, $a \ne 0$.

Example 4. Consider an IFSHG, such that $V = \{v_1, v_2, \cdots, v_6\}$ and $E_{h_j} = \{E_{h_1}, E_{h_2}, E_{h_3}, E_{h_4}\}$,

The incidence matrix and IFSHG with its corresponding is shown in Figure 7: $a = C_{v_{12}}(0.3, 0.5)$, $b = C_{v_{13}}(0.2, 0.6)$, $c = C_{v_{23}}(0.3, 0.5)$, $d = C_{v_{26}}(0.3, 0.5)$, $e = C_{v_{34}}(0.4, 0.4)$, $f = C_{v_{35}}(0.2, 0.6)$, $g = C_{v_{46}}(0.4, 0.4)$

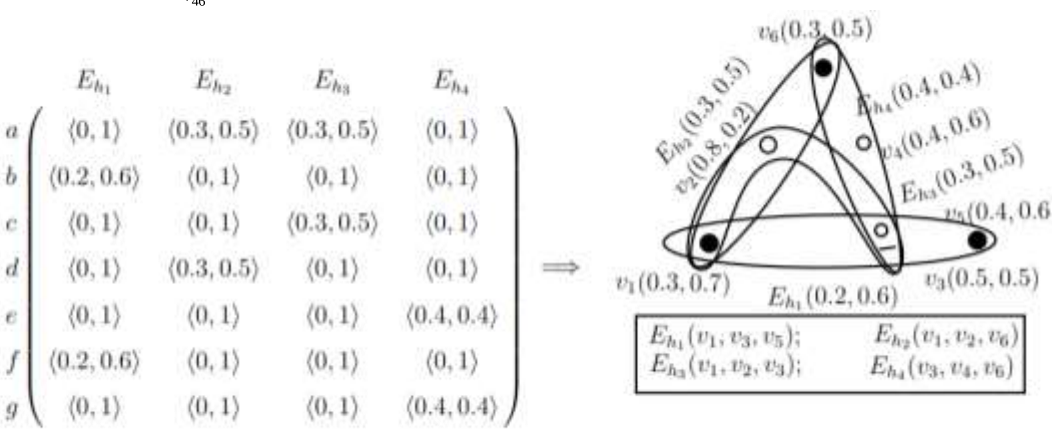


Figure 7: An Example of Elementary IFSHG

Lemma 4.1

An elementary IFSHG is an IFSHG in which all intuitionistic fuzzy hyperedges are elementary.

Definition 4.9. Let I_{sh} be an IFSHG. Let $F(I_{sh}) = \{(x_1, y_1), (x_2, y_2), \cdots, (x_n, y_n)\}$.
Then I_{sh} is called *sectionally elementary* if, for every hyperedge E_{h_j} and $\forall \langle x_i, y_i \rangle \in F(\mathrm{I}_{sh})$,

$$\mu_\alpha(E_{h_j}) = \mu_{x_i}(E_{h_j}),\ \nu_\beta(E_{h_j}) = \nu_{y_i}(E_{h_j}),\ \forall \alpha \in (x_{i+1}, x_i],\ \beta \in (y_{i+1}, y_i]$$

I_{sh} is sectionally elementary iff $\mu_\alpha(E_{h_j}),\ \nu_\beta(E_{h_j}) \in F(\mathrm{I}_{sh})$.

Example 5. Consider an IFSHG, such that $V = \{v_1, v_2, \cdots, v_7\}$ and $E_{h_j} = \{E_{h_1}, E_{h_2}, E_{h_3}, E_{h_4}\}$, given by the incidence matrix and the corresponding IFSHG with its $\langle \mu_{ij}^c, \nu_{ij}^c \rangle$ values are given in Figure 8: $a = C_{v_{16}}(0.1, 0.6)$, $b = C_{v_{23}}(0.4, 0.4)$, $c = C_{v_{24}}(0.3, 0.5)$, $d = C_{v_{35}}(0.4, 0.3)$, $e = C_{v_{37}}(0.4, 0.4)$, $f = C_{v_{45}}(0.3, 0.5)$, $g = C_{v_{56}}(0.4, 0.3)$, $h = C_{v_{67}}(0.1, 0.6)$

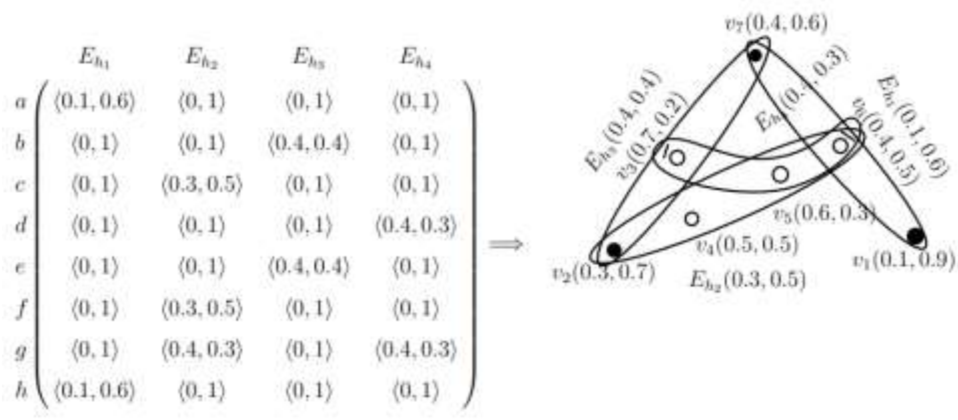


Figure 8: An Example of sectionally elementary IFSHG

Theorem 4.1. Let I_{sh} be an elementary intuitionistic fuzzy hypergraph. Then I_{sh} is support simple if and only if I_{sh} is strongly support simple.

Proof

Suppose that I_{sh} is elementary and support simple. Then $supp(E_{h_x}) = supp(E_{h_y})$. Assume without loss of generality that $h(E_{h_x}) \geq h(E_{h_y})$.

Since I_{sh} is elementary, we get that $\mu(E_{h_x}) \leq \mu(E_{h_y})$ and $\nu(E_{h_x}) \geq \nu(E_{h_y})$ and since I_{sh} is support simple, $E_{h_x} = E_{h_y}$. Therefore I_{sh} is strongly support simple.

The proof of converse part is obvious.

Definition 4.10. Let I_{sh} be an IFSHG and $C(I_{sh}) = \{I_{sh}^{x_1,y_1}, I_{sh}^{x_2,y_2}, ...I_{sh}^{x_n,y_n}\}$ is said to be *ordered* if $I_{sh}^{x_n,y_n} \subset ... \subset I_{sh}^{x_2,y_2} \subset I_{sh}^{x_1,y_1}$.

The IFSHG is said to be *simply ordered* if I_{sh} is ordered and if whenever $E_{h_j} \in I_{sh}^{x_{i+1},y_{i+1}} - I_{sh}^{x_i,y_i}$ then $E_{h_j} \not\subseteq I_{sh}^{x_i,y_i}$.

Theorem 4.2. If I_{sh} is an elementary IFSHG, then I_{sh} is ordered. Also, if I_{sh} is ordered IFSHG with $C(I_{sh}) = \{I_{sh}^{x_1,y_1}, I_{sh}^{x_2,y_2}, ...I_{sh}^{x_n,y_n}\}$ and if $I_{sh}^{x_n,y_n}$ is simple, then I_{sh} is elementary.

Definition 4.11. An IFSHG I_{sh} is called $(\mu,\nu)-$ *tempered IFSHG* of I_{sh} if there is an intuitionistic fuzzy subset, defined by $\mu_{ij} : V \to [0,1]$ and $\nu_{ij} : V \to [0,1]$ such that $E_{h_j} = \{(\mu_{E_{h_j}}(C_v),\ \nu_{E_{h_j}}(C_v)),\ \forall C_v \in E_{h_j}\}$ where

$$\mu_{E_{h_j}}(C_v) = \begin{cases} min(\mu(y) / y \in E_{h_j}) & \text{if } C_v \in E_{h_j} \\ 0 & otherwise \end{cases} \text{ and}$$

$$\nu_{E_{h_j}}(C_v) = \begin{cases} \max(\nu(y) / y \in E_{h_j}) & \text{if } C_v \in E_{h_j} \\ 1 & otherwise \end{cases}$$

Example 6. Consider an IFSHG, I_{sh} such that $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$, $E_{h_j} = \{E_{h_1}, E_{h_2}, E_{h_3}\}$, whose incidence matrix is

$$\begin{array}{c} \\ a \\ b \\ c \\ d \\ e \\ f \end{array} \begin{array}{c} \begin{array}{ccc} E_1 & E_2 & E_3 \end{array} \\ \begin{pmatrix} \langle 0,1\rangle & \langle 0,1\rangle & \langle 0.7,0.2\rangle \\ \langle 0.5,0.5\rangle & \langle 0,1\rangle & \langle 0,1\rangle \\ \langle 0,1\rangle & \langle 0,1\rangle & \langle 0.7,0.2\rangle \\ \langle 0,1\rangle & \langle 0.6,0.4\rangle & \langle 0,1\rangle \\ \langle 0.5,0.5\rangle & \langle 0,1\rangle & \langle 0,1\rangle \\ \langle 0,1\rangle & \langle 0.6,0.4\rangle & \langle 0,1\rangle \end{pmatrix} \end{array}$$

Then $E_h^{(0.5,0.5)} = \{\{b,e\},\{d,f\},\{a,c\}\}$, $E_h^{(0.6,0.4)} = \{\{d,f\},\{a,c\}\}$, $E_h^{(0.7,0.2)} = \{\{a,c\}\}$.

Define $\mu_{ij}^c : V \to [0,1]$ and $\nu_{ij}^c : V \to [0,1]$ by, $\mu_{12}(v_1,v_2) = 0.7$, $\mu_{23}(v_2,v_3) = 0.5$, $\mu_{25}(v_2,v_5) = 0.7$, $\mu_{45}(v_4,v_5) = 0.6$, $\mu_{36}(v_3,v_6) = 0.5$, $\mu_{56}(v_5,v_6) = 0.6$ and $\nu_{12}(v_1,v_2) = 0.2$, $\nu_{23}(v_2,v_3) = 0.5$, $\nu_{25}(v_2,v_5) = 0.2$, $\nu_{45}(v_4,v_5) = 0.4$, $\nu_{36}(v_3,v_6) = 0.5$, $\nu_{56}(v_5,v_6) = 0.4$.

$\mu_{ij}(E_{h_1}) = \mu_{ij}(b) \wedge \mu_{ij}(e) = 0.5$, $\mu_{ij}(E_{h_2}) = \mu_{ij}(d) \wedge \mu_{ij}(f) = 0.6$, $\mu_{ij}(E_{h_3}) = \mu_{ij}(d) \wedge \mu_{ij}(f) = 0.6$,

$\mu_{ij}(E_{h_4}) = \mu_{ij}(a) \wedge \mu_{ij}(c) = 0.7$

$\nu_{ij}(E_{h_1}) = \nu_{ij}(b) \vee \nu_{ij}(e) = 0.5$, $\nu_{ij}(E_{h_2}) = \nu_{ij}(d) \vee \nu_{ij}(f) = 0.4$, $\nu_{ij}(E_{h_3}) = \nu_{ij}(d) \vee \nu_{ij}(f) = 0.4$,

$\nu_{ij}(E_{h_4}) = \nu_{ij}(a) \vee \nu_{ij}(c) = 0.2$. Thus I_{sh} is (μ,ν) – tempered IFSHG.

Definition 4.12. The *strength* δ_{C_v} of a hyperedge E_{h_j} is the minimum membership and maximum non-membership of consecutive vertices in the hyperedge.

$$\delta_{C_v} = \{min(\mu_{ij}^c), max(\nu_{ij}^c)\}, \forall(\mu_{ij}^c, \nu_{ij}^c) > 0.$$

Theorem 4.3. An IFSHG I_{sh} is (μ,ν) – tempered IFSHG of some crisp semihypergraphiff I_{sh} is elementary, support simple and simply ordered.

Proof

Consider an IFSHG which is a (μ,ν) – tempered of some crisp hypergraph. As it is (μ,ν) – tempered, the (μ,ν) values of intuitionistic fuzzy hyperedges of I_{sh} are constant. Hence, I_{sh} is elementary.

Also, if support of two intuitionistic fuzzy hyperedges of the (μ,ν) – tempered IFSHG are equal then the intuitionistic fuzzy hyperedges are same. Hence, it is support simple.

Let $C(I_{sh}) = \{I_{sh}^{x_1,y_1}, I_{sh}^{x_2,y_2}, ...I_{sh}^{x_n,y_n}\}$. I_{sh} is elementary implies that I_{sh} is ordered.

Claim: I_{sh} is simply ordered.

Let $E_{h_j} \in I_{sh}^{x_{i+1},y_{i+1}} - I_{sh}^{x_i,y_i}$ then there is an $(v_i, v_j) \in E_h$ where $\mu_{ij}^c(v_i,v_j) = x_{i+1}$ and

$\nu_{ij}^{c}(v_i,v_j)=y_{i+1}$. Since $x_{i+1}< x_i$ and $y_{i+1}< y_i$, it follows that $(v_i,v_j)\notin \mathrm{I}_{sh}^{x_i,y_i}$ and $E_h \not\subseteq \mathrm{I}_{sh}^{x_i,y_i}$.
Thus, I_{sh} is simply ordered.

Conversely, assume I_{sh} is elementary, support simple and simply ordered.
Then, $\mathrm{I}_{sh}^{(x_i,y_i)}=\mathrm{I}_{sh_i}=(V_i,E_{h_i})$. Define $\mu_{ij}^{c}:V\to[0,1]$ and $\nu_{ij}^{c}:V\to[0,1]$ by

$$\mu_{E_{h_j}}(C_v)=\begin{cases} x_1 & \text{if } (v_i,v_j)\in E_{h_j} \\ x_i & \text{if } (v_i,v_j)\in E_{h_i}-E_{h_{i-1}}, \quad i=1,2,3,\dots,n \end{cases}$$

and

$$\nu_{E_{h_j}}(C_v)=\begin{cases} y_1 & \text{if } (v_i,v_j)\in E_{h_j} \\ y_i & \text{if } (v_i,v_j)\in E_{h_i}-E_{h_{i-1}}, i=1,2,3,\dots,n \end{cases}$$

To show that, I_{sh} is $(\mu,\nu)-$tempered IFSHG .

Let $E' \in E_{h_i}$. Since I_{sh} is elementary and support simple, there is a unique intuitionistic fuzzy hyperedge (a,b) in E_{h_j} and having support E'.

Indeed, distinct hyperedges in E_{h_j} must have distinct supports in E_{h_j}.
It is enough if we show that, for each $E' \in E_{h_i}$, $\mu_{ij}^{c}(v_i,v_j)=a$, $\nu_{ij}^{c}(v_i,v_j)=b.$

As all hyperedges are elementary and different hyperedges have different supports, from the definition of the $F(\mathrm{I}_{sh})$, we have $h(a,b)$ is equal to few member of $\langle x_i,y_i\rangle$ of $F(\mathrm{I}_{sh})$. Consequently, $E'\subseteq \mathrm{I}_{sh}^{(x_i,y_i)}$. Also if $i>1$, then $E' \in E_{h_i}-E_{h_{i-1}}$. The definition of $(\mu,\nu)-$tempered implies that for each and $\nu_{ij}^{c}\le y_i$.

Claim: $\mu_{ij}^{c}=x_i$ *and* $\nu_{ij}^{c}=y_i$ *for some* $(v_i,v_j)\in E'$.

If not, then by definition of $(\mu,\nu)-$tempered $\mu_{ij}(v_i,v_j)\ge x_{i-1}$ and $\nu_{ij}(v_i,v_j)\le y_{i-1}$ for all $(v_i,v_j)\in E'$ which implies that $E'\subseteq \mathrm{I}_{sh}^{x_{i-1},y_{i-1}}$ and so $E'\in E_{h_i}-E_{h_{i-1}}$ and from our assumption that I_{sh} is simply ordered, we know, $E'\not\subseteq \mathrm{I}_{sh}^{x_{i-1},y_{i-1}}$, a contradiction.

Hence, from the definition of μ_{ij}^{c} and ν_{ij}^{c}, we get $\mu_{ij}^{c}=a, \nu_{ij}^{c}=b.$

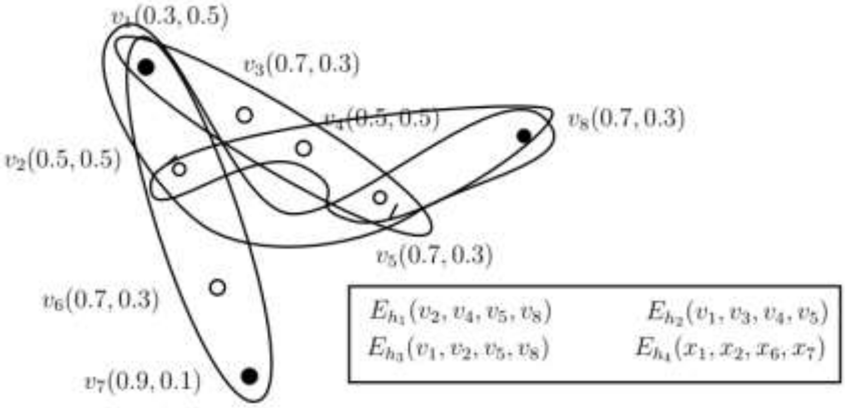
Numerical Example:

A family has four decision making members. They intend to buy a new family car by collecting its information corresponding to various features of the car.

Let V denote set of vertices which represents various features of particular brands, $V=\{v_1,v_2,\dots,v_n\}$, E_{h_j} – the hyperedges which denotes the brandedcars and C_v *be* consecutive vertices.

They consider the eight criteria such as Price (v_1), Engine (v_2), Brakes (v_3), Technology (v_4), Safety (v_5), Performance (v_6), Fuel Tank Capacity (v_7) and Seat Capacity (v_8) which has been taken into consideration as vertices. In this paper, we find the best car model from four alternative cars under consideration Amaze (*Honda*) (E_{h_1}), Ameo (*Volkswagen*) (E_{h_2}), Xcent (*Hyundai*) (E_{h_3}), Swift Dzire (*Maruti Suzuki*) (E_{h_4}).
Linguistic Variables and IF values are defined as given below:

Very low(VL) $(0,1)$, Low(L) $(0.3,0.5)$, Medium(M) $(0.5,0.5)$, Medium high(MH) $(0.5,0.4)$, High(H) $(0.7,0.3)$, Very high(VH) $(0.9,0.1)$



Consider the membership and non-membership values for each vertex:

$$v_1(0.3,0.5),\ v_2(0.5,0.5),\ v_3(0.7,0.3),\ v_4(0.5,0.5),$$
$$v_5(0.7,0.3),\ v_6(0.7,0.3),\ v_7(0.9,0.1),\ v_8(0.7,0.3)$$

The (μ_{ij}^c, ν_{ij}^c) values are given below:

	E_{h_1}	E_{h_2}	E_{h_3}	E_{h_4}
a	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0.3, 0.5\rangle$	$\langle 0.3, 0.5\rangle$
b	$\langle 0, 1\rangle$	$\langle 0.6, 0.4\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$
c	$\langle 0.7, 0.3\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$
d	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0.4, 0.4\rangle$	$\langle 0, 1\rangle$
e	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0.3, 0.4\rangle$
f	$\langle 0, 1\rangle$	$\langle 0.1, 0.5\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$
g	$\langle 0.6, 0.4\rangle$	$\langle 0.6, 0.4\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$
h	$\langle 0.2, 0.6\rangle$	$\langle 0, 1\rangle$	$\langle 0.2, 0.6\rangle$	$\langle 0, 1\rangle$
i	$\langle 0.1, 0.6\rangle$	$\langle 0, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0.4, 0.4\rangle$

From the above incidence matrix, we see that, the strength of each hyperedge is given by $(0.1,0.6),(0.1,0.5),(0.2,0.6),(0.3,0.5)$

Among the strength of hyperedge, which has the maximum membership and minimum non-membership is said to be stronger hyperedge. Here E_{h_4} is stronger compared to others. So, the family can choose Swift Dzire.

Conclusion

Fuzzy graph and Fuzzy hypergraphs have wide range of applications. Intuitionistic fuzzy concepts are developing day by day in real life applications. On extension of hypergraph we have introduced the basic definitions in IFSHG. Strong IFSHGs and Effective IFSHGs were defined with examples. We have also defined some properties like simple, core, elementary, sectionally elementary IFSHGs and $(\mu,\nu)-$tempered IFSHGs. Some properties of IFSHGs are analyzed. The author has an idea to extend the concept in DNA splicing using bipartite IFSHGs.

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